24677-A Homework 1

Saeed Bai

TOTAL POINTS

113 / 120

QUESTION 1

Pendulum Control 30 pts

1.1 Linearization 9.5 / 10

- **- 0 pts** Correct
- **- 1 pts** Linearized about $$\theta=0$$ instead of $$\pi/2$$
- **- 1 pts** Wrong "A" Matrix
- **- 1 pts** Wrong "B" Matrix
- ✓ **- 0.5 pts No input or incorrect system stability**

1.2 Controller design 10 / 10

- ✓ **- 0 pts Correct**
- **- 1.5 pts** No overshoot consideration
- **- 1.5 pts** No settling time consideration
- **- 1 pts** Wrong transfer function considered
- **- 0.5 pts** Wrong formula for solving Kp
- **- 0.5 pts** Wrong formula for solving Kd
- **- 10 pts** No attempt

1.3 Simulation 9 / 10

- **- 0 pts** Correct
- **- 0.5 pts** Wrong initial conditions setup in Simulink
- **- 1 pts** Wrong dynamics setup in Simulink
- ✓ **- 0.5 pts Incorrect system stable range**
- ✓ **- 0.5 pts Wrong feedback loop setup in Simulink**
- **- 0.5 pts** Wrong reference signal in Simulink
- **- 0.5 pts** No controller found
- **- 10 pts** No attempt

1 Use the gain block here instead like you did earlier in the path to multiply signal by a constant

2 This shouldn't loop back here, what you did here is telling the system to compare the reference using the "torque" you just looped back, which is not really an apple-to-apple comparison.

QUESTION 2

System properties 20 pts

2.1 Part a 4 / 4

- ✓ **- 0 pts Correct**
- **- 1 pts** Incorrect answer for linear or non-linear
- **- 1 pts** Incorrect answer for time-variant or time-invariant

2.2 Part b 4 / 4

- ✓ **- 0 pts Correct**
- **- 1 pts** Incorrect answer for linear or non-linear
- **- 1 pts** Incorrect answer for time-variant or time-invariant

2.3 Part c 4 / 4

- ✓ **- 0 pts Correct**
- **- 1 pts** Incorrect answer for linear or non-linear
- **- 1 pts** Incorrect answer for time-variant or time-invariant

2.4 Part d 4 / 4

- ✓ **- 0 pts Correct**
- **- 1 pts** Incorrect answer for linear or non-linear
- **- 1 pts** Incorrect answer for time-variant or time-invariant

2.5 Part e 2 / 4

- **- 0 pts** Correct
- ✓ **- 1 pts Incorrect answer for linear or non-linear**
- ✓ **- 1 pts Incorrect answer for time-variant or time-invariant**

QUESTION 3

Suspension dynamics 20 pts

3.1 System model 8.5 / 10

- **0 pts** Correct
- **0.5 pts** Incorrect "C" matrix
- ✓ **1.5 pts** **No state space representation**
- **0.5 pts** Incorrect "B" matrix
- **0.5 pts** Incorrect equation of motion

3.2 Matlab simulation 10 / 10

- ✓ **0 pts** **Correct**
- **1 pts** Wrong input for step profile
- **0.5 pts** Wrong output signal for step profile
- **1 pts** Wrong input for half sine wave profile
- **0.5 pts** Wrong output signal for half sine wave profile
- **10 pts** No attempt

QUESTION 4

Slider system 20 pts

4.1 Standard coordinates 10 / 10

- ✓ **0 pts** **Correct**
- **1 pts** Missing spring reaction force $$kx_2$$ in $$m_1$$
- **1 pts** Missing damper reaction force $$b_2\dot{x_2}$$ in $$m_1$$
- **1 pts** Incorrect states used
- **1 pts** Incorrect equation(s) derived from FBD

4.2 Modified coordinates 9 / 10

- **0 pts** Correct
- **2 pts** Incorrect transform methodology
- ✓ **1 pts** **Wrong output matrices**

QUESTION 5

5 Coordinate transformation 10 / 10

- ✓ **0 pts** **Correct**
- **1 pts** Wrong transformation matrix "M" or "inv(M)"
- **2 pts** Wrong transformation formulas
- **1 pts** Wrong "A_hat"
- **1 pts** Wrong "B_hat"
- **1 pts** Wrong "C_hat"

