CFRM 462: Introduction to Computational Finance and Econometrics Homework 2

1. a) 1 - pnorm
$$(0.1, \text{mean} = 0.05, \text{ sd} = 0.1) = 0.3085375$$

b) pnorm
$$(-0.1,\text{mean}=0.05,\text{sd}=0.1)=0.0668072$$

c)
$$pnorm(0.15,mean=0.05,sd=0.1) - pnorm(-0.05,mean=0.05,sd=0.1) = 0.6826895$$

d)
$$qnorm(.01,mean=0.05,sd=0.1) = -0.1826348$$

e) qnorm
$$(.05,\text{mean}=0.05,\text{sd}=0.1) = -0.1144854$$

f) qnorm
$$(.95,\text{mean}=0.05,\text{sd}=0.1)=0.2144854$$

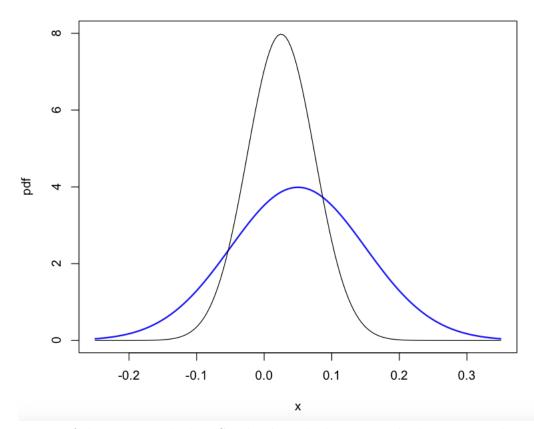
g) qnorm
$$(.99,\text{mean}=0.05,\text{sd}=0.1)=0.2826348$$

2.
$$x.vals = seq(-0.25, 0.35, length = 150)*0.1 + 0.05$$

$$plot(x.vals,dnorm(x.vals,mean = 0.05,sd = 0.1),type = "l",lwd = 2, col = "blue", xlab = "x", ylab = "pdf",main = "Combined PDF",ylim = $c(0.8)$)$$

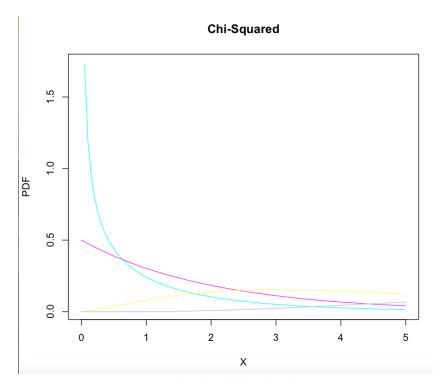
lines(x.vals,dnorm(x.vals,mean = 0.025,sd = 0.05))

Combined PDF

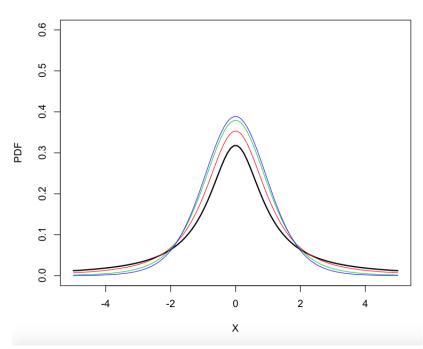


Microsoft has more risk than Starbucks, which means a higher potential return. However, Starbucks has a less risk which means your expected return will be lower.

