Solutions to Problem Set 2

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Course: Monetary and Fiscal Policy: Theory and Practice

Problem 1: Memorandum

To: Director, Research Department

From: Economist, Fed Research Team

Subject: Evaluation of the FORM Act

The FORM Act which is aimed to improve transparency and accountability by requiring the Fed to publicly outline its monetary policy rule and deviations. It states a significant governance shift but poses challenges to the Fed's independence and effectiveness.

Advantages of the FORM Act

The FORM Act could support public confidence in monetary policy by improving transparency. By requiring the Fed to specify and adhere to a clear policy rule such as a Taylor rule, the public and market participants could get better insights into the Fed's decision-making process. This predictability might reduce uncertainty which could stabilize financial markets and improve consumers planning. Furthermore, accountability to Congress and the public could incentivize the Fed to systematically assess its policies by ensuring they are data-driven and consistent with its mandate.

Potential risks and side effects

Despite its advantages the FORM Act risks undermining the Fed's independence. MP decisions often require flexibility to respond to unforeseen economic shocks. Mandating strict adherence to a pre-specified rule and justifying deviations to Congress could delay policy responses or sometimes politicize decisions by reducing the Fed's effectiveness in achieving its dual mandate of price stability and controlling unemployment rate. Additionally, while policy rules provide a valuable framework for decision-making, but real-world often requires discretion. Dynamic economic models and rules can't fully explain conditions or mostly predict future crises. Excessive reliance on a general rule could lead to suboptimal outcomes, particularly during geopolitical situation around the globe.

Conclusion and recommendation

While the FORM Act is directed on to improve transparency and accountability, these benefits must be carefully weighed against the potential risks. A middle point might involve publishing detailed policy frameworks without binding the Fed to a time-varying specific rule. Such an approach could maintain transparency while preserving the necessary discretion for effective policy implementation.

Respectfully submitted,
Sodik Umurzakov
Economist, Fed Research Department

Problem 2: The MMB task

In this task I analyzed the OW (2008) rule and compared it with a user-specified and model-specified rules, particularly in the context of the US-CMR14 model. The OW(2008) rule is:

$$i_t = 6.97 + 2.34\pi_{t+h} - 1.53u_{t+h} \tag{1}$$

where π represents inflation and u represents the unemployment rate. This rule incorporates a forward-looking component where $h = 3$.

For the **user-specified rule** the MMB modifies the formula with $h = 0$ and, according to comments in the MMB description of the OW(2008), replaces the unemployment rate with the output gap according to Okun's law:

$$-2(\mu - \tilde{\mu}) \approx (y - \bar{y}) \tag{2}$$

Using this transformation the user-specified rule is:

$$i_t = 6.97 + 2.34\pi_t - 0.756(y - \bar{y}) \tag{3}$$

Here, y is output and $(y - \bar{y})$ is the output gap. The output gap coefficient is derived by dividing -1.53 by -2, consistent with Okun's law. This comparison highlights how the modification from unemployment to the output gap affects the structure of the policy rule. Further analysis can now be conducted on IRFs to evaluate the implications of this adjustment.

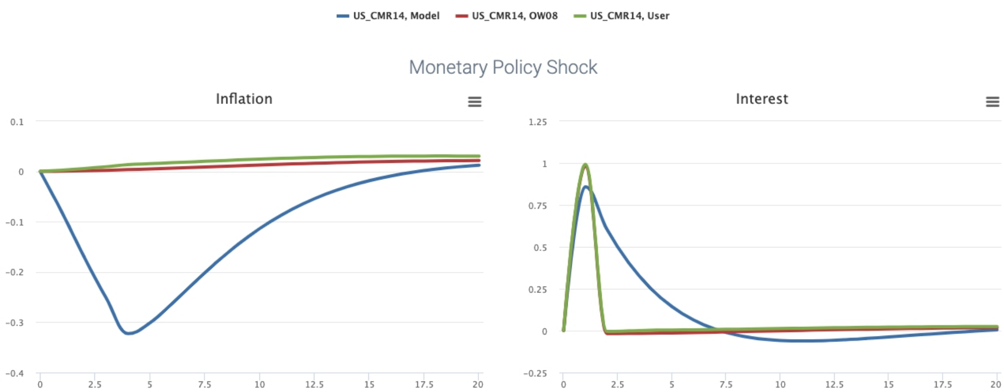

Figure 1: IRFs to monetary policy shock

In figures 1 and 2 we can see the three cases of rules within the context of the model US-CMR14. The difference between the model-specified rule and the OW (2008) rule comes from how they handle forward-looking dynamics and parameters. The model-specified rule is designed to fit the dynamics of the US-CMR14 model, so it reacts faster and adjusts more sharply, while the OW (2008) rule, using forecasts ($h = 3$), takes a smoother and slower approach to stabilize.

Both the OW (2008) and user-specified rules have similar effects on the interest rate, with both eventually returning to the steady state. The model-specified rule, however, stands out with a sharp initial spike followed by a gradual decline back to normal.

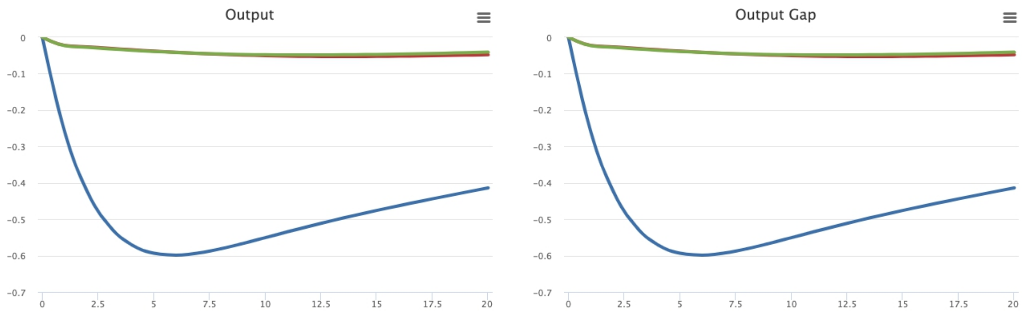

Figure 2: IRFs to monetary policy shock

When it comes to output and the output gap, all three rules show a decline, but the drop is much more pronounced under the model-specified rule, reaching around 0.6. In comparison, the OW (2008) and user-specified rules result in much smaller and almost negligible changes. Inflation shows a clear drop under the model-specified rule, exceeding 0.3 before stabilizing. Meanwhile, the OW (2008) and user-specified rules show only minor initial shifts, followed by a small rise before settling back to normal.

