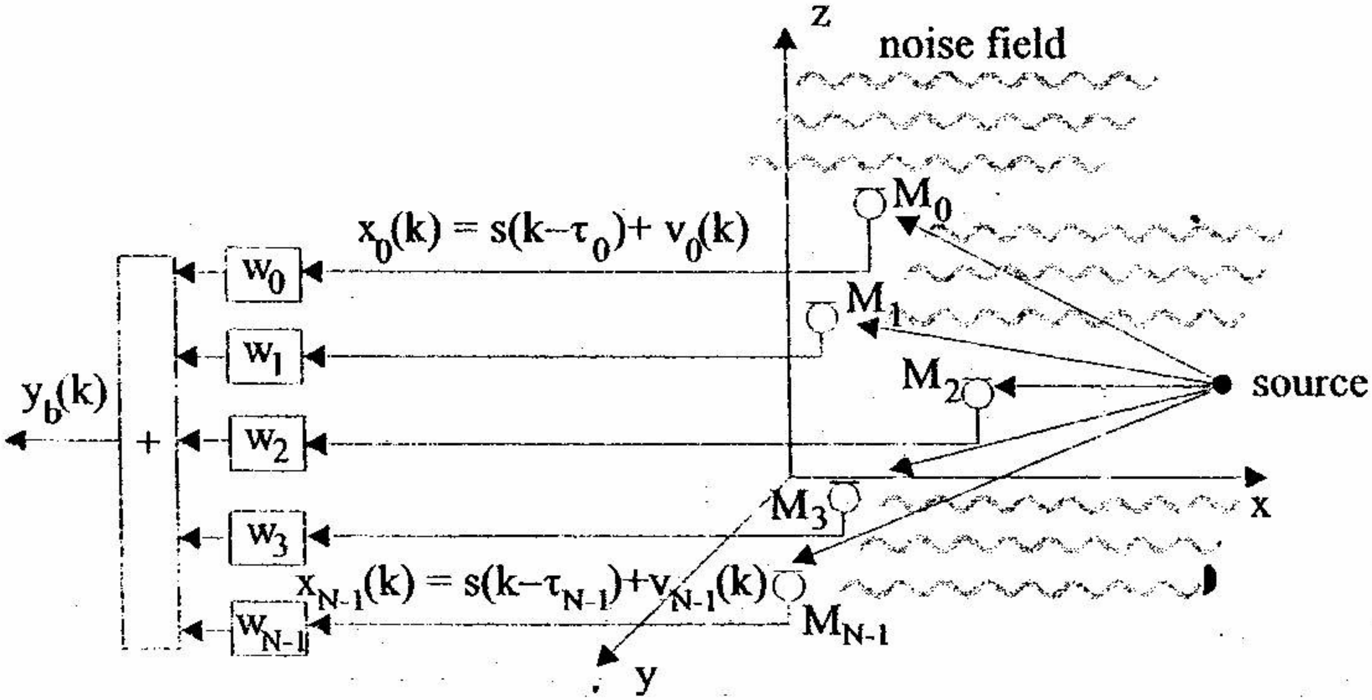
Superdirective Microphone Array

Presentation Subjects:

- Superdirective Beamformer SDB (constrained) model.
- Performance measures for beamformer models.
- Derive an optimal design for a given application.
- Measures result comparison, between the Superdirective beamformer model and other models such as delay and sum beamformer (DSB).
- Alternative Superdirective Beamformer model (unconstrained).
- Further extension and conclusion remarks.

Superdirective Beamformer model



Matrix formulation of the sensor

$$\begin{pmatrix} x_0(k) \\ x_1(k) \\ \vdots \\ x_{N-1}(k) \end{pmatrix} = \begin{pmatrix} a_0 s(k-\tau_0) \\ a_1 s(k-\tau_1) \\ \vdots \\ a_{N-1} s(k-\tau_{N-1}) \end{pmatrix} + \begin{pmatrix} v_0(k) \\ v_1(k) \\ \vdots \\ v_{N-1}(k) \end{pmatrix}$$

$$\boldsymbol{x}(k) = \boldsymbol{a} s(k-\boldsymbol{\tau}) + \boldsymbol{v}(k) \, .$$

- In the time domain we had the matrix form:

$$\boldsymbol{x}(k) = \boldsymbol{a}s(k-\boldsymbol{\tau}) + \boldsymbol{v}(k) \ .$$

- The Furrier Transform leads to:

$$\boldsymbol{X}\left(\boldsymbol{e}^{j\Omega}\right) = \boldsymbol{S}\left(\boldsymbol{e}^{j\Omega}\right)\boldsymbol{d} + \boldsymbol{V}\left(\boldsymbol{e}^{j\Omega}\right)$$

where:

$$d^T = \left[a_0 e^{-j\Omega\tau_0},\ a_1 e^{-j\Omega\tau_1}, \ldots., a_{N-1} e^{-j\Omega\tau_{N-1}}\right]$$

- Finally the output signal is obtain by:

$$Y_b\left(e^{j\Omega}\right) = \sum_{n=0}^{N-1} W_n^*\left(e^{j\Omega}\right) X_n\left(e^{j\Omega}\right) = W^H \boldsymbol{X}$$

where $W^H(j\Omega)$ is the Hermitian operator of the coefficients matrix of the beamformer.

