24677-A Homework 2

Saeed Bai

TOTAL POINTS

89 / 100

QUESTION 1

Subspaces 20 pts

1.1 Part a 5 / 5

✓ **- 0 pts Correct**

1.2 Part b 5 / 5

✓ **- 0 pts Correct**

1.3 Part c 5 / 5

✓ **- 0 pts Correct**

1.4 Part d 5 / 5

✓ **- 0 pts Correct**

QUESTION 2

2 Span 20 / 20

✓ **- 0 pts Correct**

- **2 pts** Both basis vectors are incorrect
- **1 pts** Incorrect conclusion for $$z$$
- **1 pts** Incorrect conclusion for $$u$$
- **1 pts** Incorrect conclusion for $$v$$
- **1 pts** One basis vector is incorrect

QUESTION 3

Norms and orthonormalization 20 pts

3.1 Vector norms 10 / 10

✓ **- 0 pts Correct**

- **0.5 pts** Wrong $$\infty$$-norm for $$x_1$$

3.2 Orthonormal basis 9 / 10

- **0 pts** Correct

✓ **- 1 pts One incorrect orthonormal vector**

- **2 pts** Incorrect orthonormal vectors

Your e2 is not normalized

QUESTION 4

Sliding mass 40 pts

4.1 Discrete time model 10 / 10

✓ **- 0 pts Correct**

- **1 pts** Wrong "A" matrix
- **1 pts** Wrong "B" matrix

4.2 pidTuner 10 / 10

✓ **- 0 pts Correct**

- **1 pts** Wrong state space matrices
- **2 pts** Wrong response
- **1 pts** Abnormal gain values

4.3 Matlab PID implementation 10 / 10

✓ **- 0 pts Correct**

- **1 pts** Wrong response
- **1 pts** Wrong expression for integral term
- **4 pts** No iterative calculation implemented
- **5 pts** No code submitted
- **5 pts** No response submitted
- **10 pts** No attempt

4.4 Open loop control 0 / 10

- **0 pts** Correct
- **1 pts** Wrong output response
- **1 pts** Wrong control sequence
- **5 pts** No code submitted to verify against
- **5 pts** No response and control plots to verify against
- **5 pts** No demonstration of iterative solution for DT systems

✓ **- 10 pts No attempt**

1. a) $n \times n$ skew-symmetric matrix (S)

zero vector 0 in W is $n \times n$ matrix

$$0^T = 0 = -0$$

set element $A, B \in S$

$$A^T = -A \quad \& \quad B^T = -B$$

$$(A+B)^T = A^T + B^T = -(A+B)$$

set element $A \in S$ & $\alpha \in \mathbb{R}$

$$\alpha A \in S \qquad (\alpha A)^T = \alpha A^T = -\alpha A$$

Hence, the set is a subset of W

b) $n \times n$ diagonal is $\begin{bmatrix} x_1 & 0 & \cdots & 0 \\ 0 & x_2 & & \\ \vdots & & \ddots & \\ 0 & & & x_n \end{bmatrix}$

$$tr(A+B) = tr(A) + tr(B) \qquad A+B \in W$$
$$tr(cA) = c \cdot tr(A) \qquad cA \in W$$

Hence, the set is a subset of W

c) upper diagonal matrix $\begin{bmatrix} x_1 & x_{12} & \cdots & x_{1n} \\ 0 & x_{22} & \ddots & \vdots \\ 0 & 0 & \ddots & \\ 0 & \cdots & 0 & x_{nn} \end{bmatrix}$ (S)

all zero vectors in S are in W.

$A \in S, B \in S, A+B \in W$

$A \in S \quad \alpha A \in W$

Hence, the set is a subset of W.

d) singular matrix

Ex. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ which is non-singular

Hence, the set is not a subset of W

1.1 Part a **5 / 5**

✓ **- 0 pts** **Correct**

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1.2 Part b **5 / 5**

✓ **- 0 pts** **Correct**

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set element $A, B \in S$

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1.3 Part c 5 / 5

✓ - 0 pts Correct

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Ex. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ which is non-singular

Hence, the set is not a subset of W

1.4 Part d 5 / 5

✓ - 0 pts Correct

2.