Problem 3: Optimal monetary policy at the effective LB in a simple model of inflation

Please you can find handwritten solutions to Problem 3 in this document after Problem 4 and Dynare code. Thanks;)

Problem 4: Financial frictions and fiscal policy

Problem 4.1

In figure 3 we see the IRFs to monetary policy shock of two cases: with/without financial frictions. As we see inflation reacts more strongly and persists longer when financial frictions are present. This happens because financial variables like stock prices and marginal costs act as additional channels that amplify shocks. Output also shows a bigger initial drop and takes longer to recover, showing how the interaction between financial markets and the real economy spreads monetary shocks further.

Similarly, the interest rate responses look pretty similar in both cases as it sticks closely to the monetary policy rule. However, the effects of inflation and output responds slightly nudge its path when financial frictions ($\nu > 0$) are present. Marginal costs and stock prices are particularly sensitive, clearly showing how financial frictions amplify the policy shock.

Therefore, financial frictions make monetary policy shocks hit harder and last longer, especially for inflation, output and financial variables. This highlights the importance of including these frictions in macroeconomic models when analyzing how effective monetary policy can be.

Problem 4.2

First, let's go ahead and log-linearize the aggregate resource constraint. To do this, we define the variable x_t for any variable X_t as:

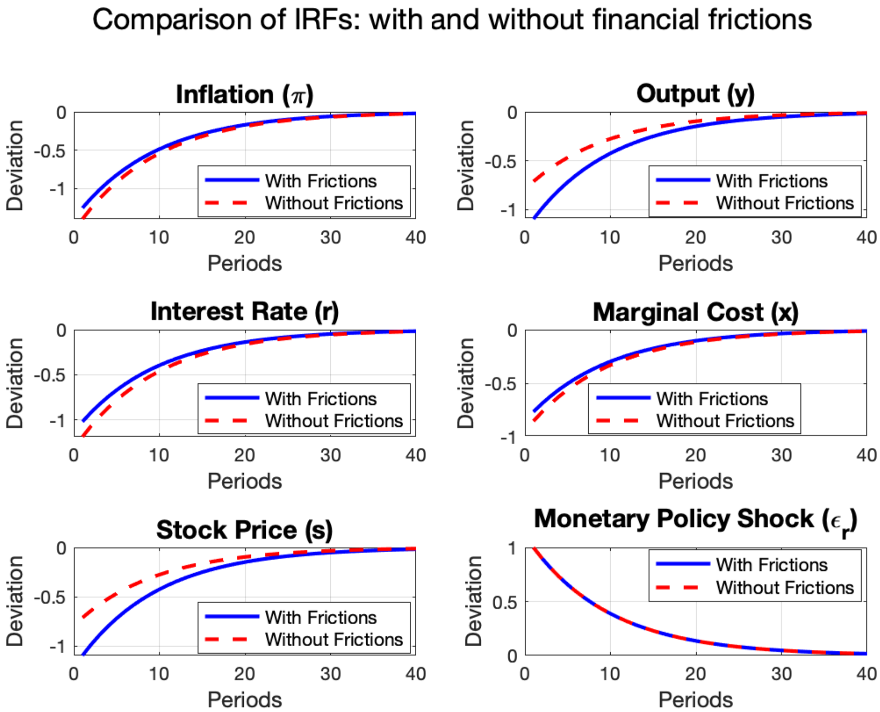

Figure 3: IRFs to monetary policy shock

$$x_t \equiv \ln(X_t) - \ln(X) \quad \Rightarrow \quad x_t = \frac{\ln(X_t)}{\ln(X)} \quad \Rightarrow \quad e^{x_t} = \frac{X_t}{X}. \tag{4}$$

Now, if we perform a Taylor approximation around $x_t = 0$ for the LHS of the first equation, we get:

$$e^{x_t} \approx 1 + (x_t - 0) = 1 + x_t. \tag{5}$$

Therefore, as derived earlier, we have:

$$X_t = X(1 + x_t). \tag{6}$$

Here, X represents the steady-state value of X_t, and x_t is simply the log-deviation of X_t from its steady-state value.

Using these expressions, we can log-linearize the aggregate resource constraint, which gives:

$$Y(1 + y_t) = C(1 + c_t) + G(1 + g_t). \tag{7}$$

Notice that the constants on both sides cancel out, then we get:

$$y_t = \frac{C}{Y} c_t + \frac{G}{Y} g_t \quad \Rightarrow \quad y_t = \frac{C}{Y} c_t + \frac{G}{Y} g_t. \tag{8}$$

Furthermore, we need to tweak the New Keynesian consumption equation to account for government expenditure. The consumption equation becomes:

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}(r_t - E_t(\pi_{t+1})). \tag{9}$$

We substitute g_t into the aggregate constraint and continue from there. This allows us to account for fiscal policy shocks within the model in a consistent way.

In figure 4 the Taylor rule shows how the economy reacts dynamically to a fiscal policy shock caused by an expansionary government spending. Inflation rises modestly at first before gradually returning back to its steady state which highlights the Taylor rule's role in stabilizing price levels. Output increases in the first periods due to the fiscal expansion but slowly returns to equilibrium over time. The interest rate adjusts immediately upward by reflecting the monetary policy's reaction to higher inflation and output. Meanwhile, consumption decreases as government spending rises, clearly showing the crowding-out effect often seen in these situations. Stock prices and marginal costs drop initially but stabilize soon after, showing the temporary impact of fiscal policy on these financial variables.

