CFRM 461: Probability and Statistics for Computational Finance Homework 3

1. (a) They are not mutually exclusive because one person can speak multiple languages.

   (b) They are not independent.

$$P(A \cap B) = P(A) \times P(B)$$
$$\implies 0.04 = 0.24 \times 0.07$$
$$\implies 0.04 \,! = 0.02$$

   (c) 24 + 7 - 4 = 27%

   (d) 24 - 4 = 20%

   (e) 100 - 27 = 73%

2. 1 - P(1st is black ∩ 2nd is black ∩ 3rd is black)

$$1 - \frac{26}{52} * \frac{25}{51} * \frac{24}{50} = 0.88$$

3. M = Miss; R = Rain

$$P(M \mid R) = 0.05$$
$$P(M \mid !R) = 0.01$$
$$P(R) = 0.28$$
$$P(R \mid M) = \frac{P(M \mid R) * P(R)}{P(M)}$$
$$P(M \cap R) = 0.05 * 0.28 = 0.014$$
$$P(M \cap !R) = 0.01 * (1 - 0.28) = 0.0072$$
$$P(M) = 0.014 + 0.0072 = 0.0212$$
$$P(R \mid M) = \frac{0.05 * 0.28}{0.0212}$$
$$\implies 0.66$$

4. (a) $\mathbb{P}(\frac{11}{36} = 1, \frac{9}{36} = 2, \frac{7}{36} = 3, \frac{5}{36} = 4, \frac{3}{36} = 5, \frac{1}{36} = 6)$

   (b)

$$F_x(x) = \left\{ \begin{array}{cc} 0, & \text{x} < 1 \\ 0.31, & 1 \leq \text{x} < 2 \\ 0.56, & 2 \leq \text{x} < 3 \\ 0.75, & 3 \leq \text{x} < 4 \\ 0.89, & 4 \leq \text{x} < 5 \\ 0.97, & 5 \leq \text{x} < 6 \\ 1, & \text{x} \geq 6 \end{array} \right\}$$

(c)

$$\frac{11}{36} * 1 + \frac{9}{36} * 2 + \frac{7}{36} * 3 + \frac{5}{36} * 4 + \frac{3}{36} * 5 + \frac{1}{36} * 6 = \frac{91}{36} = 2.53$$

(d)

$$(1 - 2.53)^2 * \frac{11}{36} + (2 - 2.53)^2 * \frac{9}{36} + (3 - 2.53)^2 * \frac{7}{36} +$$
$$(4 - 2.53)^2 * \frac{5}{36} + (5 - 2.53)^2 * \frac{3}{36} + (6 - 2.53)^2 * \frac{1}{36} = 2.03$$

5. (a) $(1 - 0.03)^6 * .03 = 0.02$

(b)

$$\lambda = 2.26 \implies P(X \leq 1) = 2.26^0 e^{-2.26} + 2.26 e^{-2.26} = 0.34$$

(c) $\sqrt{30 * .976(1 - .976)} = 0.84$