1. Monte Carlo Simulation in the CER Model

a)

Table 1: Parameter Estimates

Asset	μ_i	σ_i^2	σ_i
VBISX	0.00428	3.99e-05	0.00632
FBGRX	0.00214	3.34e-03	0.05783
GOOGL	0.00846	1.05e-02	0.10250

Table 2: Parameter Estimates Matrix

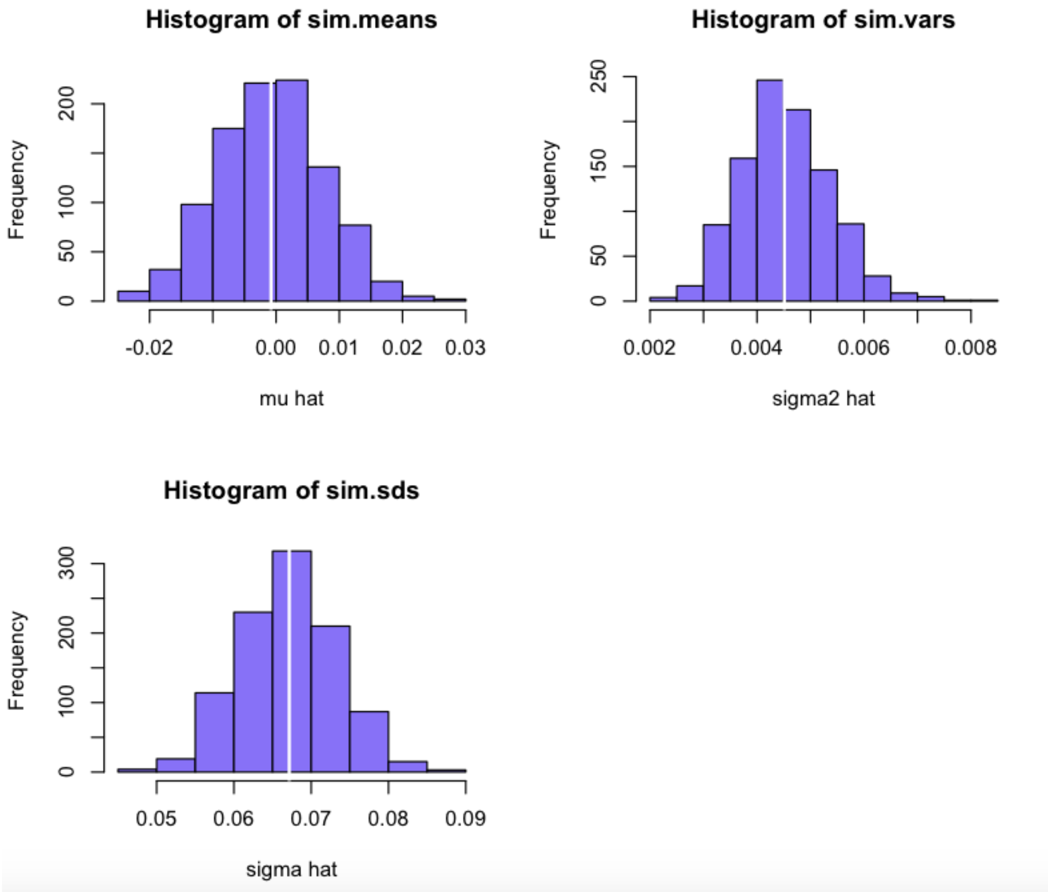
Parameters	VBISX,FBGRX	VBISX,GOOGL	FBGRX,GOOGL
σ_{ij}	-1.69e-05	-1.98e-04	3.82e-03
ρ_{ij}	-0.0464	-0.3065	0.6450

b)

```
             VBISX    FBGRX   GOOGL
muhat.vals  0.004282 0.00214 0.00846
se.muhat    0.000815 0.00747 0.01323

                 VBISX    FBGRX    GOOGL
sigma2hat.vals 3.99e-05 0.003345 0.01051
se.sigma2hat   7.28e-06 0.000611 0.00192

                VBISX    FBGRX   GOOGL
sigmahat.vals 0.006316 0.05783 0.10250
se.sigmahat   0.000577 0.00528 0.00936

            VBISX,FBGRX VBISX,GOOGL FBGRX,GOOGL
rhohat.vals     -0.0464      -0.306      0.6450
se.rhohat        0.1288       0.117      0.0754
```

μ is precise for VBISX, but not for FBGRX and GOOGL. σ_2 and σ are precise for all the assets. ρ is only precise for FBGRX, GOOGL.

c) The distribution for the mean looks slightly positively skewed, but for the most part looks normally distributed. The variance looks very positively skewed and the sd looks normally distributed.



