$\dot{x}_1 = x_1^2 - x_2$, $\dot{x}_2 = \tanh(x_1) + u_3 + x_1 x_2^2$

$\dot{x}_3 = x_1 x_2 + x_2 x_3 + (2 + \sin(x_1))\, u$

$\dot{x}_1 = x_1^2 - x_2 \Rightarrow x_2 = x_1^2 + x_1 = \varphi_1(x_1)$

$\dot{V}_1(x_1) = \frac{1}{2} x_1^2$

$\dot{x}_1 = -x_1$, $\dot{V}_1(x_1) = -x_1^2$

$z_2(x_2, x_1) = x_2 - \varphi_1(x_1) = x_2 - x_1^2 - x_1$

$(x_1 x_2^2)^{\cdot} - (\dot{x}_1 + 2\dot{x}_1 x_1) = \dot{x}_1 x_2^2 + 2x_1 x_2 \dot{x}_2 - (\dot{x}_1)(1+2x_1)$

$\dot{x}_1 = -z_2 - x_1$, $\dot{z}_2 = \dot{x}_2 - \dot{x}_1(2x_1 + 1)$

$\dot{z}_2 = \tanh(x_1) + u_3 + x_1 x_2^2 + (z_2 + x_1)(2x_1 + 1)$

$x_2 = z_2 + x_1^2 + x_1$

$\dot{z}_2 = \tanh(x_1) + u_3 + x_1 (z_2 + x_1^2 + x_1)^2 + (z_2 + x_1)(2x_1 + 1)$

$V_2(x_1, z_2) = \frac{1}{2} x_1^2 + \frac{1}{2} z_2^2 \Rightarrow \dot{V}_2 = x_1 \dot{x}_1 + z_2 \dot{z}_2$

$\dot{V}_2 = x_1(-z_2 - x_1) + z_2\left(\tanh(x_1) + u_3 + x_1 (z_2 + x_1^2 + x_1)^2 + (z_2 + x_1)(2x_1 + 1)\right)$

$\dot{z}_2 = x_1 - z_2 + z_3$

$$\varphi_2(x_1,x_2) = -\tanh(x_1) - x_1\left(z_2+x_1^2+x_1\right)^2 - (z_2+x_1)(2x_1+1) + x_1 - z_2$$

$$z_3(x_1,x_2,x_3) = x_3 - \varphi_2(x_1,x_2) \Rightarrow \dot{z}_3 = \dot{x}_3 - \dot{\varphi}_2$$

$$\dot{z}_3 = \left[x_1x_2 + x_2x_3 + (2+\sin(x_1))u\right] - \left[-\dot{x}_1\left(\frac{1}{\cosh(x_1)}\right)^2 - \dot{x}_1\left(z_2+\varphi_1\right)^2 - x_1(2)(z_2+\varphi_1)\right.$$

$$\left.(\dot{z}_2 + 2x_1\dot{x}_1 + \dot{x}_1) - (\dot{z}_2+\dot{x}_1)(2x_1+1) - (z_2+x_1)(2\dot{x}_1) - \dot{z}_2 - \dot{x}_1\right]$$

$$\dot{z}_3 = \left[x_1(z_2+\varphi_1) + (z_2+\varphi_1)(z_3+\varphi_2) + (2+\sin(x_1)u)\right] - \left[\frac{z_2+x_1}{\cosh^2(x_1)} + \underbrace{(z_2+x_1)(z_2+\varphi_1)^2 - 2x_1(z_2+\varphi_1)}_{(z_2+\varphi_1)(z_2-x_1)}\right.$$

$$\left.\underbrace{(\dot{z}_2 - (2x_1+1)(z_2+x_1))}_{-2\dot{z}_2x_1} - \underbrace{(\dot{z}_2 - z_2 - x_1)(2x_1+1)}_{-\dot{z}_2(2x_1+1)} + 2(z_2+x_1)^2 - \dot{z}_2 + (z_2+x_1)\right]$$

