A two-link planar direct-drive robot manipulator

Thursday, March 31, 2022 10:30 PM

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F_s(\dot{q}) + F_d\dot{q} + Sq = \tau$$

$$q = [q_1 \quad q_2]^T, \quad \dot{q} = [\dot{q}_1 \quad \dot{q}_2]^T, \quad \tau = [\tau_1 \quad \tau_2]^T$$

$$M(q) = \begin{bmatrix} p_1 + 2p_3 c_2(q) & p_2 + p_3 c_2(q) \\ p_2 + p_3 c_2(q) & p_2 \end{bmatrix}$$

$$V_m(q,\dot{q}) = \begin{bmatrix} p_3 s_2(q)\dot{q}_2 & -p_3 s_2(q)(\dot{q}_1 + \dot{q}_2) \\ p_3 s_2(q)\dot{q}_1 & 0 \end{bmatrix}$$

$$c_2(q) = \cos(q), \quad s_2(q) = \sin(q), \quad S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}$$

$$F_d = \begin{bmatrix} f_{d_1} & 0 \\ 0 & f_{d_2} \end{bmatrix}, \quad F_s(\dot{q}) = [f_{s_1}\tanh(\dot{q}_1), \ f_{s_2}\tanh(\dot{q}_2)]^T$$

$p_1, p_2, p_3, f_{d_1}, f_{d_2}, f_{s_1}, f_{s_2}, S_1,$ and S_2 are unkown

$$q_d = \begin{bmatrix} \pi \\ \frac{\pi}{2} \end{bmatrix}$$

$$\theta = [p_1, p_2, p_3, f_{d_1}, f_{d_2}, f_{s_1}, f_{s_2}, S_1, S_2]^T$$

$$e = q_d - q, \quad r = \dot{e} + \alpha e$$

$\hat{\theta}$ is the estimate of θ, so $\tilde{\theta} = \theta - \hat{\theta}$

$$z = [e \quad r \quad \tilde{\theta}]^T$$

$z = [e \quad r \quad \tilde{\theta}]$

$\dot{z} = [\dot{e} \quad \dot{r} \quad \dot{\tilde{\theta}}]^T = [r - \alpha e \quad \ddot{e} + \alpha \dot{e} \quad \dot{\tilde{\theta}}]^T$

$\dot{e} = -\dot{q}, \quad \ddot{e} = -\ddot{q}$

$M(q_d - e)(-\ddot{e}) + V_m(q_d - e, -\dot{e})(-\dot{e}) + F_s(-\dot{e})$

$+ F_d(-\dot{e}) + S(q_d - e) = \tau$

$\dot{e} = r - \alpha e, \quad \ddot{e} = \dot{r} - \alpha \dot{e} \Rightarrow \ddot{q} = -\dot{r} - \alpha \dot{q}$

$M(q)(-\dot{r} - \alpha \dot{q}) + V_m(q, \dot{q})\dot{q} + F_s(\dot{q}) + F_d \dot{q} + Sq = \tau$

$M(q)\dot{r} = -\alpha M(q)\dot{q} + V_m(q, \dot{q})\dot{q} + F_s(\dot{q}) + F_d \dot{q} + Sq - \tau$

$M\dot{r} = Y_1 \theta - \tau$

$$Y_1 = \begin{bmatrix} -\alpha \dot{q}_1, & -\alpha \dot{q}_2 & , -\alpha \cos(q_2)(2\dot{q}_1 + \dot{q}_2) - \sin(q_2)\dot{q}_2^2, & \dot{q}_1, & 0, & \tanh(\dot{q}_1), & 0 & , q_1, & 0 \\ 0, & -\alpha(\dot{q}_1 + \dot{q}_2), & -\alpha \cos(q_2)\dot{q}_1 + \sin(q_2)\dot{q}_1^2 & , 0, & \dot{q}_2, & 0 & , \tanh(\dot{q}_2), & 0, & q_2 \end{bmatrix}$$

Y_1 can be written as a function of (e, r) by replacing $\dot{q} = -\dot{e} = -(r - \alpha e)$, $q = q_d - e$