QUESTION 6

Robot arm 20 pts

6.1 Nonlinear state space model 10 / 10

- ✓ **0 pts** **Correct**
- **1 pts** Arithmetic or setup error
- **2 pts** Wrong derivation method

6.2 Linearization 9 / 10

- **0 pts** Correct
- ✓ **1 pts** **Missing term(s) in "A" matrix**
- **1 pts** Missing term(s) in "B" matrix
- **0.5 pts** Incorrect operating point application(s)

1.

$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{\tau}{ml^2}$$

(a) let $x_1 = \theta$ $\quad \dot{x}_1 = x_2$

$x_2 = \dot{\theta}$ $\quad \dot{x}_2 = \ddot{\theta} = \frac{\tau}{ml^2} - \frac{g}{l}\sin\theta$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{u}{ml^2} - \frac{g}{l}\sin\theta \end{bmatrix}$$

Linearize

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix}$$

when $m=1$, $l=6$ $\qquad B = \begin{bmatrix} 0 \\ \frac{1}{16} \end{bmatrix}$

b) $\%OS = 25 = e^{-\frac{3\pi}{\sqrt{1-\zeta^2}}} \cdot 100 \Rightarrow \zeta = 0.404$

2% settling time: $\frac{4}{\zeta\omega_n} = 2$ $\quad \zeta\omega_n = 2 \Rightarrow \omega_n = 4.95$

CLTF: $T(s) = \frac{PC}{1+PC}$ where $C = K_d s + K_p$, $P = \frac{1}{16s^2}$

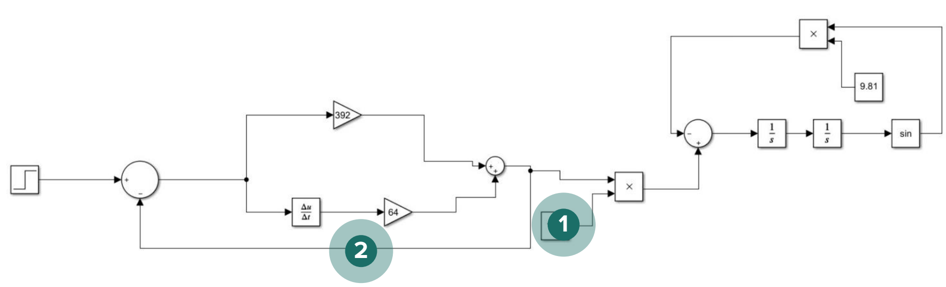
$$T(s) = \frac{\frac{1}{16s^2}\cdot(K_d s + K_p)}{1 + \frac{1}{16s^2}(K_d s + K_p)} = \frac{\frac{K_d}{16s} + \frac{K_p}{16s^2}}{\frac{16s^2 + K_d s + K_p}{16s^2}} = \frac{K_d s + K_p}{16s^2 + K_d s + K_p}$$

$16s^2 + K_d s + K_p \Rightarrow s^2 + \frac{K_d}{16}s + \frac{K_p}{16}$

$\frac{K_d}{16} = 2\zeta\omega_n \Rightarrow K_d = 64$ $\qquad \frac{K_p}{16} = \omega_n^2 \Rightarrow K_p = 392$

PD controller $\quad C = 64s + 392$

c)



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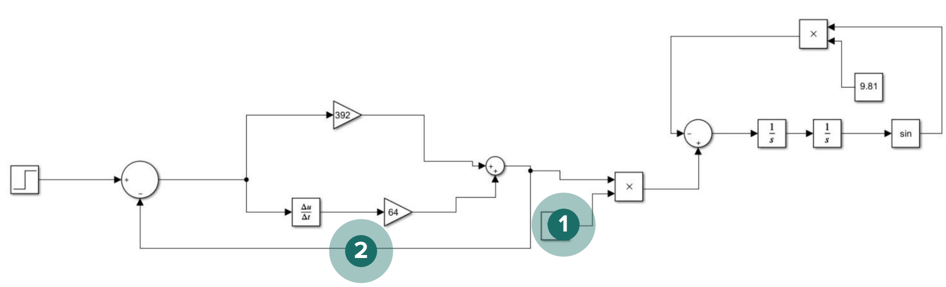