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3. > \text{mu.}\mathbf{R} = 0.04
      > sd.R = 0.09
      >
      > w0 = 10000
      >
      > q.01.R = mu.R + sd.R *qnorm(0.01)
      > \mathbf{q}.05.\mathbf{R} = \mathbf{mu.R} + \mathbf{sd.R} *\mathbf{qnorm}(0.05)
      > var.01 = abs(q.01.R * w0)
      > var.05 = abs(q.05.R * w0)
      > \mathbf{var}.01
       [1] 1693.713
      > var.05
       [1] 1080.368
4. Continuously Compounded Monthly and Annual Var
       e.01.R = exp(mu.R + sd.R * qnorm(0.01)) - 1
       e.05.R = exp(mu.R + sd.R * qnorm(0.05)) - 1
       e. Var. 01 = abs(e. 01.R * w0)
       e. Var. 05 = abs(e. 05.R * w0)
      > e.Var.01
       [1] 1558.046
      > e.Var.05
      [1] 1024.055
               b)
      > abs(e.01.R * 12 * w0)
       [1] 18696.55
      > abs(e.05.R * 12 * w0)
       [1] 12288.66
5. x. vals = seq(-5.5, length = 100)
       pos = seq(0,5, length = 100)
       plot(x.vals,dt(x.vals,1),type = "l", lwd = 2, col = 1, ylim = c(0,0.6), xl
       lines(x.vals,dt(x.vals,2),type = "l", lwd = 1, col = 2)
       lines (x. vals, dt(x. vals, 5), type = "l", lwd = 1, col = 3)
       lines(x.vals,dt(x.vals,10),type = "l", lwd = 1, col = 4)
      #Chi Squared
       plot(pos, dchisq(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, dchisq(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "l", lwd = 1, col = 5, xlab = "X", ylab = "Plot(pos, 1), type = "Plot(p
       lines (pos, dchisq (pos,2), type = "l", lwd = 1, col = 6) lines (pos, dchisq (pos,5), type = "l", lwd = 1, col = 7)
       lines(pos,dchisq(pos,10),type = "l", lwd = 1, col = 8)
```



Student T Distribution



6.
$$e.x = 1*.3 + 2*.3 + 3 * 0.4$$

 $var.x = 1*.3 + 4*.3 + 9 * .4 - E.x^2$
 $sd.x = sqrt(var.x)$

$$e.y = 1 * 0.2 + 2*0.3 + 3*0.5$$

 $var.y = 1* 0.2 + 4 * 0.3 + 9 * 0.5 - e.y^2$
 $sd.y = sqrt(var.y)$

$$\begin{array}{l} {\rm c.\,xy} = (1*1*0.1) \,+\, (1*2*0.2) \,+\, (2*1*0.1) \,+\, (2*3*0.2) \,+\, (3*2*0.1) \,+\, (9*0.3) \\ {\rm cov.\,xy} = {\rm c.\,xy} - {\rm c.\,y*e.\,x} \\ {\rm cor.\,xy} = {\rm cov.\,xy}/({\rm sd.\,x*sd.\,y}) \\ {\rm > answers} < - {\rm c}({\rm c.\,x,\,var.\,x,\,sd.\,x,\,e.\,y,\,var.\,y,\,sd.\,y}) \\ {\rm a)} \,\, 2.1000000 \,\, 0.6900000 \,\, 0.8306624 \,\, 2.3000000 \,\, 0.6100000 \,\, 0.7810250 \\ {\rm b)} \,\, 0.37, \,\, 0.5703117 \\ {\rm c)} \,\, {\rm No \,\, they \,\, are \,\, not \,\, independent.} \\ P(y=1|x=1) = \frac{P(x=1,y=1)}{P(x=1)} = \frac{.1}{.3} \neq P(y=1) = 0.2 \\ \hline 7. \,\, \# {\rm Problem} \,\, 7 \\ {\rm b.\,amzn} \,\, = \,\, 38.23 \\ {\rm b.\,costco} \,\, = \,\, 41.11 \\ {\rm s.\,amzn} \,\, = \,\, 41.29 \\ {\rm s.\,costco} \,\, = \,\, 41.74 \\ \# {\rm Simple \,\,\, Monthly \,\,\, Return} \\ {\rm r.\,amzn} \,\, = \,\, ({\rm s.\,amzn} \,\, - \,\, {\rm b.\,amzn})/{\rm b.\,amzn} \\ {\rm r.\,costco} \,\, = \,\, ({\rm s.\,costco} \,\, - \,\, {\rm b.\,costco})/{\rm b.\,costco} \\ {\rm > \,\, r.\,amzn} \\ {\rm [1]} \,\,\, 0.08004185 \\ {\rm > \,\, r.\,costco} \\ {\rm [1]} \,\,\, 0.01532474 \\ \end{array}$$

#CC Return

cc.amzn = log(s.amzn/b.amzn)

cc.costco = log(s.costco/b.costco)

> cc.amzn

[1] 0.07699979

> cc.costco

[1] 0.0152085

#dividend

$$div = 0.1$$

rd.costco = (s.costco + div)/(b.costco) - 1

rd.yield = div/b.costco

> rd.costco

[1] 0.01775724