The Monte Carlo simulations do a good job of estimating the parameters and all of the parameters are close to their true values.

```
> c(mu, mean(sim.means))
 [1] -0.000800 -0.000771
> mean(sim.means) - mu
 [1] 2.9e-05
> c(sd^2, mean(sim.vars))
 [1] 0.00451 0.00453
> mean(sim.vars) - sd^2
 [1] 2.33e-05
> c(sd, mean(sim.sds))
 [1] 0.0672 0.0670
> mean(sim.sds) - sd
 [1] -0.000117
> c(se.muhat["FBGRX"], sd(sim.means))
   FBGRX
 0.00747 0.00831
> c(se.sigma2hat["FBGRX"], sd(sim.vars))
     FBGRX
```

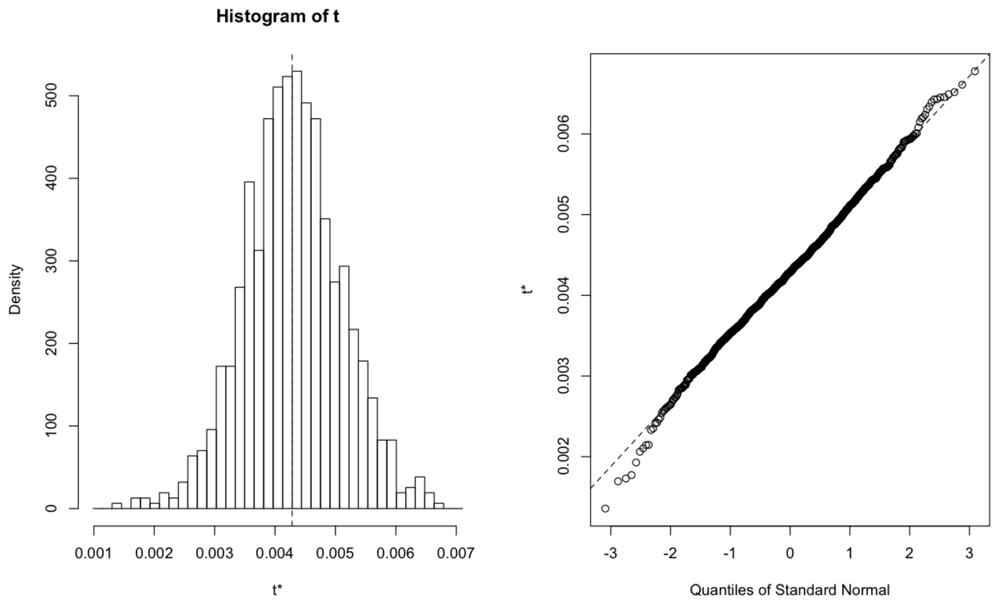
```
0.000611 0.000843
> c(se.sigmahat["FBGRX"], sd(sim.sds))
  FBGRX
0.00528 0.00625
```

