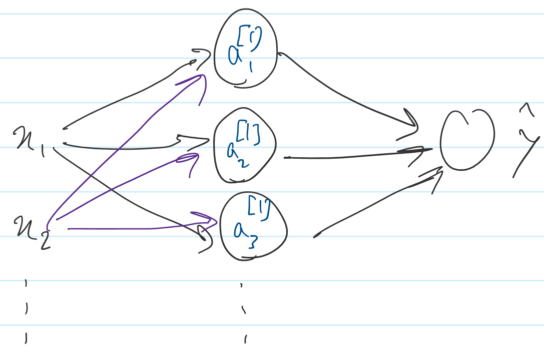
Assignment2

Saturday, October 1, 2022 4:06 AM

p1.



Forward propagation

$$z_1^{[1]} = w_1^{[1]T} \cdot x + b_1^{[1]} , \quad a_1^{[1]} = \sigma(z_1^{[1]})$$

$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]} , \quad a_2^{[1]} = \sigma(z_2^{[1]})$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

$$Z^{[1]} = \underbrace{\begin{bmatrix} — w_1^{[1]T} — \\ — w_2^{[1]T} — \\ — w_3^{[1]T} — \\ \vdots \\ — w_{n_1}^{[1]T} — \end{bmatrix}}_{n_1 \times 2} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{2\times1} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ \vdots \\ b_{n_1}^{[1]} \end{bmatrix}}_{n_1 \times 1}$$

$$A^{[1]} = \sigma(Z^{[1]}) \rightarrow A^{[1]}_{n_1 \times 1}$$

$$Z^{[2]} = \underbrace{\begin{bmatrix} — w_1^{[2]T} — \end{bmatrix}}_{1\times n_1} \underbrace{\begin{bmatrix} A_1^{[1]} \\ A_2^{[1]} \\ \vdots \\ A_{n_1}^{[1]} \end{bmatrix}}_{n_1 \times 1} + \underbrace{\begin{bmatrix} b_1^{[2]} \end{bmatrix}}_{1\times1}$$

$$\hat{y} = Z^{[2]} \rightarrow \hat{y}_{1\times1}$$

$n_1 \times 1$

$$\hat{y} = z^{[2]} \rightsquigarrow \hat{y}_{1\times 1}$$

$$\sigma(z) = \frac{1}{1+e^{-z}} \Rightarrow \frac{d\sigma(z)}{dz} = \sigma(z)(1-\sigma(z))$$

Backward propagation

$$L = \frac{1}{2m}(A^{[2]}-Y)^T(A^{[2]}-Y)$$ Mean Square Error

$$\hat{y} = A^{[2]} = z^{[2]} \Rightarrow \frac{\partial L}{\partial z^{[2]}} = \frac{\partial L}{\partial A^{[2]}}\frac{\partial A^{[2]}}{\partial z^{[2]}} = \frac{\partial L}{\partial A^{[2]}} = A^{[2]} - Y$$

$$\frac{\partial L}{\partial W^{[2]}} = \frac{1}{m}(A^{[2]}-Y)A^{[1]T}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{1}{m}\text{np.sum}(A^{[2]}-Y, \text{axis}=1, \text{keepdim}=\text{True})$$ → Sum on Columns of $A^{[2]}-Y$

$$\frac{\partial L}{\partial z^{[1]}} = W^{[2]T}(A^{[2]}-Y) * \sigma(z^{[1]})(1-\sigma(z^{[1]}))$$

$$\frac{\partial L}{\partial W^{[1]}} = \frac{1}{m}W^{[2]T}(A^{[2]}-Y)\,\sigma(z^{[1]})(1-\sigma(z^{[1]}))\,X^T$$

$$\frac{\partial L}{\partial b^{[1]}} = \frac{1}{m}\text{np.sum}\left(\frac{\partial L}{\partial z^{[1]}}, \text{axis}=1, \text{keepdim}=\text{True}\right)$$ → Sum on Columns of $\frac{\partial L}{\partial z^{[1]}}$

By calculating $\frac{\partial L}{\partial W^{[2]}}, \frac{\partial L}{\partial b^{[2]}}, \frac{\partial L}{\partial W^{[1]}}, \frac{\partial L}{\partial b^{[1]}}$ terms, backward propagation is complete.

* This algorithm is same as binary classification with logloss.

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$$L = -y \log \hat{y} - (1-y) \log(1-\hat{y})$$

$$\frac{\partial L}{\partial \hat{y}} = -\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}} = \frac{-y + \cancel{y\hat{y}} + \hat{y} - \cancel{y\hat{y}}}{\hat{y}(1-\hat{y})} = \frac{\hat{y} - y}{\hat{y}(1-\hat{y})}$$

$$\hat{y} = \underbrace{\sigma}_{\text{sigmoid}}(z) \quad \text{So} \quad \frac{\partial L}{\partial z} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z} = \left(\frac{\hat{y} - y}{\cancel{\hat{y}(1-\hat{y})}}\right)\left[\cancel{\hat{y}(1-\hat{y})}\right] = \hat{y} - y$$

The only between these two backpropagation algorithm is loss function (MSE and cross entropy) and activation functions of output layer (linear and sigmoid).
Backpropagation after these two steps show both have same update rules.