24677-A Homework 3

Saeed Bai

TOTAL POINTS

93 / 95

QUESTION 1

Change of basis 20 pts

1.1 Operator representation 10 / 10

✓ **- 0 pts** Correct

- 0.5 pts Incorrect transformation matrix multiplication order

- 0.5 pts Incorrect transformation matrix

- 0.5 pts Incorrect answer

1.2 Vector representation 10 / 10

✓ **- 0 pts** Correct

- 1 pts Incorrect transformation method

QUESTION 2

2 Ax=y solutions 10 / 10

✓ **- 0 pts** Correct

- 1 pts Incorrect particular solution

- 2 pts Incorrect null space

- 2 pts Incorrect rref matrix

- 2 pts Incorrect answer

QUESTION 3

3 Least squares solution 10 / 10

✓ **- 0 pts** Correct

- 1 pts Incorrect answer

- 2 pts Incorrect method

QUESTION 4

Eigenvalues and eigenvectors 15 pts

4.1 Part a 5 / 5

✓ **- 0 pts** Correct

- 1 pts Wrong eigenvalues

- 1 pts Wrong eigenvector

- 2 pts Incorrect methods to solve eigenvector

4.2 Part b 4 / 5

- 0 pts Correct

✓ **- 1 pts** Incorrect eigenvectors

4.3 Part c 5 / 5

✓ **- 0 pts** Correct

- 1 pts Wrong eigenvalues

- 1 pts Wrong eigenvectors

1 Arithmetic error when isolating

2 Arithmetic error when isolating

QUESTION 5

SVD Computation 10 pts

5.1 Part a 5 / 5

✓ **- 0 pts** Correct

- 1 pts No intermediate steps

- 1 pts Incorrect singular value

5.2 Part b 5 / 5

✓ **- 0 pts** Correct

- 1 pts Incorrect singular value

- 1 pts No intermediate steps

QUESTION 6

Sliding mass control 30 pts

6.1 Optimal feedforward control 15 / 15

✓ **- 0 pts** Correct

- 2 pts Wrong response

- 2 pts Wrong control input

- 5 pts Wrong methodology

6.2 Deadbeat control 14 / 15

- 0 pts Correct

- **- 1 pts** Incorrect velocity response graph
- ✓ **- 1 pts** **Incorrect control response graph**
- **- 1 pts** Incorrect position response graph
- **- 2 pts** Incorrect "K"s
- **- 7.5 pts** No pole placement methodology found

None found

1.

$$\{a_1, a_2, a_3\} = \Bigg\{ \underbrace{\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}}_{Q} \Bigg\}, \quad \{b_1, b_2, b_3\} = \Bigg\{ \underbrace{\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}}_{X} \Bigg\}$$

a)

$$A = \begin{bmatrix} 8 & -2 & -1 \\ 4 & -2 & -3 \\ 2 & -3 & -3 \end{bmatrix} \quad \text{where } A = M\hat{A}M^{-1}$$

$$Ax = M\hat{A}M^{-1}x$$

$$\hat{A} = M^{-1}AM$$

$$B = Q^{-1}X\hat{B} \quad \text{where } M = Q^{-1}X$$

$$\hat{A} = M^{-1}AM = (Q^{-1}X)^{-1}A \cdot Q^{-1}X$$

$$= \begin{bmatrix} \frac{541}{245} & -\frac{1518}{245} & -\frac{4723}{735} \\ \frac{858}{245} & -\frac{404}{245} & -\frac{5309}{735} \\ -\frac{733}{245} & -\frac{1126}{245} & \frac{598}{245} \end{bmatrix}$$

b)

$$\hat{x} = X^{-1}Qx = \begin{bmatrix} -2 & -4 & 5 \\ 3 & -3 & -2 \\ 1 & -2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & 2 \\ 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 238 \\ 116 \\ 185 \end{bmatrix}$$

1.1 Operator representation 10 / 10

- ✓ **- 0 pts** Correct
- **- 0.5 pts** Incorrect transformation matrix multiplication order
- **- 0.5 pts** Incorrect transformation matrix
- **- 0.5 pts** Incorrect answer

1.