$$\{y_1, y_2, y_3, y_4\} = \left\{ \begin{bmatrix} -1 \\ 1 \\ -5 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 7 \\ -8 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 10 \\ -11 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 15 \\ -15 \end{bmatrix} \right\}.$$

$$y = \begin{bmatrix} -1 & 2 & 3 & 5 \\ 1 & 1 & 2 & 5 \\ -5 & 7 & 10 & 15 \\ 7 & -8 & -11 & -15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & -3 & -5 \\ 0 & 3 & 5 & 10 \\ 0 & -3 & -5 & -10 \\ 0 & -15 & -25 & -50 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \textcircled{1} & -2 & 3 & -5 \\ 0 & \textcircled{3} & 5 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence, basis are $\begin{bmatrix} -1 \\ 1 \\ -5 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 7 \\ -8 \end{bmatrix}$

$$z = \begin{bmatrix} 3 \\ -1 \\ 13 \\ 17 \end{bmatrix} = a \begin{bmatrix} -1 \\ 1 \\ -5 \\ 7 \end{bmatrix} + b \begin{bmatrix} 2 \\ 1 \\ 7 \\ -8 \end{bmatrix} \Rightarrow \begin{array}{l} a = -\frac{5}{3} \\ b = \frac{2}{3} \end{array}$$

$$u = \begin{bmatrix} 4 \\ 9 \\ 12 \\ -8 \end{bmatrix} = a \begin{bmatrix} -1 \\ 1 \\ -5 \\ 7 \end{bmatrix} + b \begin{bmatrix} 2 \\ 1 \\ 7 \\ -8 \end{bmatrix} \Rightarrow \text{no } a, b \text{ found}$$

$$v = \begin{bmatrix} -1 \\ -1 \\ -3 \\ 3 \end{bmatrix} = a \begin{bmatrix} -1 \\ 1 \\ -5 \\ 7 \end{bmatrix} + b \begin{bmatrix} 2 \\ 1 \\ 7 \\ -8 \end{bmatrix} \Rightarrow \begin{array}{l} a = -\frac{1}{3} \\ b = -\frac{2}{3} \end{array}$$

2 Span 20 / 20

✓ - **0 pts** **Correct**

- **2 pts** Both basis vectors are incorrect
- **1 pts** Incorrect conclusion for $$z$$
- **1 pts** Incorrect conclusion for $$u$$
- **1 pts** Incorrect conclusion for $$v$$
- **1 pts** One basis vector is incorrect

3. $x_1 = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$ $x_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

a) 1-norm: $\|x\|_1 = \sum_{i=1}^{n} |x_i| = |2| + |-3| + |-1| = 6$

$\|x\|_1 = \sum_{i=1}^{n} |x_i| = |1| + |1| + |-1| = 3$

2-norm: x_1: $\|x\|_2 = \sqrt{2^2 + (-3)^2 + (-1)^2} = \sqrt{14}$

x_2: $\|x\|_2 = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$

∞-norm: x_1: $\max |x_{1i}| = 3$

x_2: $\max |x_{2i}| = 1$

b) space. $\left\{ \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$

$u_1 = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} \Rightarrow e_1 = \begin{bmatrix} 0.535 \\ -0.802 \\ -0.267 \end{bmatrix}$

$$u_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} - \frac{\begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}}{\begin{bmatrix} 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}} \cdot \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 3/\sqrt{3} \\ 3/\sqrt{3} \\ -3/\sqrt{3} \end{bmatrix}$$

3.1 Vector norms 10 / 10

✓ - **0 pts** Correct

- **0.5 pts** Wrong $$\infty$$-norm for $$x_1$$

3. $x_1 = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$ $x_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

a) 1-norm: $\|x\|_1 = \sum_{i=1}^{n} |x_i| = |2| + |-3| + |-1| = 6$

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$$e_2 = \begin{bmatrix} 3/\sqrt{3} \\ 3/\sqrt{3} \\ -3/\sqrt{3} \end{bmatrix}$$

3.2 Orthonormal basis 9 / 10

- **0 pts** Correct

✓ **- 1 pts** **One incorrect orthonormal vector**

- **2 pts** Incorrect orthonormal vectors

Your e2 is not normalized

4. a)

$$x[(k+1)T] = [I + A(T)]\,x(kT) + B(T)u(kT)$$

$$y(kT) = Cx(kT) + Du(kT)$$

$$\frac{x[(k+1)T] - x(kT)}{T} \approx \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

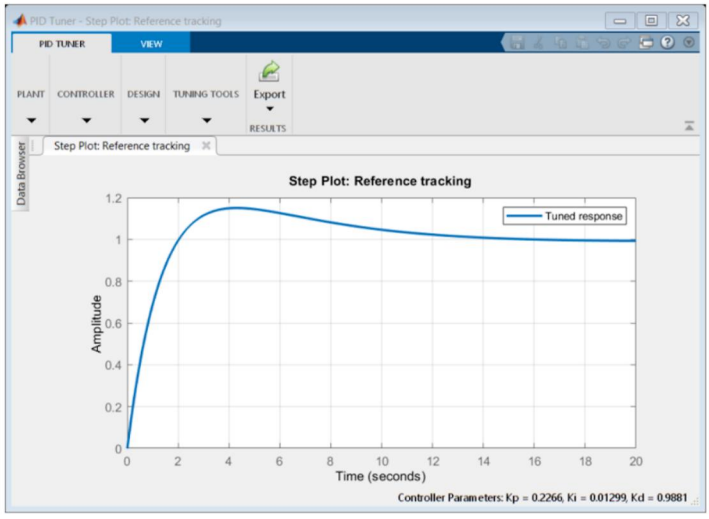
$$x[(k+1)T] = \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0.01 \end{bmatrix} u_k$$

