Probability and Statistics, Spring 2018

Homework 5

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5.1.3

$$\begin{aligned}P[x_1 \le X < x_2 \cup y_1 \le Y < y_2] &= [F_X(x_2) - F_X(x_1)] + [F_Y(y_2) - F_Y(y_1)] \\ &\quad - [F_{X,Y}(x_2, y_2) - F_{X,Y}(x_1, y_2) - (F_{X,Y}(x_2, y_1) - F_{X,Y}(x_1, y_1))] \\ &= F_X(x_2) - F_X(x_1) + F_Y(y_2) - F_Y(y_1) \\ &\quad - F_{X,Y}(x_2, y_2) + F_{X,Y}(x_1, y_2) + F_{X,Y}(x_2, y_1) - F_{X,Y}(x_1, y_1).\end{aligned}$$

5.2.1 (a) $\sum_{x=1,2,4;y=1,3} cxy = 1 \to 28c = 1, c = 1/28.$

(b) $P[Y < X] = \frac{1}{28}(2 \cdot 1 + 4 \cdot 1 + 4 \cdot 3) = \frac{18}{28} = \frac{9}{14}$.

(c) $P[Y > X] = \frac{1}{28}(1 \cdot 3 + 2 \cdot 3) = \frac{9}{28}$.

(d) $P[Y = X] = \frac{1}{28}(1 \cdot 1) = \frac{1}{28}$.

(e) $P[Y = 3] = \frac{1}{28}(1 + 2 + 4) \cdot 3 = \frac{21}{28} = \frac{3}{4}$.

5.3.4 $P_X(x) = \begin{cases} \frac{1}{21} & x = 0 \\ \frac{2}{21} & x = 1 \\ \frac{3}{21} & x = 2 \\ \frac{4}{21} & x = 3 \\ \frac{5}{21} & x = 4 \\ \frac{6}{21} & x = 5. \end{cases}$

$P_Y(y) = \begin{cases} \frac{6}{21} & y = 0 \\ \frac{5}{21} & y = 1 \\ \frac{4}{21} & y = 2 \\ \frac{3}{21} & y = 3 \\ \frac{2}{21} & y = 4 \\ \frac{1}{21} & y = 5. \end{cases}$

$E[X] = 0 + \frac{2}{21} + \frac{6}{21} + \frac{12}{21} + \frac{20}{21} + \frac{30}{21} = \frac{70}{21} = \frac{10}{3}$.

$E[Y] = 0 + \frac{5}{21} + \frac{8}{21} + \frac{9}{21} + \frac{8}{21} + \frac{5}{21} = \frac{35}{21} = \frac{5}{3}$.

5.6.1 (a)

	$d = 20$	$d = 100$	$d = 300$
$w = 1$	0.2	0.2	0.2
$w = 1$	0.1	0.2	0.1

(b) $E[D] = (0.2 + 0.1) \cdot 20 + (0.2 + 0.2) \cdot 100 + (0.2 + 0.1) \cdot 300 = 136$ (miles).

(c) $P_{W,D}(1, 20) = 0.2 \neq P_W(1) \cdot P_D(2) = 0.6 \cdot 0.3 = 0.18$. Thus W and D are not independent.

5.8.2 (a) $E[W] = E[Y/X] = \frac{1}{28} \cdot \frac{1}{1} + \frac{2}{28} \cdot \frac{1}{2} + \frac{4}{28} \cdot \frac{1}{4} + \frac{3}{28} \cdot \frac{3}{1} + \frac{6}{28} \cdot \frac{3}{2} + \frac{12}{28} \cdot \frac{3}{4} = \frac{30}{28} = \frac{15}{14}$.

(b) $r_{X,Y} = E[XY] = \frac{1}{28} \cdot 1 \cdot 1 + \frac{2}{28} \cdot 2 \cdot 1 + \frac{4}{28} \cdot 4 \cdot 1 + \frac{3}{28} \cdot 1 \cdot 3 + \frac{6}{28} \cdot 2 \cdot 3 + \frac{12}{28} \cdot 4 \cdot 3 = \frac{210}{28} = \frac{15}{2}$.

(c) $\mathrm{Cov}[X, Y] = E[(X - \mu_X)(Y - \mu_Y)] = E[(X - 3)(Y - \frac{5}{2})] = 0$.

(d) $\rho_{X,Y} = \frac{\mathrm{Cov}[X,Y]}{\sigma_X \sigma_Y} = 0$

(e) $\text{Var}[X+Y] = E[(X+Y)^2] - (E[X+Y])^2 = \frac{227}{7} - \frac{121}{4} = \frac{61}{28}$.

5.9.7 (a)

$$\begin{aligned} P[I] &= P[W > 10, T > 38] \\ &= P[W > 10] \cdot P[T > 38] \\ &= (1 - \Phi(\frac{10-7}{2})) \cdot (1 - \Phi(\frac{38-37}{1})) \\ &= 0.0668 \cdot 0.1587 = 0.0106. \end{aligned}$$

(b)

$$\begin{aligned} P[I \mid T = t] &= P[W > 10 \mid T = t] \\ &= 1 - \Phi(\frac{x-\mu}{\sigma}) \\ &= 1 - \Phi(\frac{10 - 7 - \sqrt{2}(t-37)}{\sqrt{2}}) \\ &= Q(\frac{3\sqrt{2}}{2} - t + 37). \end{aligned}$$

5.10.8 (a)

$$\begin{aligned} (1 - \Phi(\frac{t - E(T)}{\sigma_T}))^{10} &= (1 - \Phi(\frac{25-35}{5}))^{10} \\ &= (1 - \Phi(-2))^{10} \\ &= \Phi(2)^{10} = 0.9725^{10} = 0.7944. \end{aligned}$$

(b) $1 - \Phi(\frac{50-35}{5})^{10} = 1 - \Phi(3)^{10} = 0.0134$.

(c) $1 - (1 - \Phi(\frac{0-35}{5})^{10}) = 1 - \Phi(7)^{10}$.