24677-A Homework 4

Saeed Bai

TOTAL POINTS

101 / 105

QUESTION 1

Cayley-Hamilton computations 15 pts

1.1 Part a 5 / 5

✓ **- 0 pts Correct**

- 1 pts Incorrect answer

- 2 pts No intermediate steps to show usage of CH Theorem

1.2 Part b 5 / 5

✓ **- 0 pts Correct**

- 1 pts Incorrect answer

- 2 pts No intermediate steps to show usage of CH Theorem

1.3 Part c 5 / 5

✓ **- 0 pts Correct**

- 1 pts Incorrect answer

QUESTION 2

2 Closed form A^k 20 / 20

✓ **- 0 pts Correct**

- 2 pts Incorrect eigenvalues

- 1 pts Incorrect result

QUESTION 3

3 Find y(n) for 2 state discrete time system 19 / 20

- 0 pts Correct

- 1 pts Incorrect eigenvalues

✓ **- 1 pts Incorrect diagonalization**

- 3 pts Incorrect answer

- 10 pts No use of CH theorem or Diagonalization to generalize solution

- 20 pts No attempt

QUESTION 4

Two tank problem 30 pts

4.1 y(5) in CT 9 / 10

- 0 pts Correct

- 2 pts Used DT solution instead of CT

✓ **- 2 pts Incorrect answer**

- 5 pts Incorrect quantities

- 0.5 pts Incorrect "C" matrix

+ 1 Point adjustment

1 This is supposed to be -0.1

4.2 Discretization 10 / 10

✓ **- 0 pts Correct**

- 1 pts No "Cd" matrix or incorrect

- 2 pts Incorrect answer

- 10 pts No attempt

4.3 y(5) in DT 10 / 10

✓ **- 0 pts Correct**

- 2 pts Incorrect answer

- 1 pts Incorrect plot

- 3 pts Missing plot

- 10 pts No attempt

QUESTION 5

PID Control 20 pts

5.1 PID in state space 10 / 10

✓ **- 0 pts Correct**

- 1 pts Wrong B matrix

- 1 pts Incorrect answer for output y = Cx + Du

- 2 pts Incorrect answer

- 5 pts No intermediate step to show use of CCF

- 10 pts No attempt

5.2 PID response 8 / 10

- **- 0 pts** Correct
- ✓ **- 2 pts Incorrect plot**
- **- 2 pts** Incorrect conclusion
- **- 10 pts** No attempt

1. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

a) $\det \begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda(1-\lambda)^2 = 0$

$\lambda(1 - 2\lambda + \lambda^2) = 0$

$\lambda^3 - 2\lambda^2 + \lambda = 0$

CHT : $A^3 - A^2 = A^2 - A$

$A^4 - A^2 = A^3 - A^2 = A^2 - A$

$\vdots$

$A^{10} - A^9 = A^2 - A \qquad A^{10} - A^2 = 8(A^2 - A) \qquad A_{10} = 9A^2 - 8A$

$A^2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$A^{10} = 9\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} - 8\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 9 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

b) $A^{103} = 102A^2 - 101A$

$= 102\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} - 101\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 102 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

1.1 Part a **5 / 5**

✓ **- 0 pts** Correct

- 1 pts Incorrect answer

- 2 pts No intermediate steps to show usage of CH Theorem

1. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

a) $\det \begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda(1-\lambda)^2 = 0$

$\lambda(1-2\lambda+x^2) = 0$

$\lambda^3 - 2x^2 + \lambda = 0$

CHT : $A^3 - A^2 = A^2 - A$

$A^4 - A^2 = A^3 - A^2 = A^2 - A$

$\vdots$

$A^{10} - A^9 = A^2 - A \qquad A^{10} - A^2 = 8(A^2 - A) \qquad A_{10} = 9A^2 - 8A$

$A^2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$A^{10} = 9\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} - 8\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 9 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

b) $A^{103} = 102A^2 - 101A$

$= 102\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} - 101\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 102 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

