16711 HW 5 - Writeup

Saeed Bai, Mohammadreza Mousaei

TOTAL POINTS

8 / 6

QUESTION 1

Computing Manipulator EOMs 6 pts

1.1 2 / 2

- ✓ **- 0 pts** Correct
- **- 0.5 pts** Partially correct
- **- 1 pts** No M_11 value provided.

1.2 2 / 2

- ✓ **- 0 pts** Correct
- **- 0.5 pts** Partially correct
- **- 0 pts** Correct but I think there's a typo
- **- 1 pts** No answer provided
- **- 0.5 pts** Further simplification needed.

1.3 2 / 2

- ✓ **- 0 pts** Correct
- **- 0.5 pts** Partially correct
- **- 1 pts** No answer provided

1.4 2 / 0

- ✓ **+ 2 pts** Correct
- **- 0 pts** Not answered.
- **+ 0.5 pts** No result provided

Mohammadreza Mousaei
Dongwei Bai

16-711
April 8, 2021

Homework 5

The code for this assignment is provided in *HW5.m*. Please change the variable *PrintDetails* to 1, if you want the code to print the details step by step.

Problem 1

From the provided code, section Q1, the component M_{11} of the resulting inertia matrix $M(\theta)$ is:

$$\begin{aligned} M_{11} =& I_{z_1} + m_3(r_2\cos(\theta_2+\theta_3) + l_1\cos(\theta_2))^2 + I_{z_3}\cos(\theta_2+\theta_3)^2 + I_{y_3}\sin(\theta_2+\theta_3)^2 \\ &+ I_{z_2}\cos(\theta_2)^2 + I_{y_2}\sin(\theta_2)^2 + m_2 r_1^2\cos(\theta_2)^2 \end{aligned}$$

Problem 2

From the provided code, section Q2, the component C_{21} of the resulting Coriolis matrix $C(\theta,\dot{\theta})$ is:

$$\begin{aligned} C_{21} =& \dot{\theta}_1 \Bigg(\frac{m_3\sin(2\theta_2)l_1^2}{2} + m_3\sin(2\theta_2+\theta_3)l_1 r_2 + \frac{m_2\sin(2\theta_2)r_1^2}{2} + \frac{m_3\sin(2\theta_2+2\theta_3)r_2^2}{2} \\ &- \frac{I_{y_3}\sin(2\theta_2+2\theta_3)}{2} + \frac{I_{z_3}\sin(2\theta_2+2\theta_3)}{2} - \frac{I_{y_2}\sin(2\theta_2)}{2} + \frac{I_{z_2}\sin(2\theta_2)}{2} \Bigg) \end{aligned}$$

Problem 3

From the provided code, section Q3, the element N_3 of the vector $N(\theta,\dot{\theta})$ is:

$$N_3 = -g m_3 r_2 \cos(\theta_2+\theta_3)$$

Problem 4

We implemented a PD controller to control the manipulator. We have:

$$\left.\begin{aligned} \tau &= M\ddot{\theta} + C\dot{\theta} + N \\ \tau &= K_p(\theta_{des}-\theta) + K_d(\dot{\theta_{des}} - \dot{\theta}) \end{aligned}\right\} \longrightarrow \ddot{\theta} = M^{-1}\left(K_p(\theta_{des}-\theta) + K_d(\dot{\theta_{des}} - \dot{\theta}) - C\dot{\theta} - N\right)$$

Through experiments, we tuned PD values to be: $K_p = 30$ and $K_d = 10$. The termination criteria is when error drops below 10^{-4}. The trajectory results for θ, $\dot{\theta}$ and $\ddot{\theta}$ are shown in Fig. 1, respectively in each column from left to right.

1.1 2 / 2

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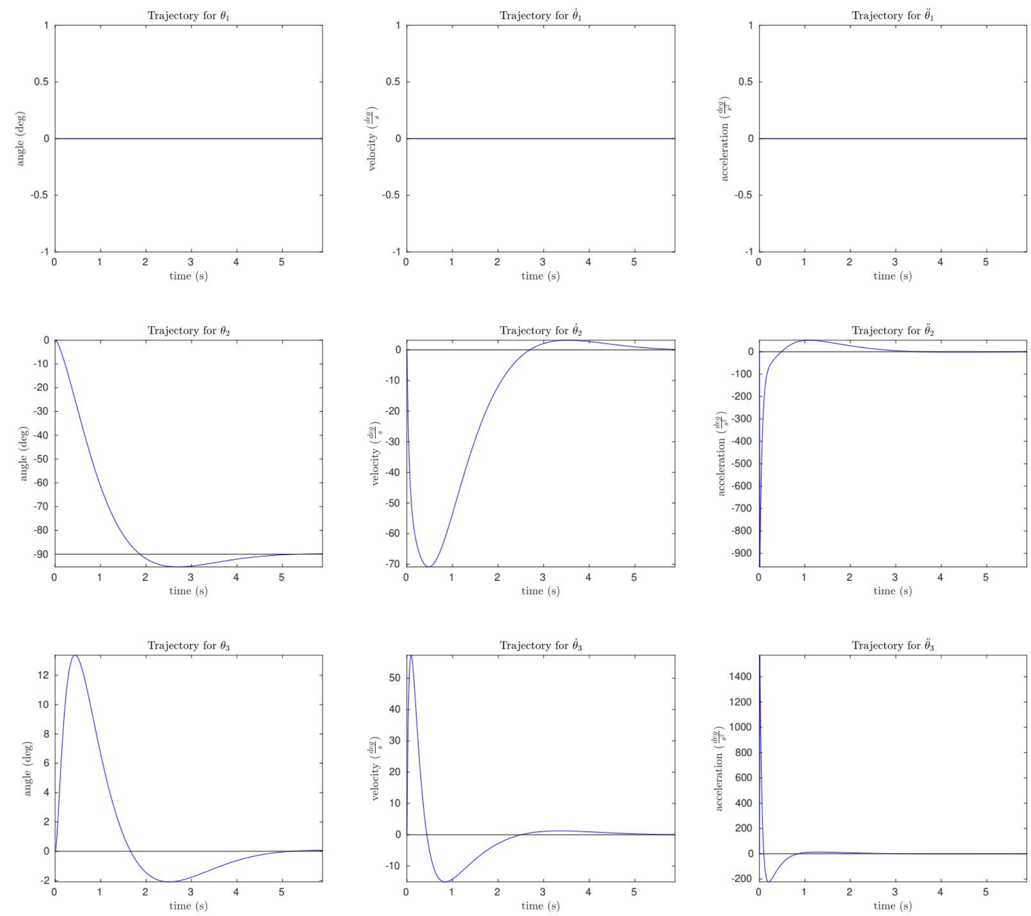

Figure 1: The resulting trajectory for θ, $\dot{\theta}$ and $\ddot{\theta}$, respectively in each column from left to right

1.4 2 / 0

✓ **+ 2 pts** Correct

- 0 pts Not answered.

+ 0.5 pts No result provided