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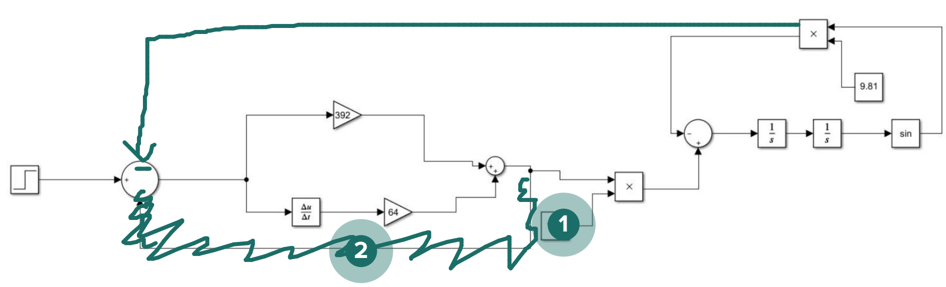

1.3 Simulation 9 / 10

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2 This shouldn't loop back here, what you did here is telling the system to compare the reference using the "torque" you just looped back, which is not really an apple-to-apple comparison.

2.

(a) $y(t) = a, \quad a \neq 0 \ \forall t$

$y_1(t) = a_1 \qquad y_2(t) = a_2 \qquad y_1(t) + y_2(t) = a_1 + a_2$

$y(t_1+t_2) = a$ so system is nonlinear

$y_1(t) + y_2(t) \neq y(t_1+t_2)$

$y(t-\tau) = a$ so time invariant

b) $y(t) = -3u(t) + 2$

$y_1(t) + y_2(t) = -3(u_1(t) + u_2(t)) + 4$

$y(t_1+t_2) = -3(u(t_1+t_2)) + 4$

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c) $y(t) = u^3(t)$

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d) $y(t) = u(t^3) \qquad \alpha H(u_1) + \beta H(u_2) = H(\alpha u_1 + \beta u_2)$

$y_1(t) + y_2(t) = u_1(t^3) + u_2(t^3) \qquad y(t_1+t_2) = u((t_1+t_2)^3)$

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e) $y(t) = e^{-t}u(t-T)$

$y_1(t) + y_2(t) = e^{-t}u_1(t-T) + e^{-t}u_2(t-T)$

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$y(t_1+t_2) \neq y_1(t) + y_2(t)$ non-linear

$y(t-\tau) = e^{-(t-\tau)}u((t-\tau)-T)$ time-invariant

2.1 Part a 4 / 4

✓ **- 0 pts** **Correct**

- 1 pts Incorrect answer for linear or non-linear

- 1 pts Incorrect answer for time-variant or time-invariant

2.

(a) $y(t) = a, \quad a \neq 0 \quad \forall t$

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$y_1(t) + y_2(t) = u_1(t^3) + u_2(t^3) \qquad y(t_1+t_2) = u((t_1+t_2)^3)$

$y(t-\tau) = u((t-\tau)^3)$ time varying linear

e) $y(t) = e^{-t}u(t-T)$

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$y(t_1+t_2) \neq y_1(t) + y_2(t)$ non-linear

$y(t-\tau) = e^{-(t-\tau)}u((t-\tau)-T)$ time-invariant

2.2 Part b 4 / 4

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- 1 pts Incorrect answer for linear or non-linear

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(a) $y(t) = a, \quad a \neq 0 \ \forall t$

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e) $y(t) = e^{-t}u(t-T)$

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$y(t-T) = -3u(t-\tau) + 2$ so time invariant

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$y(t_1 + t_2) = a$ so system is nonlinear

$y_1(t) + y_2(t) \neq y(t_1 + t_2)$

$y(t - \tau) = a$ so time invariant

b) $y(t) = -3u(t) + 2$

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2.5 Part e 2 / 4

- **- 0 pts** Correct
- ✓ **- 1 pts** **Incorrect answer for linear or non-linear**
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3.