1.2 Part b 5 / 5

✓ **- 0 pts** Correct

- 1 pts Incorrect answer

- 2 pts No intermediate steps to show usage of CH Theorem

c) from a), $\lambda_1 = 0$ $\lambda_{2,3} = 1$

$$e^{0t} = \alpha_1 0^2 + \alpha_2 0 + \alpha_3 \Rightarrow \alpha_3 = 1$$

$$e^{t} = \alpha_1 1^2 + \alpha_2 1 + \alpha_3 \Rightarrow \alpha_1 + \alpha_2 + 1 = e^t$$

$$te^{t} = 2\alpha_1 + \alpha_2 \Rightarrow 2\alpha_1 + \alpha_2 = te^t$$

$$\alpha_2 = e^t - 1 - \alpha_1$$

$$2\alpha_1 + e^t - 1 - \alpha_1 = te^t$$

$$\alpha_1 = te^t - e^t + 1 = e^t(t-1) + 1$$

$$\alpha_2 = e^t - 1 - te^t + e^t - 1 = 2e^t - te^t - 2$$

$$e^{At} = (te^t - e^t + 1)A^2 + (2e^t - te^t - 2)A + I$$

$$= \begin{bmatrix} e^t & e^t - 1 & te^t - e^t + 1 \\ 0 & 1 & e^t - 1 \\ 0 & 0 & e^t \end{bmatrix}$$

1.3 Part c 5 / 5

✓ **- 0 pts** **Correct**

- 1 pts Incorrect answer

2、

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 9 & 23 & 30 \\ -7 & -18 & -235 \end{bmatrix}.$$

$$\det \begin{vmatrix} -\lambda & 0 & 0 \\ 9 & 23-\lambda & 30 \\ -7 & -18 & -235-\lambda \end{vmatrix} = 0 \qquad -\lambda(\lambda^2 + 212\lambda - 4865) = 0$$

$$\lambda_1 = 0 \quad \lambda_2 = -106 + 3\sqrt{1789}$$

$$\lambda_3 = -106 - 3\sqrt{1789}$$

$$\lambda_1 = 0 \Rightarrow x_1 = \begin{bmatrix} 4865 \\ -1905 \\ 1 \end{bmatrix} t \qquad \lambda_2 = -106 + 3\sqrt{1789} \Rightarrow x_2 = \begin{bmatrix} 0 \\ -\frac{43+\sqrt{1789}}{6} \\ 1 \end{bmatrix} t$$

$$\lambda_3 = -106 - 3\sqrt{1789} \quad x_3 = \begin{bmatrix} 0 \\ -\frac{43-\sqrt{1789}}{6} \\ 1 \end{bmatrix} t \qquad M = \begin{bmatrix} 4865 & 0 & 0 \\ -1905 & -\frac{43+\sqrt{1789}}{6} & -\frac{43-\sqrt{1789}}{6} \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^k = M\hat{A}M^{-1} = \begin{bmatrix} 4865 & 0 & 0 \\ -1905 & -\frac{43+\sqrt{1789}}{6} & -\frac{43-\sqrt{1789}}{6} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & (-106+3\sqrt{1789})^k & 0 \\ 0 & 0 & (-106-3\sqrt{1789})^k \end{bmatrix} \begin{bmatrix} 4865 & 0 & 0 \\ -1905 & -\frac{43+\sqrt{1789}}{6} & -\frac{43-\sqrt{1789}}{6} \\ 1 & 1 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ a & b & c \\ d & e & f \end{bmatrix}$$

$$a: \frac{6041813880\sqrt{1789}(3\sqrt{1789}-106)^k + 251168730240(3\sqrt{1789}-106)^k}{2193278220} - \frac{500375\sqrt{1789}(-106-3\sqrt{1789})^k - 122304985(-106-3\sqrt{1789})^k}{365546370}$$

$$b: \frac{(43+\sqrt{1789})(3\sqrt{1789}-106)^k - (47-6\sqrt{1789})(-106-3\sqrt{1789})^k}{7\sqrt{1789}}$$

$$c: \frac{8885\sqrt{1789}(-106-3\sqrt{1789})^k + 384635(-106-3\sqrt{1789})^k - 384635(3\sqrt{1789}-106)^k - 8885\sqrt{1789}(3\sqrt{1789}-106)^k}{25138}$$

$$d: \frac{(11387\sqrt{1789}-1789)(-106-3\sqrt{1789})^k}{60924395} - \frac{(143821110\sqrt{1789}-142493850)(3\sqrt{1789}-106)^k}{3655463700}$$

e: $$\frac{6(-106-3\sqrt{1789})^k - 6(3\sqrt{1789}-106)^k}{7\sqrt{1789}}$$

f: $$\frac{(10734-43\sqrt{1789})(3\sqrt{1789}-106)^k + (43\sqrt{1789}+1789)(-106-3\sqrt{1789})^k}{12523}$$

