24677-A Project - P1

Saeed Bai

TOTAL POINTS

43.5 / 50

QUESTION 1

Linearization Model 40 pts

1.1 Linearization about arbitrary point 13 / 15

- **- 0 pts** Correct
- **- 2.5 pts** Incorrect Jacobian for A
- **- 2.5 pts** Incorrect Jacobian for B
- **- 1 pts** Incorrect C

- 2 Point adjustment

Some elements that I've glimpsed are correct, but I cannot tell if the matrix is structured correctly in the format you're presenting to me

1 This is unreadable for me...

1.2 Linearization about specified point 8.5 / 10

- **- 0 pts** Correct
- ✓ **- 1.5 pts Incorrect values in A**
- **- 1 pts** Incorrect values in B

2 Use simplify() next time to clean up long fractions

3 Incorrect

1.3 Transfer function and poles 12 / 15

- **- 0 pts** Correct
- ✓ **- 1.5 pts Incorrect poles**
- ✓ **- 1.5 pts Incorrect zeros**

No poles and zeros to verify against

QUESTION 2

2 Performance graph 10 / 10

- ✓ **- 0 pts Correct**

Exercise 1

1. Linearized state equation

from equation 2.4, get $\ddot{y}, \ddot{x}, \ddot{\psi}, \dot{X}, \dot{Y}$

$$x = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \psi \\ \dot{\psi} \\ X \\ Y \end{pmatrix} \Rightarrow \dot{x} = \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \\ \dot{X} \\ \dot{Y} \end{pmatrix}$$

$$\Rightarrow \dot{x} = \begin{pmatrix} \dot{\psi}\dot{y} + \frac{1}{m}(F - fmg) \\ -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \\ \dot{\psi} \\ \frac{2l_f C_\alpha}{I_z}\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_r C_\alpha}{I_z}\left(-\frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \\ \dot{x}\cos\psi - \dot{y}\sin\psi \\ \dot{x}\sin\psi + \dot{y}\cos\psi \end{pmatrix}$$

get A matrix using jacobian respect to $\dot{x}, \dot{y}, \psi, \dot{\psi}, X, Y$

get B matrix using jacobian respect to δ, F

Results shown in matlab code

$$y = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}}_{C} \begin{pmatrix} \dot{x} \\ y \\ \psi \\ \dot{\psi} \\ X \\ Y \end{pmatrix} + \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}_{D} u$$

2. plug in $\dot{y} = \dot{\psi} = \psi = 0$ and $\dot{x} = 6$

as well as $\delta = 0$ at this position

Results shown in matlab code

3. $G(s) = C \cdot (sI - A)^{-1} \cdot B + D$

Results shown in matlab code

Table of Contents

Project 1 Excercise 1: Model Linearization

Initializations

```
m = 4500; %mass of vehicle
f = 0.028; % rolling resistance coefficient
g = 9.81;
Ca = 20000; % Cornering stiffness of each tire
lf = 1.01; % length from front tire to the center of mass
lr = 3.32; % length from rear tire to the center of mass
Iz = 29526.2; % Yaw intertia
delT = 0.032;
syms F_input x y wheel_angle psi x_dot y_dot psi_dot X Y s
% Recreating equations in section 2.4
x_ddot = psi_dot * y_dot + 1 / m * (F_input - f*m*g);
y_ddot = -psi_dot * x_dot + 2 * Ca / m *
 (cos(wheel_angle)*(wheel_angle ...
    - (y_dot + lf*psi_dot))/x_dot) - (y_dot-lr*psi_dot)/x_dot;
psi_dot = psi_dot;
psi_ddot = (2*lf*Ca/Iz)*(wheel_angle- (y_dot+lf*psi_dot)/x_dot)-
(2*lr*Ca) ...
    /Iz*(-(y_dot-lr*psi_dot)/x_dot);
X_dot = x_dot*cos(psi)-y_dot*sin(psi);
Y_dot = x_dot*sin(psi)+y_dot*cos(psi);
% Nonlinear state equations
Xcross_dot = [x_ddot;y_ddot;psi_dot;psi_ddot;X_dot;Y_dot];
u = [1 / m * F_input;2 * Ca / m * (cos(wheel_angle)*(wheel_angle) -
 (y_dot + ...
    lf*psi_dot)/x_dot);0;(2*lf*Ca/Iz)*(wheel_angle);0;0];
```