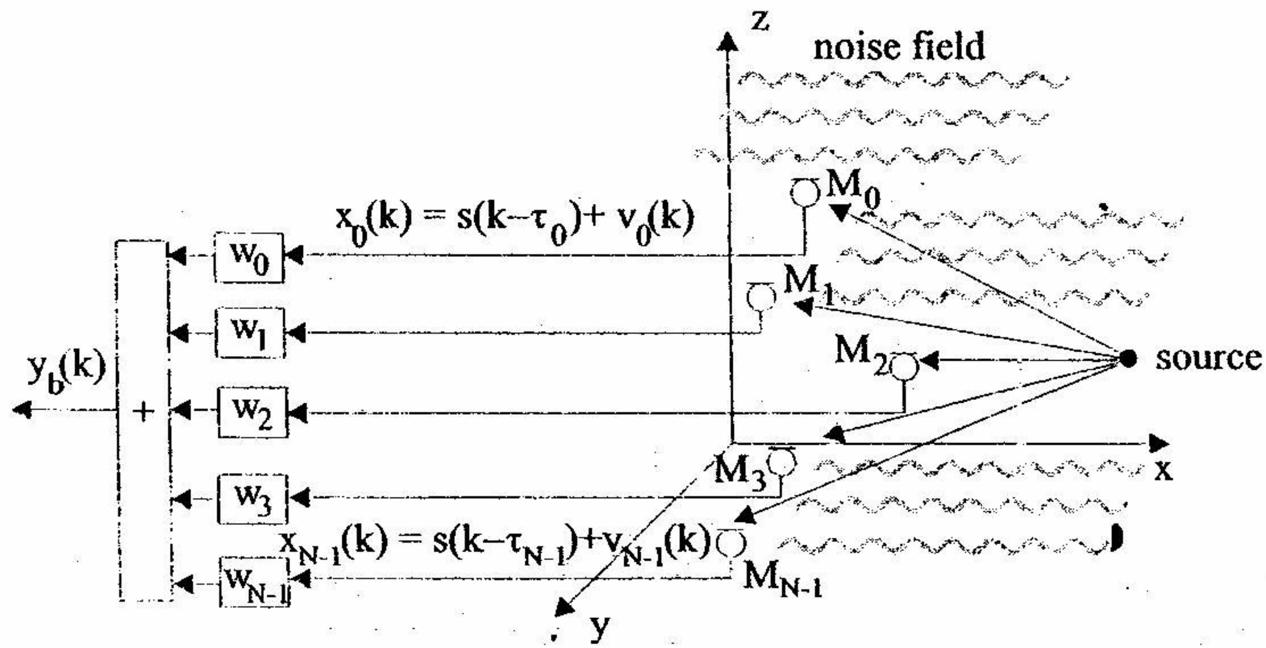


Fig. 2.1. Signal model consisting of noise field and desired source signal

$$Y_b\left(e^{j\Omega}\right)=\sum_{n=0}^{N-1} W_n^{*}\left(e^{j\Omega}\right) X_n\left(e^{j\Omega}\right)=W^H \boldsymbol{X}$$

Performance measures of Beamformer

- **Array Gain**
- **Beampattern**
- **Directivity Factor - DI**
- **Front to Back Ratio - FBR**
- **White Noise Gain - WNG**

Array Gain measure

- Definition: $$G_A = \frac{SNR_{Array}}{SNR_{Sensor}}$$
- The SNR_{Sensor}: $$SNR_{Sensor} = \frac{\phi_{ss}}{\phi_{v_a v_a}} \quad \text{where } \phi \text{ denote PSD}$$
- The SNR_{Array} (out) can be derived from (*LTI*):

$$SNR_{Array} = \frac{\phi_{Y_b Y_b}\big|_{Signal}}{\phi_{Y_b Y_b}\big|_{Noise}} = \frac{\phi_{ss}\left|W^H d\right|^2}{\phi_{v_a v_a} W^H \Phi_{VV} W} \qquad \text{where} \quad \begin{cases} \phi_{Y_b Y_b} = W^H \Phi_{XX} W \\ \Phi_{XX} = \begin{pmatrix} \phi_{X_0 X_0} & \phi_{X_0 X_1} & ,,, & \phi_{X_0 X_{N-1}} \\ \phi_{X_1 X_0} & \phi_{X_1 X_1} & ,,, & \phi_{X_1 X_{N-1}} \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{X_{N-1} X_0} & \phi_{X_{N-1} X_1} & ,,, & \phi_{X_{N-1} X_{N-1}} \end{pmatrix} \end{cases}$$

Finally we have: $$G_A = \frac{\left|W^H d\right|^2}{W^H \Phi_{VV} W}$$

Where Φ_{VV} is the cross PSD of the Noise.

$$G = \frac{\left|\boldsymbol{W}^H \boldsymbol{d}\right|^2}{\boldsymbol{W}^H \boldsymbol{\Phi}_{\boldsymbol{VV}} \boldsymbol{W}} .$$

