Digital Image Processing

Lecture #10

Ming-Sui (Amy) Lee

Course Information

- **Following Schedule**

03/28	Lecture 4	05/16	Proposal
04/04	溫書假	05/23	Lecture 9
04/11	Lecture 5	05/30	Lecture 10
04/18	Lecture 6	06/06	Lecture 11
04/25	Midterm	06/13	Demo
05/02	Lecture 7	06/20	Demo
05/09	Lecture 8	06/27	Final Package Due

Image Restoration

Image Restoration

- Attempt to reconstruct or recover an image that has been degraded
- Model the degradation and apply the inverse process

- A priori modeling
  - measurements on the physical imaging system, digitizer and display
- A posteriori modeling
  - measurements of a particular image to be restored

Image Restoration

- Forward Problem
  - Given X & H → find Y
- System Identification Problem
  - Given X & Y → find H
- Inverse Problem
  - Given H & Y → find X

⇨ Image enhancement v.s. image restoration

Image Restoration

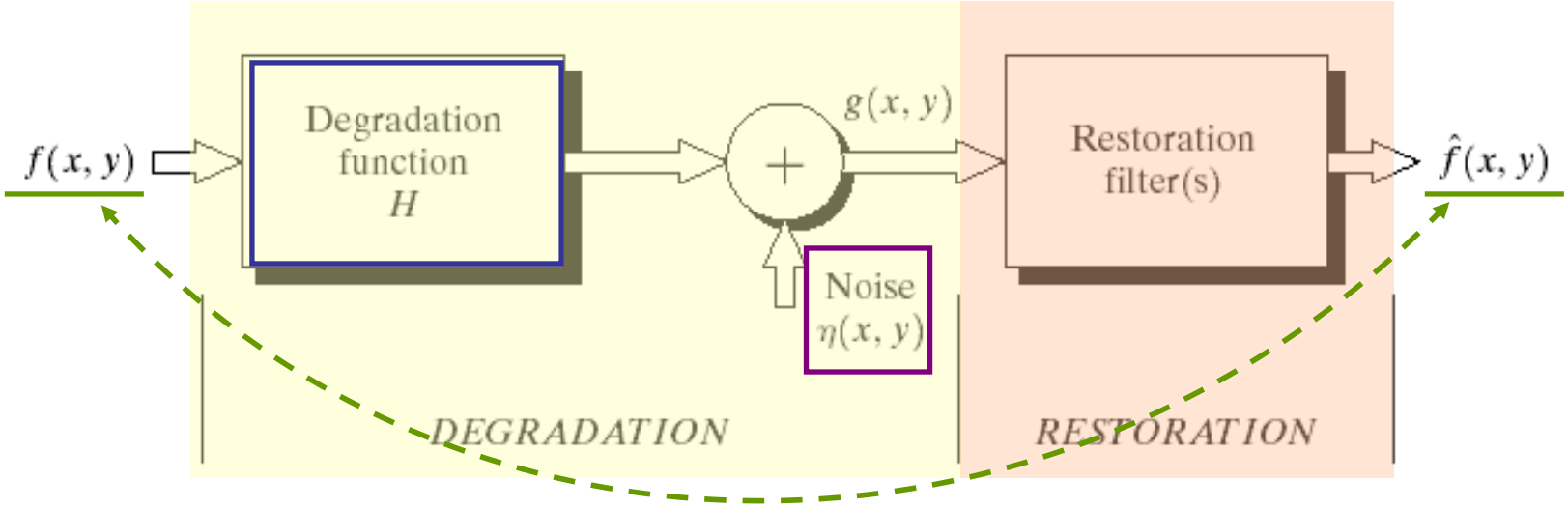
- Image Enhancement v.s. Image Restoration
  - Image enhancement
    - Largely subjective
    - No benchmark
    - Basically a heuristic procedure
  - Image restoration
    - More objective
    - With a ground truth
    - Usually formulate a criterion of goodness

Image Restoration

- Image observation models
  - An acquired image could be degraded by the sensing environment
    - Sensor noise
    - Optical system aberrations
    - Image motion blur
    - Atmospheric turbulence effects
  - Model the degradation and apply the inverse process
    - A priori modeling
    - A posteriori modeling

Image Restoration

- **Image observation models**



$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$

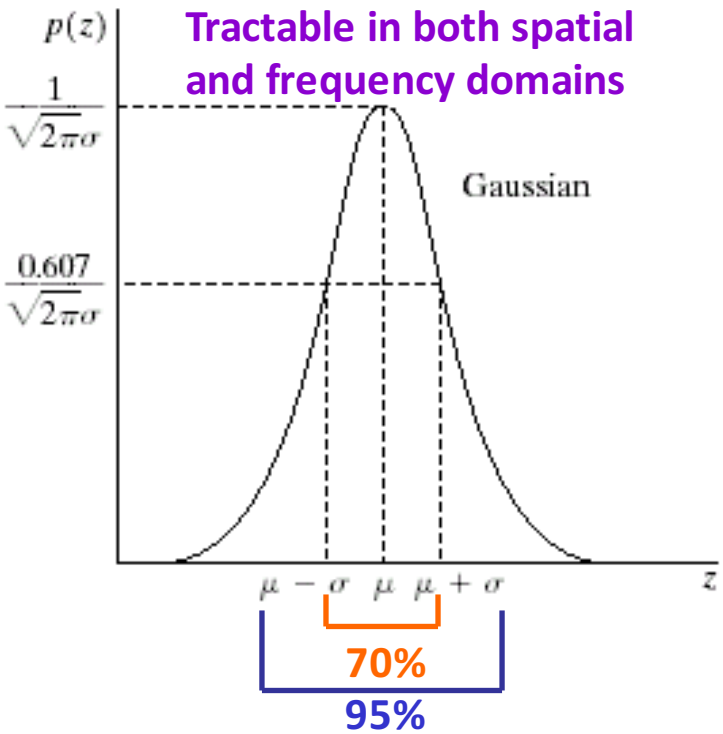
Image Restoration

- Noise models
  - Gaussian Noise
  - Rayleigh Noise
  - Gamma (Erlang) Noise
  - Exponential Noise
  - Uniform Noise
  - Impulse (Salt-and-Pepper) Noise

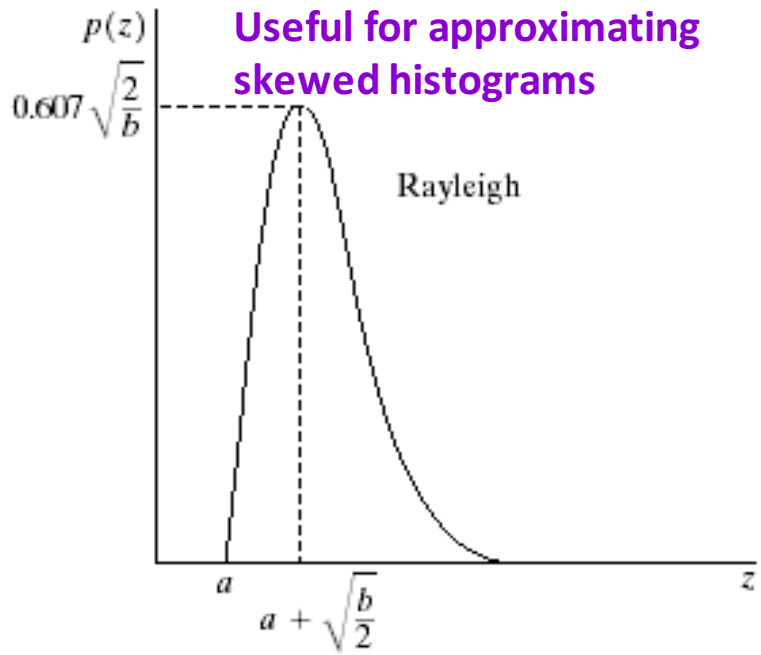
//Note// assume that noise is independent of spatial coordinates and uncorrelated w.r.t. the image itself (except periodic noise)

Image Restoration

- Gaussian noise v.s. Rayleigh noise



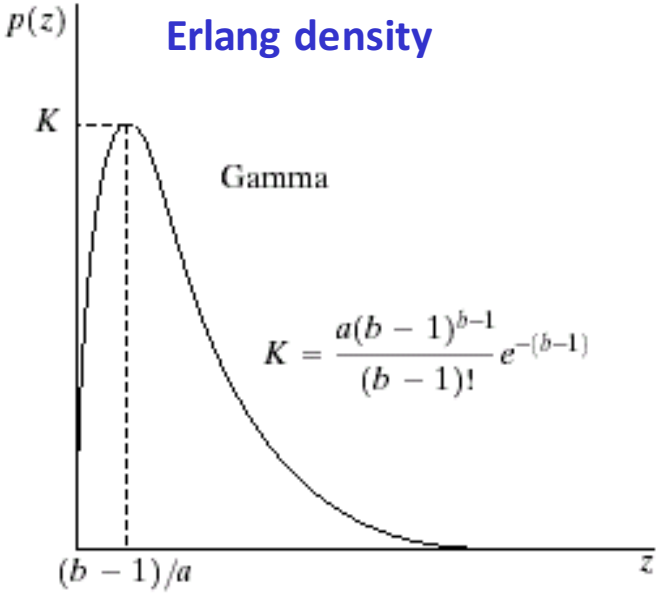
$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