2 Closed form A^k **20 / 20**

- ✓ **- 0 pts** **Correct**
- **- 2 pts** Incorrect eigenvalues
- **- 1 pts** Incorrect result

3.

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ -0.5 & -1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

$$x(n) = A_d^n x(0) + \sum_{m=0}^{n-1} A_d^{n-m-1} B u(m)$$

$$A_d = M \hat{A} M^{-1}$$

$$A_d^n = M \hat{A}^n M^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1^n \\ -0.5^n & -1^n \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -(-0.5)^{n-m-1} + (-1)^{n-m-1} & (-0.5)^{n-m-1} \\ -(-0.5)^{n-m-1} + 1 + (-1)^{n-m-1} & (-0.5)^{n-m-1} \end{bmatrix}$$

$$y(n) = \begin{bmatrix} 1 & 0 \end{bmatrix} \sum_{m=0}^{n-1} \begin{bmatrix} -(-0.5)^{n-m-1} + (-1)^{n-m-1} & (-0.5)^{n-m-1} \\ -(-0.5)^{n-m-1} + 1 + (-1)^{n-m-1} & (-0.5)^{n-m-1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} 1$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \sum_{m=0}^{n-1} \begin{bmatrix} -(-0.5)^{n-m-1} + (-1)^{n-m-1} + (-0.5)^{n-m-1} \\ -(-0.5)^{n-m-1} + (-1)^{n-m} + (-0.5)^{n-m-1} \end{bmatrix}$$

$$= \sum_{m=0}^{n-1} -(-0.5)^{n-m-1} + (-1)^{n-m} + (-0.5)^{n-m-1}$$

3 Find y(n) for 2 state discrete time system **19 / 20**

- **- 0 pts** Correct
- **- 1 pts** Incorrect eigenvalues
- ✓ **- 1 pts Incorrect diagonalization**
- **- 3 pts** Incorrect answer
- **- 10 pts** No use of CH theorem or Diagonalization to generalize solution
- **- 20 pts** No attempt

4.

$$\frac{dx_1}{dt} = -\alpha x_1 + u \qquad \dot{x}_1 = -0.1x_1 + 1 \qquad x_1(0) = 2$$

$$\frac{dx_2}{dt} = \alpha x_1 - \beta x_2 \qquad \dot{x}_2 = 0.1x_1 - 0.2x_2 \qquad x_2(0) = 1$$

$$\dot{x} = \begin{pmatrix} -0.1 & 0 \\ 0.1 & -0.2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u \qquad y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x$$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)} B u(\tau) d\tau = e^{At}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \int_0^t e^{A(t-\tau)} \begin{pmatrix} -0.1 & 0 \\ 0.1 & -0.2 \end{pmatrix} d\tau$$

$$A = \begin{pmatrix} -0.1 & 0 \\ 0.1 & -0.2 \end{pmatrix} \Rightarrow \lambda_1 = -0.1 \qquad \lambda_2 = -0.2$$

CHT: $g(\lambda) = \alpha_1(\lambda) + \alpha_2$

$$\left.\begin{array}{ll} \lambda_1 = -0.1, & e^{-0.1t} = -0.1\alpha_1 + \alpha_2 \\ \lambda_2 = -0.2 & e^{-0.2t} = -0.2\alpha_1 + \alpha_2 \end{array}\right\} \begin{array}{l} \alpha_1 = 10e^{-0.1t} - 10e^{-0.2t} \\ \alpha_2 = 2e^{-0.1t} - e^{-0.2t} \end{array}$$

$$e^{At} = \begin{pmatrix} e^{-0.1t} & 0 \\ e^{-0.1t} - e^{-0.2t} & e^{-0.2t} \end{pmatrix}$$

