Like all other assignments in the course, this assignment will be submitted via Gradescope. Please come to office hours or reach out on Piazza if you have any questions! Please use livecripts to present your Matlab work, along with screenshots of any Simulink models.

1: 20 points

The set W of $n \times n$ matrices with real entries is known to be a linear vector space. Determine which of the following sets are subspaces of W.

(a) The set of $n \times n$ skew-symmetric matrices.

(b) The set of $n \times n$ diagonal matrices.

(c) The set of $n \times n$ upper-diagonal matrices.

(d) The set of $n \times n$ singular matrices.

2: 20 points

Determine a basis for the space $W \subset \mathbb{R}^4$ spanned by the four vectors

$$\{y_1, y_2, y_3, y_4\} = \left\{ \begin{bmatrix} -1 \\ 1 \\ -5 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 7 \\ -8 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 10 \\ -11 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 15 \\ -15 \end{bmatrix} \right\}.$$

Test whether the following vectors are $\in W$.

- $z = \begin{bmatrix} 3 \\ -1 \\ 13 \\ -17 \end{bmatrix}$
- $u = \begin{bmatrix} 4 \\ 9 \\ 12 \\ -8 \end{bmatrix}$
- $v = \begin{bmatrix} -1 \\ -1 \\ -3 \\ 3 \end{bmatrix}$

3: 20 points

For the vectors below do the following.

- $x_1 = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$
- $x_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

(a) Find the 1, 2, and ∞ norms.

(b) Find two orthonormal vectors that span the same space as $\{x_1, x_2\}$. This type of problem can be used to represent the reachable subspace of a linear system (more later!).

4: 40 points

A unit mass sliding on a frictionless plane has dynamics

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u.$$

(a) Using the approximation $\dot{x}(t) \approx \frac{x[(k+1)T] - x[kT]}{T}$, find a discrete time system model for $T = 0.01$ s.

(b) Read about the *pidTuner* functionality in Matlab here: `https://www.mathworks.com/help/control/ref/pidtuner.html`. Design a PID controller that has "good" performance using the tuner ("good" is up to you) for the discrete time system. Plot the step response of the closed loop system using the *step* command.

(c) Create your PID controller in a Matlab script. Write a Matlab function that takes as arguments the current value of the error, the last value of the error, and the reference set point and sends as an output the control signal generated by your PID controller. Using your function, simulate the step response of the closed loop system by directly solving the discrete time state equations.

(d) Using the method of your choice, design an <u>open loop</u> controller (sequence of controls) that takes the mass from state $\begin{bmatrix} 0 & 0 \end{bmatrix}^T$ at $t = 0$ to state $\begin{bmatrix} 1 & 0 \end{bmatrix}^T$ at $t = 0.05$ s. Plot the response and the control sequence.