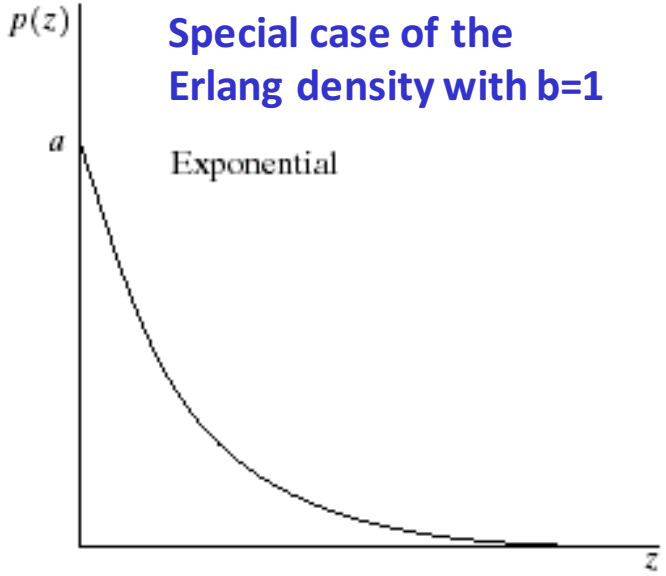


$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & for\ z \geq a \\ 0 & for\ z < a \end{cases}$$

Image Restoration

- **Gamma noise v.s. Exponential noise**



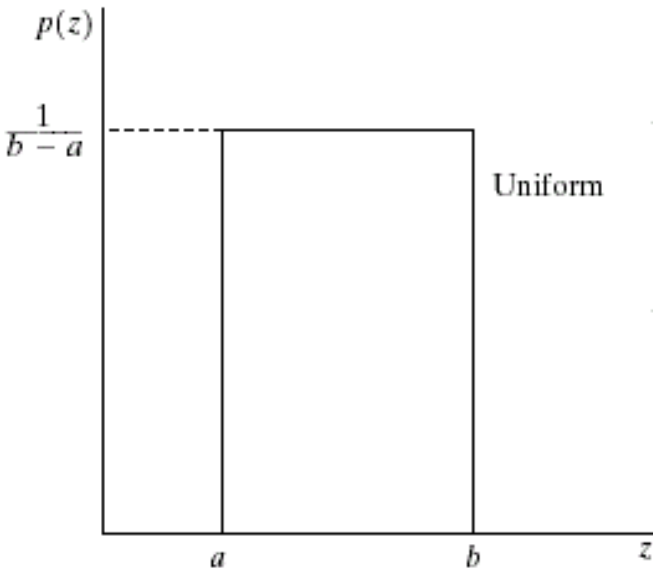


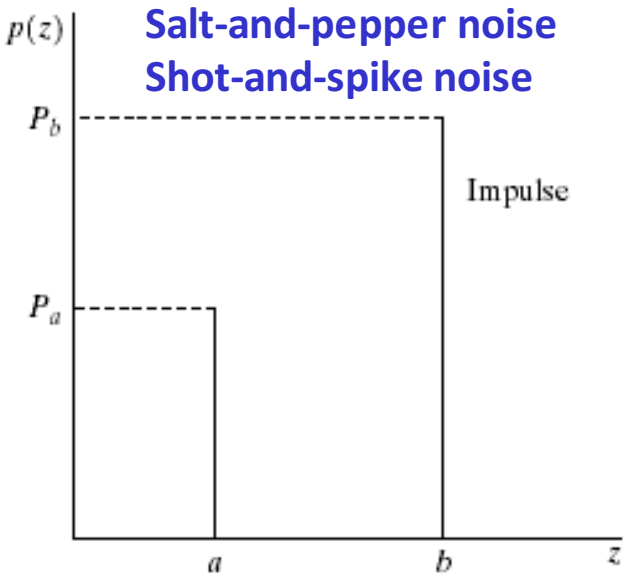
$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & for\ z \geq 0 \\ 0 & for\ z < 0 \end{cases}$$

$$p(z) = \begin{cases} ae^{-az} & for\ z \geq 0 \\ 0 & for\ z < 0 \end{cases}$$

Image Restoration

- **Uniform noise v.s. Impulse noise**





$$p(z) = \begin{cases} \dfrac{1}{b-a} & if\ a \leq\ z \leq b \\ 0 & otherwise \end{cases}$$

$$p(z) = \begin{cases} P_a & for\ z = a \\ P_b & for\ z = b \\ 0 & otherwise \end{cases}$$

Image Restoration

- Noise models
  - Gaussian Noise
    - Electronic noise and sensor noise due to poor illumination and/or high temperature
  - Rayleigh Noise
    - Helpful in characterizing noise phenomena in range imaging
  - Gamma (Erlang) Noise & Exponential Noise
    - Applications in laser imaging
  - Uniform Noise
    - As basis for random number generators
  - Impulse (Salt-and-Pepper) Noise
    - Quick transient (faulty switching) during imaging

Image Restoration

- Test pattern
  - Three-leveled gray-scale image

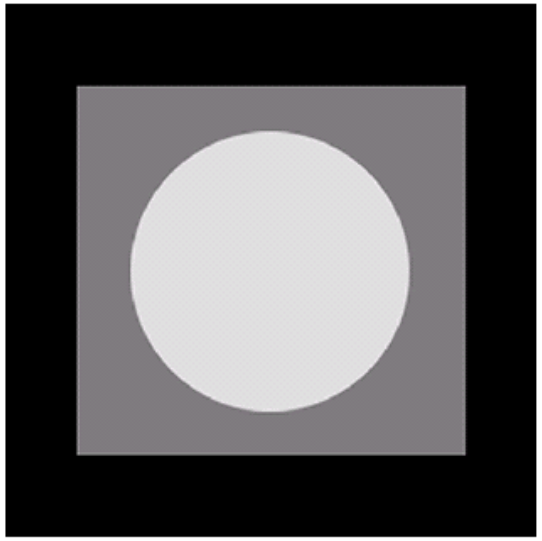


Image Restoration

- Experimental results

image

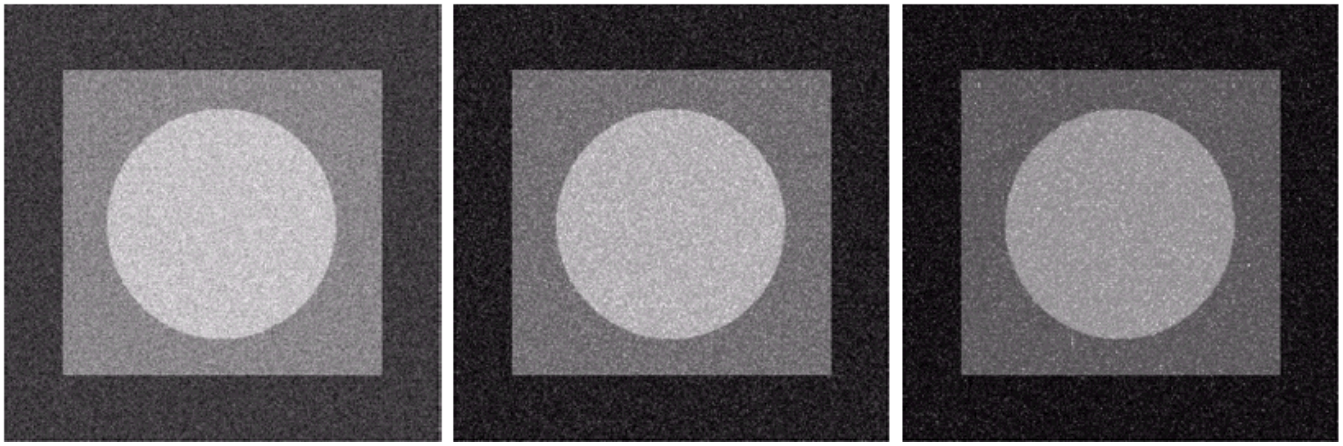
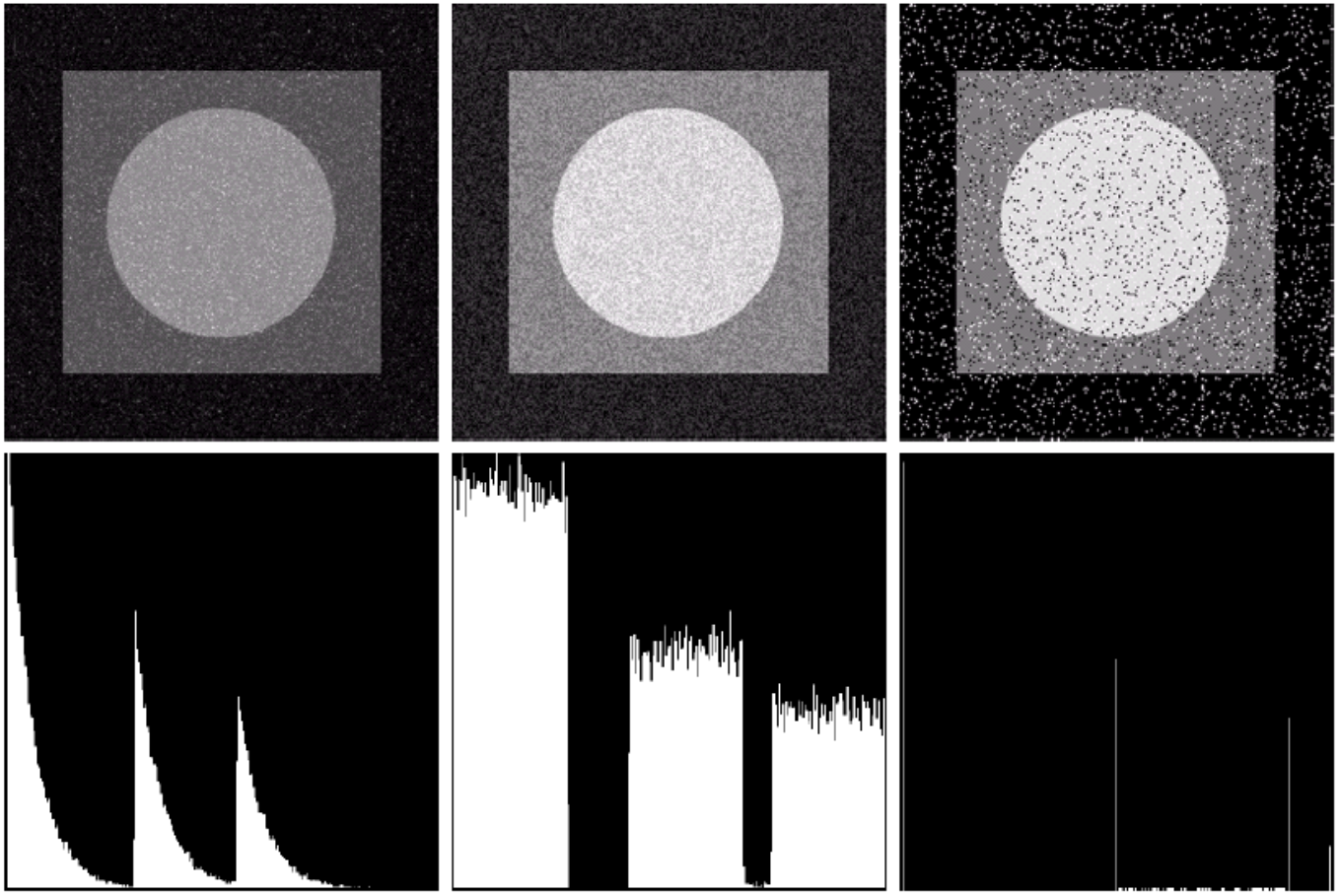