Assuming a homogeneous noise field (G_A) can be expressed in terms of the coherence matrix

$$\boldsymbol{\Gamma}_{\boldsymbol{VV}} = \begin{pmatrix} 1 & \Gamma_{V_0 V_1} & \Gamma_{V_0 V_2} & \cdots & \Gamma_{V_0 V_{N-1}} \\ \Gamma_{V_1 V_0} & 1 & \Gamma_{V_1 V_2} & \cdots & \Gamma_{V_1 V_{N-1}} \\ \vdots & \vdots & \ddots & \vdots & \\ \Gamma_{V_{N-1} V_0} & \Gamma_{V_{N-1} V_1} & \Gamma_{V_{N-1} V_2} & \cdots & 1 \end{pmatrix} ,$$

where

$$\Gamma_{V_n V_m}(e^{j\Omega}) = \frac{\Phi_{V_n V_m}(e^{j\Omega})}{\sqrt{\Phi_{V_n V_n}(e^{j\Omega}) \Phi_{V_m V_m}(e^{j\Omega})}}$$

is the coherence function

Thus,

$$G = \frac{\left|\boldsymbol{W}^H \boldsymbol{d}\right|^2}{\boldsymbol{W}^H \boldsymbol{\Gamma}_{\boldsymbol{VV}} \boldsymbol{W}} .$$

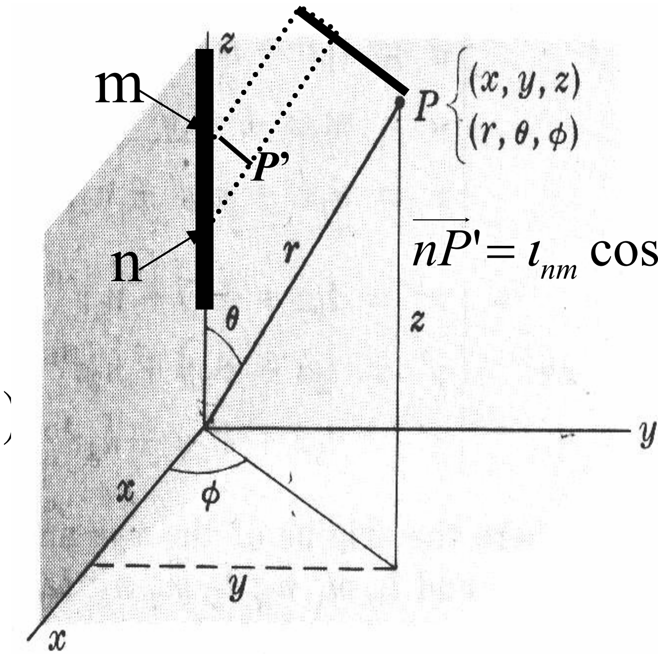
Farfield Beampattern measure

- Beampattern- The spatial-temporal Transfer function of the Array response to a wavefront with specific frequency and angle of arrival.

$$\left|H(\varphi,\theta)\right|^2\Big|_{dB} = -10\log_{10}\left(\frac{\left|W^H d\right|^2}{W^H \Gamma_{VV}\big|_{wavefront} W}\right)$$

$$\Gamma_{VnVm}\big|_{wavefront} = \exp(j\Omega\tau_{nm})$$

$$\tau_{nm} = \frac{fs}{c}\left(\iota_{x,nm}\sin(\theta)\cos(\phi) + \iota_{y,nm}\sin(\theta)\sin(\phi) + \iota_{z,nm}\cos(\theta\right.$$



- Note: in the case that the Array is aligned to the z axis there is no depending on Φ (due to rotational symmetry):

$$\tau_{nm} = \frac{fs}{c}\left(\iota_{nm}\cos(\theta)\right)$$

Directivity factor

- **Directivity factor:** describe the ability of the array to suppress Diffuse Noise and equal to the Array Gain in diffuse noise field .

Definition 1: simply applying the coherence of difuse noise into the array gain formula:

$$\mathrm{DI}\left(e^{j\Omega}\right) = 10\log 10\left(\frac{\left|W^H d\right|^2}{W^H \Gamma_{VV}\big|_{\text{Diffuse}} W}\right)$$

Where the diffuse noise coherence is:

$$\Gamma_{VnVm}\big|_{\text{Diffuse}} = \operatorname{sin c}\left\{\frac{\Omega f_s l_{nm}}{\pi c}\right\}$$

Definition 2: the ratio between the transfer function of the look-direction and the spatial integration of the TF over all directions of incoming signals.

$$\mathrm{DI}\left(e^{j\Omega}\right) = 10\log 10\left(\frac{\left|H\left(e^{j\Omega}, \varphi_0, \theta_0\right)\right|^2}{\frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi}\left|H\left(e^{j\Omega}, \varphi_0, \theta_0\right)\right|^2 \sin(\theta)\, d\varphi\, d\theta}\right)$$

- Uses: in application with diffuse noise fields like in a car or open space with no turbulence.

Front to Back Ratio (FBR)

- Formula definition:

$$\mathrm{FBR}\left(e^{j\Omega}\right)=\frac{\int_{\theta_0-\pi/2}^{\theta_0+\pi/2}\int_{\varphi_0-\pi/2}^{\varphi_0+\pi/2}\left|H\left(e^{j\Omega},\varphi_0,\theta_0\right)\right|^2\sin(\theta)\,d\varphi\,d\theta}{\int_{\theta_0+\pi/2}^{\theta_0+3\pi/2}\int_{\varphi_0+\pi/2}^{\varphi_0+3\pi/2}\left|H\left(e^{j\Omega},\varphi_0,\theta_0\right)\right|^2\sin(\theta)\,d\varphi\,d\theta}$$

- The FBR is a measure that utilizes the Beampattern by evaluating its integration ratio between the front and back of the array.

- Uses: in application that no look direction exists for example video conference and recording of a show or lecture.

White Noise Gain - WNG

- WNG- is a measure that shows the ability to suppress uncorrelated noise field (self noise of the sensor).