Everything is calculated again using Mathematica

Here, I write the results.

$$\dot{x}_1 = -z_2 - x_1$$

$$\dot{z}_2 = x_1 - z_2 + z_3 \quad , \; z_2 = x_2 - \varphi_1$$

$$\dot{z}_3 = -z_2 - z_3 \quad , \; z_3 = x_3 - \varphi_2$$

φ_1 and φ_2 are written in the above text.

$$V = 0.5x_1^2 + 0.5z_2^2 + 0.5z_3^2$$

$$\dot{V} = -x_1^2 - z_2^2 - z_3^2$$

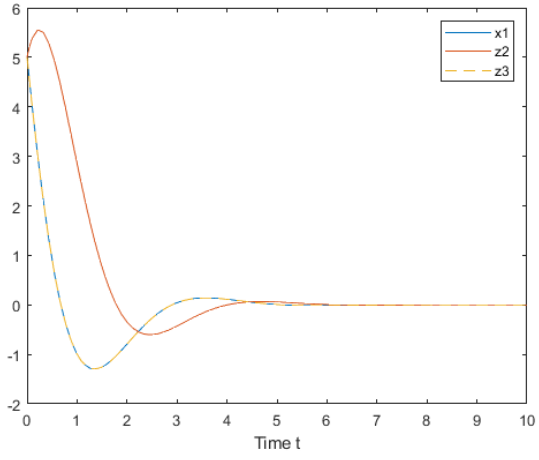
$$V(0) = 0 \; , \; V(z) > 0 \quad \forall z \in \mathbb{R}^3 - \{0\} \quad \text{PD} \qquad z = [x_1, z_2, z_3]^T$$

$\dot{V}$: $\dot{V} < -\alpha\|z\|^3$ ( ) NSD

$\dot{V}(0) = 0$, $\dot{V}(z) < 0 \quad \forall z \in \mathbb{R}^3 - \{0\}$ ND

$V(z) \to \infty$ as $\|z\| \to \infty$

So the system is GAS



HW4Anlytics

Below is the Mathematica calculation

```
In[3]:= xdot2 = Tanh[x1[t]] + x3[t] + x1[t] * x2[t]^2;
        xdot3 = x1[t] * x2[t] + x2[t] * x3[t] + (2 + Sin[x1[t]]) * u[t];
        ϕ1 = x1[t]^2 + x1[t];
        x1' = -z2[t] - x1[t];

In[7]:= x2[t] = z2[t] + ϕ1;
        zdot2 = xdot2 - D[ϕ1, t]

Out[8]= Tanh[x1[t]] + x3[t] + x1[t] (x1[t] + x1[t]^2 + z2[t])^2 -
          (-x1[t] - z2[t]) [t] - 2 x1[t] (-x1[t] - z2[t]) [t]

In[9]:= x3[t] = ϕ2 + z3[t];
        zdot2 = Tanh[x1[t]] + x3[t] + x1[t] (x1[t] + x1[t]^2 + z2[t])^2 -
            (-x1[t] - z2[t]) - 2 x1[t] (-x1[t] - z2[t]) // Simplify

Out[10]= ϕ2 + Tanh[x1[t]] + x1[t] + z2[t] +
          2 x1[t] (x1[t] + z2[t]) + x1[t] (x1[t] + x1[t]^2 + z2[t])^2 + z3[t]
```

```
(-x1[t] - z2[t]) - 2 x1[t] (-x1[t] - z2[t]) // Simplify
```

Out[10]=
$$\phi 2 + \text{Tanh}[x1[t]] + x1[t] + z2[t] + 2\,x1[t]\,(x1[t] + z2[t]) + x1[t]\,(x1[t] + x1[t]^2 + z2[t])^2 + z3[t]$$

```
In[11]:= v1 = 0.5 * x1[t]^2;
         vdot1 = D[v1, t]
```