Image Restoration

- Experimental results

image

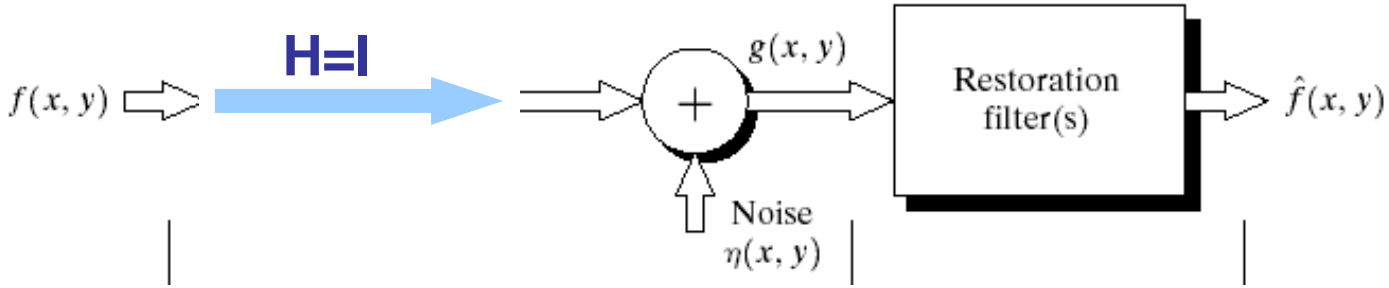
histogram



Exponential Uniform Salt & Pepper

Image Restoration

- **Restoration with only additive noise**



$$g(x,y) = f(x,y) + \eta(x,y)$$

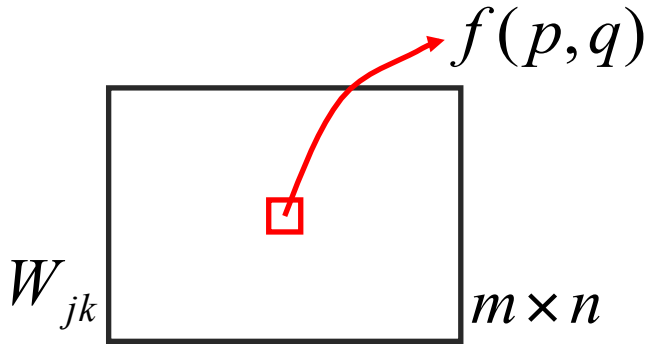
$$G(u,v) = F(u,v) + N(u,v)$$

- **Spatial filtering: three solutions**
  - **Mean filters**
  - **Order-statistics filters**
  - **Adaptive filters**

Image Restoration

- **Mean filters**
  - **Arithmetic mean filter**

$$g(j,k) = \frac{1}{mn} \sum_{(p,q)\in W_{jk}} f(p,q)$$



Smooth local variations as a result of blurring

  - **Geometric mean filter**

$$g(j,k) = \left[\prod_{(p,q)\in W_{jk}} f(p,q) \right]^{\frac{1}{mn}}$$

Tend to lose less image detail in the process

Image Restoration

- **Example**
  - **Arithmetic mean filter v.s. Geometric mean filter**

lose less image detail

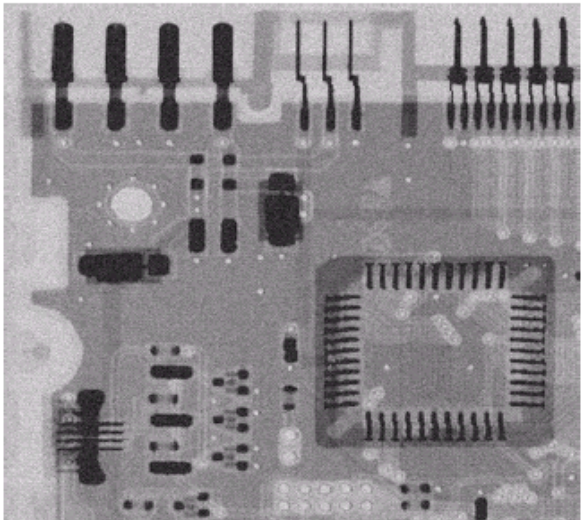
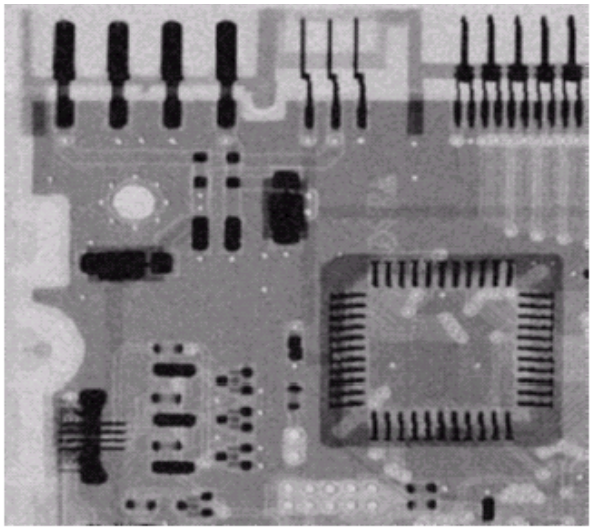
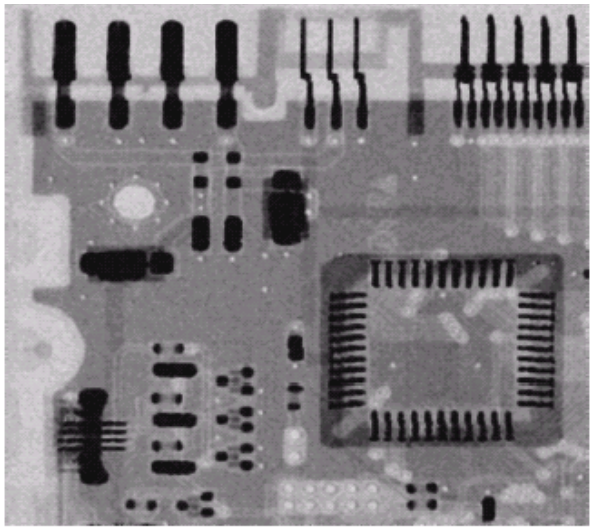


Image corrupted by additive Gaussian noise



Arithmetic mean filter Of size 3x3



Geometric mean filter Of size 3x3

Image Restoration

- **Mean filters**
  - **Harmonic mean filter**

$$g(j,k) = \frac{mn}{\sum_{(p,q)\in W_{jk}} \frac{1}{f(p,q)}}$$

→Good for salt noise
→Gaussian noise
→Fails for pepper noise

  - **Contraharmonic mean filter**

$$g(j,k) = \frac{\sum_{(p,q)\in W_{jk}} f(p,q)^{Q+1}}{\sum_{(p,q)\in W_{jk}} f(p,q)^{Q}}$$

order

→Q>0: pepper noise
Q<0: salt noise
→Q=0? →Arithmetic
→Q=-1? →Harmonic

Image Restoration

- Example
  - Contraharmonic mean filter with order=1.5

    (eliminate pepper noise)

