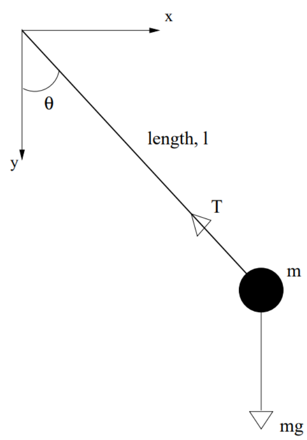
Like all other assignments in the course, this assignment will be submitted via Gradescope. Please come to office hours or reach out on Piazza if you have any questions! Please use livecripts to present your Matlab work, along with screenshots of any Simulink models.

1: 30 points

Consider the simple pendulum system shown below with a motor at the hinge that produces torque τ. The system dynamics are easy to derive from first principles.

$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{\tau}{ml^2}$$



(a) Linearize the differential equation about equilibrium point $\theta = \frac{\pi}{2}$. Analyze the stability of the system in this configuration.

(b) Assume the mass is 1 kg and the length is 4 m. Design a PD controller (maps angle error to motor torque) that places the closed loop poles such that the closed loop system nominally has a 25% overshoot and a 2 second 2% settling time.

(c) Construct the nonlinear dynamics in Simulink and apply the controller designed in part (b). Plot the initial condition response starting at rest from $\theta_0 = 45^o$. For what range of initial conditions does the PD controller stabilize the nonlinear system? No need to derive this analytically - you can use the simulation to answer this. Be sure to include an image of your Simulink model in the solution.

2: 20 points

A system has an input $u(t)$ and an output $y(t)$ which are related by the information provided below. Classify each system as linear or nonlinear and time invariant or time varying.

(a) $y(t) = a,\ a \neq 0\ \forall t$

(b) $y(t) = -3u(t) + 2$

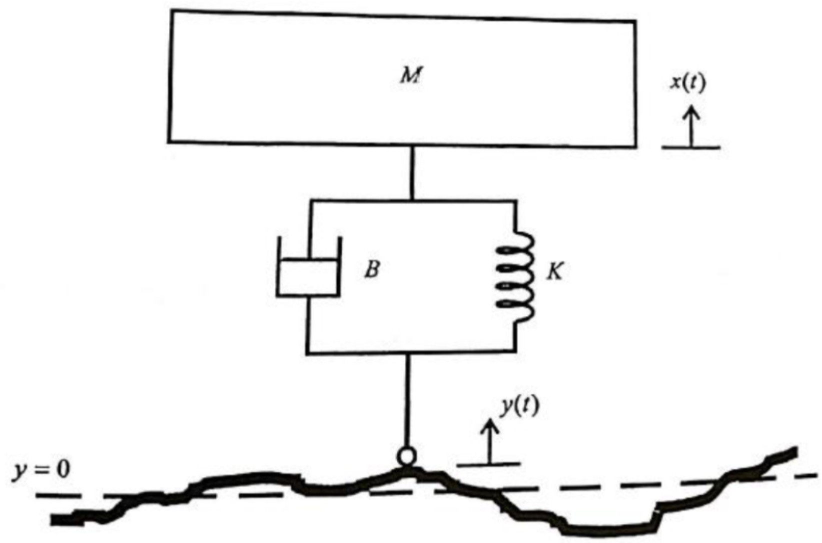
(c) $y(t) = u^3(t)$

(d) $y(t) = u(t^3)$

(e) $y(t) = e^{-t}u(t - T)$

3: 20 points

The figure below shows a model commonly used for automobile suspension analysis. In the model, the uneven ground specifies the position of the wheel's contact point. The wheel itself is not shown, as its mass is considered to be negligible relative to the mass of the rest of the car.



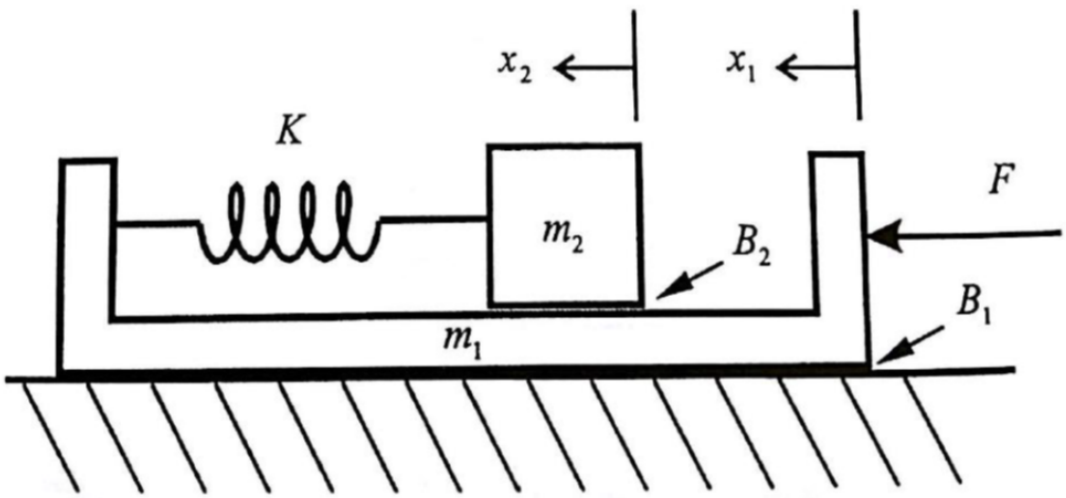
(a) Write a differential equation and a state variable description for this system, considering the height of the car $x(t)$ to be the output and the road height $y(t)$ to be the input.