$$x(t) = \begin{pmatrix} e^{-0.1t} & 0 \\ e^{-0.1t} - e^{-0.2t} & e^{-0.2t} \end{pmatrix} + \int_0^t \begin{pmatrix} e^{-0.1(t-\tau)} & 0 \\ e^{-0.1(t-\tau)} - e^{-0.2(t-\tau)} & e^{-0.2(t-\tau)} \end{pmatrix} d\tau$$

$$= \begin{pmatrix} -8e^{-0.2t} + 10 \\ -8e^{-0.2t} + 4e^{-0.2t} + 5 \end{pmatrix}$$

$$y(t) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x(t) = \begin{pmatrix} -8e^{-0.2t} + 10 \\ -8e^{-0.2t} + 4e^{-0.2t} + 5 \end{pmatrix}$$

when $t = 5$ $\quad y(5) = \begin{pmatrix} -8e^{-1} + 10 \\ -8e^{-1} + 4e^{-1} + 5 \end{pmatrix}$

4.1 y(5) in CT **9 / 10**

- **- 0 pts** Correct
- **- 2 pts** Used DT solution instead of CT
- ✓ **- 2 pts Incorrect answer**
- **- 5 pts** Incorrect quantities
- **- 0.5 pts** Incorrect "C" matrix

+ 1 Point adjustment

1 This is supposed to be -0.1

b)

$$x(k+1) = A_d x(k) + B_d u(k)$$

$$y(k) = C_d x(k) + \cancel{D_d u(k)}$$

$$A_d = e^{AT} = \begin{pmatrix} e^{-0.1} & 0 \\ e^{-0.1} - e^{-0.2} & e^{-0.2} \end{pmatrix}$$

$$B_d = \int_0^T e^{A\tau} d\tau B = \begin{pmatrix} 10e^{-0.1} \\ 10e^{-0.1} - 5e^{-0.2} \end{pmatrix}\Bigg|_0^T = \begin{pmatrix} -10e^{-0.1} + 10 \\ -10e^{-0.1} + 5e^{-0.2} + 5 \end{pmatrix}$$

$$C_d = C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$x(k+1) = \begin{pmatrix} e^{-0.1} & 0 \\ e^{-0.1} - e^{-0.2} & e^{-0.2} \end{pmatrix} x(k) + \begin{pmatrix} -10e^{-0.1} + 10 \\ -10e^{-0.1} + 5e^{-0.2} + 5 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x(k)$$

4.2 Discretization 10 / 10

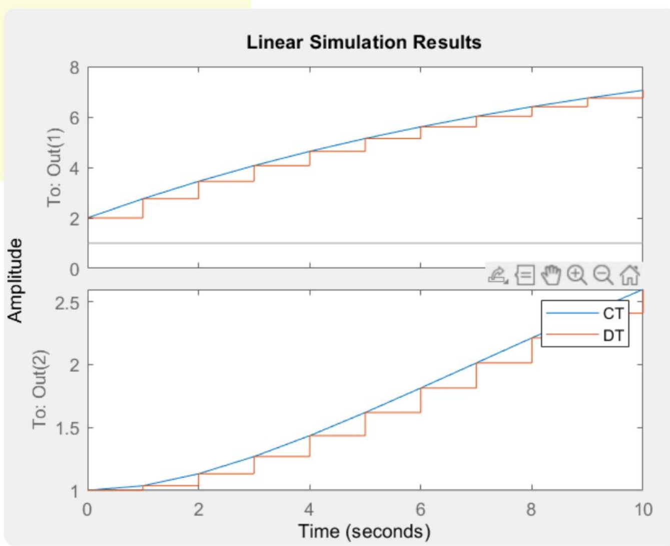
- ✓ **- 0 pts** **Correct**
- **- 1 pts** No "Cd" matrix or incorrect
- **- 2 pts** Incorrect answer
- **- 10 pts** No attempt

c) $y(5) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(5)$

$$y(5) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-0.1} & 0 \\ e^{-0.1} - e^{-0.2} & e^{-0.2} \end{pmatrix}^5 + \sum_{m=0}^{4} Ad^{k-m-1} B \cancel{u(5)}$$