In the end, the Taylor rule manages to smooth out the fluctuations in key macroeconomic indicators by ensuring the system returns to steady state after shock. This shows how effective the rule is in handling volatility from fiscal policy changes and supporting overall economic stability.

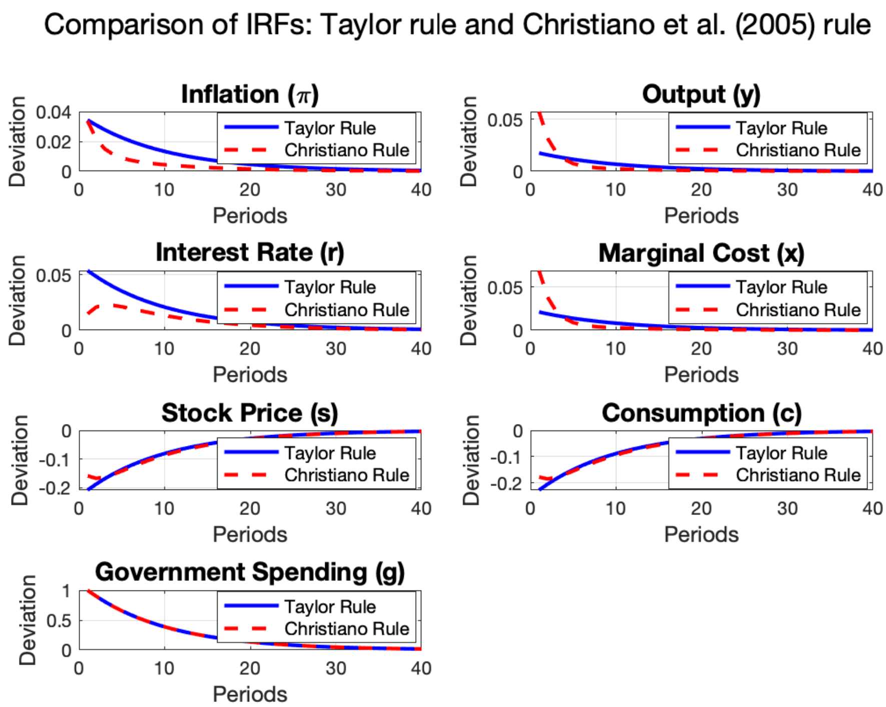

Figure 4: IRFs to fiscal policy shock

Problem 4.3

In figure 4, we can see the comparison between the Taylor rule and Christiano et al. (2005) rule highlights key differences in how the economy responds to a fiscal policy shock. Under the Christiano rule, inflation and output adjust more gradually compared to the Taylor rule, reflecting the added inertia from incorporating the lagged interest rate term. Stock prices and marginal costs stabilize faster under the Christiano rule, while the Taylor rule generates a more pronounced immediate adjustment. Overall, the Christiano rule introduces smoother dynamics but at the cost of slower stabilization in some macroeconomic variables, showcasing a trade-off between responsiveness and stability.

Problem 4.4

In our analysis we included government expenditure without explicitly introducing a government budget constraint, as the model was treated within a partial equilibrium framework. However, if we aim to explore the broader context or determine under what assumptions such an omission is acceptable, it becomes essential to address the limitations of this approach. Specifically, when transitioning to a general equilibrium model, the inclusion of the government budget constraint is necessary to ensure the model's consistency and its ability to capture the interdependence of fiscal policies and other macroeconomic variables.

Problem 4.5

To get the log-linear representation of this equation, we first define x_t for any variable X_t like this:

$$x_t = \ln(X_t) - \ln(X) \quad \Rightarrow \quad x_t = \frac{\ln(X_t)}{\ln(X)} \quad \Rightarrow \quad e^{x_t} = \frac{X_t}{X}. \tag{10}$$

Next, we apply a Taylor approximation around $x_t = 0$ to the left-hand side of the equation:

$$e^{x_t} \approx 1 + (x_t - 0) = 1 + x_t. \tag{11}$$

So, from this, we get:

$$X_t = X(1 + x_t). \tag{12}$$

Here, X is the steady-state value of X_t, and x_t represents the log-deviations of X_t from its steady-state value. If X_t^{-1} is raised to minus one, we then have:

$$X_t^{-1} = X^{-1} e^{-x_t}. \tag{13}$$

Following the same steps as before and using another Taylor approximation, we get:

$$X_t^{-1} = X^{-1}(1 - x_t). \tag{14}$$

Now, using these definitions, we can log-linearize the government's budget constraint:

$$G(1+g_t) + B(1+b_{t-1})\Pi^{-1}(1-\pi_t) = T(1+t_t) + B(1+b_t)R^{-1}(1-r_t). \tag{15}$$

Expanding and rearranging terms, we get:

$$G+Gg_t+B\Pi^{-1}+B\Pi^{-1}b_{t-1}-B\Pi^{-1}\pi_t=T+Tt_t+BR^{-1}+BR^{-1}b_t-BR^{-1}r_t. \tag{16}$$

By ignoring the cross-products of the log deviations (since they are small), and noting that constants on both sides are equal in steady state, we can cancel them out. Assuming $R=\Pi$ in the steady state and dividing both sides by Y, we finally get:

$$\frac{G}{Y}g_t+\frac{B}{Y}(b_{t-1}-\pi_t)=\frac{T}{Y}t_t+\beta\frac{B}{Y}(b_t-r_t). \tag{17}$$

Problem 4.6

The figure 5 shows the results of including of the government budget constraint and fiscal rule which significantly effects the dynamics of the variables in response to a fiscal policy shock. The increase in government expenditure leads to a rise in government debt and lump-sum taxes with taxes peaking shortly after the shock before slowly returning to the steady state. Consumption initially declines due to the crowding-out effect caused by higher taxes and borrowing, but it slowly recovers over time. Inflation and output both rise in response to the fiscal shock, with inflation exhibiting a more pronounced and immediate increase. Overall, the fiscal rule dampens the long-run effects of the shock which ensures stability in debt and taxes while allowing the economy to adjust gradually.