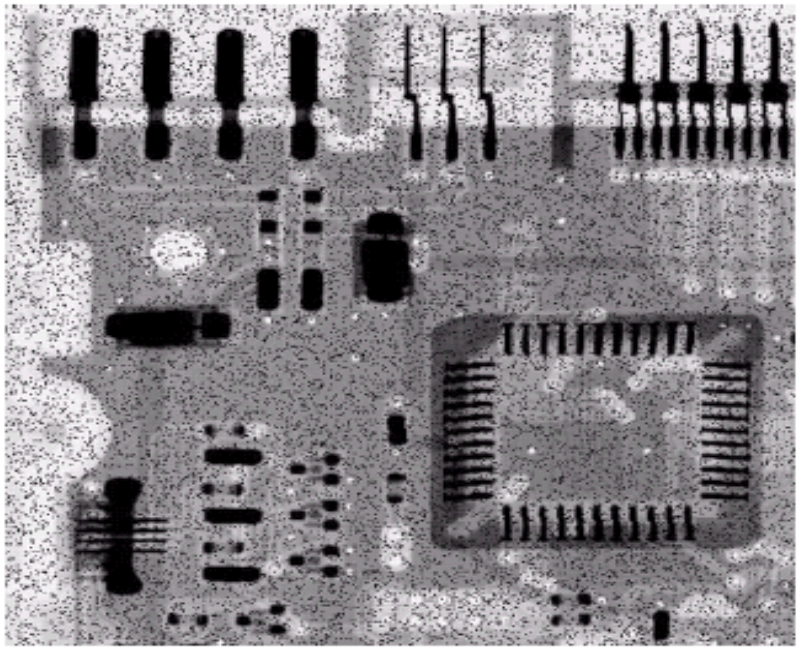
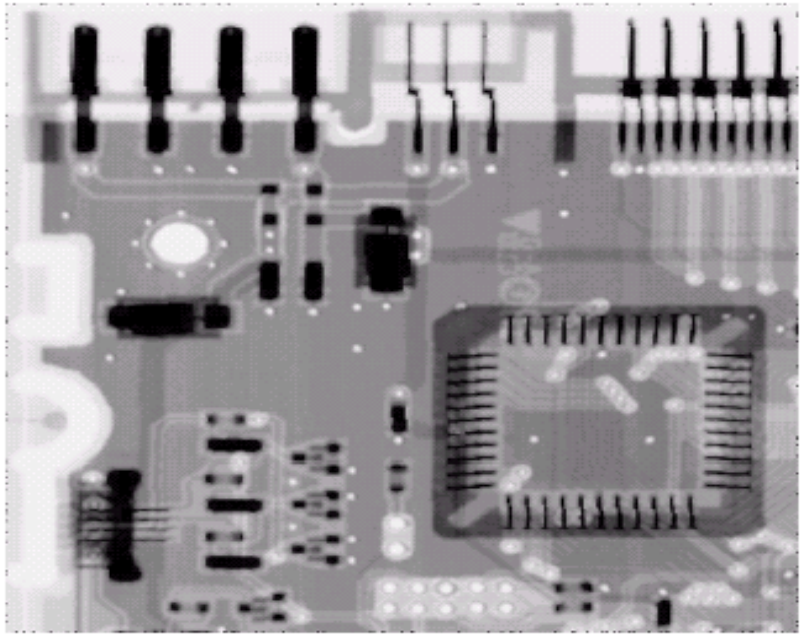


Image corrupted by pepper noise



3x3 contraharmonic filter of order 1.5

Image Restoration

- Example
  - Contraharmonic mean filter with order=-1.5 (eliminate salt noise)

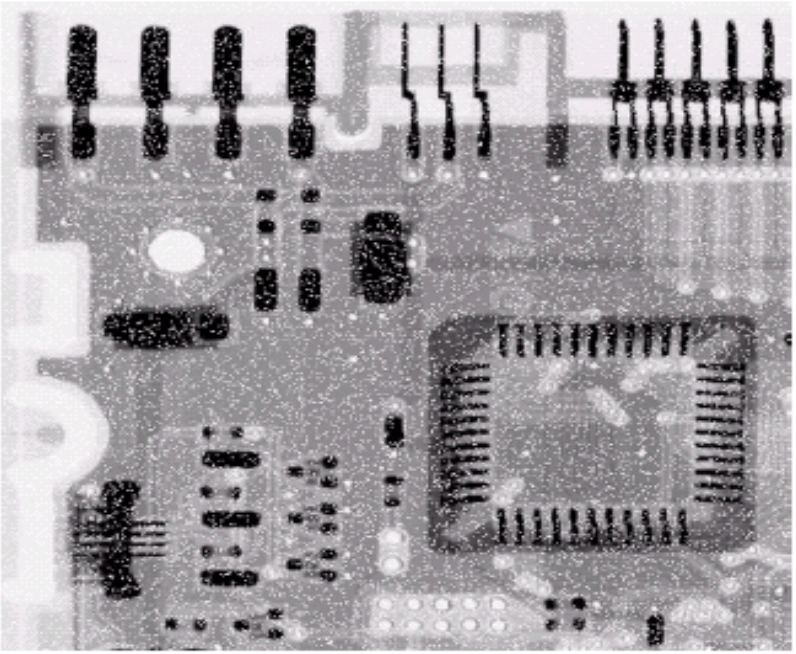
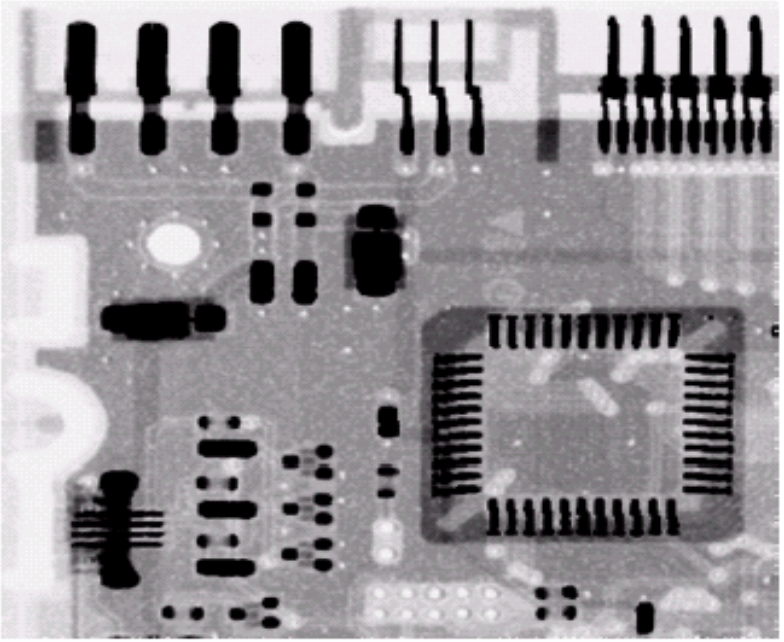


Image corrupted by salt noise

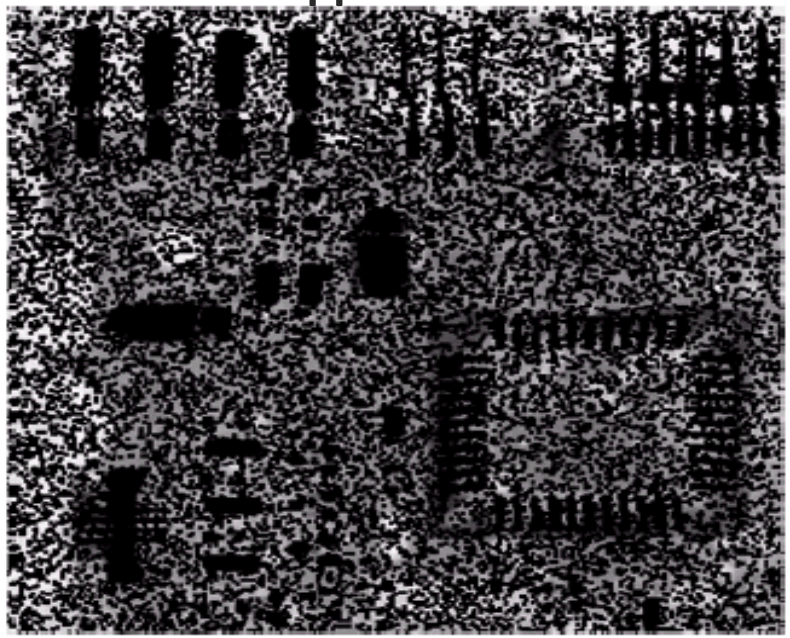


3x3 contraharmonic filter of order -1.5

Image Restoration

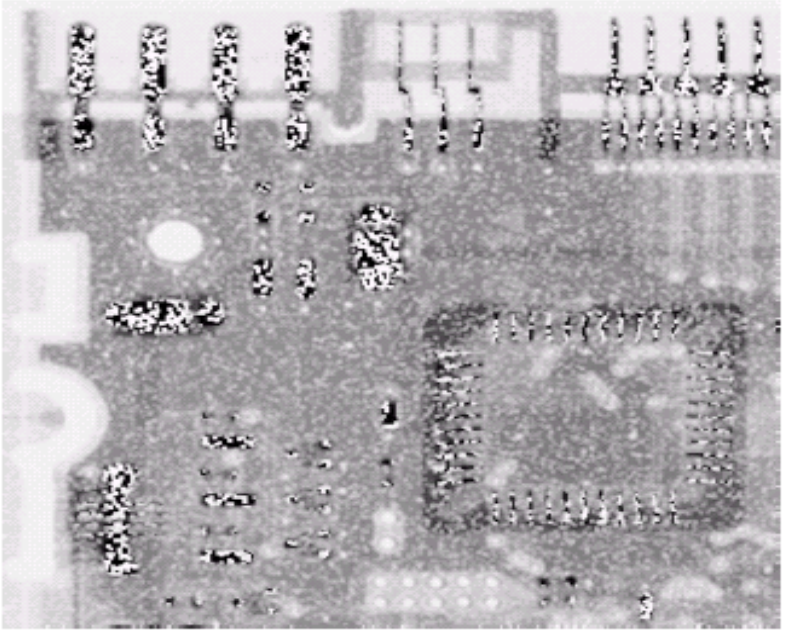
- Contraharmonic mean filters
  - Choose wrong sign of the filter

