ECON 424 Homework 7

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Due: Friday, August 11th at 11:59pm via Canvas

Readings

- Book chapters and class slides on Portfolio Theory with Matrix Algebra and Portfolio Theory with No Short Sales. Ruppert and Matteson, Chapter 16.
- Notes on using the solver in Excel (on Canvas). This will be very helpful
- Example spreadsheet: 3firmClassExample.xls

Programs and Data

- Data for this lab are on the class homework page in the file econ424lab7returns.csv
- Example Excel spreadsheet: 3firmExample.xls
- R script files: econ424lab7.r, portfolioTheoryMatrix.Rmd and portfolioTheoryNoShorts.Rmd.

Exercises

Using the monthly closing price data on the four Northwest stocks (Boeing, Microsoft, Nordstrom and Starbucks) over the period February 1995 - January 2000, you will estimate expected returns, variances and covariances to be used as inputs to the Markowitz

algorithm. You will compute efficient portfolios allowing for short-sales and efficient portfolios not allowing for short sales. For computing efficient portfolios, you will use matrix algebra formulas to simplify the running of the solver in Excel. This is a long homework assignment and you should start early.

On the class web page is the script file econ424lab7.r, which walks you through the portfolio theory calculations using the R functions in the IntroCompFinR package. The Excel file 3firmExample.xls gives a spreadsheet template for the portfolio calculations. Use this spreadsheet as a template for your analysis in this lab (and also for the class project). For this lab I want you to do all of the calculations in an Excel spreadsheet. You can check your results using the R code. As in the previous labs, copy and paste all statistical results and graphs into a MS Word document while you work (or create an R markdown file), and add any comments and answer all questions in this document. Please make sure to add comments to your output. Don?t just turn in the numbers and graphs! See the previous solutions on the class homework page for examples of the types of comments that I am looking for.

Return data on Boeing, Nordstrom, Starbucks and Microsoft stock are in the Excel file 424lab7returns.csv on the class webpage.

1. Using the return data on Boeing, Nordstrom, Starbucks and Microsoft in the matrix ret.mat, estimate the parameters μ_i, σ_i^2, σ_i, σ_{ij}, and ρ_{ij} of the constant expected return (CER) model:

$$\begin{aligned} R_{it} &= \mu_i + \epsilon_{it},\ t = 1, ..., T \\ \epsilon_{it} &\sim\ iid\ N(0, \sigma_i^2) \\ cov(\epsilon_{it}, \epsilon_{jt}) &= \sigma_{ij} \end{aligned}$$

where R_{it} denotes the simple returns on asset i (i =Boeing, Nordstrom, Starbucks,

and Micrsoft). Compute estimated standard errors for the means and volatilities and briefly comment. Arrange these estimates and standard errors nicely in a table.

2. Show the estimated risk-return tradeoff of these assets (i.e., plot the means on the y-axis and the standard deviations on the horizontal axis). Briefly comment.

a. Assuming a risk free rate of 0.005 (0.5% per month or about 6% per year) compute the Sharpe ratios for each asset. Which asset has the highest Sharpe ratio?

3. Compute the global minimum variance portfolio allowing short-sales. The minimization problem is

$$\min_m \sigma_p^2 = m'\Sigma m \text{ subject to}$$
$$m'1 = 1$$

where m is the vector of portfolio weights and Σ is the covariance matrix. Briefly comment on the weights. Compute the expected return and standard deviation and add the points to the risk return graph.

4. Of the four stocks, determine the stock with the largest estimated expected return. Use this maximum average return as the target return for the computation of an efficient portfolio allowing for short-sales. That is, find the minimum variance portfolio that has an expected return equal to this target return. The minimization problem is

$$\min_m \sigma_p^2 = x'\Sigma x \text{ subject to}$$
$$x'1 = 1$$
$$x'\mu = \mu_0$$

where x is the vector of portfolio weights, μ is the vector of expected returns and μ_0

is the target expected return. Are there any negative weights in this portfolio? Compute the expected return, variance and standard deviation of this portfolio. Finally, compute the covariance between the global minimum variance portfolio and the above efficient portfolio using the formula $cov(R_{p,m}, R_{p,x}) = m'\Sigma m$.

5. Repeat questions 3 and 4 but this time do not allow short sales. That is, add the following constraint: $x_i \geq 0$ for $i = 1, ..., 4$.

6. Using the fact that all efficient portfolios that allow short-sales can be written as a convex combination of two efficient portfolios, compute efficient portfolios as convex combinations of the global minimum variance portfolio computed from question 3 and the efficient portfolio computed in question 4. That is, compute

$$z = \alpha \cdot m + (1 - \alpha) \cdot x$$

for values of α between 1 and -1 (e.g., make a grid for $\alpha = 1, 0.9, ..., -1$). Compute the expected return, variance and standard deviation of these portfolios.

7. Plot the Markowitz bullet based on the efficient portfolios you computed in question 6. On the plot, indicate the location of the minimum variance portfolio and the location of the efficient portfolio found in question 4.

8. Compute the tangency portfolio assuming the risk-free rate is 0.005 ($rf = 0.5\%$) per month. That is, solve

$$\max_t \text{ slope} = \frac{\mu_p - r_f}{\sigma_p} \text{ subject to}$$

$$\mu_p = t'\mu$$

$$\sigma_p = (t'\Sigma t)^{1/2}$$

$$1 = t'1$$

where t denotes the portfolio weights in the tangency portfolio. Are there any negative weights in the tangency portfolio? If so, interpret them.

9. Repeat question 8 but this time do not allow short sales. That is, add the following constraint: $x_i \geq 0$ for $i = 1, ..., 4$.

10. On the graph with the Markowitz bullet, plot the efficient portfolios that are combinations of T-bills and the tangency portfolio. Indicate the location of the tangency portfolio on the graph.

11. Find the efficient portfolio of combinations of T-bills and the tangency portfolio that has the same SD value as Starbucks. What is the expected return on this portfolio? Indicate the location of this portfolio on your graph of the Markowitz bullet.

12. Suppose you have \$50,000 to invest over a period of time. For each of the four stocks and the global minimum variance portfolio, compute the 1% and 5% value-at-risk (VaR) for a one month investment.