24677-A Project - P2

Saeed Bai

TOTAL POINTS

46 / 50

QUESTION 1

Exercise 1 25 pts

1.1 Controllability and Observability 9.5 / 10

- **0 pts** Correct
- **2 pts** Incorrect part A
- **2 pts** Incorrect part B
- **2 pts** Incorrect part C

- 0.5 Point adjustment

The components for part C are all correct, but you needed to give me a conclusion for the entire system, not just the subcomponents.

1.2 State Space Realization 5 / 5

✓ **- 0 pts Correct**

- **1.5 pts** Incorrect state matrices
- **1 pts** Incorrect output matrices

1.3 State Feedback 5 / 5

✓ **- 0 pts Correct**

- **2 pts** Incorrect gains

1.4 Observer Design 5 / 5

✓ **- 0 pts Correct**

- **2 pts** Incorrect gains

QUESTION 2

Exercise 2 25 pts

2.1 Controllability vs. Speed 5 / 5

✓ **- 0 pts Correct**

2.2 Controllability Plots 4 / 5

- **0 pts** Correct

✓ **- 1 pts Incorrect pole graphs**

1 Unexpected shape, it should be a logarithmic shape almost

2.3 Observability 2.5 / 5

- **0 pts** Correct

✓ **- 2.5 pts No conclusion on measurement with only heading error**

2.4 Pole Placement Design 5 / 5

✓ **- 0 pts Correct**

Incomplete reasoning

2.5 Observer Design 5 / 5

✓ **- 0 pts Correct**

1. $\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{B} u, \quad y = \underbrace{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}}_{C} x$

$$M = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 3 \end{bmatrix} \qquad \det(M) = -1 \neq 0$$

Controllable

$$N = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -2 \\ 2 & 5 & 4 \end{bmatrix} \qquad \det(N) = 1 \neq 0$$

Observable

2. $\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}}_{A} x + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x$

$$\lambda I - A = 0 \quad \Rightarrow \quad \lambda = 0, 1, -2$$

$\lambda = 1$

$$r(1I - A \;\vdots\; B) = \text{rank} \left[\begin{array}{ccc|cc} 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & -2 & 2 & 0 & 0 \end{array}\right] = 3 \neq 0$$

Controllable

$$r\begin{pmatrix} 1I - A \\ \hdashline C \end{pmatrix} = \text{rank} \left[\begin{array}{ccc} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -2 & 2 \\ \hdashline 1 & 0 & 1 \end{array}\right] = 3 \neq 0$$

Observable

$\lambda = 0$

$$r(-A \;\vdots\; B) = \text{rank} \left[\begin{array}{ccc|cc} 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 \end{array}\right] = 3 \neq 0$$

Controllable

$$r\begin{pmatrix} 1I - A \\ \hdashline C \end{pmatrix} = \text{rank} \left[\begin{array}{ccc} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & -2 & 1 \\ \hdashline 1 & 0 & 1 \end{array}\right] = 3 \neq 0$$

Observable

$\lambda = -2$

$$r(-2I - A \;\vdots\; B) = \text{rank} \left[\begin{array}{ccc|cc} -2 & -1 & 0 & 0 & 1 \\ 0 & -2 & -1 & 1 & 0 \end{array}\right] = 3$$

Controllable

$$\begin{bmatrix} 0 & -2 & -1 & \vdots & 0 & 0 \end{bmatrix}$$

$$r\begin{pmatrix} -2I-A \\ \hline C \end{pmatrix} = \text{rank}\begin{bmatrix} -2 & -1 & 0 \\ 0 & -2 & -1 \\ 0 & -2 & -1 \\ 1 & 0 & 1 \end{bmatrix} = 3 \neq 0$$

Observable

3.

$$\dot{x} = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 2 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 2 & 2 & 1 & 3 & -1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} x$$

$\lambda = 2$ $\lambda = 2$ $\lambda = 1$ $\lambda = 1$

$\lambda = 2$: $\hat{B}^2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \Rightarrow r(\hat{B}^2) = 3$ Controllable

$\hat{C}^2 = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow r(\hat{C}^2) = 2$ not observable

$\lambda = 1$: $\hat{B}^1 = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow r(\hat{B}^1) = 2$ controllable

$\hat{C}^1 = \begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \Rightarrow r(\hat{C}^1) = 1$ not observable

1.1 Controllability and Observability 9.5 / 10

- **- 0 pts** Correct
- **- 2 pts** Incorrect part A
- **- 2 pts** Incorrect part B
- **- 2 pts** Incorrect part C

- 0.5 Point adjustment

The components for part C are all correct, but you needed to give me a conclusion for the entire system, not just the subcomponents.