Pepper noise



3x3 contraharmonic filter
of order -1.5

Salt noise



3x3 contraharmonic filter
of order 1.5

Image Restoration

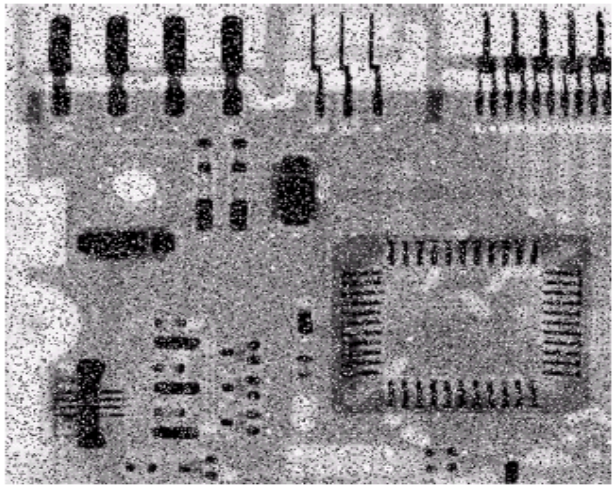
- **Order-statistics filters**
  - **with response that is based on ordering the pixels contained in the image area encompassed by the filter**
  - **Median filter**

$$g(j,k) = \underset{(p,q)\in W_{jk}}{median}\{f(p,q)\}$$

    - **Effective in the presence of impulse noise**
    - **Less blurring than smoothing filters**

Image Restoration

- Example
  - Median filter



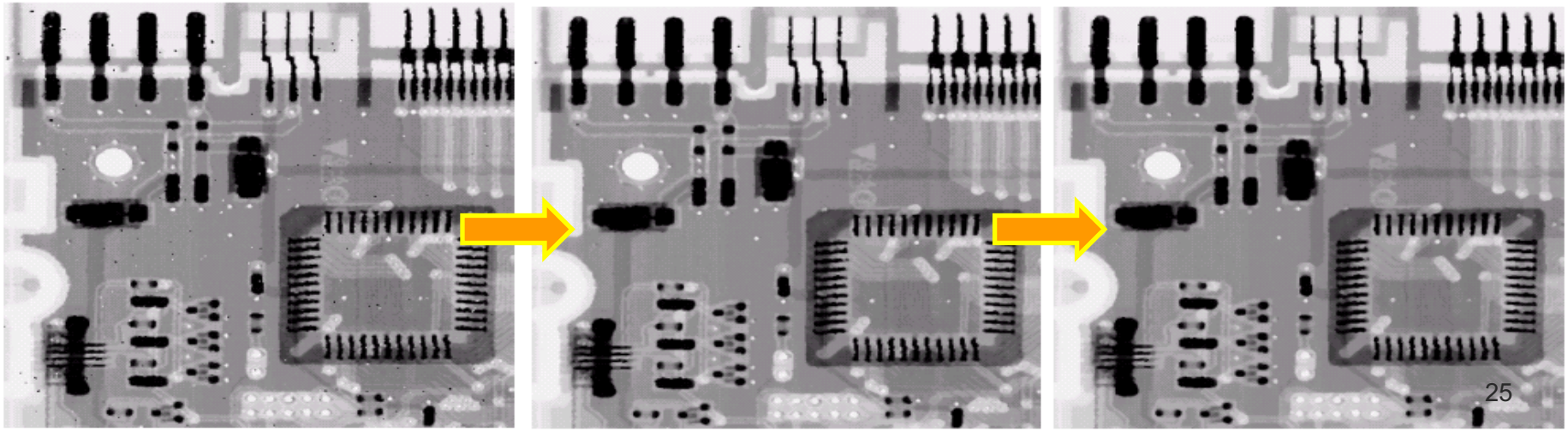


Image Restoration

- **Order-statistics filters**
  - **Max and min filters**

$$g(j,k) = \max_{(p,q)\in W_{jk}} \{f(p,q)\}$$ → **Remove pepper noise**

$$g(j,k) = \min_{(p,q)\in W_{jk}} \{f(p,q)\}$$ → **Remove salt noise**

  - **Midpoint filter**

$$g(j,k) = \frac{1}{2}\left[\max_{(p,q)\in W_{jk}} \{f(p,q)\} + \min_{(p,q)\in W_{jk}} \{f(p,q)\}\right]$$

    - **Combine order statistics and averaging**
    - **Best for randomly distributed noise, eg. Gaussian or uniform noise**

Image Restoration

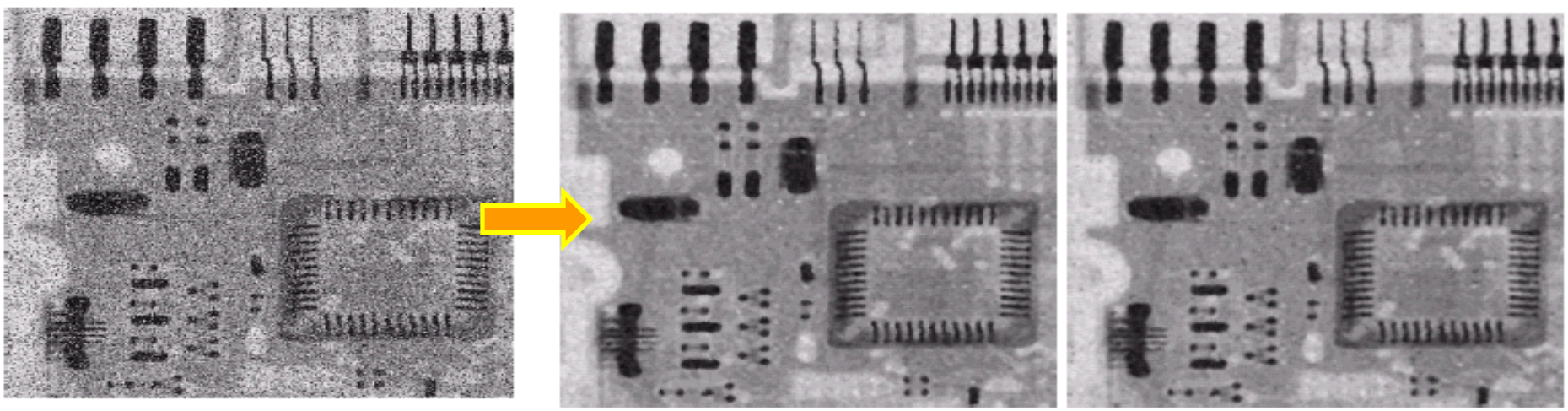
- **Order-statistics filters**
  - **Alpha-trimmed mean filter**
    - **Suppose we delete d/2 lowest and d/2 highest gray-level values of $f(p,q)$ in the neighborhood W_{jk}**

$$g(j,k) = \frac{1}{mn-d} \sum_{(p,q)\in W_{jk}} f_r(p,q)$$

    - **where $f_r(p,q)$ represents the remaining $mn-d$ pixels**
    - ***d* ranges from 0 to $mn-1$**
    - **Useful for removing multiple types of noise**