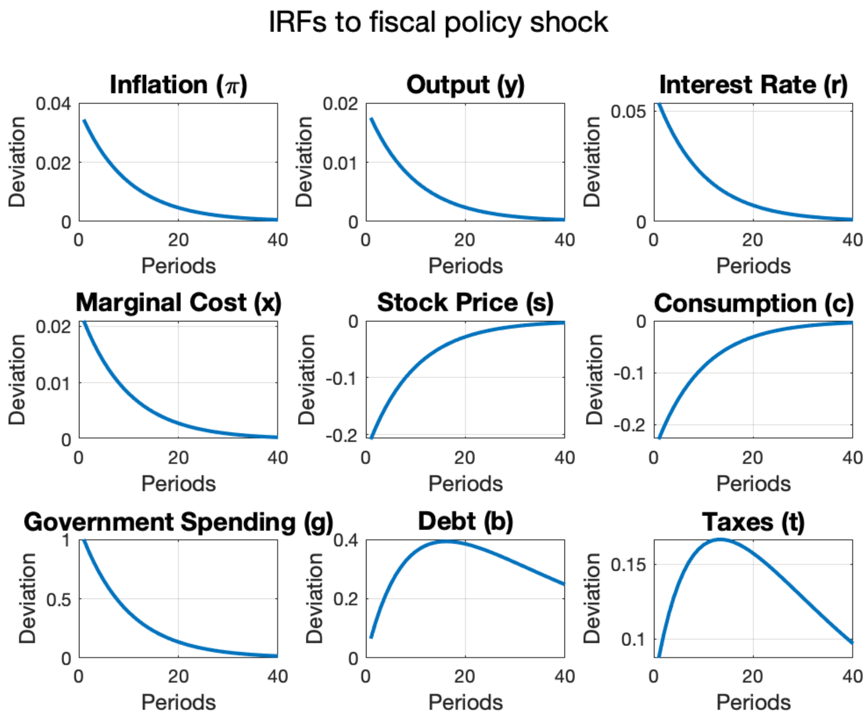

Figure 5: IRFs to fiscal policy shock

MATLAB (Dynare) code

```
%%%%%%%%%%%%%%%%%%%%%Problem Set 2, Problem 4%%%%%%%%%%%%%%%%%%%%
% Performed by: Sodik Umurzakov
```

```
%%%%%%%%Problem 4.1 With financial frictions%%%%%%%%%%%%%%%%%%%
%Endo variables
var pi y r x s eps_pi eps_r;

%Exo variables
varexo eps_pi_shock eps_r_shock;

% Set parameter value
parameters beta lambda kappa sigma eta nu phi_pi phi_y rho;

% Set initial values of parameters
beta = 0.99;
lambda = 0.66;
kappa = ((1-lambda)*(1-beta*lambda))/lambda;
sigma = 1;
eta = 1.2;
nu = 0.5; %with financial frictions
phi_pi = 1.5;
phi_y = 0.125;
rho = 0.9;

model;
    %Inflation
    pi = beta*pi(+1) + kappa*x + eps_pi;

    %Output
    y = y(+1) - (1/sigma)*(r - pi(+1));

    %Interest rate
    r = phi_pi*pi + phi_y*y + eps_r;

    %Marginal cost
    x = eta*y - nu*s;

    %Stock price
    s = (1-beta)*y(+1) + beta*s(+1) - (r - pi(+1));

    %AR(1) process of inflation
    eps_pi = rho*eps_pi(-1) + eps_pi_shock;

    %AR(1) process of interest rate
    eps_r = rho*eps_r(-1) + eps_r_shock;
end;

```

```
%Set values in steady state
initval;
    pi = 0.0;
    y = 0.0;
    r = 0.0;
    x = 0.0;
    s = 0.0;
    eps_pi = 0.0;
    eps_r = 0.0;
end;

shocks;
    var eps_pi_shock = 0;
    var eps_r_shock = 1;
end;

stoch_simul(order=1, irf=40);

% Save IRF data to a MAT file
irfs_with = oo_.irfs; % Extract the IRF data
save('IRFs_with_frictions.mat', 'irfs_with');
```

Listing 1: Problem 4.1 With financial frictions

```
%%%%%%%%%%%%%%%%%%%%%Problem Set 2, Problem 4%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%Problem 4.1 Without financial frictions%%%%%%%%%%%%%%%%%%%%%
%Endo variables
var pi y r x s eps_pi eps_r;

%Exo variables
varexo eps_pi_shock eps_r_shock;

% Set parameter value
parameters beta lambda kappa sigma eta nu phi_pi phi_y rho;

% Set initial values of parameters
beta = 0.99;
lambda = 0.66;
kappa = ((1-lambda)*(1-beta*lambda))/lambda;
sigma = 1;
eta = 1.2;
nu = 0; %without financial frictions
phi_pi = 1.5;
phi_y = 0.125;
rho = 0.9;
```

```
model;
    %Inflation
    pi = beta*pi(+1) + kappa*x + eps_pi;

    %Output
    y = y(+1) - (1/sigma)*(r - pi(+1));

    %Interest rate
    r = phi_pi*pi + phi_y*y + eps_r;

    %Marginal cost
    x = eta*y - nu*s;

    %Stock price
    s = (1-beta)*y(+1) + beta*s(+1) - (r - pi(+1));

    %AR(1) process of inflation
    eps_pi = rho*eps_pi(-1) + eps_pi_shock;

    %AR(1) process of interest rate
    eps_r = rho*eps_r(-1) + eps_r_shock;
end;

%Set values in steady state
initval;
    pi = 0.0;
    y = 0.0;
    r = 0.0;
    x = 0.0;
    s = 0.0;
    eps_pi = 0.0;
    eps_r = 0.0;
end;

shocks;
    var eps_pi_shock = 0;
    var eps_r_shock = 1;
end;

stoch_simul(order=1, irf=40);

% Save IRF data to a MAT file
irfs_without = oo_.irfs; % Extract the IRF data
```

```
save('IRFs_without_frictions.mat', 'irfs_without');
```