$$\{a_1, a_2, a_3\} = \left\{ \overbrace{\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}}^{Q} \right\}, \quad \{b_1, b_2, b_3\} = \left\{ \overbrace{\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}}^{X} \right\}$$

a)

$$A = \begin{bmatrix} 8 & -2 & -1 \\ 4 & -2 & -3 \\ 2 & -3 & -3 \end{bmatrix} \quad \text{where} \quad A = M\hat{A}M^{-1}$$

$$Ax = M\hat{A}M^{-1}x$$

$$\hat{A} = M^{-1}AM$$

$B = Q^{-1}X\hat{B}$ where $M = Q^{-1}X$

$\hat{A} = M^{-1}AM = (Q^{-1}X)^{-1}A \cdot Q^{-1}X$

$$= \begin{bmatrix} \frac{541}{245} & -\frac{1518}{245} & -\frac{4723}{735} \\ \frac{858}{245} & -\frac{404}{245} & -\frac{5309}{735} \\ -\frac{733}{245} & -\frac{1126}{245} & \frac{598}{245} \end{bmatrix}$$

b)

$$\hat{x} = X^{-1}Qx = \begin{bmatrix} -2 & -4 & 5 \\ 3 & -3 & -2 \\ 1 & -2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & 2 \\ 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 238 \\ 116 \\ 185 \end{bmatrix}$$

1.2 Vector representation 10 / 10

✓ **- 0 pts** Correct

- 1 pts Incorrect transformation method

2

$$A = \begin{bmatrix} 2 & 3 & 1 & 4 & -9 \\ 1 & 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & 2 & -5 \\ 2 & 2 & 2 & 3 & -8 \end{bmatrix}, \quad y = \begin{bmatrix} 17 \\ 6 \\ 8 \\ 14 \end{bmatrix}$$

$$[A \mid y] = \left(\begin{array}{ccccc|c} 2 & 3 & 1 & 4 & -9 & 17 \\ 1 & 1 & 1 & 1 & -3 & 6 \\ 1 & 1 & 1 & 2 & -5 & 8 \\ 2 & 2 & 2 & 3 & -8 & 14 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & -3 & 6 \\ 0 & 1 & -1 & 2 & -3 & 5 \\ 0 & 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{array}{l} x_1 + x_2 + x_3 + x_4 - 3x_5 = 6 \\ x_2 - x_3 + 2x_4 - 3x_5 = 5 \\ x_4 - 2x_5 = 2 \end{array}$$

$$x_4 = 2 + 2x_5$$

$$x_2 = 5 + x_3 + 3x_5 - 4 - 4x_5 = 1 + x_3 - x_5$$

$$x_1 = 6 - x_2 - x_3 - x_4 + 3x_5 = 6 - 1 - x_3 + x_5 - x_3 - 2 - 2x_5 + 3x_5$$

$$= 3 - 2x_3 + 2x_5$$

Rewrite :

$$\begin{bmatrix} 3 - 2x_3 + 2x_5 \\ 1 + x_3 - x_5 \\ x_3 \\ 2 + 2x_5 \\ x_5 \end{bmatrix} \Rightarrow x_3 \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 2 \\ -1 \\ 0 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

2 Ax=y solutions 10 / 10

✓ **- 0 pts** **Correct**

- 1 pts Incorrect particular solution

- 2 pts Incorrect null space

- 2 pts Incorrect rref matrix

- 2 pts Incorrect answer

3.

$$-2 = x_1 - 2x_2$$
$$5 = x_1 - 2x_2$$
$$1 = -2x_1 + x_2$$
$$-3 = x_1 - 3x_2.$$

$$\Rightarrow \underbrace{\begin{bmatrix} 1 & -2 \\ 1 & -2 \\ -2 & 1 \\ 1 & -3 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 \\ 5 \\ 1 \\ -3 \end{bmatrix}}_{B}$$

$$A^T = \begin{bmatrix} 1 & 1 & -2 & 1 \\ -2 & -2 & 1 & -3 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 1 & -2 & 1 \\ -2 & -2 & 1 & -3 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ -2 & 1 \\ 1 & -3 \end{bmatrix} = \begin{pmatrix} 7 & -9 \\ -9 & 18 \end{pmatrix}$$

$$A^T B = \begin{bmatrix} 1 & 1 & -2 & 1 \\ -2 & -2 & 1 & -3 \end{bmatrix} \begin{bmatrix} -2 \\ 5 \\ 1 \\ -3 \end{bmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

$$A^T A x = A^T B$$

$$\begin{bmatrix} 7 & -9 \\ -9 & 18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \Rightarrow \begin{array}{l} x_1 = 0 \\ x_2 = 2/9 \end{array} \qquad \hat{x} = \begin{bmatrix} 0 \\ 2/9 \end{bmatrix}$$