a) $M\ddot{x} + B(\dot{x} - \dot{y}) + K(x - y) = 0$

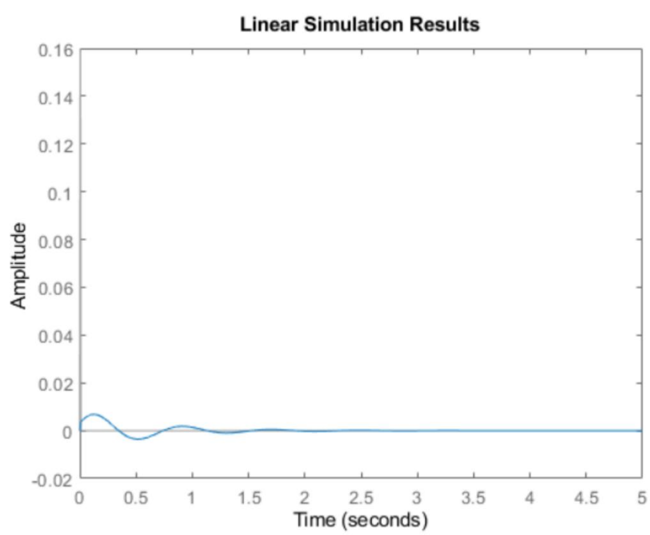
$$M\ddot{x} + B\dot{x} + Kx = B\dot{y} + Ky$$

$$G(s) = \frac{Bs + K}{Ms^2 + Bs + K}$$

b) 24677 LCS HW- 01

```
%Q3 Response of the system
M = 300; %kg
K = 20000; %N/m
B = 1000; %Ns/m
Stepsize = 0.15; %m
t = 0:0.0001:0.01;
T = 0:0.0001:5;
y = 0.08* sin(100*pi*t)+abs(0.08* sin(100*pi*t));
dl = length(T) - length(y);
tmp = zeros(1,dl);
y = [y tmp];
system = tf([B,K],[M,B,K]);
output_step = step(Stepsize * system);
plot(output_step);
lsim(system,y,T);
%Q6 Linearizing
% syms x1 x2 x3 x4 u1 u2 m1 m2 i1 i2 l1 g d
%
% f_x_u = [x2;
%          (u2+m2*x1*x4^2-m2*g*sin(x3))/m2;
%           x4;
%          (u1-2*m2*x1*x4*x2-(m1*l1+m2*x1)*g*cos(x3))/(m1*l1^2+i1+i2+m2*x1^2)];
% output1 = jacobian(f_x_u,[x1,x2,x3,x4]);
% output2 = jacobian(f_x_u,[u1,u2]);
% new_output1 = subs(output1,[x1,x2,x3,x4,u1,u2],[d,0,0,0,0,0])
% new_output2 = subs(output2,[x1,x2,x3,x4,u1,u2],[d,0,0,0,0,0])
```

Sine:



3.1 System model 8.5 / 10

- **- 0 pts** Correct
- **- 0.5 pts** Incorrect "C" matrix
- ✓ **- 1.5 pts** **No state space representation**
- **- 0.5 pts** Incorrect "B" matrix
- **- 0.5 pts** Incorrect equation of motion

3.

a) $M\ddot{x} + B(\dot{x} - \dot{y}) + K(x - y) = 0$

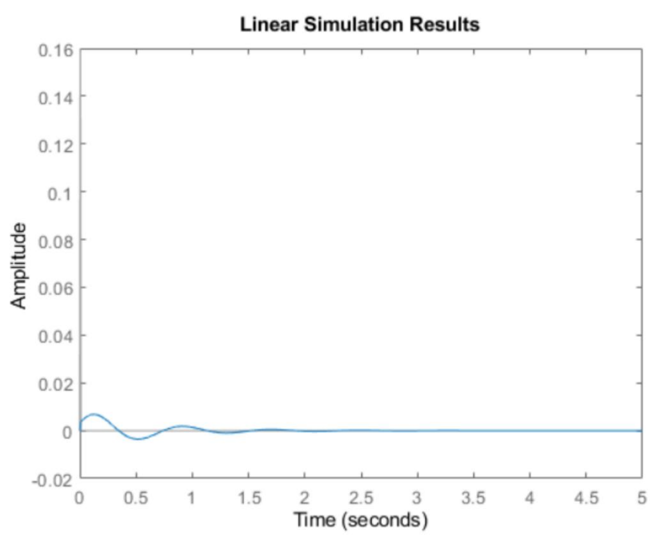
$M\ddot{x} + B\dot{x} + kx = B\dot{y} + ky$

