A two-link planar direct-drive robot manipulator

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$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F_s(\dot{q}) + F_d\dot{q} + Sq = \tau$$

$$q = [q_1 \quad q_2]^T, \quad \dot{q} = [\dot{q}_1 \quad \dot{q}_2]^T, \quad \tau = [\tau_1 \quad \tau_2]^T$$

$$M(q) = \begin{bmatrix} p_1 + 2p_3 c_2(q) & p_2 + p_3 c_2(q) \\ p_2 + p_3 c_2(q) & p_2 \end{bmatrix}$$

$$V_m(q,\dot{q}) = \begin{bmatrix} p_3 s_2(q)\dot{q}_2 & -p_3 s_2(q)(\dot{q}_1 + \dot{q}_2) \\ p_3 s_2(q)\dot{q}_1 & 0 \end{bmatrix}$$

$$c_2(q) = \cos(q), \quad s_2(q) = \sin(q), \quad S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}$$

$$F_d = \begin{bmatrix} f_{d_1} & 0 \\ 0 & f_{d_2} \end{bmatrix}, \quad F_s(\dot{q}) = [f_{s_1}\tanh(\dot{q}_1), \; f_{s_2}\tanh(\dot{q}_2)]^T$$

$p_1, p_2, p_3, f_{d_1}, f_{d_2}, f_{s_1}, f_{s_2}, S_1$, and S_2 are unkown

$$q_d = \begin{bmatrix} \pi \\ \frac{\pi}{2} \end{bmatrix}$$

$$\theta = [p_1, p_2, p_3, f_{d_1}, f_{d_2}, f_{s_1}, f_{s_2}, S_1, S_2]^T$$

$$e = q_d - q, \quad r = \dot{e} + \alpha e$$

$\hat{\theta}$ is the estimate of θ, so $\tilde{\theta} = \theta - \hat{\theta}$

$$z = [e \quad r \quad \tilde{\theta}]^T$$

$z=[e\ \ r\ \ \tilde{\theta}]$

$\dot{z}=[\dot{e}\ \ \dot{r}\ \ \dot{\tilde{\theta}}]^T=[r-\alpha e\ \ \ \ddot{e}+\alpha\dot{e}\ \ \ \dot{\tilde{\theta}}]^T$

$\dot{e}=-\dot{q},\ \ \ddot{e}=-\ddot{q}$

$M(q_d-e)(-\ddot{e})+V_m(q_d-e,-\dot{e})(-\dot{e})+F_s(-\dot{e})$

$+F_d(-\dot{e})+S(q_d-e)=\tau$

$\dot{e}=r-\alpha e,\ \ddot{e}=\dot{r}-\alpha\dot{e}\Rightarrow\ddot{q}=-\dot{r}-\alpha\dot{q}$

$M(q)(-\dot{r}-\alpha\dot{q})+V_m(q,\dot{q})\dot{q}+F_s(\dot{q})+F_d\dot{q}+Sq=\tau$

$M(q)\dot{r}=-\alpha M(q)\dot{q}+V_m(q,\dot{q})\dot{q}+F_s(\dot{q})+F_d\dot{q}+Sq-\tau$

$M\dot{r}=Y_1\theta-\tau$

$$Y_1=\begin{bmatrix}-\alpha\dot{q}_1, & -\alpha\dot{q}_2, & -\alpha\cos(q_2)(2\dot{q}_1+\dot{q}_2)-\sin(q_2)\dot{q}_2^2, & \dot{q}_1, & 0, & \tanh(\dot{q}_1), & 0, & q_1, & 0\\ 0, & -\alpha(\dot{q}_1+\dot{q}_2), & -\alpha\cos(q_2)\dot{q}_1+\sin(q_2)\dot{q}_1^2, & 0, & \dot{q}_2, & 0, & \tanh(\dot{q}_2), & 0, & q_2\end{bmatrix}$$

Y_1 can be written as a function of (e,r) by replacing $\dot{q}=-\dot{e}=-(r-\alpha e)$, $q=q_d-e$