- Applying the uncorrelated characterize leads to a unity coherent matrix:

$$\Gamma_{\mathrm{VV}}\Big|_{\mathrm{uncorr}} = I$$

Inserting the coherent matrix into the G_A equation leads to:

$$\mathrm{WNG}\left(e^{j\Omega}\right) = \frac{\left|\mathrm{W}^{\mathrm{H}}\mathrm{d}\right|^{2}}{\mathrm{W}^{\mathrm{H}}\mathrm{W}}$$

- Uses: in application that have self sensor noise or other uncorrelated noise. for example recording in acoustic room.

Design of Superdirective Beamformers

- **Motivation:**

1. Minimized the PSD of the beamformer output.
2. Undistorted signal response in the look direction:

$$W^{H}d = 1$$

Therefore we have to solve the following constrained minimization problem:

$$\min_{W} \ \phi_{Y_b Y_b} = W^{H}\Phi_{XX}W \qquad \text{subject to} \ \ W^{H}d = 1$$

- we are interesting only in optimal suppression of the noise, the Solution depend only on the noise PSD and the look-direction, and known as Minimum Variance Distortionless Response (MVDR):

$$W = \frac{\Phi_{VV}^{-1}d}{d^{H}\Phi_{VV}^{-1}d}; \qquad \text{in homogeneous noise field } W = \frac{\Gamma_{VV}^{-1}d}{d^{H}\Gamma_{VV}^{-1}d}$$

- The design procedure reduces to choice of the theoretical noise field and the array structure according to the desire *d* vector.

- **DSB – SDB over WNG measure (WNG design)**

Comparison

SDB

$$\Gamma_{VV}\big|_{uncorr} = I$$

$$W = \frac{\Gamma_{VV}{}^{-1} d}{d^H \Gamma_{VV}{}^{-1} d} = \frac{1}{N} d$$

$$WNG\left(e^{j\Omega}\right) = \frac{\left|W^H d\right|^2}{W^H W} \approx N$$

Delay and Sum

$$SNR_{Sensor} = \frac{\phi_{ss}}{\phi_{v_a v_a}}$$

$$SNR_{array} = \frac{N^2 \phi_{ss}^2}{N \phi_{v_a v_a}^2}$$

$$WNG\left(e^{j\Omega}\right) \approx N$$

- The DSB known as optimal in WNG improvement and reaches to N, the SDB also optimal and improve the SNR by the same factor N. Other models such as Chebycheff window worsen the performance subject to WNG.

• Diffuse noise design

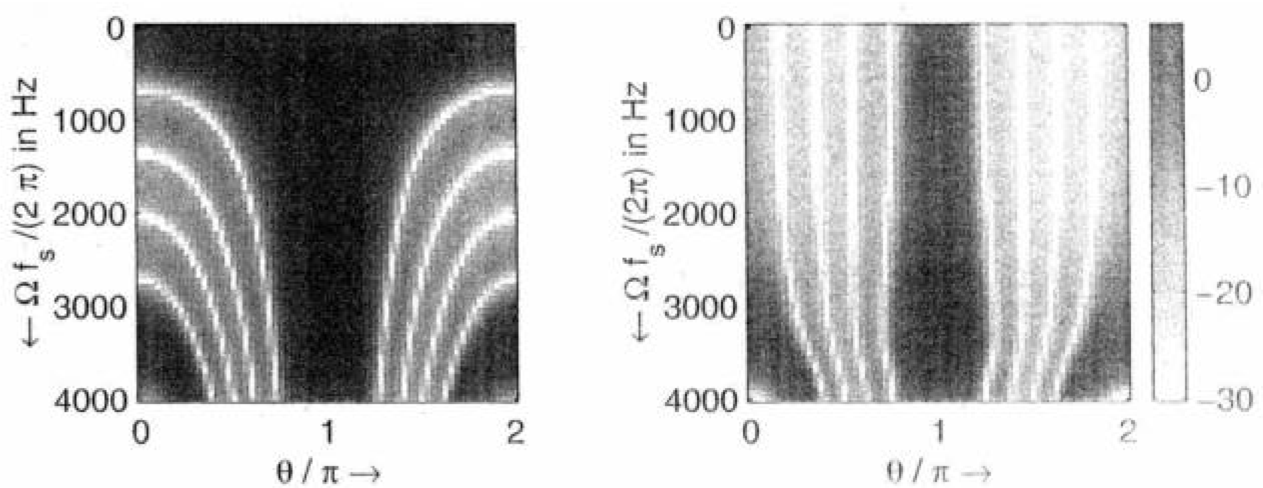
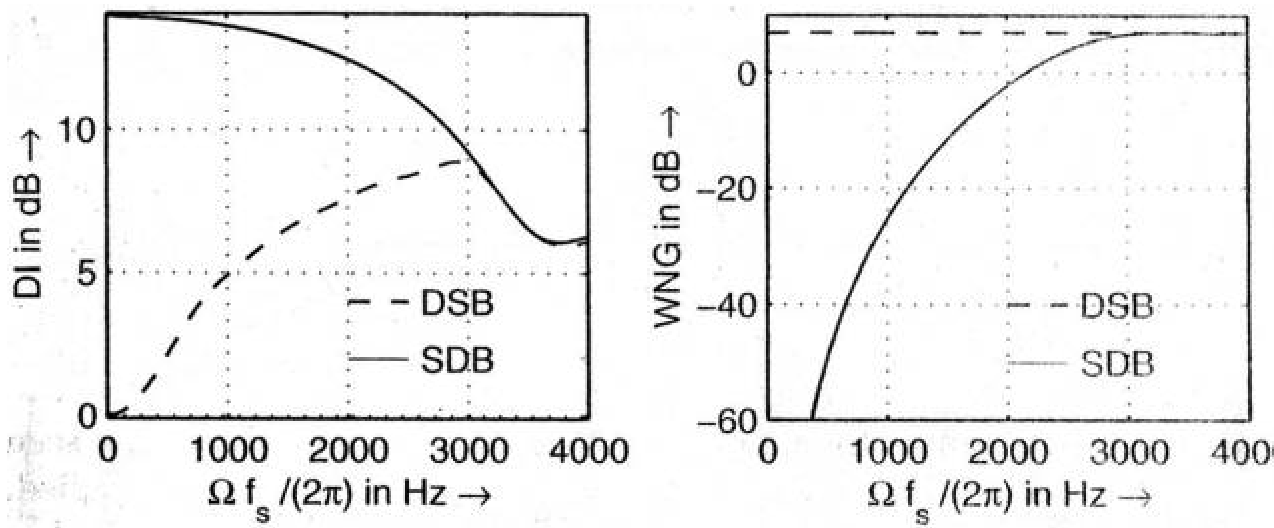