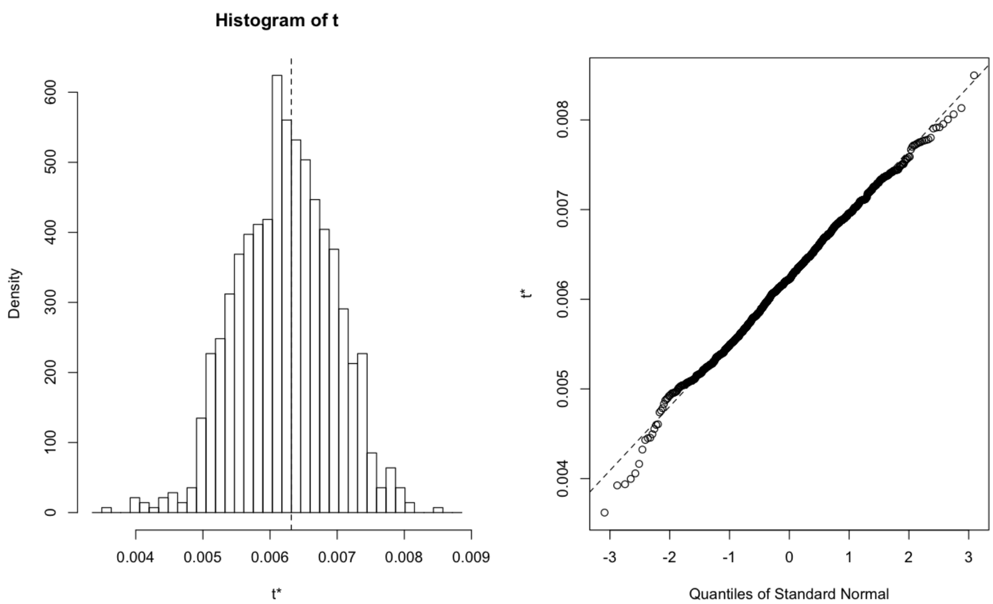
2. Bootstrapping

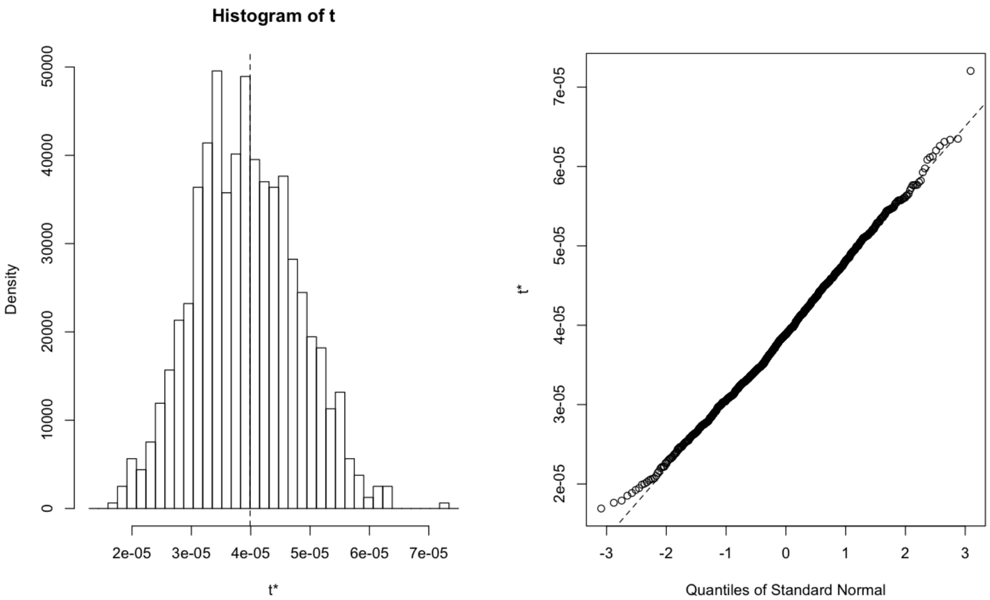
a) All of the SE below are very close to the analytic solutions.

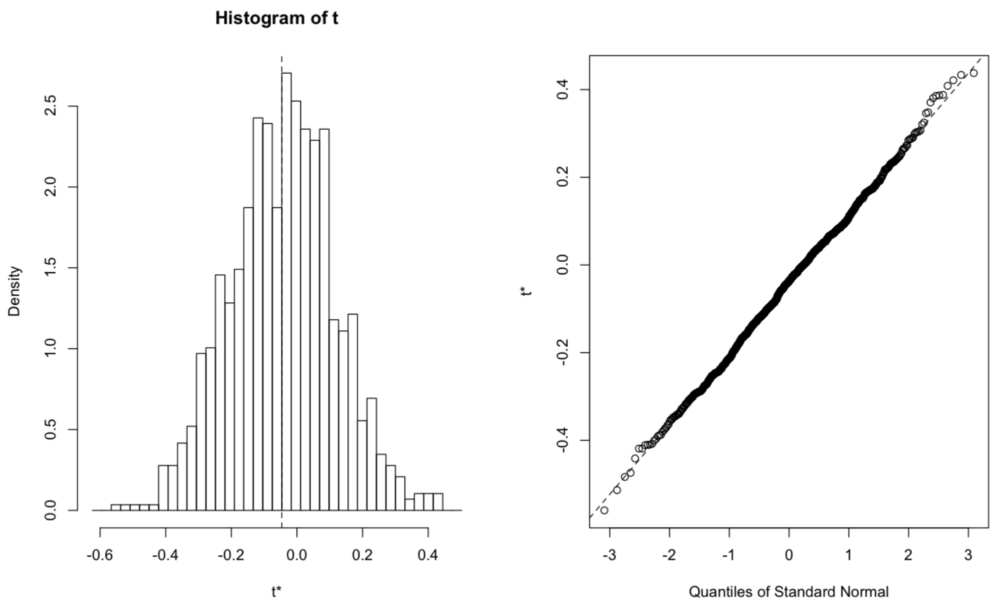
```
boot(data = VBISX, statistic = mean.boot, R = 999)
Bootstrap Statistics :
    original  bias     std. error
t1*  0.00428 1.6e-05     0.000807
> se.muhat["VBISX"]
  VBISX
0.000815

boot(data = VBISX, statistic = sd.boot, R = 999)

Bootstrap Statistics :
    original    bias     std. error
t1*  0.00632 -8.42e-05     0.000714
> se.sigmahat["VBISX"]
  VBISX
0.000577

boot(data = VBISX, statistic = var.boot, R = 999)

Bootstrap Statistics :
    original    bias     std. error
t1* 3.99e-05 -7.31e-07     8.63e-06
> se.sigma2hat["VBISX"]
  VBISX
7.28e-06

boot(data = ret.mat[, c("VBISX", "FBGRX")], statistic = rho.boot,
    R = 999)


Bootstrap Statistics :
    original  bias     std. error
t1*  -0.0464  0.0036         0.16
> se.rhohat[1]
VBISX,FBGRX
      0.129
```