Problem 1

```
A = jacobian(Xcross_dot,[x_dot;y_dot;psi;psi_dot;X;Y])
B = jacobian(u,[wheel_angle;F_input])
C = [1 0 0 0 0 0;
     0 0 1 0 0 0];
D = [0 0;0 0];
```

```
A =

[
```

```
      0,                                     psi_dot,
                 0,
 y_dot, 0, 0]
[                          (80*cos(wheel_angle)*((101*psi_dot)/100 -
 wheel_angle + y_dot))/(9*x_dot^2) - ((83*psi_dot)/25 - y_dot)/x_dot^2
 - psi_dot, - (80*cos(wheel_angle))/(9*x_dot) - 1/x_dot,
                        0, 83/(25*x_dot) - (404*cos(wheel_angle))/
(45*x_dot) - x_dot, 0, 0]
[

      0,                                           0,
                 0,
     1, 0, 0]
[(2531980201121385*((83*psi_dot)/25 - y_dot))/
(562949953421312*x_dot^2) + (3081084341123613*((101*psi_dot)/100
 + y_dot))/(2251799813685248*x_dot^2),   7046836463361927/
(2251799813685248*x_dot),                                  0,
 -3673659225542684193/(225179981368524800*x_dot), 0, 0]
[

         cos(psi),                                  -sin(psi), -
 y_dot*cos(psi) - x_dot*sin(psi),
                    0, 0, 0]
[

         sin(psi),                                   cos(psi),
 x_dot*cos(psi) - y_dot*sin(psi),
                    0, 0, 0]


B =

[                                                            0,
 1/4500]
[(80*cos(wheel_angle))/9 - (80*wheel_angle*sin(wheel_angle))/9,
 0]
[                                                            0,
 0]
[                             3081084341123613/2251799813685248,
 0]
[                                                            0,
 0]
[                                                            0,
 0]
```

Problem 2

```
x_dot = 6;
y_dot = 0;
psi_dot = y_dot;
psi = y_dot;
wheel_angle = 0;
```

1.1 Linearization about arbitrary point **13 / 15**

- **- 0 pts** Correct
- **- 2.5 pts** Incorrect Jacobian for A
- **- 2.5 pts** Incorrect Jacobian for B
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- 2 Point adjustment

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Exarsise 1

1. Linearized state equation

from equation 2.4, get $\ddot{y}, \ddot{x}, \ddot{\psi}, \dot{X}, \dot{Y}$

$$x = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \psi \\ \dot{\psi} \\ X \\ Y \end{pmatrix} \Rightarrow \dot{x} = \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \\ \dot{X} \\ \dot{Y} \end{pmatrix}$$

$$\Rightarrow \dot{x} = \begin{pmatrix} \dot{\psi}\dot{y} + \frac{1}{m}(F - fmg) \\ -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \\ \dot{\psi} \\ \frac{2l_f C_\alpha}{I_z}\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_r C_\alpha}{I_z}\left(-\frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right) \\ \dot{x}\cos\psi - \dot{y}\sin\psi \\ \dot{x}\sin\psi + \dot{y}\cos\psi \end{pmatrix}$$

get A matrix using jacobian respect to $\dot{x}, \dot{y}, \psi, \dot{\psi}, X, Y$

get B matrix using jacobian respect to δ, F

Results shown in matlab code

$$y = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}}_{C} \begin{pmatrix} \dot{x} \\ y \\ \psi \\ \dot{\psi} \\ X \\ Y \end{pmatrix} + \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}_{D} u$$