Listing 2: Problem 4.1 Without financial frictions

```
%%%%%%%%%%%%%%%%%%%Problem Set 2, Problem 4%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%Problem 4.2%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Endo variables
var pi y r x s c g eps_pi eps_r;

%Exo variables
varexo eps_pi_shock eps_r_shock eps_g_shock;

% Set parameter value
parameters beta lambda kappa sigma eta nu phi_pi phi_y rho;

% Set initial values of parameters
beta = 0.99;
lambda = 0.66;
kappa = ((1-lambda)*(1-beta*lambda))/lambda;
sigma = 1;
eta = 1.2;
nu = 0;
phi_pi = 1.5;
phi_y = 0.125;
rho = 0.9;

model;
    %Inflation
    pi = beta*pi(+1) + kappa*x + eps_pi;

    %Interest rate Taylor rule
    r = phi_pi*pi + phi_y*y + eps_r;

    %Marginal cost
    x = eta*y - nu*s;

    %Stock price
    s = (1-beta)*y(+1) + beta*s(+1) - (r - pi(+1));

    %Consumption
    c = c(+1) - (r - pi(+1))/sigma;

    %Output
    y = 0.8*c+0.2*g;

```

```
    %AR(1) process of inflation
    eps_pi = rho*eps_pi(-1) + eps_pi_shock;

    %AR(1) process of interest rate
    eps_r = rho*eps_r(-1) + eps_r_shock;

    %AR(1) process of government spending
    g = rho *g(-1) + eps_g_shock;
end;

%Set values in steady state
initval;
    pi = 0.0;
    y = 0.0;
    r = 0.0;
    x = 0.0;
    s = 0.0;
    c = 0.0;
    g = 0.0;
    eps_pi = 0.0;
    eps_r = 0.0;
end;

shocks;
    var eps_pi_shock = 0;
    var eps_r_shock = 0;
    var eps_g_shock = 1;
end;

stoch_simul(order=1, irf=40);
% Save IRF data to a MAT file
irfs_taylor = oo_.irfs; % Extract the IRF data
save('IRFs_Taylor.mat', 'irfs_taylor');
```

Listing 3: Problem 4.2 IRFs to fiscal policy shock

```
%%%%%%%%%%%%%%%%%%%%%%%%Problem Set 2, Problem 4%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%Problem 4.3%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Endo variables
var pi y r_alter x s c g eps_pi eps_r;

%Exo variables
varexo eps_pi_shock eps_r_shock eps_g_shock;

% Set parameter value
```

```
parameters beta lambda kappa sigma eta nu phi_pi phi_y rho;

% Set initial values of parameters
beta = 0.99;
lambda = 0.66;
kappa = ((1-lambda)*(1-beta*lambda))/lambda;
sigma = 1;
eta = 1.2;
nu = 0; %according to the conditions in the PS2_problem4.2
phi_pi = 1.5;
phi_y = 0.125;
rho = 0.9;

model;
    %Inflation
    pi = beta*pi(+1) + kappa*x + eps_pi;

    %Alternative Interest rate rule Cristiano et al (2005)
    r_alter = 0.8*r_alter(-1) + 0.3*pi + 0.08*y + eps_r;

    %Marginal cost
    x = eta*y - nu*s;

    %Stock price
    s = (1-beta)*y(+1) + beta*s(+1) - (r_alter - pi(+1));

    %Consumption
    c = c(+1) - (r_alter - pi(+1))/sigma;

    %Output
    y = 0.8*c+0.2*g;

    %AR(1) process of inflation
    eps_pi = rho*eps_pi(-1) + eps_pi_shock;

    %AR(1) process of interest rate
    eps_r = rho*eps_r(-1) + eps_r_shock;

    %AR(1) process of government spending
    g = rho *g(-1) + eps_g_shock;
end;

%Set values in steady state
initval;
```

```
    pi = 0.0;
    y = 0.0;
    r_alter = 0.0;
    x = 0.0;
    s = 0.0;
    c = 0.0;
    g = 0.0;
    eps_pi = 0.0;
    eps_r = 0.0;
end;

shocks;
    var eps_pi_shock = 0;
    var eps_r_shock = 0;
    var eps_g_shock = 1;
end;

stoch_simul(order=1, irf=40);
% Save IRF data to a MAT file
irfs_christiano = oo_.irfs; % Extract the IRF data
save('IRFs_Christiano.mat', 'irfs_christiano');
```

Listing 4: Problem 4.3 Christiano et al (2005) rule

```
%%%%%%%%%%%%%%%%%%Problem Set 2, Problem 4%%%%%%%%%%%%%%%%%
% Performed: Sodik Umurzakov
%%%%%%%%%%%%%%%%%%%%%Problem 4.6%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Endo variables
var pi y r x s c g b t eps_pi eps_r;

%Exo variables
varexo eps_pi_shock eps_r_shock eps_g_shock;

% Set parameter value
parameters beta lambda kappa sigma eta nu phi_pi phi_y rho phi_b
   phi_g;

% Set initial values of parameters
beta = 0.99;
lambda = 0.66;
kappa = ((1-lambda)*(1-beta*lambda))/lambda;
sigma = 1;
eta = 1.2;
nu = 0; %according to the conditions in the PS2_problem4.2 ( e.g.
   without financial frictions)
```

```
phi_pi = 1.5;
phi_y = 0.125;
phi_b = 0.043;
phi_g = 0.124;
rho = 0.9;

model;
    %Inflation
    pi = beta*pi(+1) + kappa*x + eps_pi;

    %Interest rate (original)
    r = phi_pi*pi + phi_y*y + eps_r;

    %Marginal cost
    x = eta*y - nu*s;

    %Stock price
    s = (1-beta)*y(+1) + beta*s(+1) - (r - pi(+1));

    %Consumption
    c = c(+1) - (r - pi(+1))/sigma;

    %Output
    y = 0.8*c+0.2*g;

    %Government BC
    0.2*g + 3.6*(b(-1) - pi) = 0.4*t + beta*3.6*(b - r);

    %Lump-sum tax
    0.4*t = phi_b*3.6*b + phi_g*0.2*g;

    %AR(1) process of inflation
    eps_pi = rho*eps_pi(-1) + eps_pi_shock;

    %AR(1) process of interest rate
    eps_r = rho*eps_r(-1) + eps_r_shock;

    %AR(1) process of government spending
    g = rho *g(-1) + eps_g_shock;
end;

%Set values in steady state
initval;
    pi = 0.0;
```

```
    y = 0.0;
    r = 0.0;
    x = 0.0;
    s = 0.0;
    c = 0.0;
    g = 0.0;
    b = 0.0;
    t = 0.0;
    eps_pi = 0.0;
    eps_r = 0.0;
end;

shocks;
    var eps_pi_shock = 0;
    var eps_r_shock = 0;
    var eps_g_shock = 1;
end;

stoch_simul(order=1, irf=40);

% Save IRF data to a MAT file
irfs = oo_.irfs; % Extract the IRF data
save('IRFs_data.mat', 'irfs');
```