b) The mean looks relatively normally distributed, except it is slightly negatively skewed and the qq plot has non-normal behavior in the lower quantiles. The sd does not look symmetric about the mean and exhibits non-normal behavior in the lower quantiles of the qq plot. The var looks almost bimodal and exhibits non-normal behavior in the lower quantiles of the qq plot. Rho looks the most normal because it is almost symmetric and the qq plot is very linear.









c) The bootstrap estimate suggests only a small loss and error of 131, also the confidence interval is fairly small

```
boot(data = VBISX, statistic = ValueAtRisk.boot, R = 999)


Bootstrap Statistics :
    original   bias     std. error
t1*     −609    9.76          131

boot.ci(boot.out = VBISX.VaR.boot, conf = 0.95, type = c("norm",
    "perc"))
```

```
Intervals :
Level       Normal              Percentile
95%    (-888, -354)    (-875, -335)
```

3. Class Project

    a) The parameter estimates for mu are much less accurate than sigma, which is clear from the wide confidence intervals that include both negative and positive numbers as well as the magnitude of the SE compared to mu. Sigma and sigma2 are relatively accurate, the SE is very small. However, sigma2 has a much larger confidence interval, which suggests the se is not very accurate.

```
#Compute se for mean
nobs = nrow(ret.mat)
se.muhat = sd.vals/sqrt(nobs)

# compute approx 95% confidence intervals
mu.lower = muhat.vals - 2*se.muhat
mu.upper = muhat.vals + 2*se.muhat

#Sigma SE
se.sigma = sd.vals/sqrt(2*nobs)

sigma.lower = sd.vals - 2 * se.sigma
sigma.upper = sd.vals + 2 * se.sigma

#Var SE
se.sigma2 = sd.vals/sqrt(nobs/2)

sigma2.lower = var.vals - 2 * se.sigma2
sigma2.upper = var.vals + 2 * se.sigma2

#Cov SE

#Cor SE
se.rho = (1-rhohat.vals)^2/sqrt(nobs)

rho.lower = rhohat.vals - 2 * se.rho
rho.upper = rhohat.vals + 2 * se.rho

#Combine the SE
rbind(se.muhat,se.sigma,se.sigma2,se.rho)

vfinx    presx    hlemx    vbllx    fshbx    vpacx
se.muhat   0.00425  0.00611  0.00619  0.00306  0.000290  0.00501
se.sigma   0.00301  0.00432  0.00438  0.00216  0.000205  0.00354
se.sigma2  0.00601  0.00864  0.00875  0.00432  0.000410  0.00708
```

```
rbind(mu.lower,mu.upper,sigma.lower,sigma.upper)

                 vfinx     presx     hlemx     vbllx     fshbx     vpacx
mu.lower      0.00303  -0.00457  -0.00935  0.000462  0.000726  -0.00538
mu.upper      0.02003   0.01987   0.01541  0.012689  0.001885   0.01465
sigma.lower   0.03005   0.04320   0.04377  0.021616  0.002050   0.03540
sigma.upper   0.04207   0.06047   0.06128  0.030262  0.002870   0.04955
sigma2.lower -0.01072  -0.01459  -0.01475 -0.007973 -0.000814  -0.01235
sigma2.upper  0.01332   0.01997   0.02027  0.009319  0.000826   0.01596
```