2. plug in $\dot{y} = \dot{\psi} = \psi = 0$ and $\dot{x} = 6$

as well as $\delta = 0$ at this position

Results shown in matlab code

3. $G(s) = C \cdot (sI - A)^{-1} \cdot B + D$

Results shown in matlab code

```
      0,                                    psi_dot,
                0,
 y_dot, 0, 0]
[                          (80*cos(wheel_angle)*((101*psi_dot)/100 -
 wheel_angle + y_dot))/(9*x_dot^2) - ((83*psi_dot)/25 - y_dot)/x_dot^2
 - psi_dot, - (80*cos(wheel_angle))/(9*x_dot) - 1/x_dot,
                        0, 83/(25*x_dot) - (404*cos(wheel_angle))/
(45*x_dot) - x_dot, 0, 0]
[

      0,                                          0,
                0,
     1, 0, 0]
[(2531980201121385*((83*psi_dot)/25 - y_dot))/
(562949953421312*x_dot^2) + (3081084341123613*((101*psi_dot)/100
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(2251799813685248*x_dot),                                 0,
 -367365922554268419 3/(225179981368524800*x_dot), 0, 0]
[

         cos(psi),                                  -sin(psi), -
 y_dot*cos(psi) - x_dot*sin(psi),
                    0, 0, 0]
[

         sin(psi),                                   cos(psi),
 x_dot*cos(psi) - y_dot*sin(psi),
                    0, 0, 0]


B =

[                                                         0,
 1/4500]
[(80*cos(wheel_angle))/9 - (80*wheel_angle*sin(wheel_angle))/9,
 0]
[                                                         0,
 0]
[                             3081084341123613/2251799813685248,
 0]
[                                                         0,
 0]
[                                                         0,
 0]
```

Problem 2

```
x_dot = 6;
y_dot = 0;
psi_dot = y_dot;
psi = y_dot;
wheel_angle = 0;
```

```
A_new = subs(A)
B_new = subs(B)
```

```
A_new =

[0,                                 0, 0,
          0, 0, 0]
[0,                            -89/54, 0,
 -9373/[illegible]50, 0, 0]
[0,                                 0, 0,
          1, 0, 0]
[0, 2348945487787309/4503599627370496, 0,
 -1224553075180894731/450359962737049600, 0, 0]
[1,                                 0, 0,
          0, 0, 0]
[0,                                 1, 6,
          0, 0, 0]


B_new =

[                               0, 1/4500]
[                            80/9,      0]
[                               0,      0]
[3081084341123613/2251799813685248,      0]
[                               0,      0]
[                               0,      0]
```

Problem 3

```
G_s = C *inv((s*eye(length(A_new))-A_new))* B_new + D;
G_s = vpa(G_s,2)
```

```
G_s =

[
                                                                  0, 2.2e-4/s]
[(6.2e+17*(54.0*s + 89.0))/(2.4e+19*s^3 + 1.1e+20*s^2 + 2.0e+20*s) +
 1.1e+20/(2.4e+19*s^3 + 1.1e+20*s^2 + 2.0e+20*s),          0]
```

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1.2 Linearization about specified point **8.5 / 10**

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 -1224553075180894731/450359962737049600, 0, 0]
[1,                                  0, 0,
          0, 0, 0]
[0,                                  1, 6,
          0, 0, 0]


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[                               0, 1/4500]
[                            80/9,      0]
[                               0,      0]
[3081084341123613/2251799813685248,     0]
[                               0,      0]
[                               0,      0]
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```