$$G(s) = \frac{Bs + k}{Ms^2 + Bs + K}$$

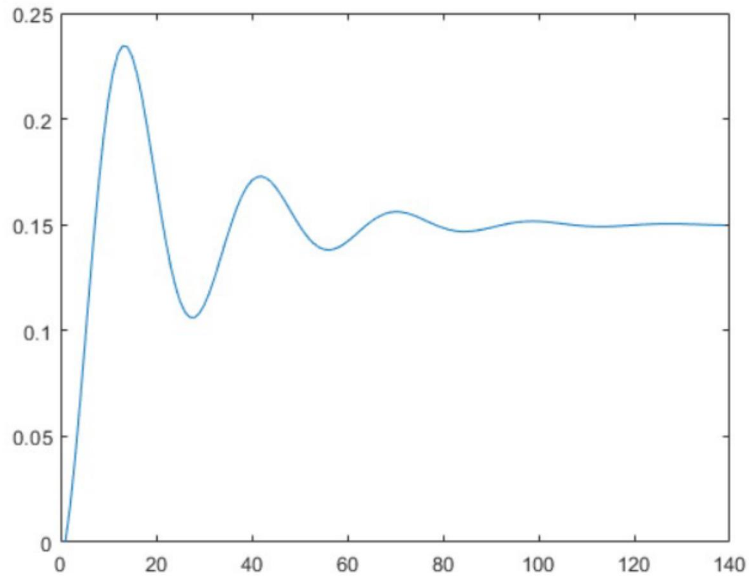
b) 24677 LCS HW- 01

```
%Q3 Response of the system
M = 300; %kg
K = 20000; %N/m
B = 1000; %Ns/m
Stepsize = 0.15; %m
t = 0:0.0001:0.01;
T = 0:0.0001:5;
y = 0.08* sin(100*pi*t)+abs(0.08* sin(100*pi*t));
dl = length(T) - length(y);
tmp = zeros(1,dl);
y = [y tmp];
system = tf([B,K],[M,B,K]);
output_step = step(Stepsize * system);
plot(output_step);
lsim(system,y,T);
%Q6 Linearizing
% syms x1 x2 x3 x4 u1 u2 m1 m2 i1 i2 l1 g d
%
% f_x_u = [x2;
%         (u2+m2*x1*x4^2-m2*g*sin(x3))/m2;
%          x4;
%         (u1-2*m2*x1*x4*x2-(m1*l1+m2*x1)*g*cos(x3))/(m1*l1^2+i1+i2+m2*x1^2)];
% output1 = jacobian(f_x_u,[x1,x2,x3,x4]);
% output2 = jacobian(f_x_u,[u1,u2]);
% new_output1 = subs(output1,[x1,x2,x3,x4,u1,u2],[d,0,0,0,0,0])
% new_output2 = subs(output2,[x1,x2,x3,x4,u1,u2],[d,0,0,0,0,0])
```

Sine:



Step :



3.2 Matlab simulation 10 / 10

✓ **- 0 pts** **Correct**

- 1 pts Wrong input for step profile

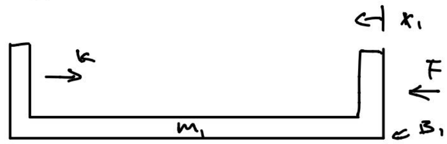
- 0.5 pts Wrong output signal for step profile

- 1 pts Wrong input for half sine wave profile

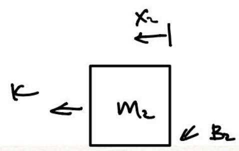
- 0.5 pts Wrong output signal for half sine wave profile

- 10 pts No attempt

4.



$$m_1\ddot{x}_1 = -B_1\dot{x}_1 - K(x_1-x_2) - B_2(\dot{x}_1-\dot{x}_2) + F$$



$$m_2\ddot{x}_2 = B_2(\dot{x}_1-\dot{x}_2) + K(x_1-x_2)$$

(a) State variables $\mathbf{x} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}$ and output variable $y = x_2$.

