# ECON 424 Homework 2

# Summer 2017

Anthony Sanford

Department of Economics, University of Washington

Email: sanfoan@uw.edu

Due: Friday, July 7 at 11:59pm via Canvas

## Instructions

In this lab you will become more familiar with random variables and probability distributions. Try to do all of the calculations and plots in R. You can also do everything in Excel too. You will find the examples in probReview.R and probReview.xls to be helpful for some of the exercises that follow.

## Review Exercises

1. Suppose X is a normally distributed random variable with mean 0.05 and variance $(0.10)^2$. Compute the following:

- $Pr(X > 0.10)$

- $Pr(X < -0.10)$

- $Pr(-0.05 < X < 0.15)$

- 1% quantile, $q_{.01}$

- 5% quantile, $q_{.05}$

- 95% quantile, $q_{.95}$

- 99% quantile, $q_{.99}$

Hint: you can use the R functions pnorm() and qnorm() to answer these questions.

2. Let $X$ denote the monthly return on Microsoft Stock and let Y denote the monthly return on Starbucks stock. Assume that $X \sim N(0.05, (0.10)^2)$ and $Y \sim N(0.025, (0.05)^2)$.

- Using a grid of values between -0.25 and 0.35, plot the normal curves for $X$ and $Y$. Make sure that both normal curves are on the same plot.

- Comment on the risk-return tradeoffs for the two stocks.

3. Let R denote the simple monthly return on Microsoft stock and let W0 denote initial wealth to be invested over the month. Assume that $R \sim N(0.04, (0.09)^2)$ and that $W_0 =\$100,000$

- Determine the 1% and 5% value-at-risk (VaR) over the month on the investment. That is, determine the loss in investment value that may occur over the next month with 1% probability and with 5% probability.

4. Let r denote the continuously compounded monthly return on Microsoft stock and let W0 denote initial wealth to be invested over the month. Assume that $r \sim N(0.04, 0.09)^2)$ and that $W_0 =\$100,000$.

- Determine the 1% and 5% value-at-risk (VaR) over the month on the investment. That is, determine the loss in investment value that may occur over the next month with 1% probability and with 5% probability. (Hint: compute the 1% and 5% quantile from the Normal distribution for r and then convert continuously compounded return quantile to a simple return quantile using the transformation $(R = e^r - 1)$

- Determine the 1% and 5% value-at-risk (VaR) over the year on the investment. Hint: to answer this question, you must determine the normal distribution that applies to the annual (12 month) continuously compounded return. This was done as an example in class.

5. In this question, you will examine the chi-square and Student?s t distributions.

   - On the same graph, plot the probability curves of chi-squared distributed random variables with 1, 2, 5 and 10 degrees of freedom. Use different colors and line styles for each curve.

   - On the same graph, plot the probability curves of Student?s t distributed random variables with 1, 2, 5 and 10 degrees of freedom. Also include the probability curve for the standard normal distribution. Use different colors and line styles for each curve.

6. Consider the following joint distribution of X and Y:

| X/Y | 1 | 2 | 3 |
|-----|-----|-----|-----|
| 1 | 0.1 | 0.2 | 0 |
| 2 | 0.1 | 0 | 0.2 |
| 3 | 0 | 0.1 | 0.3 |

   (a) Find the marginal distributions of X and Y. Using these distributions, compute E[X], var(X), SD(X), E[Y], var(Y) and SD(Y).

   (b) Compute COV(X,Y) and CORR(X,Y)

   (c) Are X and Y independent? Fully justify your answer.

7. Consider a one month investment in two Northwest stocks: Amazon and Costco. Suppose you buy Amazon and Costco at the end of September at $P_{A,t-1} = \$38.23$,

$P_{C,t-1} = \$41.11$ and then sell at the end of the October for $P_{A,t} = \$41.29$, $P_{C,t} = \$41.74$. (Note: these are actual closing prices for 2004 taken from Yahoo!)

(a) What are the simple monthly returns for the two stocks?

(b) What are the continuously compounded returns for the two stocks?

(c) Suppose Costco paid a \$0.10 per share cash dividend at the end of October. What is the monthly simple total return on Costco? What is the monthly dividend yield?

(d) Suppose the monthly returns on Amazon and Costco from question (a) above are the same every month for 1 year. Compute the simple annual returns as well as the continuously compounded annual returns for the two stocks.

(e) At the end of September, 2004, suppose you have \$10,000 to invest in Amazon and Costco over the next month. If you invest \$8000 in Amazon and \$2000 in Costco, what are your portfolio shares, $x_A$ and $x_C$.

(f) Continuing with the previous question, compute the monthly simple return and the monthly continuously compounded return on the portfolio. Assume that Costco does not pay a dividend.

8. Consider an investment in a foreign stock (e.g. a stock trading on the London stock exchange) by a U.S. national (domestic investor). The domestic investor takes U.S. dollars, converts them to the foreign currency (e.g. British Pound) via the exchange rate (price of foreign currency in U.S. dollars) and then purchases the foreign stock using the foreign currency. When the stock is sold, the proceeds in the foreign currency must then be converted back to the domestic currency. To be more precise, consider the information in the table below:

| Time | Cost of 1 Pound | Value of UK Shares | Value in U.S. \$ |
|------|-----------------|--------------------|------------------|
| 0 | \$1.50 | £40 | $1.5 \cdot 40 = 60$ |
| 1 | \$1.30 | £45 | $1.3 \cdot 45 = 58.5$ |

(a) Compute the simple rate of return, $R_e$, from the prices of foreign currency.

(b) Compute the simple rate of return, $R_{uk}$, from the UK stock prices.

(c) Compute the simple rate of return, $R_{us}$, from the prices in US dollars.

(d) . What is the relationship between $R_{us}$, $R_{uk}$ and $R_e$?

9. Let $R_t$ denote the simple monthly return and assume that $R_t \sim iidN(\mu, \sigma^2)$. Consider the 2-period simple return $R_t(2) = (1 + R_t)(1 + R_{t-1}) - 1$.

(a) Assuming that $cov(R_t, R_{t-1}) = 0$, show that $E[R_t R_{t-1}] = \mu^2$. Hint: Use $cov(R_t, R_{t-1}) = E[R_t R_{t-1}] - E[R_t]E[R_{t-1}]$.

(b) Show that $E[R_t(2)] = (1 + \mu)^2 - 1$

(c) Is $R_t(2)$ normally distributed? Why or why not?

# R Exercises: Simulating Data from Bivariate distributions

Let X and Y be distributed bivariate normal with

$\mu_x = 0.05$, $\mu_y = 0.025$, $\sigma_x = 0.10$, $\sigma_y = 0.05$

(a) Using R package mvtnorm function rmvnorm(), simulate 100 observations from the bivariate distribution with $\rho_{xy} = 0.9$ . Using the plot() function create a scatterplot of the observations and comment on the direction and strength of the linear association. Using the function pmvnorm(), compute the joint probability $Pr(X \leq 0, Y \leq 0)$.

(b) Using R package mvtnorm function rmvnorm(), simulate 100 observations from the bivariate distribution with $\rho_{xy} = -0.9$. Using the plot() function create a scatterplot of the observations and comment on the direction and strength of the linear association. Using the function pmvnorm(), compute the joint probability $Pr(X \leq 0, Y \leq 0)$.

(c) Using R package mvtnorm function rmvnorm(), simulate 100 observations from the bivariate distribution with $\rho_{xy} = 0$. Using the plot() function create a scatterplot of the observations and comment on the direction and strength of the linear association. Using the function pmvnorm(), compute the joint probability $Pr(X \leq 0, Y \leq 0)$.