3 Least squares solution 10 / 10

✓ **- 0 pts** **Correct**

- 1 pts Incorrect answer

- 2 pts Incorrect method

4. a) $\begin{bmatrix} 1 & -2 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ $\begin{vmatrix} 1-\lambda & -2 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{vmatrix} = 0$

$\lambda_1 = 1$, $\lambda_{2,3} = \dfrac{3 \pm \sqrt{13}}{2}$

when $\lambda = 1$

$$\begin{pmatrix} 0 & -2 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \left.\begin{aligned} -2x_2 &= 0 \\ -x_1 + \cancel{x_2} - x_3 &= 0 \\ -x_2 &= 0 \end{aligned}\right\} \quad x_1 = -x_3 \quad u_1 = x\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

when $\lambda = \dfrac{3+\sqrt{13}}{2}$

$$\begin{pmatrix} \frac{-1-\sqrt{13}}{2} & -2 & 0 \\ -1 & \frac{1-\sqrt{13}}{2} & -1 \\ 0 & -1 & \frac{-1-\sqrt{13}}{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \left.\begin{aligned} \frac{-1-\sqrt{13}}{2}x_1 - 2x_2 &= 0 \\ -x_1 + \frac{1-\sqrt{13}}{2}x_2 - x_3 &= 0 \\ -x_2 + \frac{-1-\sqrt{13}}{2}x_3 &= 0 \end{aligned}\right\} \quad u_2 = x\begin{pmatrix} 1 \\ (-1-\sqrt{13})/4 \\ 1/2 \end{pmatrix}$$

when $\lambda = \dfrac{3-\sqrt{13}}{2}$

$$\begin{pmatrix} \frac{-1+\sqrt{13}}{2} & -2 & 0 \\ -1 & \frac{1+\sqrt{13}}{2} & -1 \\ 0 & -1 & \frac{-1+\sqrt{13}}{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \left.\begin{aligned} \frac{-1+\sqrt{13}}{2}x_1 - 2x_2 &= 0 \\ -x_1 + \frac{1+\sqrt{13}}{2}x_2 - x_3 &= 0 \\ -x_2 + \frac{-1+\sqrt{13}}{2}x_3 &= 0 \end{aligned}\right\} \quad u_3 = x\begin{pmatrix} 1 \\ (-1+\sqrt{13})/4 \\ 1/2 \end{pmatrix}$$

b) $\begin{bmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ -3 & -3 & -5 \end{bmatrix}$ $\begin{vmatrix} 1-\lambda & 3 & 3 \\ 3 & 1-\lambda & 3 \\ -3 & -3 & -5-\lambda \end{vmatrix} = 0 \Rightarrow \lambda_1 = 1,\ \lambda_2 = -2$

when $\lambda = 1$

$$\begin{pmatrix} 0 & 3 & 3 \\ 3 & 0 & 3 \\ -3 & -3 & -6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \left.\begin{aligned} 3x_2 + 3x_3 &= 0 \\ 3x_1 + 3x_3 &= 0 \\ -3x_1 - 3x_2 - 6x_3 &= 0 \end{aligned}\right\} \quad x_1 = x_2 = x_3 \quad u_1 = x\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

when $\lambda = -2$

$$\begin{pmatrix} -1 & 3 & 3 \\ 3 & -1 & 3 \\ -3 & -3 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \left.\begin{aligned} -x_1 + 3x_2 + 3x_3 &= 0 \\ 3x_1 - x_2 + 3x_3 &= 0 \\ -3x_1 - 3x_2 - 3x_3 &= 0 \end{aligned}\right\} \quad \begin{aligned} x_2 &= \frac{3}{-2}x_3 \\ x_1 &= -1.5x_3 \end{aligned} \quad u_1 = \lambda\begin{pmatrix} -3/2 \\ -3/2 \\ 1 \end{pmatrix}$$