2.

$$G(s) = \begin{bmatrix} \frac{1}{s} & \frac{s+3}{s+1} \\ \frac{1}{s+3} & \frac{s}{s+1} \end{bmatrix}. \qquad G(\infty) = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$G_{sp} = \begin{bmatrix} \frac{1}{s} & \frac{s+3-(s+1)}{s+1} \\ \frac{1}{s+3} & \frac{s-(s+1)}{s+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{2}{s+1} \\ \frac{1}{s+3} & \frac{-1}{s+1} \end{bmatrix}$$

$$d(s) = s(s+3)(s+1) = (s^2+3s)(s+1) = \frac{1}{s^3+4s^2+3s}$$

$$G_{sp} = \frac{1}{s^3+4s^2+3s} \begin{bmatrix} s^2+4s+3 & 2s^2+6s \\ s^2+s & -s^2-3s \end{bmatrix}$$

$$N_1(s) = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}, \quad N_2(s) = \begin{bmatrix} 4 & 6 \\ 1 & -3 \end{bmatrix}, \quad N_3(s) = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad A = \left[\begin{array}{cc|cc|cc} -4 & 0 & -3 & 0 & 0 & 0 \\ 0 & -4 & 0 & -3 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \left[\begin{array}{cc|cc|cc} 1 & 2 & 4 & 6 & 3 & 0 \\ 1 & -1 & 1 & -3 & 0 & 0 \end{array}\right] \qquad D = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

3. $K = [\,1 \quad 6 \quad 2\,]$

Details in code

4. $L = [\,1.4653 \quad 3.6042\,]$

Details in code

1.2 State Space Realization 5 / 5

✓ **- 0 pts** **Correct**

- 1.5 pts Incorrect state matrices

- 1 pts Incorrect output matrices

2. $$G(s)=\begin{bmatrix} \frac{1}{s} & \frac{s+3}{s+1} \\ \frac{1}{s+3} & \frac{s}{s+1} \end{bmatrix}.$$ $$G(\infty) = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$G_{sp} = \begin{bmatrix} \frac{1}{s} & \frac{s+3-(s+1)}{s+1} \\ \frac{1}{s+3} & \frac{s-(s+1)}{s+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{2}{s+1} \\ \frac{1}{s+3} & \frac{-1}{s+1} \end{bmatrix}$$

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$$C = \left[\begin{array}{cc|cc|cc} 1 & 2 & 4 & 6 & 3 & 0 \\ 1 & -1 & 1 & -3 & 0 & 0 \end{array}\right] \qquad D = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

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1.3 State Feedback 5 / 5

✓ **- 0 pts** **Correct**

- 2 pts Incorrect gains

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4. $L = [\ 1.4653 \quad 3.6042\ ]$

Details in code

1.4 Observer Design 5 / 5

✓ **- 0 pts** **Correct**

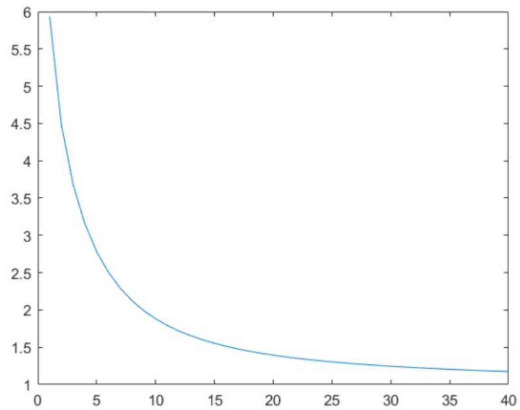
- 2 pts Incorrect gains

2.1

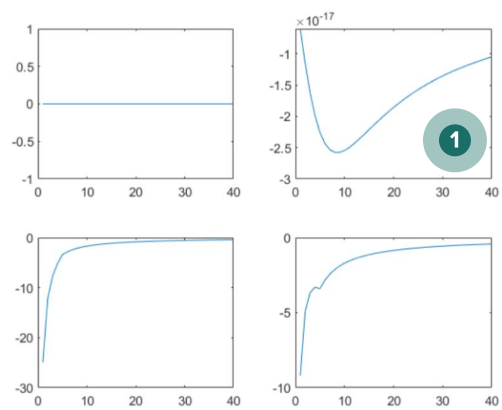
```
Rank1 = 4
Rank2 = 4
Rank3 = 4
```

Based on the rank, controllable

2.2 (a)



2.2 (b)




```
The system are both controllable and stable
```

2.3

```
Rank21 = 4
Rank22 = 4
Rank23 = 4
```

Based on rank obtained

Observable

2.1 Controllability vs. Speed 5 / 5

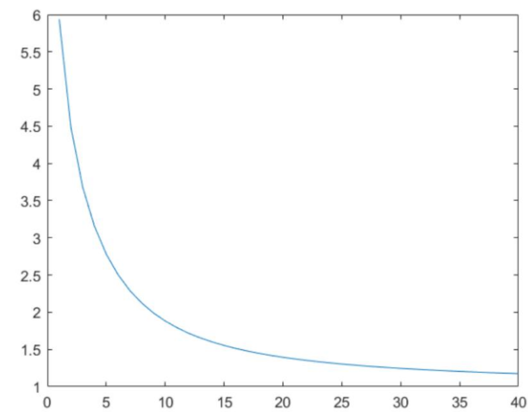
✓ **- 0 pts** **Correct**

2. 1.

