Like all other assignments in the course, this assignment will be submitted via Gradescope. Please come to office hours or reach out on Piazza if you have any questions! Please use livecripts to present your Matlab work, along with screenshots of any Simulink models.

1: 15 points

Given

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) Find A^{10}.

(b) Find A^{103}.

(c) Find e^{At}.

2: 20 points

Find a closed-form expression for A^k for $k \geq 1$ where

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 9 & 23 & 30 \\ -7 & -18 & -235 \end{bmatrix}.$$

3: 20 points

For the following system, find an expression for $y(n)$ if the input $u(k) = 1 \; \forall k \geq 0$ starting at $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$.

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ -0.5 & -1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

4: 30 points

Let $x_1(t)$ be the water level in Tank 1 and $x_2(t)$ be the water level in Tank 2 . Let α be the rate of outflow from Tank 1 and β be rate of outflow from Tank 2 . Let u be the supply of water to the system. The system can be modelled into the following differential equations:

$$\frac{dx_1}{dt} = -\alpha x_1 + u$$

$$\frac{dx_2}{dt} = \alpha x_1 - \beta x_2$$

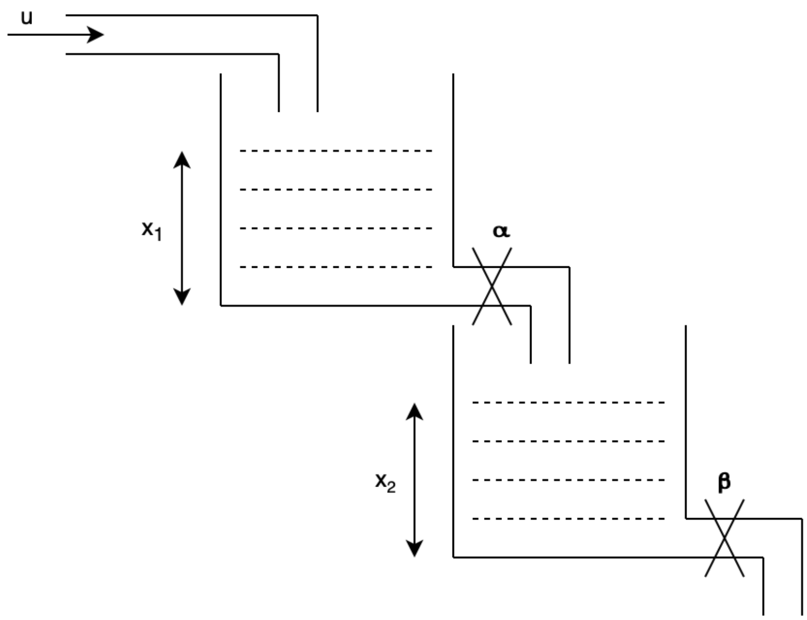

Figure 1: Tank Problem

Given $\alpha = 0.1$, $\beta = 0.2$, $u = 1$, $x_1(0) = 2$, $x_2(0) = 1$, find the water level in both tanks after 5s.

(a) Find $y(5)$ for the CT system. Solve with the Cayley-Hamilton theorem. You may use a calculator but do not directly use programming.

(b) Find the discretized state space representation using sample time $T = 1s$.

(c) Find $y(5)$ of the discrete time system. Also plot signals $y(t)$ for both CT and DT systems in the same figure.

5: 20 points

A PID controller in its classical form is not realizable because it has more poles than zeros. To implement the controller, we add a pole to the system at "high" frequency to get a bi-proper system, i.e.

$$C(s) = \frac{K_d s^2 + K_p s + K_i}{s\,(\tau s + 1)}$$

with τ small.

(a) Find a state space realization for the modified (bi-proper) PID controller.

(b) Find and graph the solution for the controller response (starting at zero initial state) given an input of $u = 1$ for $K_d = 10$, $K_p = 100$, $K_i = 1$, and $\tau = 0.001$. The response should grow without bound - why?