Out[12]= `1. x1[t] (-x1[t] - z2[t]) [t]`

```
In[13]:= v2 = v1 + 0.5 * z2[t]^2;
          z2'[t] = zdot2;
         D[v2, t]
```

Out[15]=
$$1.\,z2[t]\,\left(\phi 2 + \text{Tanh}[x1[t]] + x1[t] + z2[t] + 2\,x1[t]\,(x1[t] + z2[t]) + x1[t]\,(x1[t] + x1[t]^2 + z2[t])^2 + z3[t]\right) + 1.\,x1[t]\,(-x1[t] - z2[t])\,[t]$$

```
In[16]:= vdot2 = z2[t] (x1[t] - z2[t]) + x1[t] (-x1[t] - z2[t]) // Simplify
```

Out[16]= $-x1[t]^2 - z2[t]^2$

```
In[17]:= ϕ2 = -Tanh[x1[t]] - x1[t] (x1[t] + x1[t]^2 + z2[t])^2 -
             (x1[t] + z2[t]) - 2 x1[t] (x1[t] + z2[t]) - z2[t] + x1[t];
```

```
In[18]:= zdot3 = xdot3 - D[ϕ2, t]
```

Out[17]=
$$(2 + \text{Sin}[x1[t]])\,u[t] + x1[t]\,(x1[t] + x1[t]^2 + z2[t]) + (x1[t] + x1[t]^2 + z2[t])\,(\phi 2 + z3[t])$$

```
In[19]:= v3 = v2 + 0.5 * z3[t]^2
          z3'[t] = zdot3;
          x3[t] = ϕ2 + z3[t];
          FactorTerms[D[v3, t], z3] // Simplify
```

