Like all other assignments in the course, this assignment will be submitted via Gradescope. Please come to office hours or reach out on Piazza if you have any questions! Please use livecripts to present your Matlab work, along with screenshots of any Simulink models.

1: 20 points

Consider two bases for space $\mathbb{R}^3$.

$$\{a_1, a_2, a_3\} = \left\{ \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} \right\}, \quad \{b_1, b_2, b_3\} = \left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} \right\}$$

If a linear operator is given in the $\{a_1, a_2, a_3\}$ basis by

$$A = \begin{bmatrix} 8 & -2 & -1 \\ 4 & -2 & -3 \\ 2 & -3 & -3 \end{bmatrix}$$

(a) Find the representation for this operator in the basis $\{b_1, b_2, b_3\}$.

(b) If we are given a vector $x = \begin{bmatrix} 2 & -1 & -4 \end{bmatrix}^T$ in the basis $\{a_1, a_2, a_3\}$, determine the representation for x in the basis $\{b_1, b_2, b_3\}$.

2: 10 points

For the pair of matrices

$$A = \begin{bmatrix} 2 & 3 & 1 & 4 & -9 \\ 1 & 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & 2 & -5 \\ 2 & 2 & 2 & 3 & -8 \end{bmatrix}, \quad y = \begin{bmatrix} 17 \\ 6 \\ 8 \\ 14 \end{bmatrix}$$

Determine all the possible solutions to the system $Ax = y$.

3: 10 points

Find the best solution, in the least-squared error sense, to the equations

$$\begin{aligned} -2 &= x_1 - 2x_2 \\ 5 &= x_1 - 2x_2 \\ 1 &= -2x_1 + x_2 \\ -3 &= x_1 - 3x_2. \end{aligned}$$

4: 15 points

Find the eigenvalues and corresponding eigenvectors of the following matrices.

(a) $\begin{bmatrix} 1 & -2 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ -3 & -3 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & 0 \end{bmatrix}$

5: 10 points

Compute the singular values of the following matrices.

(a) $\begin{bmatrix} -1 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 2 \\ 2 & 4 \end{bmatrix}$

6: 30 points

We again revisit the problem of controlling the sliding mass in discrete time. With a sampling period of 0.01 s, the plant dynamics are given by

$$x_{k+1} = \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0.01 \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k$$

(a) Using Matlab, find the optimal feedforward control sequence (in the sense of $\|u\|_2$) that drives the state from $x_0 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ to $x_{10} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. Plot the response of the system and the optimal control input on separate graphs.

(b) Let's now take a feedback approach to the same problem, using a control of the form $u = \underbrace{\begin{bmatrix} k_1 & k_2 \end{bmatrix}}_{K} x$. As we will see later, the closed loop system is stable $\iff$ the eigenvalues of $A + BK$ satisfy $|\lambda_i| < 1 \; \forall i$. Find the values k_1 and k_2 that place both eigenvalues at $\lambda_1 = \lambda_2 = 0$ (this is known as <u>deadbeat</u> control). Plot the response of the closed loop system and the control usage in this case.