Image Restoration

- Example
  - Median and alpha-trimmed filters
    - Image corrupted by both uniform and salt-and-pepper noise



median Alpha-trimmed

Image Restoration

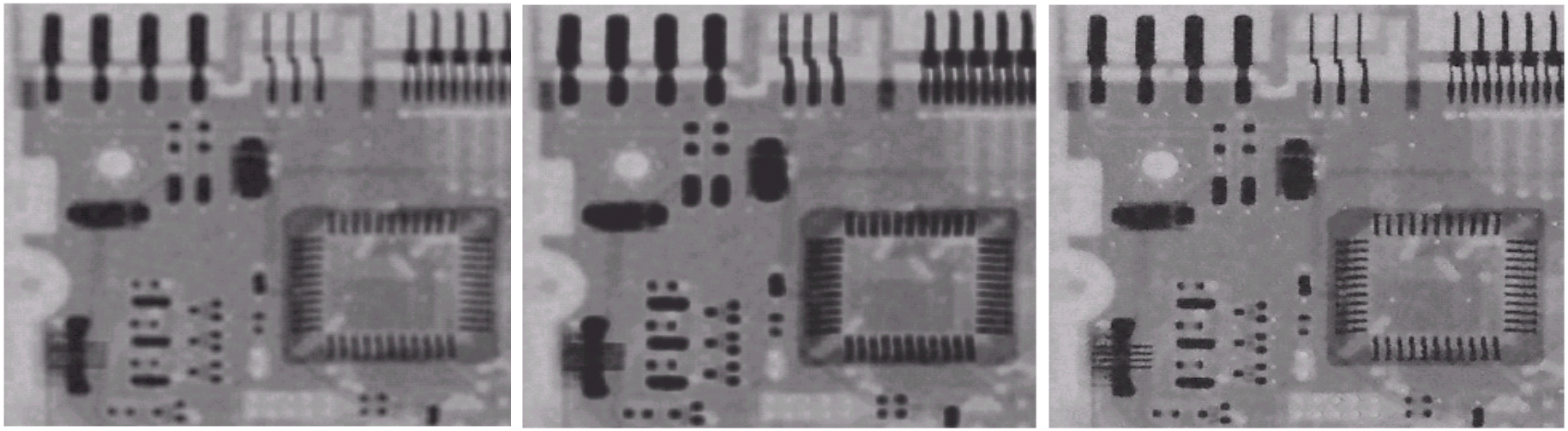
- **Adaptive filters**
  - **Adaptive, local noise reduction filter**

    (Adaptive mean filter)
    - σ_η^2 : noise variance
    - σ_L^2 : local variance of the pixels in S_{xy}
    - m_L : local mean of the pixels in S_{xy}
  - If $\sigma_L^2 >> \sigma_\eta^2$, return a value close to $f(j,k)$

    $$g(j,k) = f(j,k) - \frac{\sigma_\eta^2}{\sigma_L^2}\left(f(j,k) - m_L\right)$$
  - If $\sigma_L^2 \approx \sigma_\eta^2$, return arithmetic mean of the pixels in S_{xy}

Image Restoration

- Example
  - Adaptive noise reduction filter
    - Compared with arithmetic and geometric mean filters



arithmetic geometric adaptive noise reduction

Image Restoration

- **Adaptive filters**
  - **Adaptive median filter**

$z_{\min}$ = minimum gray level value in $W_{j,k}$

$z_{\max}$ = maximum gray level value in $W_{j,k}$

z_{med} = median of gray levels in $W_{j,k}$

z_{jk} = gray level at coordinates (j,k)

$W_{\max}$ = maximum allowed size of W_{jk}

$Level \quad A:$

$A_1 = z_{med} - z_{\min}$

$A_2 = z_{med} - z_{\max}$

$if \quad A_1 > 0 \quad \text{and} \quad A_2 < 0, \quad \text{go to } Level \quad B$

$else \quad \text{increase the window } size$

$if \quad \text{window size} \le W_{\max}, \quad \text{repeat Level } A$

$else \quad \text{output} \quad z_{jk}$

$Level \quad B:$

$B_1 = z_{jk} - z_{\min}$

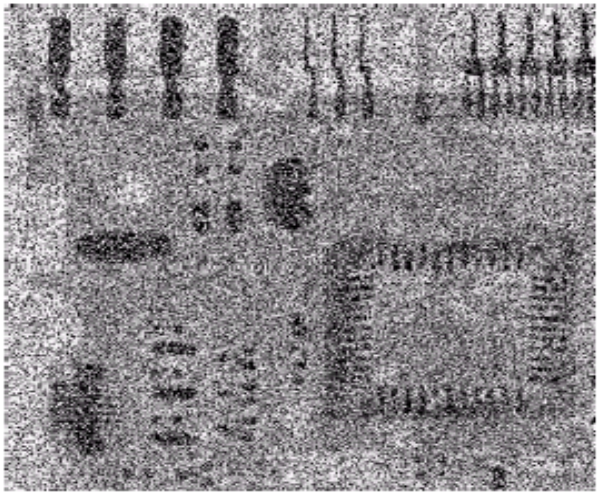
$B_2 = z_{jk} - z_{\max}$

$if \quad B_1 > 0 \quad \text{and} \quad B_2 < 0, \quad \text{output} \quad z_{jk}$

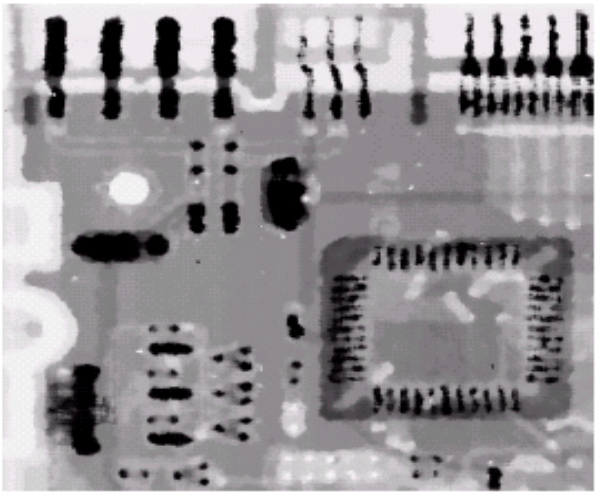
$else \quad \text{output} \quad z_{med}$

Image Restoration

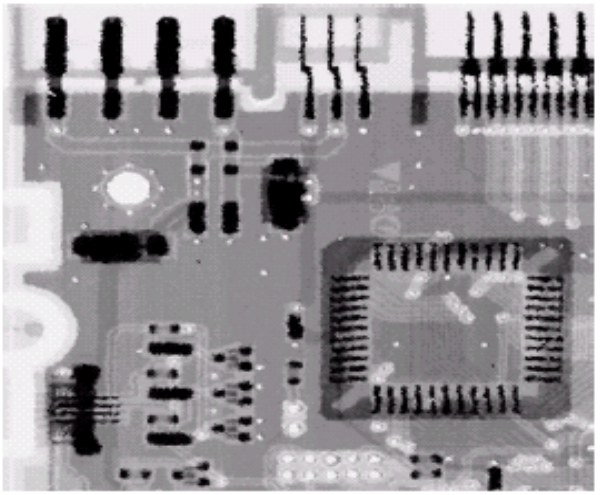
- Example
  - Adaptive median filter



salt-and-pepper



median



adaptive median

→Handle impulse noise with larger probabilities
→Preserve detail while smoothing nonimpulse noise
→Change the size of filter

Image Restoration

- Periodic Noise Reduction by Frequency Domain Filtering
  - Example

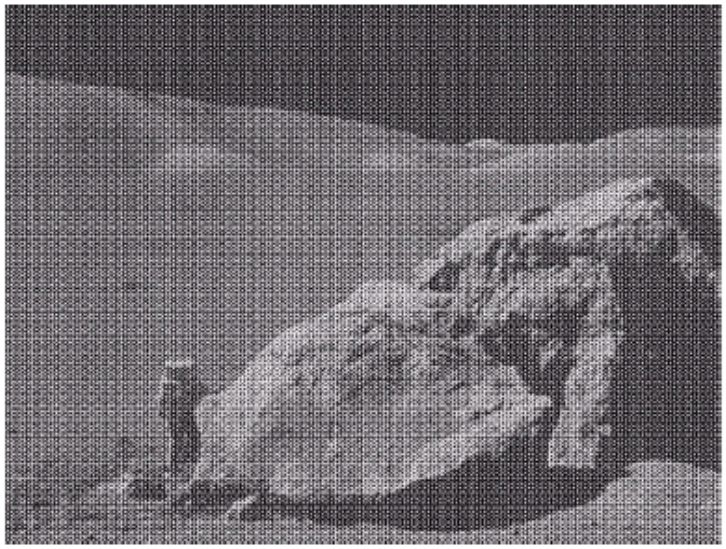
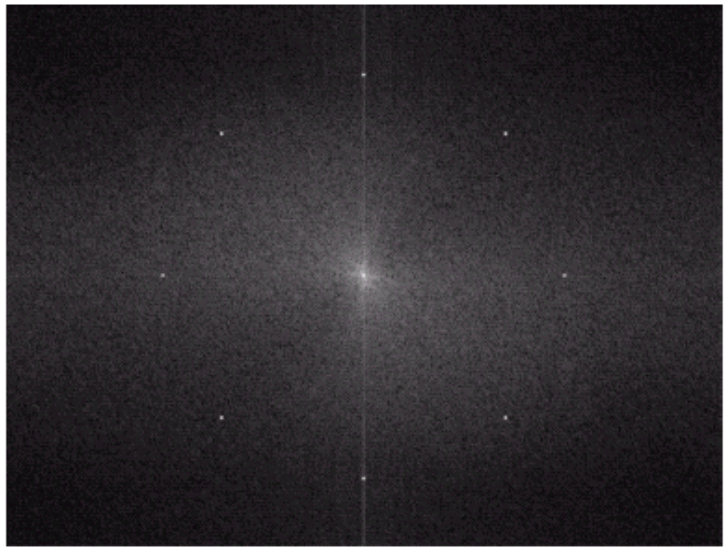