(b) Using the parameters $M = 300$ kg, $K = 20000$ N/m, and $B = 1000$ N·s/m, use Matlab to plot the response of the system to

- $y(t)$ is a step of size 0.15 m (simulates hitting a curb). HINT: You can use the *step* command.
- $y(t)$ is a 1/2 sine wave of amplitude 0.08 m and frequency 50 Hz (simulates hitting a speed bump at 30 mph). Be sure to pad your $y(t)$ with zeros after the 1/2 sine to let the system ring out. HINT: You can use the *lsim* command.

4: 20 points

For the mechanical system shown below, friction between surfaces is modeled as viscous damping with damping coefficients denoted by B_i. Use the principles of dynamics to find the equations of motion, then create state space realizations using the following states.



(a) State variables $\mathbf{x} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}$ and output variable $y = x_2$.

(b) State variables $\mathbf{x} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 - x_1 \\ \dot{x}_2 - \dot{x}_1 \end{bmatrix}$ and output variable $y = x_2 - x_1$.

5: 10 points

Consider the state space system given by

$$\dot{\mathbf{x}} = \begin{bmatrix} 18 & 9 & 13 \\ 50 & 23 & 35 \\ -65 & -31 & -46 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} u(t)$$

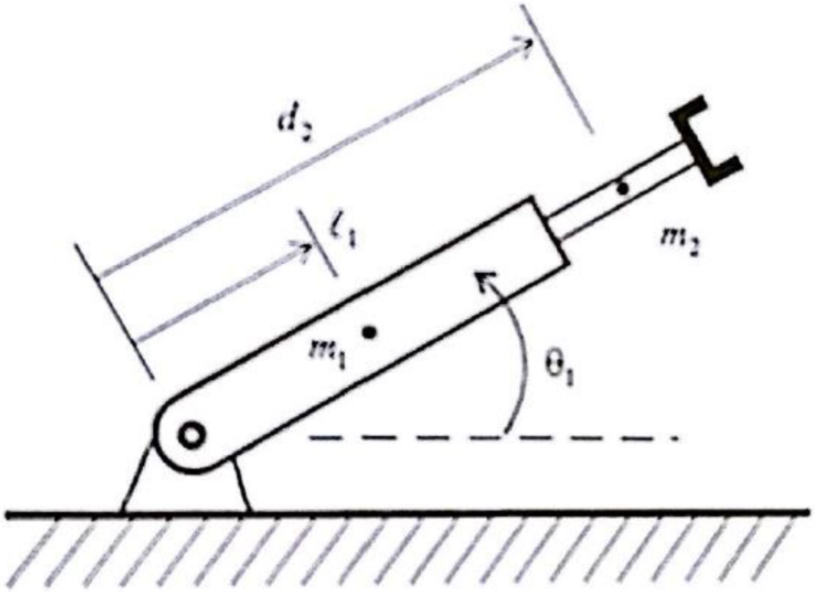
$$y(t) = \begin{bmatrix} 5 & -5 & 5 \end{bmatrix} \mathbf{x}.$$

Write the system equations in terms of the new state variables

$$\hat{\mathbf{x}} = \begin{bmatrix} -4x_1 - 2x_2 - 3x_3 \\ 15x_1 + 7x_2 + 10x_3 \\ -5x_1 - 2x_2 - 3x_3 \end{bmatrix}.$$

6: 20 points

The robot shown in the figure below has the equations of motion given. Symbols $m_1,\ m_2,\ I_1,\ I_2,\ l_1,$ and g are constant parameters, representing the characteristics of the rigid body links. Quantities θ_1 and d_2 are the coordinate variables and are functions of time. The robot is actuated by two inputs τ_1 and τ_2, which control the rotational and prismatic joints.



$$\left(m_1 l_1^2 + I_1 + I_2 + m_2 d_2^2\right)\ddot{\theta}_1 + 2m_2 d_2 \dot{\theta}_1 \dot{d}_2 + (m_1 l_1 + m_2 d_2)\, g\cos\theta_1 = \tau_1$$
$$m_2 \ddot{d}_2 - m_2 d_2 \dot{\theta}_1^2 + m_2 g \sin\theta_1 = \tau_2$$

(a) Write the differential equations in state space form.

(b) Linearize the system the operating point $\theta_1 = \dot{\theta}_1 = \dot{d}_2 = \ddot{d}_2 = 0$ and $d_2 = \bar{d}$, a constant.