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{m_1} & \frac{-B_1-B_2}{m_2} & \frac{k}{m_1} & \frac{B_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & \frac{B_2}{m_2} & \frac{-k}{m_2} & \frac{-B_2}{m_2} \end{bmatrix}}^{A} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \overbrace{\begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix}}^{B} u$$

$$y = [\underbrace{0\ \ 0\ \ 1\ \ 0}_{C}]\, x$$

(b) State variables $\mathbf{x} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 - x_1 \\ \dot{x}_2 - \dot{x}_1 \end{bmatrix}$ and output variable $y = x_2 - x_1$.

$S_1 = x_1 \quad S_2 = \dot{x}_1 \quad S_3 = x_1$

$$\hat{x} = M^{-1}x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$M^{-1}AM = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{m_2B_2+m_1(-B_1-B_2)}{m_1m_2} & \frac{k}{m_1} & \frac{B_2}{m_1} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m_1B_1+m_1B_2-B_2m_1}{m_1m_2} & \frac{-km_1-km_2}{m_1m_2} & \frac{-m_2B_2-m_1B_2}{m_1m_2} \end{bmatrix}$$

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{m_2B_2+m_1(-B_1-B_2)}{m_1m_2} & \frac{k}{m_1} & \frac{B_2}{m_1} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m_1B_1+m_1B_2-B_2m_1}{m_1m_2} & \frac{-km_1-km_2}{m_1m_2} & \frac{-m_2B_2-m_1B_2}{m_1m_2} \end{bmatrix}\hat{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

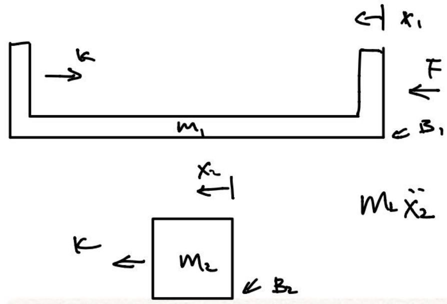
$$M^{-1}B = \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ \frac{-1}{m_1} \end{bmatrix}$$

$$y = [0\ \ 0\ \ 1\ \ 0]\,\hat{x}$$

4.1 Standard coordinates 10 / 10

- ✓ **- 0 pts** Correct
- **- 1 pts** Missing spring reaction force $$kx_2$$ in $$m_1$$
- **- 1 pts** Missing damper reaction force $$b_2\dot{x_2}$$ in $$m_1$$
- **- 1 pts** Incorrect states used
- **- 1 pts** Incorrect equation(s) derived from FBD

4.



$$m_1\ddot{x}_1 = -B_1\dot{x}_1 - K(x_1-x_2) - B_2(\dot{x}_1-\dot{x}_2) + F$$

$$m_2\ddot{x}_2 = B_2(\dot{x}_1-\dot{x}_2) + K(x_1-x_2)$$

(a) State variables $\mathbf{x} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}$ and output variable $y = x_2$.

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{m_1} & \frac{-B_1-B_2}{m_2} & \frac{k}{m_1} & \frac{B_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & \frac{B_2}{m_2} & \frac{-k}{m_2} & \frac{-B_2}{m_2} \end{bmatrix}}^{A} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \overbrace{\begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix}}^{B} u$$

$$y = \underbrace{[0 \;\; 0 \;\; 1 \;\; 0]}_{C}\, x$$

(b) State variables $\mathbf{x} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 - x_1 \\ \dot{x}_2 - \dot{x}_1 \end{bmatrix}$ and output variable $y = x_2 - x_1$.