```
> rd.yield
[1] 0.002432498
#annualized
annual.amzn = (1+r.amzn)^12 - 1
annual.costco = (1+r.costco)^12 - 1
> annual.amzn
[1] 1.519341
> annual.costco
[1] 0.2002166
annual.amzn.cc = 12 * cc.amzn
annual.costco.cc = 12 * cc.costco
> annual.amzn.cc
[1] 0.9239975
> annual.costco.cc
[1] 0.182502
x_a = 0.8
x_c = 0.2
r.combined = x_a * r.amzn + x_c * r.costco
cc.combined = log(1+r.combined)
> r.combined
[1] 0.06709843
> cc.combined
[1] 0.06494322
> R_e
[1] -0.1333333
> R_uk
[1] 0.125
> R_us
[1] -0.025
                             R_e = \frac{e_1 - e_0}{e_0}
```

$$R_{e} = \frac{e_{1} - e_{0}}{e_{0}}$$

$$R_{u}k = \frac{uk_{1} - uk_{0}}{uk_{0}}$$

$$R_{us} = (1 + R_{e})(1 + R_{u}k) - 1$$

8.

$$cov(R_t, R_{t-1}) = E[R_t R_{t-1}] - E[R_t] E[R_{t-1}]$$

$$0 = E[R_t R_{t-1}] - E[R_t] E[R_{t-1}]$$

$$E[R_t R_{t-1}] = E[R_t] E[R_{t-1}]$$

$$R_t \sim N(\mu, \sigma^2) \therefore E[R_t] = \mu$$

$$\implies E[R_t R_{t-1}] = \mu^2$$

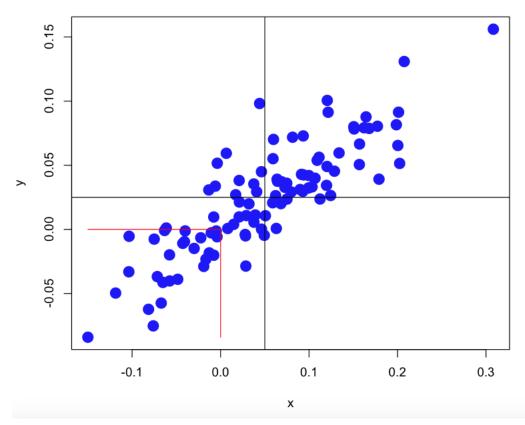
$$E[R_t(2)] = E[(1 + R_t)(1 + (R_{t-1}) - 1]$$

$$(1 + E[R_t])(1 + E[R_{t-1}]) - 1$$

$$\implies (1 + \mu)^2 - 1$$

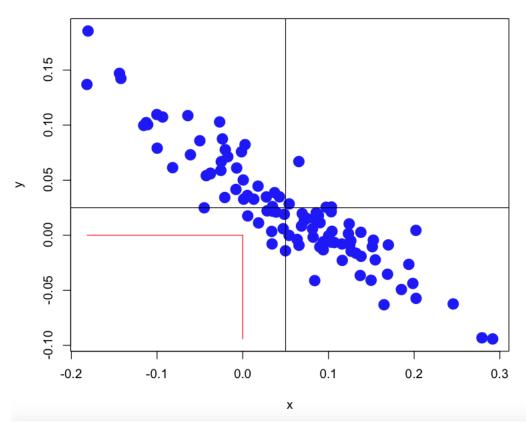
No $R_t(2)$ is not normally distributed since it is the product of two normal distribution.

Bivariate normal: rho=0.9



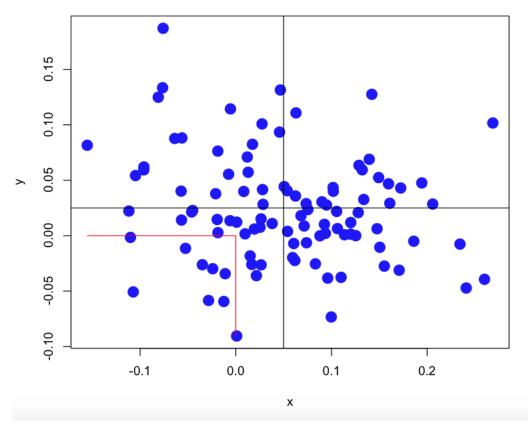
The graph has a positive linear relationship and the joint probability is 0.245

Bivariate normal: rho=-0.9



The graph has a negative linear relationship and the joint probability is 0.000803

Bivariate normal: rho=0



The graph has no correlation and the joint probability is 0.0952