4.1 Part a 5 / 5

✓ **- 0 pts** **Correct**

- 1 pts Wrong eigenvalues

- 1 pts Wrong eigenvector

- 2 pts Incorrect methods to solve eigenvector

4\. a) $\begin{bmatrix} 1 & -2 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ $\qquad \begin{vmatrix} 1-\lambda & -2 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{vmatrix} = 0$

$$\lambda_1 = 1 \quad , \quad \lambda_{2,3} = \frac{3 \pm \sqrt{13}}{2}$$

when $\lambda = 1$

$$\begin{pmatrix} 0 & -2 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \left.\begin{array}{r} -2x_2 = 0 \\ -x_1 + \cancel{x_2} - x_3 = 0 \\ -x_2 = 0 \end{array}\right\} \quad x_1 = -x_3 \quad u_1 = x\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

when $\lambda = \frac{3+\sqrt{13}}{2}$

$$\begin{pmatrix} \frac{-1-\sqrt{13}}{2} & -2 & 0 \\ -1 & \frac{1-\sqrt{13}}{2} & -1 \\ 0 & -1 & \frac{-1-\sqrt{13}}{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \left.\begin{array}{r} \frac{-1-\sqrt{13}}{2}x_1 - 2x_2 = 0 \\ -x_1 + \frac{1-\sqrt{13}}{2}x_2 - x_3 = 0 \\ -x_2 + \frac{-1-\sqrt{13}}{2}x_3 = 0 \end{array}\right\} \quad u_2 = x\begin{pmatrix} 1 \\ (-1-\sqrt{13})/4 \\ 1/2 \end{pmatrix}$$

when $\lambda = \frac{3-\sqrt{13}}{2}$

$$\begin{pmatrix} \frac{-1+\sqrt{13}}{2} & -2 & 0 \\ -1 & \frac{1+\sqrt{13}}{2} & -1 \\ 0 & -1 & \frac{-1+\sqrt{13}}{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \left.\begin{array}{r} \frac{-1+\sqrt{13}}{2}x_1 - 2x_2 = 0 \\ -x_1 + \frac{1+\sqrt{13}}{2}x_2 - x_3 = 0 \\ -x_2 + \frac{-1+\sqrt{13}}{2}x_3 = 0 \end{array}\right\} \quad u_3 = x\begin{pmatrix} 1 \\ (-1+\sqrt{13})/4 \\ 1/2 \end{pmatrix}$$

b) $\begin{bmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ -3 & -3 & -5 \end{bmatrix}$ $\qquad \begin{vmatrix} 1-\lambda & 3 & 3 \\ 3 & 1-\lambda & 3 \\ -3 & -3 & -5-\lambda \end{vmatrix} = 0 \Rightarrow \lambda_1 = 1 , \lambda_2 = -2$

when $\lambda = 1$

$$\begin{pmatrix} 0 & 3 & 3 \\ 3 & 0 & 3 \\ -3 & -3 & -6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \left.\begin{array}{r} 3x_2 + 3x_3 = 0 \\ 3x_1 + 3x_3 = 0 \\ -3x_1 - 3x_2 - 6x_3 = 0 \end{array}\right\} \quad x_1 = x_2 = x_3 \quad u_1 = x\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

when $\lambda = -2$

$$\begin{pmatrix} -1 & 3 & 3 \\ 3 & -1 & 3 \\ -3 & -3 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \left.\begin{array}{r} -x_1 + 3x_2 + 3x_3 = 0 \\ 3x_1 - x_2 + 3x_3 = 0 \\ -3x_1 - 3x_2 - 3x_3 = 0 \end{array}\right\} \quad \begin{array}{l} x_2 = \frac{3}{-2}x_3 \\ x_1 = -1.5x_3 \end{array} \quad u_1 = \lambda\begin{pmatrix} -3/2 \\ -3/2 \\ 1 \end{pmatrix}$$

4.2 Part b **4 / 5**

- **- 0 pts** Correct
- ✓ **- 1 pts Incorrect eigenvectors**

(c) $\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & 0 \end{bmatrix}$

$$\begin{vmatrix} -\lambda & 1 \\ -\omega_n^2 & -\lambda \end{vmatrix} = 0 \qquad \lambda^2 + \omega_n^2 = 0$$

$$\lambda_1 = \omega_n i \qquad \lambda_2 = -\omega_n i$$

when $\lambda = \omega_n i$

$$\begin{pmatrix} -\omega_n i & 1 \\ -\omega_n^2 & -\omega_n i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{aligned} -\omega_n i x_1 + x_2 &= 0 \\ -\omega_n^2 x_1 - \omega_n i x_2 &= 0 \end{aligned} \qquad u_1 = x\begin{pmatrix} 1 \\ \omega_n i \end{pmatrix}$$

when $\lambda = -\omega_n i$

$$\begin{pmatrix} \omega_n i & 1 \\ -\omega_n^2 & \omega_n i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{aligned} \omega_n i x_1 + x_2 &= 0 \\ -\omega_n^2 x_1 + \omega_n i x_2 &= 0 \end{aligned} \qquad u_2 = x\begin{pmatrix} 1 \\ -\omega_n i \end{pmatrix}$$