$$= \begin{pmatrix} 5.15 \\ 1.62 \end{pmatrix}$$

```
%Problem 4
%Initializations
a=0.1;
b=0.2;
u=1;
x_ini = [2;1];
t_b = 1;
%CT system
A = [-a 0;a -b];
B = [1;0];
C = [1 0;0 1];
D = 0;
CT = ss(A,B,C,D)
%DT system
AA = [exp(-0.1) 0;exp(-0.1)-exp(-0.2) exp(-0.2)];
BB = [-10*exp(-0.1) + 10;-10*exp(-0.1)+5*exp(-0.2)+5];
CC = C;
DD = 0;
DT = ss(AA,BB,CC,DD,t_b);
%Plotting
t = 0:1:10;  % 201 points
u = ones(size(t));
lsim(CT, u, t, x_ini)
legend('CT', 'DT')
hold on
lsim(DT, u, t, x_ini)
```



4.3 y(5) in DT 10 / 10

- ✓ **- 0 pts** **Correct**
- **- 2 pts** Incorrect answer
- **- 1 pts** Incorrect plot
- **- 3 pts** Missing plot
- **- 10 pts** No attempt

5. a) Transferring to z:

$$C(s) = \frac{\frac{K_d s^2}{2} + \frac{K_p s}{2} + \frac{K_i}{2}}{s^2 + \frac{s}{2}} \Rightarrow \quad u = \frac{K_d}{2}\ddot{z} + \frac{K_p}{2}\dot{z} + \frac{K_i}{2}z$$

$$x = \ddot{z} + \tfrac{1}{2}\dot{z}$$

State space:

$$\dot{m} = Am + Bx = \begin{pmatrix} 0 & 1 \\ 0 & -\frac{1}{2} \end{pmatrix} m + \begin{pmatrix} 0 \\ 1 \end{pmatrix} x$$

$$y = Cm + Dx = \left(\frac{K_i}{2} \;\vdots\; \frac{K_p}{2} - \frac{K_d}{2^2} \right) m + \frac{K_d}{2} x$$

b) $$A = \begin{pmatrix} 0 & 1 \\ 0 & -1000 \end{pmatrix} \qquad \lambda_1 = 0 \;, \quad \lambda_2 = -1000$$

CHT: $\quad g(\lambda) = \alpha_1(\lambda) + \alpha_0$

$$e^0 = \alpha_0 = 1$$

$$e^{-1000t} = \alpha_1(-1000) + 1 \qquad \alpha_1 = \frac{-e^{-1000t} + 1}{1000}$$

$$e^{At} = \begin{pmatrix} 0 & \frac{1 - e^{-1000t}}{1000} \\ 0 & e^{-1000t} \end{pmatrix}$$

$$m(t) = e^{At}\cancel{m(0)} + \int_0^t e^{A(t-\tau)} B\, x(\tau)\, d\tau$$

$$= \int_0^t \begin{pmatrix} 1 - e \\ e^{-1000(t-\tau)} \end{pmatrix} d\tau = \begin{pmatrix} (t + e^{-1000t} - 1)/1000 \\ 1 - e^{-1000t}/1000 \end{pmatrix}$$

$$y(t) = Cm(t) + Dx = (1000 \quad -9900000) \begin{pmatrix} t + e^{-1000t} - 1 \\ 1 - e^{-1000t} \end{pmatrix} \frac{1}{1000} + 0.0001$$

$$= t + e^{-1000t} - 1 - 9900 + 9900e^{-1000t} + 0.0001$$

$$= t + e^{-1000t}(9901) - 9900.99$$

Unbounded because as $t \to \infty$, $y \to \infty$

5.1 PID in state space 10 / 10

- ✓ **- 0 pts** **Correct**
- **- 1 pts** Wrong B matrix
- **- 1 pts** Incorrect answer for output y = Cx + Du
- **- 2 pts** Incorrect answer
- **- 5 pts** No intermediate step to show use of CCF
- **- 10 pts** No attempt