Listing 5: Problem 4.6

```
%% Plot IFRs for all problems

% IRF data for both cases
load('IRFs_with_frictions.mat');
load('IRFs_without_frictions.mat');

% Time horizon for IRFs
horizon = 1:40;

% Variable names and corresponding titles
variables = {'pi_eps_r_shock', 'y_eps_r_shock', 'r_eps_r_shock', '
   x_eps_r_shock', 's_eps_r_shock', 'eps_r_eps_r_shock'};
titles = {'Inflation (\pi)', 'Output (y)', 'Interest Rate (r)', ...
          'Marginal Cost (x)', 'Stock Price (s)', 'Monetary Policy
             Shock (\epsilon_r)', };

% Number of variables
num_vars = length(variables);

```

```
% Create a figure
figure;

% Loop through variables and plot for both cases
for i = 1:num_vars
    subplot(ceil(num_vars / 2), 2, i);
    % Plot with financial frictions
    plot(horizon, irfs_with.(variables{i}), 'b-', 'LineWidth', 2, '
       DisplayName', 'With Frictions');
    hold on;
    % Plot without financial frictions
    plot(horizon, irfs_without.(variables{i}), 'r--', 'LineWidth', 2,
        'DisplayName', 'Without Frictions');
    title(titles{i}, 'FontSize', 14);
    xlabel('Periods', 'FontSize', 12);
    ylabel('Deviation', 'FontSize', 12);
    grid on;
    legend('show', 'FontSize', 10, 'Location', 'Best');
    hold off;
end

% Add a title
sgtitle('Comparison of IRFs: with and without financial frictions', '
   FontSize', 16);

% Save the plot
saveas(gcf, 'IRFs_Comparison.png');


%%
%Problem 2.2-2.3

% IRF data for both cases
load('IRFs_Taylor.mat');
%irfs_taylor = oo_.irfs;
load('IRFs_Christiano.mat');
%irfs_christiano = oo_.irfs;

% Time horizon for IRFs
horizon = 1:40;

% Variable names and corresponding titles
variables_christiano = {'pi_eps_g_shock', 'y_eps_g_shock', '
   r_alter_eps_g_shock', 'x_eps_g_shock', ...
```

```
                 's_eps_g_shock', 'c_eps_g_shock', 'g_eps_g_shock'};
titles = {'Inflation (\pi)', 'Output (y)', 'Interest Rate (r)', 'Marginal Cost (x)', ...
             'Stock Price (s)', 'Consumption (c)', 'Government Spending (g)'};

variables = {'pi_eps_g_shock', 'y_eps_g_shock', 'r_eps_g_shock', 'x_eps_g_shock', ...
                 's_eps_g_shock', 'c_eps_g_shock', 'g_eps_g_shock'};
% Number of variables
num_vars = length(variables);

% Create a figure
figure;

% Loop through variables and plot for both cases
for i = 1:num_vars
    subplot(ceil(num_vars / 2), 2, i);
    % Plot for Taylor Rule
    plot(horizon, irfs_taylor.(variables{i}), 'b-', 'LineWidth', 2, 'DisplayName', 'Taylor Rule');
    hold on;
    % Plot for Christiano et al. Rule
    plot(horizon, irfs_christiano.(variables_christiano{i}), 'r--', 'LineWidth', 2, 'DisplayName', 'Christiano Rule');
    title(titles{i}, 'FontSize', 14);
    xlabel('Periods', 'FontSize', 12);
    ylabel('Deviation', 'FontSize', 12);
    grid on;
    legend('show', 'FontSize', 10, 'Location', 'Best');
    hold off;
end

% Add a global title
sgtitle('Comparison of IRFs: Taylor rule and Christiano et al. (2005) rule', 'FontSize', 16);

% Save the plot as a high-resolution image
saveas(gcf, 'IRFs_Comparison_Taylor_Christiano.png');


%%
%Problem 4.6

```

```matlab
% IRF data
load('IRFs_data.mat');

% Time horizon for IRFs
horizon = 1:40;

% Variable names and corresponding titles
variables = {'pi_eps_g_shock', 'y_eps_g_shock', 'r_eps_g_shock', 'x_eps_g_shock', ...
             'stock_eps_g_shock', 'c_eps_g_shock', 'g_eps_g_shock', 'b_eps_g_shock', ...
             't_eps_g_shock'};
titles = {'Inflation (\pi)', 'Output (y)', 'Interest Rate (r)', 'Marginal Cost (x)', ...
          'Stock Price (s)', 'Consumption (c)', 'Government Spending (g)', ...
          'Debt (b)', 'Taxes (t)'};

% Number of variables
num_vars = length(variables);

% Create a figure
figure;

% Dynamically create subplots for each variable
for i = 1:num_vars
    subplot(ceil(num_vars / 3), 3, i); % Adjust layout dynamically
    plot(horizon, oo_.irfs.(variables{i}), 'LineWidth', 2);
    title(titles{i}, 'FontSize', 14);
    xlabel('Periods', 'FontSize', 12);
    ylabel('Deviation', 'FontSize', 12);
    grid on;
end

% Adjust tittle
sgtitle('IRFs to fiscal policy shock', 'FontSize', 16);
%Save
saveas(gcf, 'IRFs_to_fiscal_policy.png');
```

Listing 6: Plot IFRs for all problems

In the next page you can find handwritten solutions...