Fig. 2.2. Left: beampattern of a delay-and-sum beamformer. Right: beampattern of an optimal array for isotropic noise (superdirective beamformer). ($l = 5$ cm, $N = 5$, endfire steering direction)

- The look direction gain is unmodified at all frequencies in both models.
- The SDB attenuate sources coming other then the look direction over the all frequencies range more then the DSB.
- No suppression on low frequencies in the DSB.
- $\iota < \lambda/2$ not fulfill in high frequencies -therefore we can see spatial aliasing in both models.

f_s = 8kHz; l = 5 ; N = 5 ;diffuse Noise coherence;



- DI measure -
  low frequencies - Reaches 0 in the DSB and N^2 at SDB.
  higher frequencies - Reaches N for both (the coherence tend to 0; sinc function).

- WNG (or self noise) measure -
  DSB - frequency independency (same gain N).
  SDB - low frequencies : noise boost.
  higher frequencies : same as the DSB behavior.

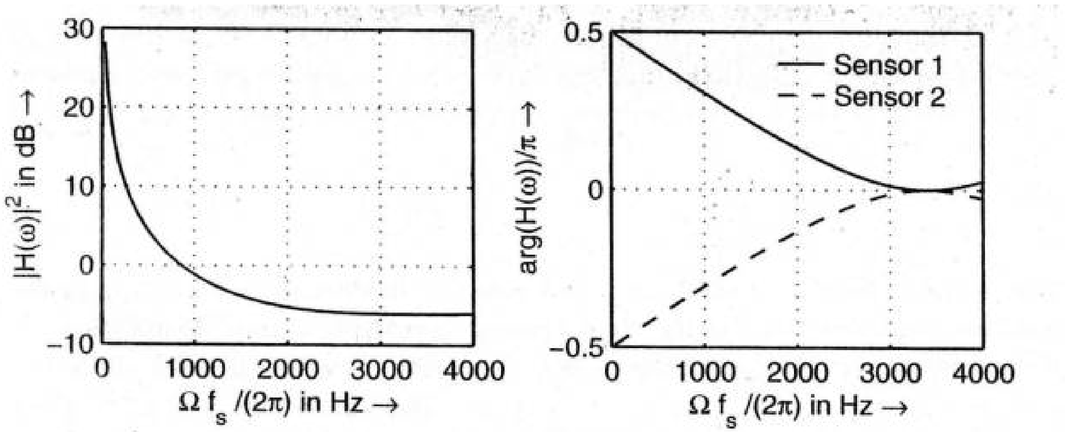


Fig. 2.4. Coefficients of a two channel SDB, left: Magnitude, right: Phase ($l = 5$ cm, $N = 2$, endfire steering direction)

- Due to the filter phase correlated signal will be decrease

Therefore, to avoid signal attenuate, the magnitude of the filter should compensated this behavior. Uncorrelated noise are not decrease because they arrived in delay of π therefore the magnitude compensation will amplify them. Therefore the performance of SDB subject to WNG are worsen.

This behavior accrue in the low frequencies and in the high frequencies The magnitude of the filter reducess to -6dB , due to less correlation in the high frequencies.

SDB with WNG constrain

In order to solve the self noise amplification, we need to find the coefficient's matrix under the WNG constrained.
The solution involves adding a factor μ to the diagonal of the coherence matrix:

$$W = \frac{(\Gamma_{VV} + \mu I)^{-1} d}{d^H (\Gamma_{VV} + \mu I)^{-1} d}$$

or its mathematically equivalent form:

$$W = \frac{\Gamma_{VV}{}^{-1} d}{d^H \Gamma_{VV}{}^{-1} d} \quad \text{where}: \Gamma_{VnVm} = \frac{\sin c\left\{\dfrac{\Omega f_s l_{nm}}{\pi c}\right\}}{1 + \dfrac{\sigma^2}{\phi_{VV}}}$$