4.3 Part c **5 / 5**

✓ **- 0 pts** **Correct**

- 1 pts Wrong eigenvalues

- 1 pts Wrong eigenvectors

1 Arithmetic error when isolating

2 Arithmetic error when isolating

5、 (a) $\begin{bmatrix} -1 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 2 \\ 2 & 4 \end{bmatrix}$

a) $A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} -1 & 2 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}$ $\quad A^TA = \begin{bmatrix} 5 & -2 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = X$

$|X - \lambda I| = 0 \Rightarrow \lambda_1 = 0, \ \lambda_2 = 1, \ \lambda_3 = 6$

singular values: $0, \ 1, \ \sqrt{6}$

b) $B = \begin{bmatrix} -1 & 2 \\ 2 & 4 \end{bmatrix}$ $\quad B^T = \begin{bmatrix} -1 & 2 \\ 2 & 4 \end{bmatrix}$ $\quad B^TB = \begin{bmatrix} 5 & 6 \\ 6 & 20 \end{bmatrix} = X$

$|X - \lambda I| = 0 \Rightarrow \lambda_{1,2} = \dfrac{25 \pm 3\sqrt{41}}{2}$

singular values: $\sqrt{\dfrac{25 \pm 3\sqrt{41}}{2}}$

5.1 Part a **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** No intermediate steps
- **- 1 pts** Incorrect singular value

5. (a) $\begin{bmatrix} -1 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 2 \\ 2 & 4 \end{bmatrix}$

a) $A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} -1 & 2 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}$ $\quad A^TA = \begin{bmatrix} 5 & -2 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = X$

$|X - \lambda I| = 0 \Rightarrow \lambda_1 = 0,\ \lambda_2 = 1,\ \lambda_3 = 6$

singular values: $0,\ 1,\ \sqrt{6}$

b) $B = \begin{bmatrix} -1 & 2 \\ 2 & 4 \end{bmatrix}$ $\quad B^T = \begin{bmatrix} -1 & 2 \\ 2 & 4 \end{bmatrix}$ $\quad B^TB = \begin{bmatrix} 5 & 6 \\ 6 & 20 \end{bmatrix} = X$

$|X - \lambda I| = 0 \Rightarrow \lambda_{1,2} = \frac{25 \pm 3\sqrt{41}}{2}$

singular values: $\sqrt{\frac{25 \pm 3\sqrt{41}}{2}}$

5.2 Part b 5 / 5

- ✓ **- 0 pts** **Correct**
- **- 1 pts** Incorrect singular value
- **- 1 pts** No intermediate steps

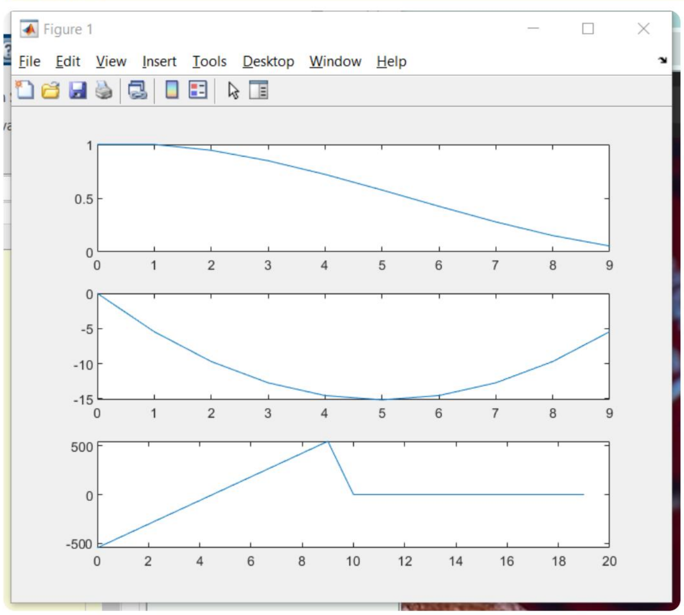
6.

