Question 5-Assignment 3

Page 656: 7b

Functions

NewtonMethodnLinear.m | functionEquation.m | jacobian.m

```
function Y = functionEquation(x)
    Y = zeros(2,1);

    % the two defined functions
    Y(1) = log(x(1)*x(1)+x(2)*x(2))-sin(x(1)*x(2))-log(2)-log(pi);
    Y(2) = exp(x(1)-x(2))+cos(x(1)*x(2));
```

Jacobian

NewtonMethodnLinear.m | functionEquation.m | jacobian.m

```
function Y = jacobian(x)
    Y = zeros(2,2);

    % the jacobian of the two functions
    Y(1,1) = (2*x(1))/(x(1)*x(1)+x(2)*x(2))-x(2)*cos(x(1)*x(2));
    Y(1,2) = (2*x(2))/(x(1)*x(1)+x(2)*x(2))-x(1)*cos(x(1)*x(2));

    Y(2,1) = exp(x(1)-x(2))-x(2)*sin(x(1)*x(2));
    Y(2,2) = -1*exp(x(1)-x(2))-x(1)*sin(x(1)*x(2));
```

Main Method

NewtonMethodnLinear.m | functionEquation.m | jacobian.m

```
F = @functionEquation;
J = @jacobian;

% The procedure from the textbook (page 653)
x0 = [2;2]; % initial guess
A = J(x0);
z0 = -F(x0);
y0 = linsolve(A,z0);
x1 = x0 + y0;
norm(y0,2);

for j = 1:15
    x0 = x1;
    A = J(x0);
    z0 = -F(x0);
    y0 = linsolve(A,z0);
    x1 = x0 + y0;
    err = norm(y0,2);

    fprintf(' %3d %15.10f %15.10f %15.10f \n',j,x1(1),x1(2),err);
    if err < 0.000001 % error bound
        break;
    end
end
```

Outputs

Tolerance: 10^{-5}

Initial guess x=[2,2]

```
x0 =

     2
     2

   1    1.8300800407    1.7090238434    0.2686356127
   2    1.7755574721    1.7684117059    0.0806202748
   3    1.7724654734    1.7724385985    0.0050770385
   4    1.7724538511    1.7724538507    0.0000191757
   5    1.7724538509    1.7724538509    0.0000000003
```

Initial guess x=[-2,-2]

```
x0 =

    -2
    -2

   1   -1.7090238434   -1.8300800407    0.2686356127
   2   -1.7684117059   -1.7755574721    0.0806202748
   3   -1.7724385985   -1.7724654734    0.0050770385
   4   -1.7724538507   -1.7724538511    0.0000191757
   5   -1.7724538509   -1.7724538509    0.0000000003
```

Initial guess x=[10.2]

```
x0 =

    10
     2

   1    9.2169292151    3.2203257084    0.7539830049
   2    6.9433837596    1.8985113850    2.6298673054
   3    6.3535262605    2.3075134025    0.7177844520
   4    5.4080884103    2.3794514512    0.9481707713
   5    4.8726908278    2.8703794186    0.7264026710
   6    5.7670842672    3.8228092642    1.3065459177
   7    5.0586124998    3.9561218595    0.7209053289
   8    5.1908688548    4.3906327762    0.4541932190
   9    5.0777858611    3.7754197417    0.6255196571
  10    4.6203202512    4.4361800316    0.8036659412
  11    0.6314813141    6.8418149595    4.6581021320
  12    1.6801080542   -3.9091570175   10.8019913206
  13    2.0574994525   -2.5272898856    1.4324737477
  14   16.2195862023   12.6801749730   20.7805603518
  15   14.8010098840   10.9110649481    2.2676218934
>>
```

Does not converge to a solution. Unstable

Tolerance: 10^{-8}

Initial guess x=[10,10]

```
   1 -205.3059624655  339.8124842368  390.8820618857
   2 -1215.4228295657 -1332.0574205507 1953.3266659047
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  2.492180e-48.
> In NewtonMethodnLinear (line 16)
   3 -1215.9060060075 -1331.5405969925    0.7075069358
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  4.218419e-48.
> In NewtonMethodnLinear (line 16)
   4 -1216.3744994524 -1331.0090904374    0.7085092279
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  2.515333e-48.
> In NewtonMethodnLinear (line 16)
   5 -1216.8058737141 -1330.4404646990    0.7137359343
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  5.668996e-47.
> In NewtonMethodnLinear (line 16)
   6 -1217.2887627786 -1329.9233537636    0.7075207193
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.444170e-46.
> In NewtonMethodnLinear (line 16)
   7 -1217.7720250347 -1329.4066160197    0.7075028651
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.413984e-46.
> In NewtonMethodnLinear (line 16)
   8 -1218.2643390623 -1328.8989300473    0.7071903197
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.010264e-45.
> In NewtonMethodnLinear (line 16)
   9 -1218.7486574701 -1328.3832484551    0.7074544683
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.498329e-45.
> In NewtonMethodnLinear (line 16)
  10 -1219.2171617804 -1327.8517527653    0.7085082617
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.528561e-46.
> In NewtonMethodnLinear (line 16)
  11 -1219.9589145118 -1327.5935054968    0.7854226673
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.491869e-44.
> In NewtonMethodnLinear (line 16)
  12 -1220.4461767493 -1327.0807677342    0.7073362010
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.020204e-44.
> In NewtonMethodnLinear (line 16)
  13 -1220.9544056248 -1326.5889966097    0.7072025373
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.691403e-43.
> In NewtonMethodnLinear (line 16)
  14 -1221.4388260198 -1326.0734170048    0.7074499616
'arning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  4.322001e-43.
> In NewtonMethodnLinear (line 16)
  15 -1221.9236439463 -1325.5582349312    0.7074326757
>
```