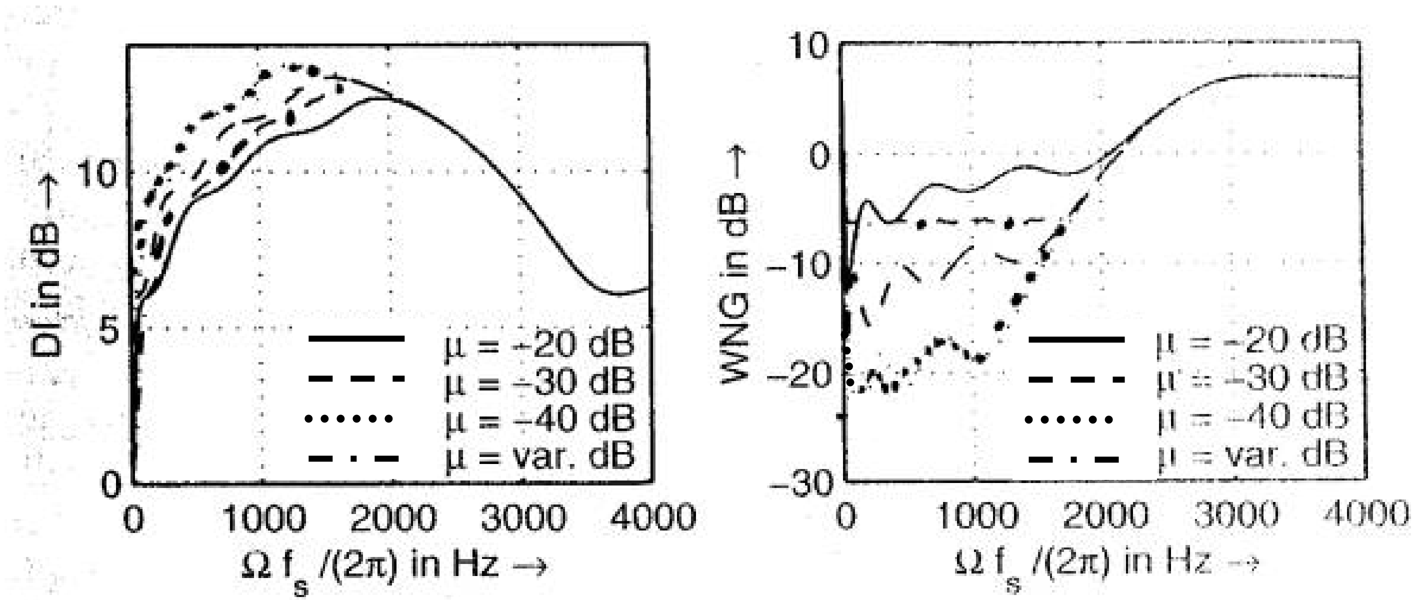


Fig. 2.5. Left: Directivity index (DI) for different constrained designs. Right: White noise gain (WNG) for different constrained designs. (l = 5 cm, N = 5, endfire steering direction)

- The WNG was set to –6dB.
- The WNG constraint design facilitates a good compromise between the DI and WNG measures.

Note: The "var" factor stand for variable factor.

• Front to Back optimal ratio design

- The regular SDB design for FBR is assuming isotropic noise field:

$$\Gamma_{\mathrm{VnVm}} = \mathrm{J}_0\left(\frac{\omega \imath_{\mathrm{nm}}}{\mathrm{c}}\right)$$

- an alternative design is assuming uncorrelated noise field, only in the back-half of the array, coming from infinite sources. this assumption in the case of two dimensional leads to a noise field with function:

$$\mathrm{f}\left(\mathrm{e}^{\mathrm{j}\Omega},\theta_0\right)=\frac{1}{\pi}\int_{\theta_0+\pi/2}^{\theta_0+3\pi/2}\exp(\mathrm{j}\Omega \mathrm{f_s} \mathrm{c}^{-1}\imath\cos(\theta))\mathrm{d}\theta$$

Inserting this function results into the coherence function yields a very good practical result.

* Note: in both cases we should insert the WNG constraint.

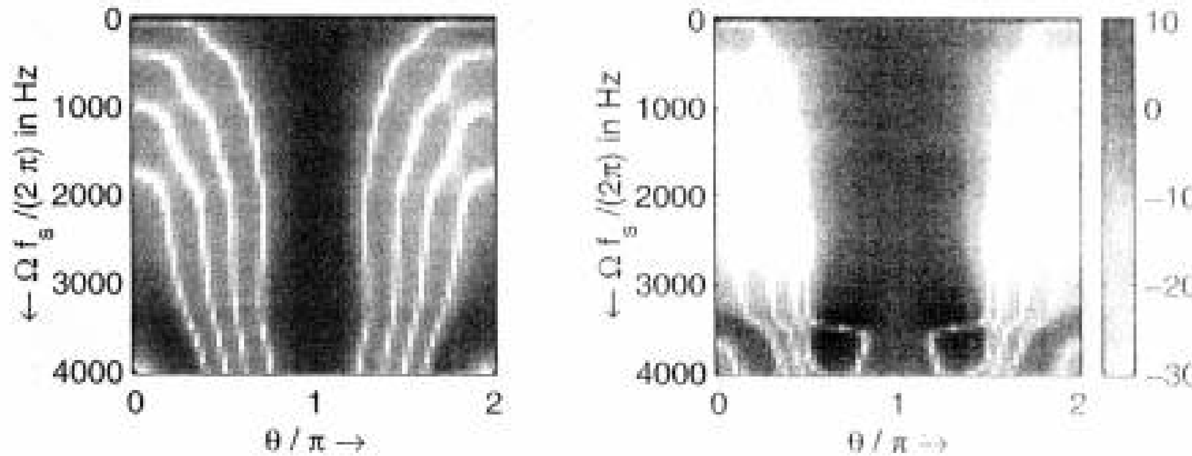


Fig. 2.6. Left: beampattern of a **constrained** superdirective beamformer. Right: beampattern of a constrained **beamformer**, designed with (2.36). ($l = 5$ cm, $N = 5$, $\mu = 0.01$, endfire steering direction)