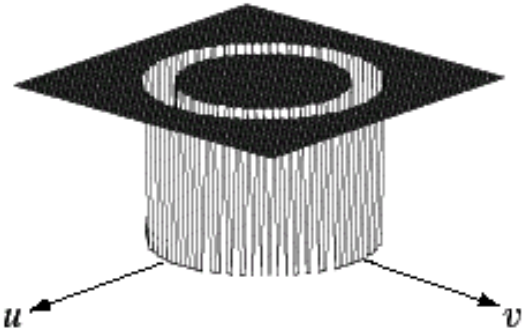
Image corrupted by sinusoidal noise



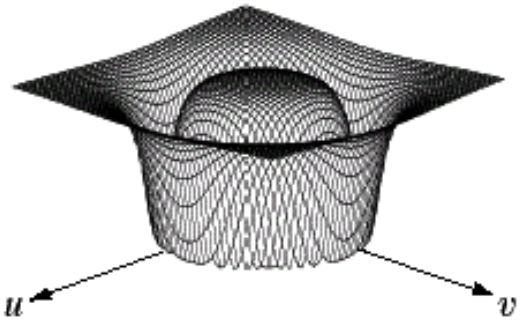
Spectrum of the corrupted image

Image Restoration

- Periodic Noise Reduction by Frequency Domain Filtering
  - Band-reject Filters



ideal

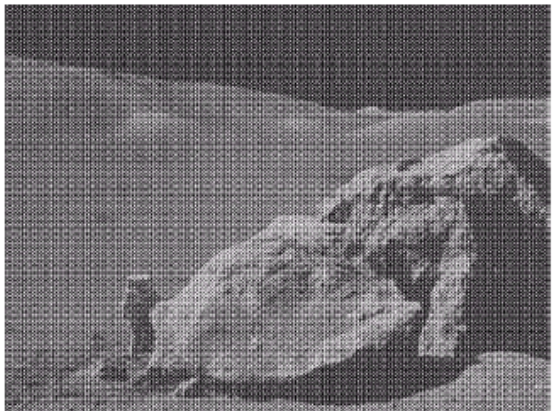


Butterworth (of order 1)

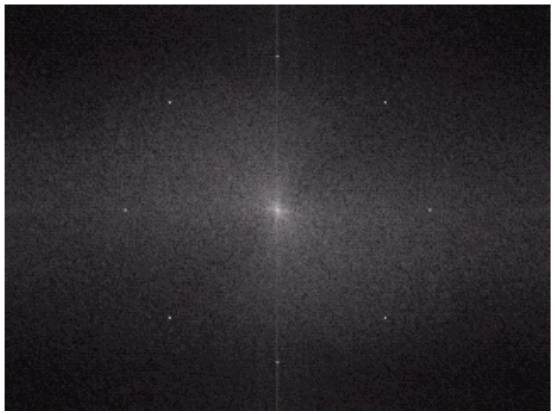
Gaussian

Image Restoration

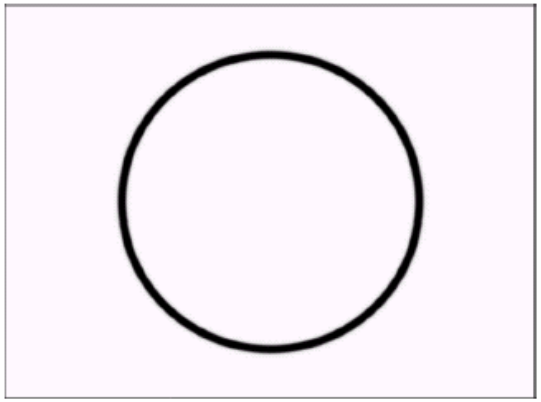
- Example



Degraded image



Spectrum



Butterworth (of order 1)

Restored image

Image Restoration

- **Periodic Noise Reduction by Frequency Domain Filtering**
  - **Notch Filters**

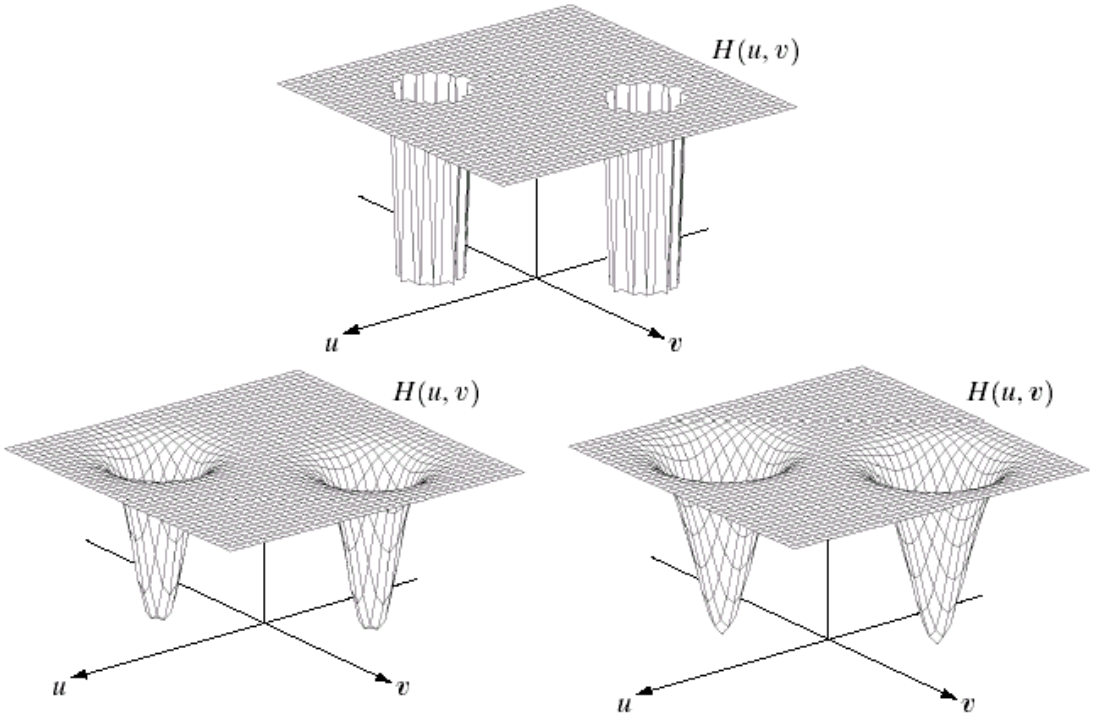
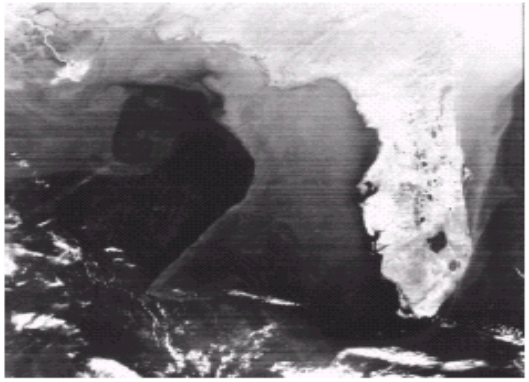
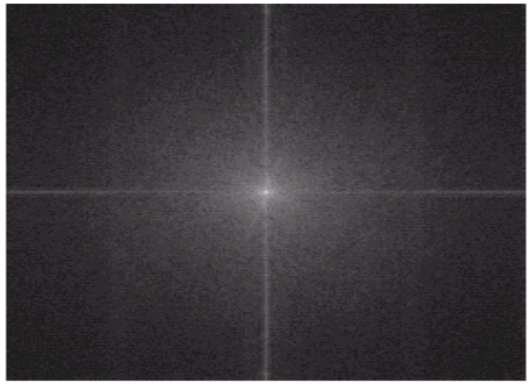