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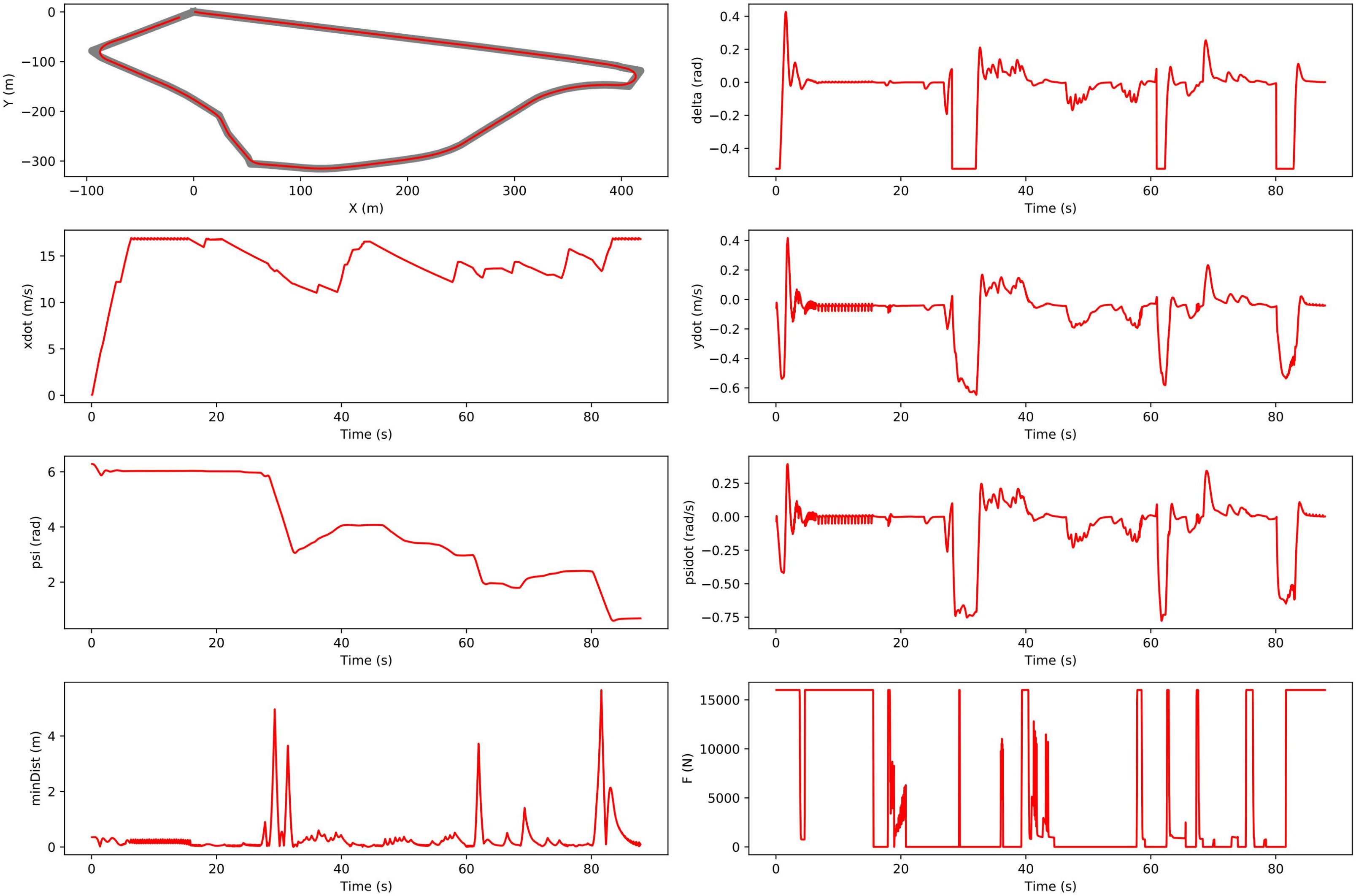
1.3 Transfer function and poles 12 / 15

- **- 0 pts** Correct
- ✓ **- 1.5 pts Incorrect poles**
- ✓ **- 1.5 pts Incorrect zeros**

No poles and zeros to verify against



0
−100
−200
−300
Y (m)
−100
0
100
200
300
400
X (m)
0.4
0.2
0.0
−0.2
−0.4
delta (rad)
0
20
40
60
80
Time (s)
15
10
5
0
xdot (m/s)
0
20
40
60
80
Time (s)
0.4
0.2
0.0
−0.2
−0.4
−0.6
ydot (m/s)
0
20
40
60
80
Time (s)
6
4
2
psi (rad)
0
20
40
60
80
Time (s)
0.25
0.00
−0.25
−0.50
−0.75
psidot (rad/s)
0
20
40
60
80
Time (s)
4
2
0
minDist (m)
0
20
40
60
80
Time (s)
15000
10000
5000
0
F (N)
0
20
40
60
80
Time (s)

2 Performance graph 10 / 10

✓ **- 0 pts** **Correct**