5. a) Transferring to z:

$$C(s) = \frac{\frac{K_d s^2}{2} + \frac{K_p s}{2} + \frac{K_i}{2}}{s^2 + \frac{s}{2}} \Rightarrow \begin{array}{l} u = \frac{K_d}{2}\ddot{z} + \frac{K_p}{2}\dot{z} + \frac{K_i}{2} z \\ x = \ddot{z} + \frac{1}{2}\dot{z} \end{array}$$

State space:

$$\dot{m} = Am + Bx = \begin{pmatrix} 0 & 1 \\ 0 & -\frac{1}{2} \end{pmatrix} m + \begin{pmatrix} 0 \\ 1 \end{pmatrix} x$$

$$y = Cm + Dx = \left(\frac{K_i}{2} \;\vdots\; \frac{K_p}{2} - \frac{K_d}{2^2} \right) m + \frac{K_d}{2} x$$

b) $$A = \begin{pmatrix} 0 & 1 \\ 0 & -1000 \end{pmatrix} \qquad \lambda_1 = 0, \quad \lambda_2 = -1000$$

CHT: $g(\lambda) = \alpha_1 \lambda + \alpha_0$

$$e^0 = \alpha_0 = 1$$

$$e^{-1000t} = \alpha_1(-1000) + 1 \qquad \alpha_1 = \frac{-e^{-1000t} + 1}{1000}$$

$$e^{At} = \begin{pmatrix} 0 & \frac{1 - e^{-1000t}}{1000} \\ 0 & e^{-1000t} \end{pmatrix}$$

$$m(t) = e^{At} \cancel{m(0)} + \int_0^t e^{A(t-\tau)} B x(\tau)\, d\tau$$

$$= \int_0^t \begin{pmatrix} 1 - e \\ e^{-1000(t-\tau)} \end{pmatrix} d\tau = \begin{pmatrix} (t + e^{-1000t} - 1)/1000 \\ 1 - e^{-1000t}/1000 \end{pmatrix}$$

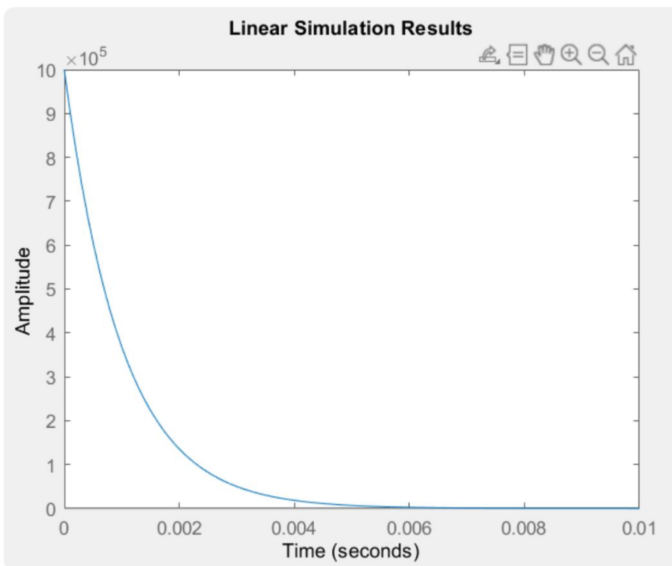
$$y(t) = Cm(t) + Dx = (1000 \quad -9900000) \begin{pmatrix} t + e^{-1000t} - 1 \\ 1 - e^{-1000t} \end{pmatrix} \frac{1}{1000} + 0.0001$$

$$= t + e^{-1000t} - 1 - 9900 + 9900 e^{-1000t} + 0.0001$$

$$= t + e^{-1000t}(9901) - 9900.99$$

Unbounded because as $t \to \infty$, $y \to \infty$

```
%Problem 5
clear all;
%Initialization
uu = 1;
Kd =10;
Kp =100;
Ki =1;
Kd = 1000;
tau =0.001;
%State space
A = [0 1;0 -1/tau];
B = [0;1];
C = [Ki/tau Kp/tau-Kd/tau^2];
D = Kd/tau;
sys = ss(A,B,C,D);
t = 0:0.0001:0.01;
u = ones(size(t));
lsim(sys, u, t)
```



5.2 PID response 8 / 10

- **- 0 pts** Correct
- ✓ **- 2 pts Incorrect plot**
- **- 2 pts** Incorrect conclusion
- **- 10 pts** No attempt