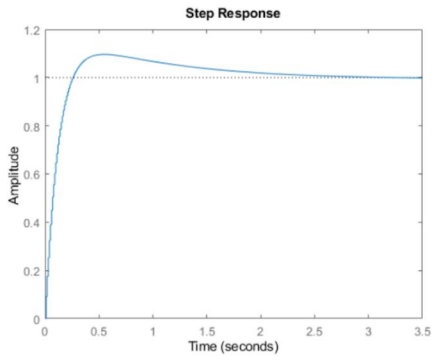
$$y(kT) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k$$

b)

```
A = [1 0.01;0 1];
B = [0;0.01];
C = [1 0];
D = 0;
ts = 0.01;
[b a] = ss2tf(A,B,C,D);
sys = tf(b,a,ts);

BD = pidTuner(H,'PID'); %Getting proper Kp Ki Kd
K_p = 10.6988;
K_i = 3.1334;
K_d = 9.1327;
CC  = pid(K_p,K_i,K_d,D,ts);
G  = CC * sys;
CLTF = feedback(G,1);
step(CLTF);
```





4.1 Discrete time model **10 / 10**

✓ **- 0 pts** **Correct**

- 1 pts Wrong "A" matrix

- 1 pts Wrong "B" matrix

4. a)

$$x[(k+1)T] = [I + A(T)]\,x(kT) + B(T)u(kT)$$

$$y(kT) = Cx(kT) + Du(kT)$$

$$\frac{x[(k+1)T] - x(kT)}{T} \approx \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u$$

$$x[(k+1)T] = \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0.01 \end{bmatrix} u_k$$

$$y(kT) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k$$

b)

```
A = [1 0.01;0 1];
B = [0;0.01];
C = [1 0];
D = 0;
ts = 0.01;
[b a] = ss2tf(A,B,C,D);
sys = tf(b,a,ts);

BD = pidTuner(H,'PID'); %Getting proper Kp Ki Kd
K_p = 10.6988;
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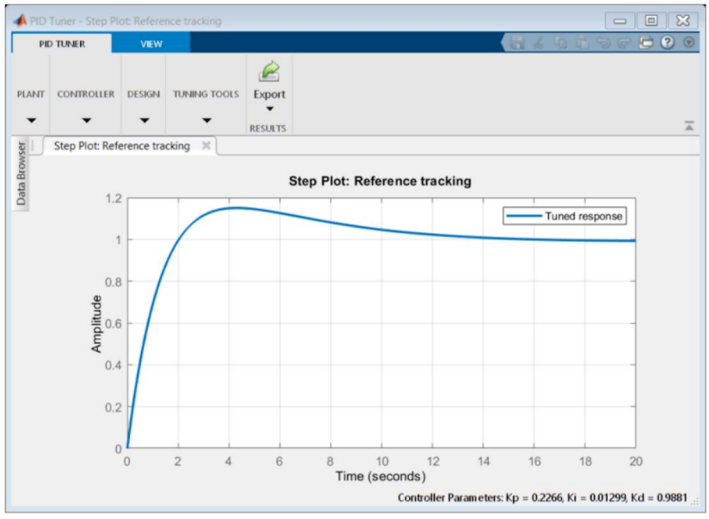
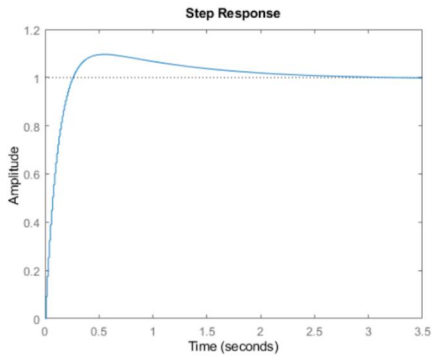




4.2 pidTuner 10 / 10

✓ **- 0 pts** **Correct**

- 1 pts Wrong state space matrices

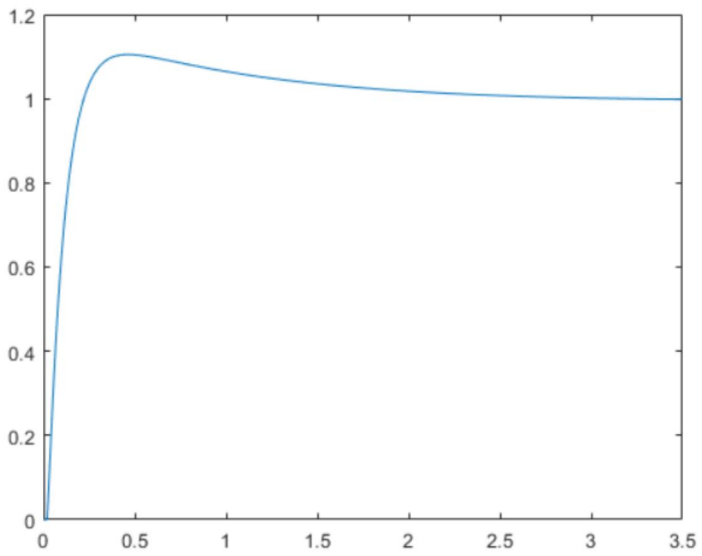
- 2 pts Wrong response

- 1 pts Abnormal gain values

c)

Part c