Problem 3. Optimal MP at the effective LB

$$\pi_t = -a(i_t - i^*) + b q_t + \pi_{t-1} + e_t \quad b \sim N(\bar{b}, \sigma_b), \; e_t \sim N(0, \sigma_e)$$

Loss function: $L(\pi_t) = E_t(\pi_t - \pi^*)^2$, Apply $Var(y) = E[y^2] - E[y]^2$

3.1.a. $$L(\pi_t) = Var(\pi_t - \pi^*) + \left[E_t[\pi_t - \pi^*]\right]^2 = Var(\pi_t) + \left[E_t(\pi_t - \pi^*)\right]^2$$

Plug π_t into Loss function:

$$L(\pi_t) = Var(-\alpha(i_t - i^*) + b q_t + \pi_{t-1} + e_t) + \left[E_t(-\alpha(i_t - i^*) + b q_t + \pi_{t-1} + e_t - \pi^*)\right]^2$$

As we see π^*, i_t, q_t - controlled by central bank and they are deterministic. So, these terms are do not contribute to the variance and can be excluded from Var. term. Similarly parameters as α and target interest rate i^*, and lag of inflation (π_{t-1}) are also deterministic. So, b and ε_t remain:

$$L(\pi_t) = Var(b q_t) + Var(e_t) + \left[E_t(-\alpha(i_t - i^*) + b q_t + \pi_{t-1} + e_t - \pi^*)\right]^2$$
$$= q_t^2 \sigma_b + \sigma_\varepsilon + \left[E_t(-\alpha(i_t - i^*) + b q_t + \pi_{t-1} + e_t - \pi^*)\right]^2.$$

Since we are not at LB, then FOC wrt i_t, q_t:

$$\frac{\partial L}{\partial i_t} = -2\alpha(-\alpha(i_t - i^*) + \bar{b} q_t + \pi_{t-1} + \cancel{e_t} - \pi^*) = 0, \text{ since } E_t(e_t) = 0$$

$$\frac{\partial L}{\partial q_t} = 2 q_t \sigma_b + 2\bar{b}(-\alpha(i_t - i^*) + \bar{b} q_t + \pi_{t-1} - \pi^*) = 0$$

$\boxed{3.1.8.}$

From FOC of i_t: $\cancel{2}\alpha^{\cancel{2}}(i_t - i^*) = \cancel{2}\cancel{\alpha}(\bar{b}q_t + \pi_{t-1} - \pi^*)$

$$\alpha i_t - \alpha i^* = \bar{b}q_t + \pi_{t-1} - \pi^*$$

$$\boxed{i_t = \frac{\bar{b}q_t + \pi_{t-1} - \pi^*}{\alpha} + i^*}$$

And FOC of q_t:

$$\cancel{2}q_t\sigma_\varepsilon + \cancel{2}\bar{b}^2 q_t = -2\bar{b}(-\alpha(i_t - i^*) + \pi_{t-1} - \pi^*)$$

$$q_t(\bar{b}^2 + \sigma_\varepsilon) = -\bar{b}(-\alpha(i_t - i^*) + \pi_{t+1} + \pi^*)$$

$$q_t = -\frac{1}{(\bar{b}^2 + \sigma_\varepsilon)}\bar{b}\left(-\alpha(i_t - i^*) + \pi_{t-1} - \pi^*\right)$$

Now plug i_t:

$$q_t = -\frac{1}{(\bar{b}^2 + \sigma_\varepsilon)}\bar{b}\left[-\cancel{\alpha}\left(\frac{\bar{b}q_t + \pi_{t-1} - \pi^*}{\cancel{\alpha}} + \cancel{i^*} - \cancel{i^*}\right) + \pi_{t-1} - \pi^*\right]$$

$$= -\frac{1}{(\bar{b}^2 + \sigma_\varepsilon)}\bar{b}\left[-\bar{b}q_t - \cancel{\pi_{t-1}} + \cancel{\pi^*} + \cancel{\pi_{t-1}} - \cancel{\pi^*}\right]$$

$$\cancel{q_t} = -\frac{\bar{b}^2 q_t}{(\bar{b}^2 + \sigma_\varepsilon)} = -\cancel{q_t} + \frac{\bar{b}^2 q_t}{\sigma_\varepsilon}$$

$$\frac{\bar{b}^2 q_t}{\sigma_\varepsilon} = 0 \quad \Rightarrow \quad \boxed{q_t = 0}$$

And $\boxed{i_t = \frac{1}{\alpha}(\pi_{t-1} - \pi^*) + i^*}$

3.1.C.

We observe that the optimal level of quantitative easing ($q_t = 0$) occurs because interest rate is not at its LB under the initial assumption. In this case, policy makers tend to rely solely on the interest rate as their main monetary policy tool. The key reason of this preference lies in the relative uncertainty associated with the parameters governing these ~~p~~ instruments. Specifically α is less uncertain than β in QE.

The key difference is that for estimating α we have ~~enou~~ enougth data points comparing to QE, β, therefore estimating α is more accurate. Moreover, QE is relatively new instrument. It has been used as extensively as the nominal interest rate, which about the limited data points to estimate β.

As a result, central bankers are prefer interest rate to use as a primary instrument.

3.1.d.

