Like all other assignments in the course, this assignment will be submitted via Gradescope. Please come to office hours or reach out on Piazza if you have any questions! Please use livecripts to present your Matlab work, along with screenshots of any Simulink models.

1: 20 points

An LTI system is described by the equations

$$\dot{x} = \begin{bmatrix} a & 0 \\ 1 & -1 \end{bmatrix} x.$$

Use Lyapunov's direct method to determine the range of variable a for which the system is asymptotically stable.

3: 30 points

For the three systems given below, determine the stability (i.e. Lyapunov, asymptotic, or BIBO).

(a)

$$x(k+1) = \begin{bmatrix} 1 & 0 \\ -0.5 & 0.5 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 5 & 5 \end{bmatrix} x(k)$$

(b)

$$\dot{x} = \begin{bmatrix} -7 & -2 & 6 \\ 2 & -3 & -2 \\ -2 & -2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix} x$$

(c) Provide answers for both DT and CT interpretations of the following matrices.

$$A = \begin{bmatrix} 2 & -5 \\ -4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad D = 0$$

4: 20 points

Use Lyapunov's stability theorem to determine the stability of the following system.

$$\dot{x}_1 = x_2 - x_1 x_2^2$$
$$\dot{x}_2 = -x_1^3$$

HINT: Consider the Lyapunov function $V(x_1, x_2) = x_1^4 + 2x_2^2$.

5: 20 points

Use instability criterion with the function $V(x_1, x_2) = x_1^2 - x_2^2$ to prove that the origin of the following system is unstable.

$$\dot{x}_1 = 3x_1 + x_2^3$$
$$\dot{x}_2 = -x_2 + x_1^2$$

6: 20 points

Consider the equation of motion for a simple pendulum

$$\ddot{\theta} + \frac{g}{L}\sin\theta = 0$$

(a) Using the total energy of the system as a Lyapunov function, show that $\theta_0 = 0$ is stable in the sense of Lyapunov.

(b) Using the system energy as a Lyapunov function, show that the equilibrium point $\theta_0 = \pi$ is unstable. For this you will need to use a change of variables $x = \theta - \pi$ to give an equivalent system with $x_0 = 0$ the relevant equilibrium point.