```
T = ts;
T_max = 3.5; %Time interval
T_t = 0: ts : 3.5;
SE = size(T_t);
x = zeros(2,length(T_t));
y = zeros(SE);
err = zeros(SE+1);
u_d = err;
u_i = err;
u_p = err;
u_add = err;
add_err = 0;
for i = 1 : length(T_t)
    x(:,i+1) = A * x(:,i) + B * u_add(i);
    y(i) = C * x(:,i);
    err(i+1) = 1 - y(i);
    add_err = add_err + err(i+1);
    u_d(i+1) = K_d / T * (err(i+1) - err(i));
    u_i(i+1) = K_i * T * add_err;
    u_p(i+1) = K_p * err(i);
    u_add(i+1) = u_d(i+1) + u_i(i+1) + u_p(i+1);
end
plot(T_t,y)
```



d)

Part d

```
Td = 0.05;
T_d = 0:0.01:Td;
%x(1)=Ax(0)+Bu(0)
%x(2)=Ax(1)+Bu(1)=A(Ax(0)+Bu(0))+Bu(1)
```

4.3 Matlab PID implementation 10 / 10

✓ **- 0 pts** **Correct**

- 1 pts Wrong response

- 1 pts Wrong expression for integral term

- 4 pts No iterative calculation implemented

- 5 pts No code submitted

- 5 pts No response submitted

- 10 pts No attempt

c)

Part c

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y = zeros(SE);
err = zeros(SE+1);
u_d = err;
u_i = err;
u_p = err;
u_add = err;
add_err = 0;
for i = 1 : length(T_t)
    x(:,i+1) = A * x(:,i) + B * u_add(i);
    y(i) = C * x(:,i);
    err(i+1) = 1 - y(i);
    add_err = add_err + err(i+1);
    u_d(i+1) = K_d / T * (err(i+1) - err(i));
    u_i(i+1) = K_i * T * add_err;
    u_p(i+1) = K_p * err(i);
    u_add(i+1) = u_d(i+1) + u_i(i+1) + u_p(i+1);
end
plot(T_t,y)
```

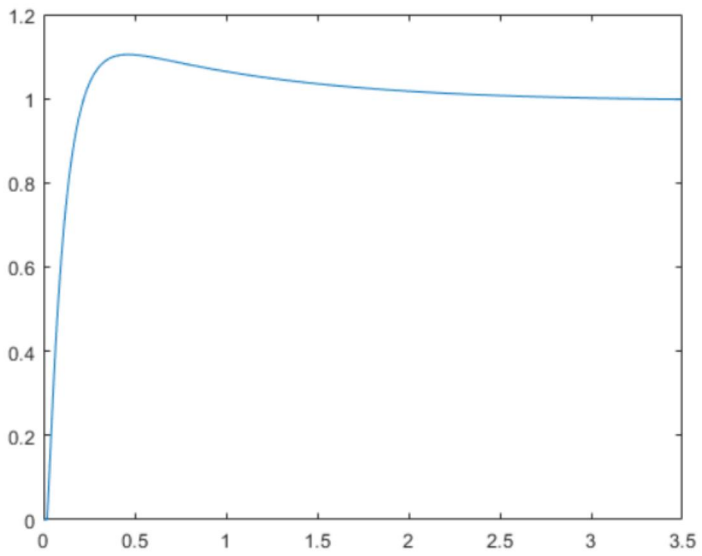

d)

Part d

```
Td = 0.05;
T_d = 0:0.01:Td;
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%x(2)=Ax(1)+Bu(1)=A(Ax(0)+Bu(0))+Bu(1)
```

4.4 Open loop control 0 / 10

- **0 pts** Correct
- **1 pts** Wrong output response
- **1 pts** Wrong control sequence
- **5 pts** No code submitted to verify against
- **5 pts** No response and control plots to verify against
- **5 pts** No demonstration of iterative solution for DT systems

✓ - **10 pts** **No attempt**