We can design the controller if we estimate

θ such that errors will be zero as $t\to\infty$ or t_f

$z=[e\ \ r\ \ \tilde{\theta}]$,

$$\dot{z} = \begin{bmatrix} \dot{e} \\ \dot{r} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} r - \alpha e \\ M^{-1}(Y_1\theta - \tau) \\ \dot{\tilde{\theta}} \end{bmatrix}$$

$$V(z) = \frac{1}{2} e^T e + \frac{1}{2} r^T M(q_d - e) r + \frac{1}{2} \tilde{\theta}^T \tilde{\theta}$$

$$\underbrace{\frac{\partial V(z)}{\partial z}}_{1\times 13} = \left[\underbrace{\frac{\partial V}{\partial e}}_{1\times 2}, \underbrace{\frac{\partial V}{\partial r}}_{1\times 2}, \underbrace{\frac{\partial V}{\partial \theta}}_{1\times 9} \right] = \left[e^T + \frac{1}{2} r^T \frac{\partial}{\partial e}\left(M(q_d - e)\right) r, \; r^T M(q_d - e), \; \tilde{\theta}^T \right]$$

$$\dot{V}(z) = \frac{\partial V}{\partial z} \dot{z} = \left(e^T + \frac{1}{2} r^T \frac{\partial}{\partial e}\left(M(q_d - e)\right) r \right)(r - \alpha e) + r^T M(q_d - e) \underbrace{\dot{r}}_{M^{-1}(Y_1\theta - \tau)} + \tilde{\theta}^T \dot{\tilde{\theta}}$$

$$\dot{V}(z) = -e^T \alpha e + r^T \left(e + Y_2 \theta + Y_1 \theta - \tau \right) + \tilde{\theta}^T \dot{\tilde{\theta}}$$

$$\boxed{\tau = \left(Y_1(e,r) + Y_2(e,r) \right) \hat{\theta} + e}$$

$$\dot{V}(z) = -e^T \alpha e + r^T \left(Y_1(e,r) + Y_2(e,r) \right) \tilde{\theta} + \tilde{\theta}^T \dot{\tilde{\theta}}$$

$$\dot{\tilde{\theta}} = \left(Y_1^T(e,r) + Y_2^T(e,r) \right) r$$

$$\boxed{\dot{\hat{\theta}} = -\left(Y_1^T(e,r) + Y_2^T(e,r) \right) r}$$

$$Y_2(e,r) = \frac{1}{2} \frac{\partial}{\partial e}\left(M(q_d - e) r \right)(r - \alpha e)$$

$$r = \dot{e} + \alpha e = -\dot{q} + \alpha(q_d - q)$$

$$Mr = \begin{bmatrix} \left(p_1 + 2p_3\cos(q_2)\right)\left(\alpha(q_{d_1} - q_1) - \dot{q}_1\right) + \left(p_2 + p_3\cos(q_2)\right)\left(\alpha(q_{d_2} - q_2) - \dot{q}_2\right) \\ p_2 + p_3\cos(q_2)\left(\alpha(q_{d_1} - q_1) - \dot{q}_1\right) + p_2\left(\alpha(q_{d_2} - q_2) - \dot{q}_2\right) \end{bmatrix}$$

$$\frac{\partial}{\partial e}(Mr) = -\frac{\partial}{\partial q}(Mr) = \begin{bmatrix} -\frac{\partial}{\partial q_1}, & -\frac{\partial}{\partial q_2} \end{bmatrix} \begin{bmatrix} (Mr)_1 \\ (Mr)_2 \end{bmatrix}$$

$$\frac{\partial}{\partial e}(Mr) = \begin{bmatrix} \alpha\left(p_1 + 2p_3\cos(q_2)\right) & \alpha\left(p_2 + p_3\cos(q_2)\right) + p_3\sin(q_2)\left(2\left(\alpha(q_{d_1} - q_1) - \dot{q}_1\right) + \left(\alpha(q_{d_2} - q_2) - \dot{q}_2\right)\right) \\ \alpha\left(p_2 + p_3\cos(q_2)\right) & \alpha p_2 + p_3\sin(q_2)\left(\alpha(q_{d_1} - q_1) - \dot{q}_1\right) \end{bmatrix}$$

$\underset{2\times 9}{Y_2}\,\underset{9\times 1}{\Theta} = -\frac{1}{2}\underset{2\times 2}{\frac{\partial}{\partial e}(Mr)}\,\underset{2\times 1}{\dot{q}}$ can be written as a function of e and r.

$$Y_2 = -0.5\begin{bmatrix} \alpha\dot{q}_1, & \alpha\dot{q}_2, & 2\alpha\cos(q_2)\dot{q}_1 + \alpha\cos(q_2)\dot{q}_2 + \sin(q_2)\dot{q}_2\left(2\left(\alpha(q_{d_1} - q_1) - \dot{q}_1\right) + \left(\alpha(q_{d_2} - q_2) - \dot{q}_2\right)\right), & 0, & 0, & 0, & 0, & 0, & 0 \\ 0, & \alpha(\dot{q}_1 + \dot{q}_2), & \alpha\cos(q_2)\dot{q}_1 + \sin(q_2)\dot{q}_2\left(\alpha(q_{d_1} - q_1) - \dot{q}_1\right), & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}$$