$$x_{k+1} = \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0.01 \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k$$

a)

```
%% HW3 Problem 6
%Initialization
x0 = [1;0];
x10 = [0;0];
A = [1 0.01;0 1];
B = [0;0.01];
C = [1 0];
tmax = 10;
t = 0:1:tmax-1;
x = zeros(2,tmax);
xcl = zeros(2,tmax);
% a)
M = [A^9*B A^8*B A^7*B A^6*B A^5*B A^4*B A^3*B A^2*B A^1*B A^0*B];
a = x10-A^10*x0;
u = M'*inv(M*M')*a;
x(:,1) = x0;
for i = 2: tmax
    x(:,i) = A * x(:,i-1) + B*u(i-1);
end
subplot(3,1,1);
plot(t,x(1,:));
subplot(3,1,2);
plot(t,x(2,:));
u = [u' zeros(1,tmax)];
t = 0:1:19;
subplot(3,1,3);
plot(t,u);
```



6.1 Optimal feedforward control 15 / 15

✓ **- 0 pts** **Correct**

- 2 pts Wrong response

- 2 pts Wrong control input

- 5 pts Wrong methodology

b) $A + Bk = \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.01 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0.01k_1 & 0.01k_2 \end{bmatrix} = \begin{bmatrix} 1 & 0.01 \\ 0.01k_1 & 1+0.01k_2 \end{bmatrix}$$

$$\det \begin{vmatrix} 1-\lambda & 0.01 \\ 0.01k_1 & 1+0.01k_2-\lambda \end{vmatrix} = 0$$

$$0.0001k_1 - (1-\lambda)(1+0.01k_2-\lambda) = 0$$

$$0.0001k_1 - (1 + 0.01k_2 - \lambda - \lambda - 0.01\lambda k_2 + \lambda^2) = 0$$

$$0.0001k_1 - 1 - 0.01k_2 + 2\lambda + 0.01\lambda k_2 - \lambda^2 = 0$$

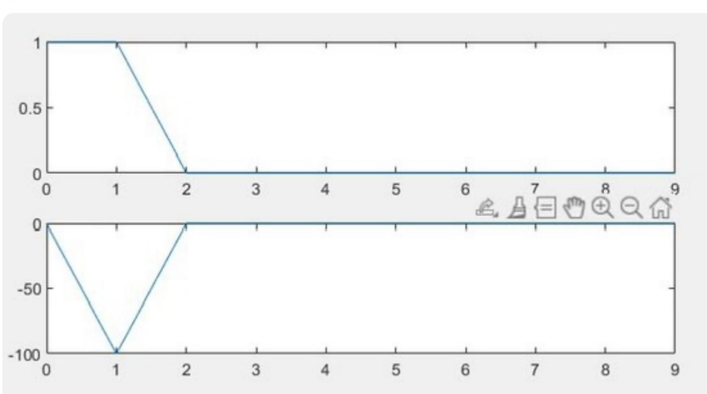
$$\lambda^2 - 2\lambda - 0.01\lambda k_2 = 0.0001k_1 - 1 - 0.01k_2$$

$$\lambda(\lambda - 2 - 0.01k_2) = 0.0001k_1 - 1 - 0.01k_2$$

to let $\lambda_1 = 0$, $\lambda_2 = 0$ $\quad -2 - 0.01k_2 = 0 \quad k_2 = -200$

$\Rightarrow 0.0001k_1 - 1 - 0.01k_2 = 0 \Rightarrow k_1 = -10000$

```
% b)
k = -[10000 200]; %from calculation
xcl(:,1) = x0;
for i = 2: tmax
    ucl = k * xcl(:,i-1);
    xcl(:,i) = A * xcl(:,i-1) + B*ucl;
end
subplot(3,1,1);
plot(t,xcl(1,:));
subplot(3,1,2);
plot(t,xcl(2,:));
```



6.2 Deadbeat control 14 / 15

- **- 0 pts** Correct
- **- 1 pts** Incorrect velocity response graph
- ✓ **- 1 pts Incorrect control response graph**
- **- 1 pts** Incorrect position response graph
- **- 2 pts** Incorrect "K"s
- **- 7.5 pts** No pole placement methodology found

None found