Out[18]= $0.5\,x1[t]^2 + 0.5\,z2[t]^2 + 0.5\,z3[t]^2$

Out[21]=
$$1.\,z2[t]\,\left(\phi 2 + \text{Tanh}[x1[t]] + x1[t] + z2[t] + 2\,x1[t]\,(x1[t] + z2[t]) + x1[t]\,(x1[t] + x1[t]^2 + z2[t])^2 + z3[t]\right) + 1.\,z3[t]\,\left((2 + \text{Sin}[x1[t]])\,u[t] + (x1[t] + x1[t]^2 + z2[t])\,(\phi 2 + x1[t] + z3[t])\right) + 1.\,x1[t]\,(-x1[t] - z2[t])\,[t]$$

```
In[22]:= u[t] = -(1 / (2 + Sin[x1[t]])) * (+x1[t] (x1[t] + x1[t]^2 + z2[t]) + (x1[t] + x1[t]^2 + z2[t])
          (-Tanh[x1[t]] - 2 z2[t] - 2 x1[t] (x1[t] + z2[t]) - x1[t] (x1[t] + x1[t]^2 + z2[t])^2) +
        2 (x1[t] - z2[t] + z3[t]) + Sech[x1[t]]^2 (-x1[t] - z2[t]) +
        2 (x1[t] + z2[t]) (-x1[t] - z2[t]) + (x1[t] + x1[t]^2 + z2[t])^2 (-x1[t] - z2[t]) +
        2 x1[t] (x1[t] - z2[t] + z3[t] + (-x1[t] - z2[t])) + 2 x1[t] (x1[t] + x1[t]^2 + z2[t])
         (x1[t] - z2[t] + z3[t] + (-x1[t] - z2[t]) + 2 x1[t] (-x1[t] - z2[t])) + z2[t] + z3[t]);

In[23]:= FactorTerms[D[v3, t], z3] // Simplify

Out[23]= 1. z2[t] (ϕ2 + Tanh[x1[t]] + x1[t] + z2[t] +
          2 x1[t] (x1[t] + z2[t]) + x1[t] (x1[t] + x1[t]^2 + z2[t])^2 + z3[t]) +
        1. z3[t] (3 x1[t]^6 + x1[t]^7 + 4 z2[t]^2 + z2[t]^3 + x1[t]^5 (8 + 3 z2[t]) +
          x1[t]^4 (9 + 11 z2[t]) + x1[t]^3 (3 + 21 z2[t] + 3 z2[t]^2 - 2 z3[t]) +
          x1[t]^2 (2 + ϕ2 + Tanh[x1[t]] + 13 z2[t] + 9 z2[t]^2 + z2[t]^3 - 2 z2[t] (-5 + z3[t]) - z3[t]) -
          3 z3[t] + z2[t] (1 + ϕ2 + Sech[x1[t]]^2 + Tanh[x1[t]] + z3[t])) + 1. x1[t] (-x1[t] - z2[t]) [t]

In[24]:= z2[t] (x1[t] - z2[t] + z3[t]) + x1[t] (-x1[t] - z2[t]) +
          z3[t] (5 x1[t]^5 + z2[t]^3 - z3[t] + Sech[x1[t]]^2 (-x1[t] - z2[t]) +
            z2[t]^2 (2 + (-x1[t] - z2[t])) + z2[t] (-1 + Sech[x1[t]]^2 + 2 (-x1[t] - z2[t])) +
            x1[t]^4 (8 + 5 z2[t] + 5 (-x1[t] - z2[t])) + x1[t]^3 (3 + 14 z2[t] + 8 (-x1[t] - z2[t])) +
            x1[t] (Sech[x1[t]]^2 + 5 z2[t]^2 + 4 (-x1[t] - z2[t]) + z2[t] (6 + 4 (-x1[t] - z2[t]))) +
            x1[t]^2 (4 + 6 z2[t]^2 + 3 (-x1[t] - z2[t]) + z2[t] (7 + 6 (-x1[t] - z2[t])))) // Simplify

Out[24]= -x1[t]^2 - z2[t]^2 - z3[t]^2

In[25]:= zdot3 = (2 + Sin[x1[t]]) u[t] + x1[t] (x1[t] + x1[t]^2 + z2[t]) + (x1[t] + x1[t]^2 + z2[t])
          (-Tanh[x1[t]] - 2 z2[t] - 2 x1[t] (x1[t] + z2[t]) - x1[t] (x1[t] + x1[t]^2 + z2[t])^2) +
        2 (x1[t] - z2[t] + z3[t]) + Sech[x1[t]]^2 (-x1[t] - z2[t]) +
        2 (x1[t] + z2[t]) (-x1[t] - z2[t]) + (x1[t] + x1[t]^2 + z2[t])^2 (-x1[t] - z2[t]) +
        2 x1[t] (x1[t] - z2[t] + z3[t] + (-x1[t] - z2[t])) + 2 x1[t] (x1[t] + x1[t]^2 + z2[t])
         (x1[t] - z2[t] + z3[t] + (-x1[t] - z2[t]) + 2 x1[t] (-x1[t] - z2[t]))

Out[25]= -z2[t] - z3[t]

In[27]:= u[t] // Simplify

Out[27]= 1 / (2 + Sin[x1[t]]) (3 x1[t]^6 + x1[t]^7 + (1 + Sech[x1[t]]^2 + Tanh[x1[t]]) z2[t] + 4 z2[t]^2 + z2[t]^3 +
          x1[t]^5 (8 + 3 z2[t]) + x1[t]^4 (9 + 11 z2[t]) + x1[t]^3 (2 + 21 z2[t] + 3 z2[t]^2 - 2 z3[t]) +
          x1[t]^2 (1 + Tanh[x1[t]] + 13 z2[t] + 9 z2[t]^2 - 2 z3[t]) +
          x1[t] (-2 + Sech[x1[t]]^2 + Tanh[x1[t]] + 9 z2[t]^2 + z2[t]^3 + z2[t] (9 - 2 z3[t]) - 2 z3[t]) -
          3 z3[t])
```