Initial guess x=[-10,10]

```
    1 -339.8124842368  205.3059624656  390.8820618858
    2 1332.0574880736 1215.4228703608 1953.3267447944
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  2.650430e-48.
> In NewtonMethodnLinear (line 16)
    3 1331.5402499929 1215.9056322801    0.7075268920
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  2.551238e-48.
> In NewtonMethodnLinear (line 16)
    4 1331.0319941853 1216.3973764725    0.7072031651
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  2.083738e-47.
> In NewtonMethodnLinear (line 16)
    5 1330.5146439664 1216.8800262536    0.7075323740
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  5.289050e-47.
> In NewtonMethodnLinear (line 16)
    6 1329.9977309357 1217.3631132229    0.7075112022
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  4.288157e-47.
> In NewtonMethodnLinear (line 16)
    7 1329.4928541672 1217.8582364544    0.7071404144
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  2.435566e-46.
> In NewtonMethodnLinear (line 16)
    8 1328.9792880072 1218.3446702944    0.7073670062
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.139868e-45.
> In NewtonMethodnLinear (line 16)
    9 1328.4524168346 1218.8177991218    0.7081271919
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  2.056942e-45.
> In NewtonMethodnLinear (line 16)
   10 1327.9235559502 1219.2889382374    0.7082837717
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  4.942354e-45.
> In NewtonMethodnLinear (line 16)
   11 1327.3939683293 1219.7593506165    0.7083437404
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  2.218812e-44.
> In NewtonMethodnLinear (line 16)
   12 1326.8782720240 1220.2436543112    0.7074551208
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  4.470964e-44.
> In NewtonMethodnLinear (line 16)
   13 1326.3641515428 1220.7295338300    0.7073887022
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  9.441205e-44.
> In NewtonMethodnLinear (line 16)
   14 1325.8532576929 1221.2186399801    0.7072745944
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.266262e-43.
> In NewtonMethodnLinear (line 16)
   15 1325.3528530490 1221.7182353362    0.7071070127
x >>
```

Initial guess x=[-50,50]

```
   1 3860750401370626.0000000000 3860750401373430.5000000000 1911.9451300729
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  4.549611e-18.
> In NewtonMethodnLinear (line 16)
   2 3860750401368984.5000000000 3860750401375072.0000000000 2321.5844000727
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  9.209327e-18.
> In NewtonMethodnLinear (line 16)
   3 3860750401368173.5000000000 3860750401375883.0000000000 1146.9141219488
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  3.358817e-18.
> In NewtonMethodnLinear (line 16)
   4 3860750401370397.0000000000 3860750401373659.5000000000 3144.6513028162
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  4.077619e-19.
> In NewtonMethodnLinear (line 16)
   5 3860750401352080.5000000000 3860750401391976.0000000000 25903.1209238882
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  5.498548e-18.
> In NewtonMethodnLinear (line 16)
   6 3860750401353439.0000000000 3860750401390617.5000000000 1920.9266999282
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  3.132312e-18.
> In NewtonMethodnLinear (line 16)
   7 3860750401351054.5000000000 3860750401393002.0000000000 3372.0486201696
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  5.630519e-18.
> In NewtonMethodnLinear (line 16)
   8 3860750401349728.0000000000 3860750401394328.5000000000 1875.9030716810
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  5.310687e-18.
> In NewtonMethodnLinear (line 16)
   9 3860750401348321.5000000000 3860750401395735.0000000000 1988.8775282583
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  2.377349e-18.
> In NewtonMethodnLinear (line 16)
  10 3860750401351463.0000000000 3860750401392593.5000000000 4442.8930281643
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  8.403058e-18.
> In NewtonMethodnLinear (line 16)
  11 3860750401352352.0000000000 3860750401391704.5000000000 1256.9599694975
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.330448e-17.
> In NewtonMethodnLinear (line 16)
  12 3860750401351790.5000000000 3860750401392266.0000000000  793.8907683113
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  5.787321e-19.
> In NewtonMethodnLinear (line 16)
  13 3860750401338885.5000000000 3860750401405171.0000000000 18250.7704427080
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  3.532089e-18.
> In NewtonMethodnLinear (line 16)
  14 3860750401336771.0000000000 3860750401407285.5000000000 2990.3852523476
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  1.274925e-17.
> In NewtonMethodnLinear (line 16)
  15 3860750401336185.0000000000 3860750401407871.5000000000  828.4647915355
```

Please note: I downloaded Alg 10.1 in C and edited the code to the given functions and jacobians. Some of the initial guesses were diverging, so I thought that there was something wrong with their implementation. To test this hypothesis, I used the MATLAB implementation given on this link: https://docs.google.com/viewer?a=v&pid=sites&srcid=ZGVmYXVsdGRvbWFpbnxudW1lcmljYWxhbmFseXNpczFidXJkZW58Z3g6NjEyNWYzMmQzYjc5ODQwOQ

I tried this implementation (pasted above) which lead to the same results. One interesting thing to note is that this can be associated with the fact that the gaussian elimination results in a singular matrix.