- The new design increase the main lobe but suppresses all the signals coming from the rear

- The new design boost high frequencies coming from the look-direction. this effect can cause unnatural coloring of the signal and residual noise, therefore the new design is more sensitive to parameters tuning.

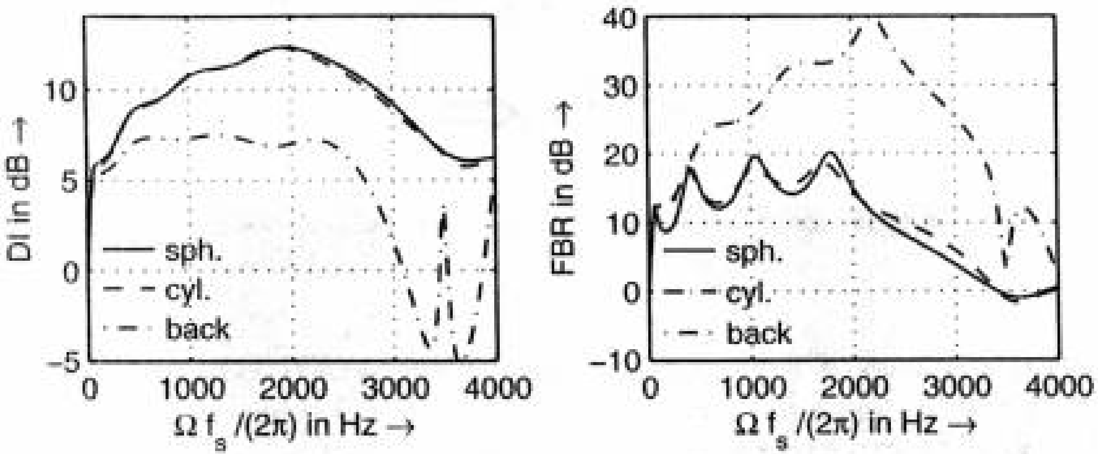


Fig. 2.7. Left: Directivity index (DI) for three optimal designs. Right: Front-to-back ratio (FBR) for three optimal designs. ($l = 5$ cm, $N = 5$, $\mu = 0.01$, endfire steering direction)

- The spherical and cylindrical design leads to much similar results
  In both measures, however the new design is completely different.

- The new design's DI performance is worsen, but the FBR performance improved significantly especially in the mid-frequencies range

- The new design's is mush sensitive to spatial aliasing.

• <u>Design for measured noise field</u>

• in some application we have priory knowledge that can improve the performance.

Example:

a given complex coherence function, between two sensors, of a noise source coming from known farfield angle θ:

$$\mathrm{Re}\{\Gamma_{\mathrm{XnXm}}(\omega)\} = \cos(\Omega \mathrm{f_s} \cos(\theta) \iota_{\mathrm{nm}})$$
$$\mathrm{Im}\{\Gamma_{\mathrm{XnXm}}(\omega)\} = -\sin(\Omega \mathrm{f_s} \cos(\theta) \iota_{\mathrm{nm}})$$

Inserting the complete coherence matrix in the SDB model and again applying the WNG constrain lades to a null (0) at θ direction.

Alternative form

- The inputs signal are first – time alignment as shown in the figure. This effect the optimal weights since **d** became the unit vector and the noise coherent function change.

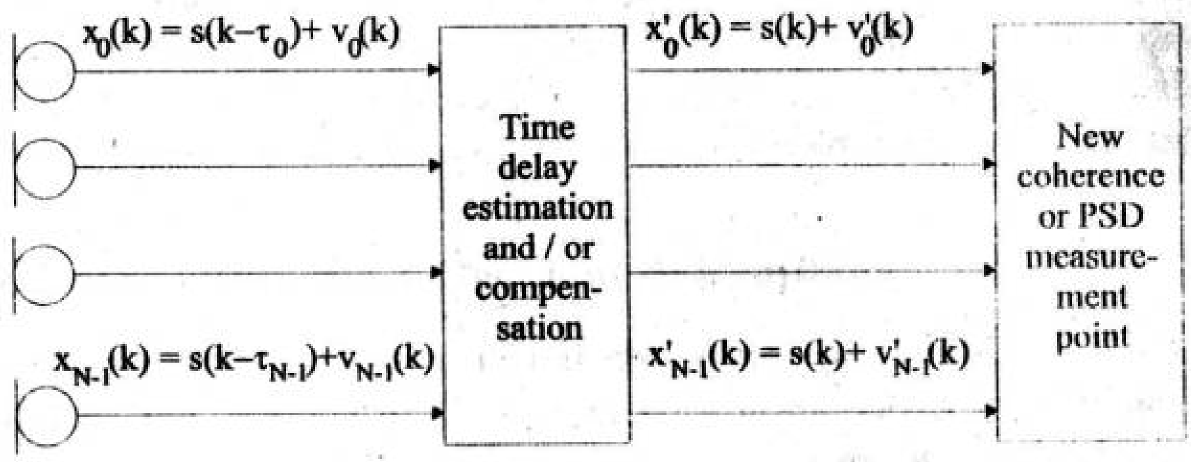


Fig. 2.8. Signal model after time delay compensation

$$\boldsymbol{W}\Big|_{\mathrm{ta}} = \frac{\mathbf{1}^T(\boldsymbol{\Gamma}'_{\boldsymbol{VV}} + \mu \boldsymbol{I})^{-1}}{\mathbf{1}^T(\boldsymbol{\Gamma}'_{\boldsymbol{VV}} + \mu \boldsymbol{I})^{-1}\mathbf{1}}.$$

