Normal forms: Let $A(P_1, P_2, P_3, \ldots P_n)$ be statement formula then the construction of truth table may not be practical always. So, we consider alternate procedure known as reduction to normal form.

(i) Disjunction Normal form (DNF): A statement form which consists of disjunction between conjunction is called DNF

Ex: (1) $(p \wedge q) \vee r$ → disjunction

(2) $(p \wedge \sim q) \vee (\sim p \wedge r) \vee (p \wedge \sim q)$

conjunction disjunction

prb: ① $(p \to q) \wedge (\sim p \wedge q)$ reduce to DNF

$\Rightarrow (\sim p \vee q) \wedge (\sim p \wedge q)$ [$\because p \to q \Leftrightarrow \sim p \vee q$]

$\Rightarrow (\sim p \wedge (\sim p \wedge q)) \vee (q \wedge (\sim p \wedge q))$ (using distributive law $(p \vee q) \wedge r = (p \wedge r) \vee (q \wedge r)$)

$((\sim p \wedge \sim p) \wedge q) \vee [(q \wedge q) \wedge \sim p]$

$\Rightarrow (\sim p \wedge q) \vee (q \wedge \sim p)$ [$\because \sim p \wedge \sim p \equiv \sim p$, $q \wedge q \equiv q$]

conjunction disjunction conjunction

∴ It is reduced to the form DNF

(i.e disjunction b/w the conjunction.

(2)

Spanning Tree: Conjunction Normal form (CNF):

A Statement which consists of ~~Con~~ Conjunction b/w disjunction is called CNF.

Ex: (i) $p \wedge q$ (ii) $(\sim p \vee q) \wedge (\sim p \vee r)$

Example: Obtain CNF ~~of the~~ form $(p \wedge q) \vee (\sim p \wedge q \wedge r)$

Sol: $(p \wedge q) \vee (\sim p \wedge q \wedge r)$

$\Rightarrow (p \vee (\sim p \wedge q \wedge r)) \wedge (q \vee (\sim p \wedge q \wedge r))$

$\Rightarrow [\underbrace{(p \vee \sim p)}_{T_0} \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge (q \vee q) \wedge (q \vee r)]$

$\Rightarrow (p \vee q) \wedge (p \vee r) \wedge [(q \vee \sim p) \wedge q \wedge (q \vee r)]$ $[\because T_0 \wedge p \equiv p$, $q \vee q \equiv q]$

The above statement is conjunction b/w disjunction.

prbs: obtain DNF of $p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$

Sol:

$\Leftrightarrow p \vee (\sim p \rightarrow (q \vee (\sim q \vee \sim r)))$ $[\because p \rightarrow q \equiv \sim p \vee q$

$\Leftrightarrow p \vee (p \vee (q \vee (\sim q \vee \sim r)))$ $[\because \sim p \rightarrow q \equiv p \vee q]$

$\Leftrightarrow p \vee (p \vee [(q \vee \sim q) \vee \sim r]]$ $[\because q \vee \sim q = q$

$\Leftrightarrow p \vee (p \vee (q \vee \sim r))$ [Associative $p \vee (q \vee r) = (p \vee q) \vee r$

$\Rightarrow (p \vee p) \vee (q \vee \sim r)$ $[\because p \vee p = p$

$\Rightarrow p \vee (q \vee \sim r)$

$\Rightarrow p \vee q \vee \sim r$ is DNF

3) obtain CNF of $(p \to q) \wedge (q \vee (p \wedge r))$ - (1)
and determine whether or not it is tautology.

Sol w.k.T $p \to q \Leftrightarrow \sim p \vee q$

$\Rightarrow (\sim p \vee q) \wedge (q \vee (p \wedge r))$

$\Rightarrow (\sim p \vee q) \wedge ((q \vee p) \wedge (q \vee r))$

$\Rightarrow (\sim p \vee q) \wedge (q \vee p) \wedge (q \vee r)$

verify tautology by Truth table.

p	q	$p \to q$	$p \wedge r$	$q \vee (p \wedge r)$	$(p \to q) \wedge (q \vee (p \wedge r))$

Ans: not a tautology.