$S_1 = X_1 \quad S_2 = \dot{X}_1 \quad S_3 = X_1$

$$\hat{x} = M^{-1}x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$M^{-1}AM = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{m_2B_2 + m_1(-B_1-B_2)}{m_1m_2} & \frac{k}{m_1} & \frac{B_2}{m_1} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m_1B_1 + m_1B_2 - B_2m_2}{m_1m_2} & \frac{-km_1-km_2}{m_1m_2} & \frac{-m_2B_2-m_1B_2}{m_1m_2} \end{bmatrix}$$

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{m_2B_2 + m_1(-B_1-B_2)}{m_1m_2} & \frac{k}{m_1} & \frac{B_2}{m_1} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m_1B_1 + m_1B_2 - B_2m_2}{m_1m_2} & \frac{-km_1-km_2}{m_1m_2} & \frac{-m_2B_2-m_1B_2}{m_1m_2} \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$M^{-1}B = \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ \frac{-1}{m_1} \end{bmatrix}$$

$$y = [0 \;\; 0 \;\; 1 \;\; 0]\, \hat{x}$$

4.2 Modified coordinates 9 / 10

- **- 0 pts** Correct
- **- 2 pts** Incorrect transform methodology
- ✓ **- 1 pts Wrong output matrices**

5. $$\hat{x} = \begin{bmatrix} -4x_1 - 2x_2 - 3x_3 \\ 15x_1 + 7x_2 + 10x_3 \\ -5x_1 - 2x_2 - 3x_3 \end{bmatrix} = \begin{bmatrix} -4 & -2 & -3 \\ 15 & 7 & 10 \\ -5 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$M\dot{\hat{x}} = AM\hat{x} + Bu$$

$$y = CM\hat{x} + \cancel{Du}$$

$$\Rightarrow \quad \dot{\hat{x}} = M^{-1}AM\hat{x} + M^{-1}Bu$$

$$M^{-1}AM = \begin{bmatrix} -4 & -2 & -3 \\ 15 & 7 & 10 \\ -5 & -2 & -3 \end{bmatrix} \begin{bmatrix} 18 & 9 & 13 \\ 50 & 23 & 35 \\ -65 & -31 & -46 \end{bmatrix} \begin{bmatrix} -4 & -2 & -3 \\ 15 & 7 & 10 \\ -5 & -2 & -3 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$CM = \begin{bmatrix} 5 & -5 & 5 \end{bmatrix} \begin{bmatrix} -4 & -2 & -3 \\ 15 & 7 & 10 \\ -5 & -2 & -3 \end{bmatrix}^{-1} = \begin{bmatrix} -45 & -25 & -40 \end{bmatrix}$$

$$M^{-1}B = \begin{bmatrix} -4 & -2 & -3 \\ 15 & 7 & 10 \\ -5 & -2 & -3 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix}$$

$$\dot{\hat{x}} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \hat{x} + \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix} u \qquad y = \begin{bmatrix} -45 & -25 & -40 \end{bmatrix} \hat{x}$$

5 Coordinate transformation 10 / 10

✓ **- 0 pts** **Correct**

- 1 pts Wrong transformation matrix "M" or "inv(M)"

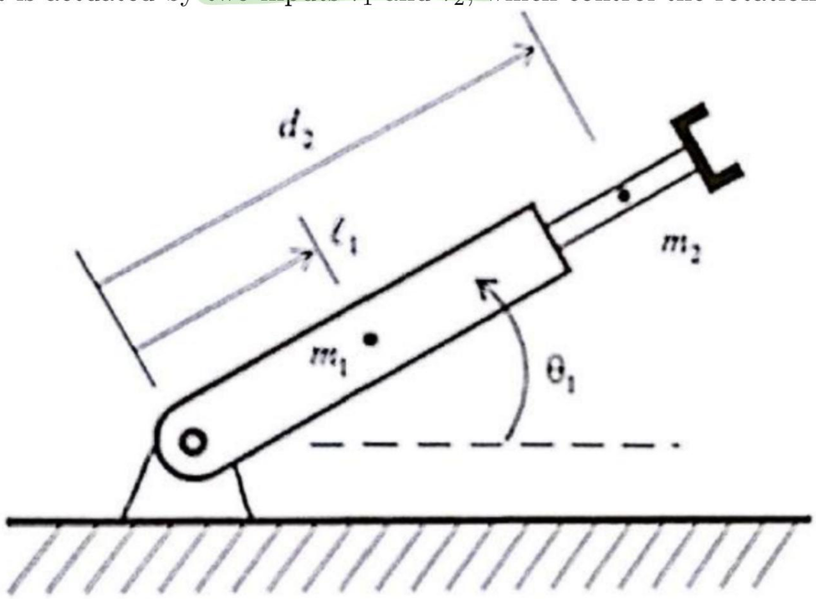
- 2 pts Wrong transformation formulas

- 1 pts Wrong "A_hat"