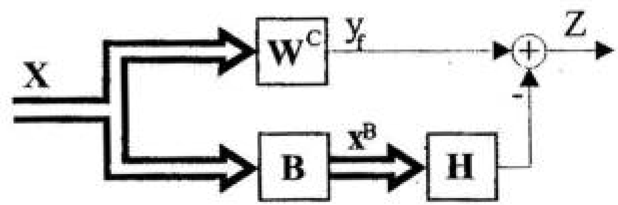


Fig. 2.9. Schematic description of the decomposition of the optimal weight vector into two orthogonal parts

<u>Griffith and Jim</u>- The problem now can be decompose into two orthogonal parts:

Part1: represents the constraints.

Part2: represents the unconstrained coefficients to minimized the noise PSD at the output.

W^C - fulfill the constraints. In standard beamformer $W^C = \frac{1}{N}$

B - blocking matrix.

H - the optimal vector, minimized the output noise PSD.

- The last model also called generalized sidelobe canceler (GSC) whose solution is given as:

$$\mathrm{H} = \left(\mathrm{B}\Gamma_{\mathrm{VV}}'\mathrm{B}^{\mathrm{H}}\right)^{-1}\mathrm{B}\Gamma_{\mathrm{VV}}'\mathrm{W}^{\mathrm{C}}$$

This design solution equivalent to the previous SDB design, however it has some advantages:

- require one less filter.
- a DSB output is available.
- integration with post-filtering process to achieve more noise reduction.

Gradient microphone

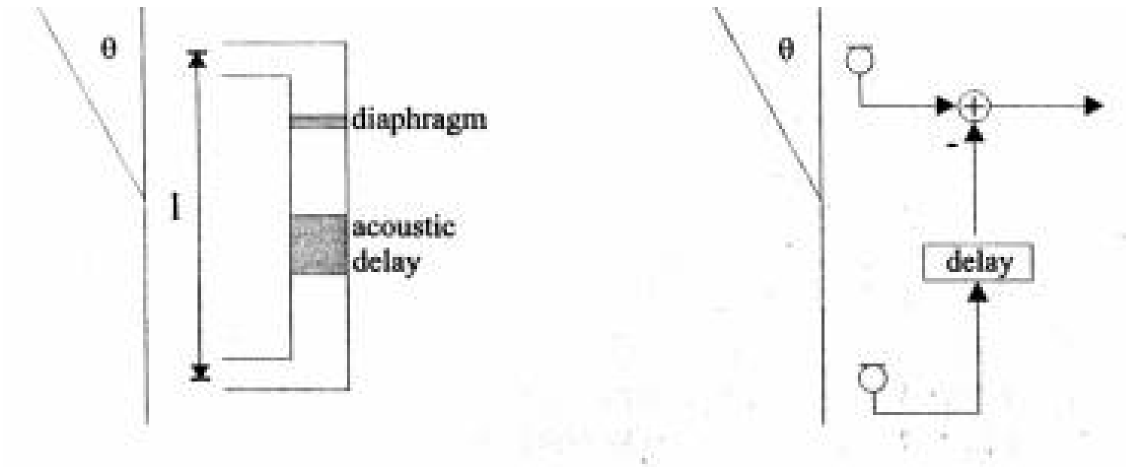


Fig. 2.10. Schematic description of a first order gradient microphone

The output of the gradient microphone is given as:

$$E(\omega,\theta) = P_0\omega\left(\tau + c^{-1}\imath\cos(\theta)\right)$$

where

τ - is the acoustic delay and P_0 is the source amplitude.

τ - can leads to different SD design: cardioid, supercardioid and hypercardioid.

First order hypercardioid- comparison to SDB

- This hypercardioid is the analogue equivalent to the SDB of two sensor.

comparison

- At low frequencies the two systems react more or less equally.
- At higher frequencies , if the assumption of small spacing is not valid, the hypercardioid cancels the desired signal completely in some of the frequencies , therefore selecting a hypercardioid enquiring consideration of the application bandwidth.
- Physical structure: the hypercardioid is much smaller.
- The digital system require A\D and D\A converters.
- The digital system is flexible and more easy integrate with post filtering.

conclusion

- The design of so-called SDB in theoretically well known noise fields reduced to solving a single equation.
- Despite of the distortionless constrain we need to subject the solution to WNG<m constrain – to avoid boost of uncorrelated noise signal. (compromise between Directivity and WNG)
- We have shown that in a careful design (the SDB model is very sensitive to parameter accurate) the SDB improves all the beamformer measures. The main cause for this improvement is that the measures connected directly to the coherent function.
- We have seen the relationship between the SDB, adaptive beamformer and GSC.

The end