Fist we will plug optimal value of i_t and q_t into inflation (π_t) then take expectation:

$$\pi_t = -\alpha\left[\frac{1}{\alpha}(\pi_{t-1} - \pi^*) + \cancel{i^*} - \cancel{i^*}\right] + \beta \cdot 0 + \pi_{t-1} + \varepsilon_t$$

$$= -\cancel{\pi_{t-1}} + \pi^* + \cancel{\pi_{t-1}} + \varepsilon_t$$

$$= \pi^* + \varepsilon_t$$, now take expection of π_t.

$$E_t(\pi_t) = E_t[\pi^* + \varepsilon_t]$$, since $E_t(\varepsilon_t) = 0$

$$= \pi^* + E_t(\varepsilon_t)$$

$$\boxed{E_t(\pi_t) = \pi^*}$$

Therefore, the expected inflation today is equal to the inflation target of the Central Bank. Inflation target is know because we take exp. at time t.

$\boxed{3.2.a.}$ Now, we are at the LB, so i_t^{LB}, we have to see that CB can't reduce interest rate further. Given this constant, we have to focus on minimizing Loss function taking derivative w.r.t q_t.

$$L(\pi_t) = q_t^2 \sigma_\beta + \sigma_\varepsilon + \left(-\alpha(i_t^{LB} - i^*) + \bar{\beta} q_t + \pi_{t-1} + \pi^*\right)^2$$

FOC:

$$\frac{\partial L}{\partial q_t} = 2 q_t \sigma_\beta + 2\bar{\beta}\left(-\alpha(i_t^{LB} - i^*) + \bar{\beta} q_t + \pi_{t-1} - \pi^*\right) = 0$$

$\boxed{3.2.b}$

From FOC of q_t:

$$2 q_t \sigma_\beta + 2\bar{\beta}^2 q_t + 2\bar{\beta}\left(-\alpha(i_t^{LB} - i^*) + \pi_{t-1} - \pi^*\right) = 0$$

$$q_t = -\frac{1}{(\sigma_\beta + \bar{\beta}^2)} \bar{\beta}\left(-\alpha(i_t^{LB} - i^*) + \pi_{t-1} - \pi^*\right)$$

$$\boxed{q_t = \frac{1}{(\sigma_\beta - \bar{\beta}^2)} \bar{\beta}\left(\alpha(i_t^{LB} - i^*) - \pi_{t-1} + \pi^*\right)} \text{ and } \boxed{i_t = i_t^{LB}}$$

$\boxed{3.2.c}$ As we are in a case $i_t = i_t^{LB}$, the CB can't reduce nominal interest rate further, therefore, the only usable instrument is QE. Central bankers focus on optimizing QE to find optimal instrument for inflation stabilization. Since, the nominal interest rate is not an option to do any more.

$\boxed{3.2.d}$ Now, first take expectation of π_t, then plug the value of q_t:

$$E_t(\pi_t) = E_t\left(-\alpha\left(i_t^{LB} - i^*\right) + b\, q_t + \pi_{t-1} + e_t\right)$$

$$= E_t\left(-\alpha\left(i_t^{LB} - i^*\right)\right) + E_t(b\, q_t) + E_t(\pi_{t-1}) + E_t(e_t)$$

$$= -\alpha\left(i_t^{LB} - i^*\right) + \bar{b}\, q_t + \pi_{t-1}.$$

Then plug q_t into $E_t(\pi_t)$:

$$E_t(\pi_t) = -\alpha\left(i_t^{LB} - i^*\right) + \bar{b}\left[\frac{1}{(\sigma_b + \bar{b}^2)}\, \bar{b}\left(+\alpha\left(i_t^{LB} - i^*\right) - \pi_{t-1} + \pi^*\right)\right] + \pi_{t-1}$$

$$= -\alpha\left(i_t^{LB} - i^*\right) + \frac{\bar{b}^2}{(\sigma_b + \bar{b}^2)}\left(\alpha\left(i_t^{LB} - i^*\right)\right) - \frac{\bar{b}^2}{(\sigma_b + \bar{b}^2)}\pi_{t-1} + \frac{\bar{b}^2}{(\sigma_b + \bar{b}^2)}\pi^* + \pi_{t-1}$$

$$= \alpha\left(i_t^{LB} - i^*\right)\left[\frac{\cancel{\bar{b}^2} + \sigma_b - \cancel{\bar{b}^2}}{(\sigma_b + \bar{b}^2)}\right] + \left[\frac{-\cancel{\bar{b}^2} + \sigma_b + \cancel{\bar{b}^2}}{(\sigma_b + \bar{b}^2)}\right]\pi_{t-1} + \frac{\bar{b}^2}{(\sigma_b + \bar{b}^2)}\pi^*$$

$$E_t(\pi_t) = \left[\alpha\left(i^* - i_t^{LB}\right) + \pi_{t-1}\right]\frac{\sigma_b}{(\sigma_b + \bar{b}^2)} + \frac{\bar{b}^2}{(\sigma_b + \bar{b}^2)}\pi^*$$

Note that the expected inflation depends on lag of inflation.

*) If $i_t^{LB} = i_t^*$, $i_t^{LB} - i_t^* = 0$, then:

$$\boxed{E_t(\pi_t) = \frac{\sigma_b}{(\sigma_b + \bar{b}^2)}\pi_{t-1} + \frac{\bar{b}^2}{(\sigma_b + \bar{b}^2)}\pi^*}$$

*) if $\pi_{t-1} = \pi^*$, then $\boxed{E_t(\pi_t) = \pi^*}$, as we know target value, ~~at~~ because we take expectation at time t.

3.3.

When we take into account the case where output gap also influence on inflation, it becomes clear that FOCs from the initial analysis remain unchanged. This is because there is no any link ~~to the output gap~~ between output gap and other variables (i_t, δ, π_{t-1}, q_t).

In practice, while output gap does effect inflation, but it doesn't have direct effect on CB's decisions regarding the interest rate and QE. Due to CB's main focus on deviations in inflation as we can see in Loss function. Although output gap has overall effect on economy, but ~~they doesn't~~ it doesn't significantly effects the bank's approaches to manage this type of MP tools.