- 1 pts Wrong "B_hat"

- 1 pts Wrong "C_hat"

robot is actuated by two inputs τ_1 and τ_2, which control the rotational and prismatic joints.



$$\left(m_1 l_1^2 + I_1 + I_2 + m_2 d_2^2\right) \ddot{\theta}_1 + 2 m_2 d_2 \dot{\theta}_1 \dot{d}_2 + \left(m_1 l_1 + m_2 d_2\right) g \cos\theta_1 = \tau_1$$
$$m_2 \ddot{d}_2 - m_2 d_2 \dot{\theta}_1^2 + m_2 g \sin\theta_1 = \tau_2$$

(a) Write the differential equations in state space form.

(b) Linearize the system the operating point $\theta_1 = \dot{\theta}_1 = \dot{d}_2 = \ddot{d}_2 = 0$ and $d_2 = \bar{d}$, a constant.

part a $\dot{x} = f(x, u)$

part b $\dot{x} = Ax + Bu$

a) $x_1 = d_2$ $x_2 = \dot{d}_2$ $x_3 = \theta_1$ $x_4 = \dot{\theta}_1$

$$\dot{\begin{bmatrix} d_2 \\ \dot{d}_2 \\ \theta_1 \\ \dot{\theta}_1 \end{bmatrix}} = \begin{bmatrix} x_2 \\ \dfrac{u_2 + m_2 x_1 x_4^2 - m_2 g \sin x_3}{m_2} \\ x_4 \\ \dfrac{u_1 - 2 m_2 x_1 x_4 x_2 - (m_1 l_1 + m_2 x_1) g \cos x_3}{m_1 l_1^2 + I_1 + I_2 + m_2 x_1^2} \end{bmatrix}$$

6.1 Nonlinear state space model 10 / 10

- ✓ **- 0 pts** **Correct**
- **- 1 pts** Arithmetic or setup error
- **- 2 pts** Wrong derivation method

b)

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```
%Q6 Linearizing
syms x1 x2 x3 x4 u1 u2 m1 m2 i1 i2 l1 g d

f_x_u = [x2;(u2+m2*x1*x4^2-m2*g*sin(x3))/m2;x4;(u1-2*m2*x1*x4*x2+(m1*l1+m2*x2)*g*cos(x3))/(m1*l1^2+i1+i2+m2*x1^2)];
output1 = jacobian(f_x_u,[x1,x2,x3,x4]);
output2 = jacobian(f_x_u,[u1,u2]);
new_output1 = subs(output1,[x1,x2,x3,x4,u1,u2],[d,0,0,0,0,0])
new_output2 = subs(output2,[x1,x2,x3,x4,u1,u2],[d,0,0,0,0,0])
```

```
new_output1 =

[                                                0,                                     1,  0, 0]
[                                                0,                                     0, -g, 0]
[                                                0,                                     0,  0, 1]
[-(2*d*g*l1*m1*m2)/(m2*d^2 + m1*l1^2 + i1 + i2)^2, (g*m2)/(m2*d^2 + m1*l1^2 + i1 + i2),  0, 0]


new_output2 =

[                             0,    0]
[                             0, 1/m2]
[                             0,    0]
[1/(m2*d^2 + m1*l1^2 + i1 + i2),    0]
```

6.2 Linearization 9 / 10

- **- 0 pts** Correct
- ✓ **- 1 pts Missing term(s) in "A" matrix**
- **- 1 pts** Missing term(s) in "B" matrix
- **- 0.5 pts** Incorrect operating point application(s)