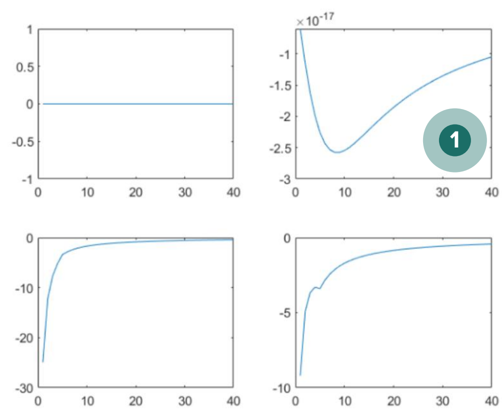
```
Rank1 = 4
Rank2 = 4
Rank3 = 4
```

Based on the rank. controllable

2.2 (a)



2.2 (b)

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2.2 Controllability Plots 4 / 5

- **- 0 pts** Correct
- ✓ **- 1 pts Incorrect pole graphs**

1 Unexpected shape, it should be a logarithmic shape almost

2. 1.

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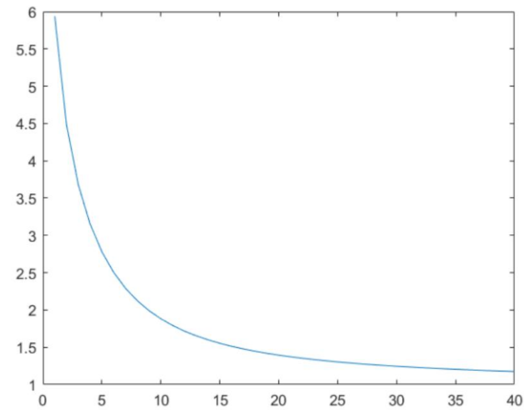

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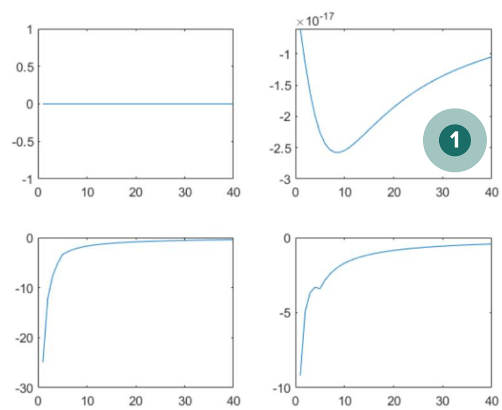


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The system are both controllable and stable
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```
Rank21 = 4
Rank22 = 4
Rank23 = 4
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Based on rank obtained

Observable

2.3 Observability 2.5 / 5

- **- 0 pts** Correct
- ✓ **- 2.5 pts** **No conclusion on measurement with only heading error**

```
p = 1×4 complex
    -5.0000 + 0.0000i  -3.0000 + 0.0000i  -1.0000 + 0.5000i ...
```

2.4

First pole design is unstable, vehicle did not folllow the trajectory

```
p = 1×4 complex
    -3.0000 + 0.0000i  -5.0000 + 0.0000i  -1.0000 + 0.5000i ...
```

Second pole design is based on the first one while changing the first two pole vehicle results in the same as first pole design

```
p = 1×4
      -5    -3    -2    -1
```

Remove imaginary poles results in the vehicle following the trajectory but osci

Thinking of changing p(2) which correspond to e2

```
p = 1×4
     -5.0000   -3.0000   -1.0000   -0.1000
```

p(4) reduced to -0.1 since p(3) need to be reduced and no repeated poles are al

Results are better turning is not ideal

```
p = 1×4
    -15.0000  -25.0000   -1.5000   -0.0010
```

This is the final pole design

2.5

The final error derivative states has:

error_1 representing result from substraction of Y difference and X difference respect to the lookahead position

error_1dot is the same as introduced which is ydot + xdot * error_2

error_2 represents the difference between psi and psi respecting lookahead position

error_2dot is psidot

2.4 Pole Placement Design 5 / 5

✓ - 0 pts Correct

Incomplete reasoning

```
p = 1x4 complex
    -5.0000 + 0.0000i  -3.0000 + 0.0000i  -1.0000 + 0.5000i ...
```

2.4

First pole design is unstable, vehicle did not folllow the trajectory

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error_2 represents the difference between psi and psi respecting lookahead position

error_2dot is psidot

2.5 Observer Design 5 / 5

✓ **- 0 pts** **Correct**