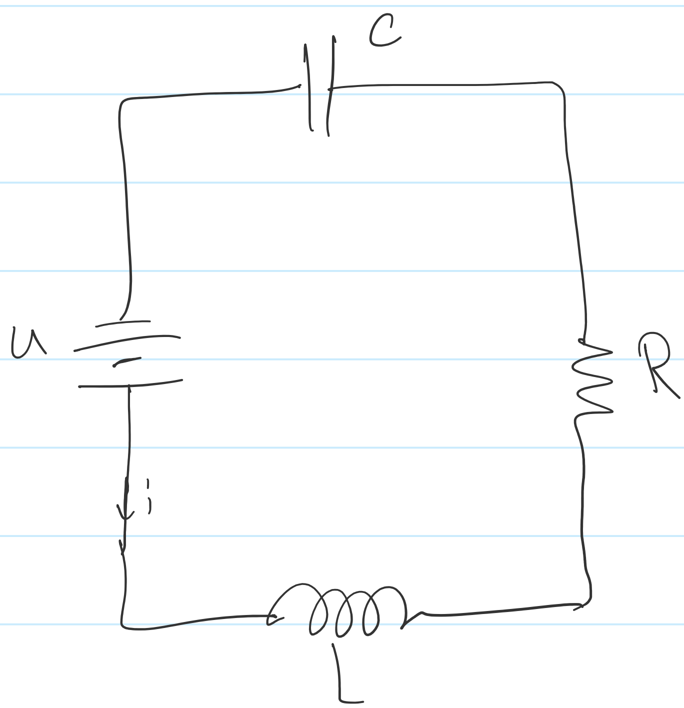
RLC Circuit

Thursday, March 31, 2022 10:00 PM



$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = u$$

$R>0$, $L>0$, and $C>0$ are unknown

$q \to 5C$, $i \to 0$ as $t \to \infty$

Controller does not depend on unknowns.

$e = q_d - q$, $q_d = 5$, $r = \dot{e} + \alpha e$, $\alpha > 0$

$$\theta = \begin{bmatrix} \frac{1}{C} \\ R \\ L \end{bmatrix}, \quad \tilde{\theta} = \theta - \hat{\theta}, \quad z = \begin{bmatrix} e \\ r \\ \tilde{\theta} \end{bmatrix}$$

$$\dot{e} = -\dot{q}, \quad \ddot{e} = -\ddot{q}, \quad \dot{e} = r - \alpha e, \quad \ddot{e} = \dot{r} - \alpha(r - \alpha e)$$

$$L(-\ddot{e}) + R(-\dot{e}) + \frac{1}{C}(q_d - e) = u$$

$$L(-\dot{r} + \alpha r - \alpha^2 e) + R(-r + \alpha e) + \frac{1}{C}(q_d - e) = u$$

$$L\dot{r} = (L\alpha - R)r + \left(-L\alpha^2 + R\alpha - \frac{1}{C}\right)e + \frac{q_d}{C} - u$$

$$\boxed{L\dot{r} = Y(e,r)\,\theta - u, \quad \dot{e} = r - \alpha e}$$

$$Y(e,r) = \left[q_d - e, \; -r + \alpha e, \; \alpha r - \alpha^2 e\right]$$

We can design the controller if we estimate θ such that errors will be zero as $t \to \infty$ or t_f

$$z = \begin{bmatrix} e & r & \tilde{\theta} \end{bmatrix},$$

$$\dot{z} = \begin{bmatrix} r - \alpha e \\ \ddot{e} + \alpha \dot{e} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} r - \alpha e \\ -\frac{1}{L}\left(R(r - \alpha e) + \frac{1}{C}e - \frac{1}{C}q_d + u\right) + \alpha(r - \alpha e) \\ \dot{\tilde{\theta}} \end{bmatrix}$$

$$V(z) = \frac{1}{2}e^2 + \frac{1}{2}r^2 + \frac{1}{2}\tilde{\theta}^T\tilde{\theta}$$

$$\dot{V}(z) = \frac{\partial V}{\partial z}\dot{z} = \left[\frac{\partial V}{\partial e}, \frac{\partial V}{\partial r}, \frac{\partial V}{\partial \tilde{\theta}}\right]\dot{z}$$

$$\Rightarrow \dot{V}(z) = \left[e, Lr, \tilde{\theta}^T\right]\dot{z}$$

$$\dot{V}(z) = e(r - \alpha e) + r\left(Y(e,r)\theta - u\right) + \tilde{\theta}^T\dot{\tilde{\theta}}$$

$$u = Y(e,r)\hat{\theta} + e$$

$$\dot{V} = e(r - \alpha e) \ldots \tilde{\theta}^T\dot{\tilde{\theta}}$$

$$\dot{V}(z) = e(r - \alpha e) + r\left(Y(e,r)\tilde{\theta} - e\right) + \dot{\tilde{\theta}}^T \tilde{\theta}$$

$$\dot{V}(z) = -\alpha e^2 + rY(e,r)\tilde{\theta} + \dot{\tilde{\theta}}^T \tilde{\theta}$$

$\boxed{\text{Design: } \dot{\tilde{\theta}}^T = -rY(e,r)}$ or $\dot{\tilde{\theta}} = -rY^T(e,r)$

So $\dot{V}(z) = -\alpha e^2$, $\boxed{\dot{\hat{\theta}} = -\dot{\tilde{\theta}} = rY^T(e,r)}$