We can design the controller if we estimate

θ such that errors will be zero as $t \to \infty$ or t_f

$z = [e \quad r \quad \tilde{\theta}]$,

$$\dot{z} = \begin{bmatrix} \dot{e} \\ \dot{r} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} r - \alpha e \\ M^{-1}\left(Y_1 \theta - \tau\right) \\ \dot{\tilde{\theta}} \end{bmatrix}$$

$$V(z) = \frac{1}{2} e^T e + \frac{1}{2} r^T M(q_d - e)\, r + \frac{1}{2} \tilde{\theta}^T \tilde{\theta}$$

$$\underbrace{\frac{\partial V(z)}{\partial z}}_{1\times 13} = \left[\underbrace{\frac{\partial V}{\partial e}}_{1\times 2}, \underbrace{\frac{\partial V}{\partial r}}_{1\times 2}, \underbrace{\frac{\partial V}{\partial \theta}}_{1\times 9} \right] = \left[e^T + \frac{1}{2} r^T \frac{\partial}{\partial e}\left(M(q_d - e)\right) r,\ r^T M(q_d - e),\ \tilde{\theta}^T \right]$$

$$\dot{V}(z) = \frac{\partial V}{\partial z} \dot{z} = \left(e^T + \frac{1}{2} r^T \frac{\partial}{\partial e}\left(M(q_d - e)\right) r \right)(r - \alpha e) + r^T M(q_d - e) \underbrace{\dot{r}}_{M^{-1}(Y_1 \theta - \tau)} + \tilde{\theta}^T \dot{\tilde{\theta}}$$

$$\dot{V}(z) = -e^T \alpha e + r^T\left(e + Y_2 \theta + Y_1 \theta - \tau\right) + \tilde{\theta}^T \dot{\tilde{\theta}}$$

$$\boxed{\tau = \left(Y_1(e,r) + Y_2(e,r) \right) \hat{\theta} + e}$$

$$\dot{V}(z) = -e^T \alpha e + r^T \left(Y_1(e,r) + Y_2(e,r) \right) \tilde{\theta} + \tilde{\theta}^T \dot{\tilde{\theta}}$$

$$\dot{\tilde{\theta}} = \left(Y_1^T(e,r) + Y_2^T(e,r) \right) r$$

$$\boxed{\dot{\hat{\theta}} = -\left(Y_1^T(e,r) + Y_2^T(e,r) \right) r}$$

$$Y_2(e,r) = \frac{1}{2} \frac{\partial}{\partial e}\left(M(q_d - e)\, r \right) (r - \alpha e)$$

$$r = \dot{e} + \alpha e = -\dot{q} + \alpha(q_d - q)$$

$$Mr = \begin{bmatrix} (p_1 + 2p_3\cos(q_2))(\alpha(q_{d_1}-q_1)-\dot{q}_1) + (p_2+p_3\cos(q_2))(\alpha(q_{d_2}-q_2)-\dot{q}_2) \\ p_2+p_3\cos(q_2)(\alpha(q_{d_1}-q_1)-\dot{q}_1) + p_2(\alpha(q_{d_2}-q_2)-\dot{q}_2) \end{bmatrix}$$

$$\frac{\partial}{\partial e}(Mr) = -\frac{\partial}{\partial q}(Mr) = \begin{bmatrix} -\frac{\partial}{\partial q_1}, & -\frac{\partial}{\partial q_2} \end{bmatrix} \begin{bmatrix} (Mr)_1 \\ (Mr)_2 \end{bmatrix}$$

$$\frac{\partial}{\partial e}(Mr) = \begin{bmatrix} \alpha(p_1+2p_3\cos(q_2)) & \alpha(p_2+p_3\cos(q_2)) + p_3\sin(q_2)\left(2(\alpha(q_{d_1}-q_1)-\dot{q}_1) + (\alpha(q_{d_2}-q_2)-\dot{q}_2)\right) \\ \alpha(p_2+p_3\cos(q_2)) & \alpha p_2 + p_3\sin(q_2)(\alpha(q_{d_1}-q_1)-\dot{q}_1) \end{bmatrix}$$

$\underset{2\times 9}{Y_2}\underset{1\times 9}{\theta} = \frac{1}{2}\underset{2\times 2}{\frac{\partial}{\partial e}(Mr)}\underset{2\times 1}{\dot{q}}$ can be written as a function of e and r.

$$Y_2 = 0.5\begin{bmatrix} \alpha\dot{q}_1, & \alpha\dot{q}_2, & 2\alpha\cos(q_2)\dot{q}_1 + \alpha\cos(q_2)\dot{q}_2 + \sin(q_2)\dot{q}_2\left(2(\alpha(q_{d_1}-q_1)-\dot{q}_1) + (\alpha(q_{d_2}-q_2)-\dot{q}_2)\right), & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & \alpha(\dot{q}_1+\dot{q}_2), & \alpha\cos(q_2)\dot{q}_1 + \sin(q_2)\dot{q}_2(\alpha(q_{d_1}-q_1)-\dot{q}_1), & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}$$