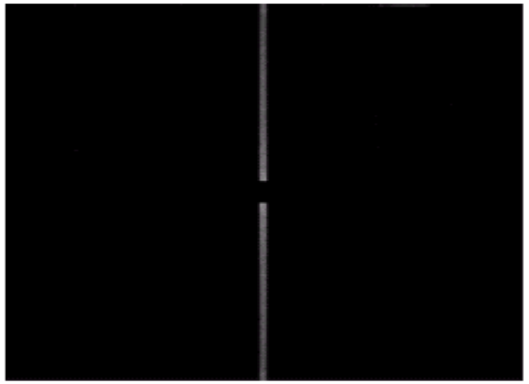
Image Restoration

- Example



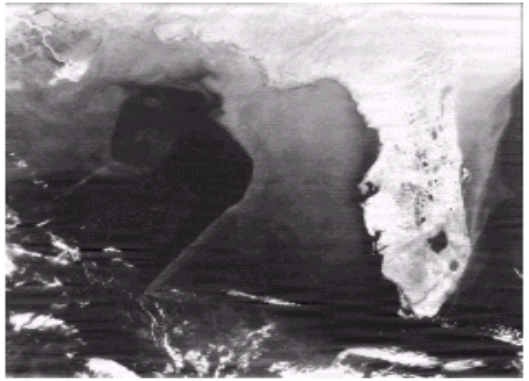
Fourier spectrum





Notch filter

Noise pattern

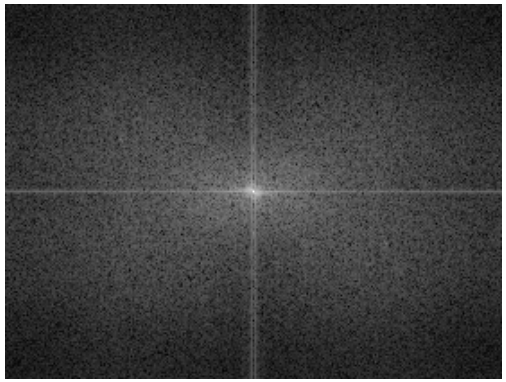


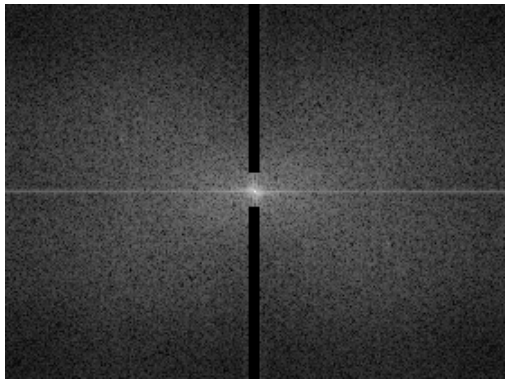
Result

Image Restoration

- Example

Fourier spectrum





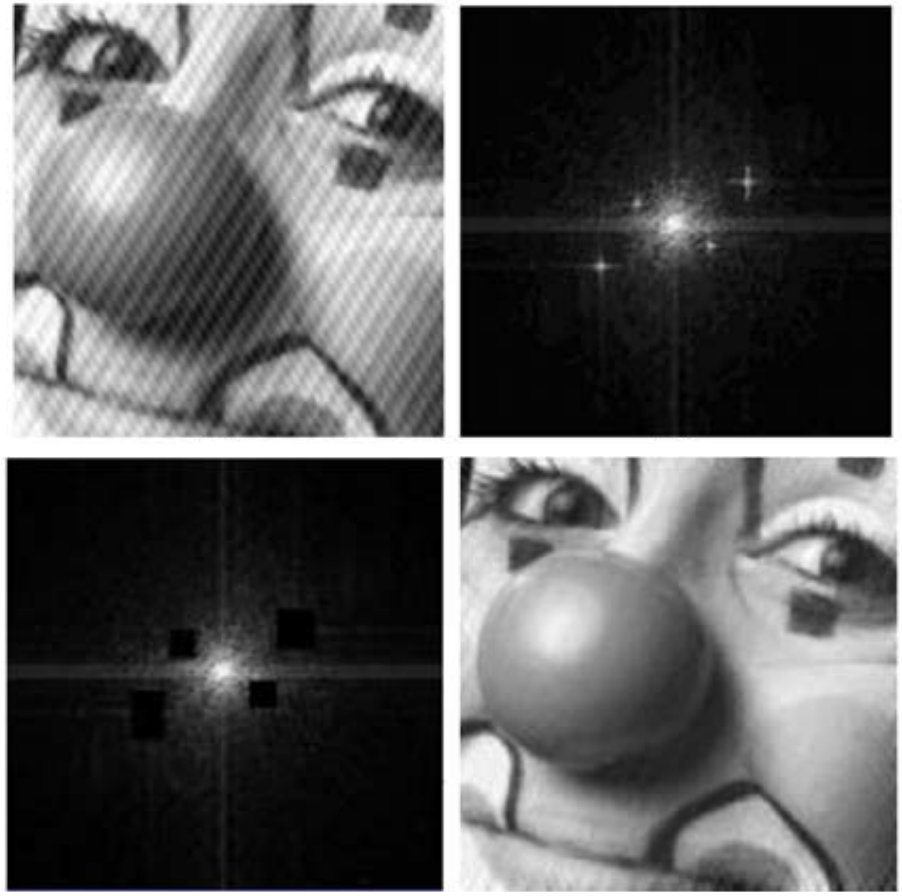
Notch filter

Noise pattern

Result

Image Restoration

- **Example**



Fourier spectrum

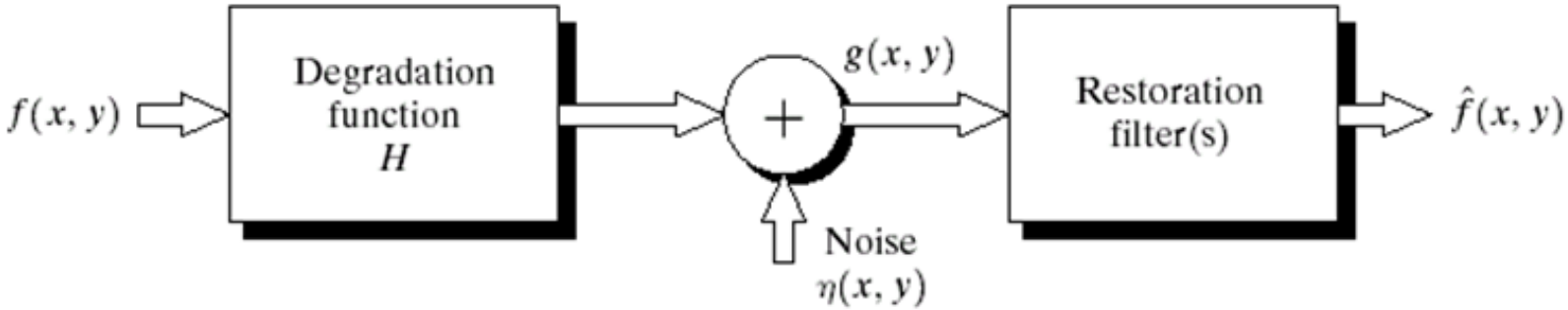
Notch filter

Result

Moallemi, Payman, and Majid Behnampourii. "Adaptive Optimum Notch Filter for Periodic Noise Reduction in Digital Images." *Amirkabir International Journal of Electrical & Electronics Engineering* 42.1 (2010): 1-7.

Image Restoration

- **Inverse Filter**



$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$

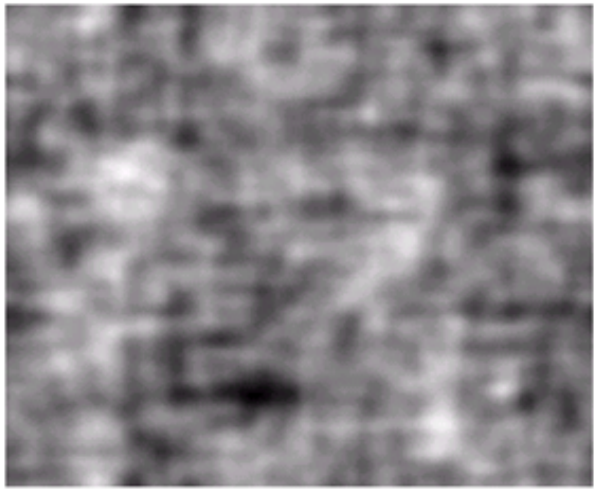
$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Image Restoration

- **Examples**

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

degraded image

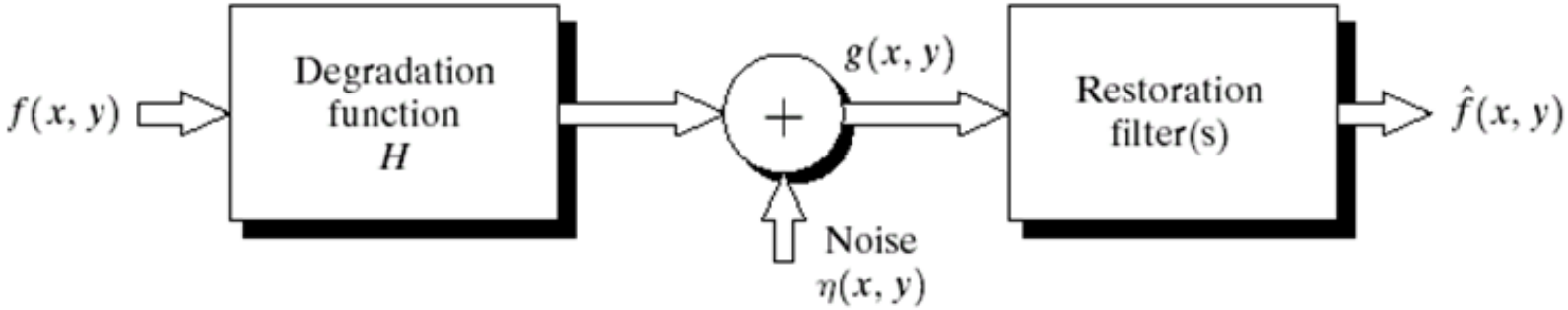


full inverse filter

limited inverse filter

Image Restoration

- **Wiener Filter**



$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$
$$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$
$$\hat{F}(u,v) = H_{Wiener}(u,v)\,G(u,v)$$

- **The objective is to find $H_{Wiener}(u,v)$ with minima MSE between original image and received one.**

$$\varepsilon = E[(F(u,v) - \hat{F}(u,v))^2]$$

Image Restoration

- **Wiener Filter**

$$
\begin{aligned}
\varepsilon &= E[(F-\hat{F})^2] \\
&= E[(F - H_{Wiener}G)^2] \\
&= E[(F - H_{Wiener}(HF+N))^2] \\
&= E[((1-HH_{Wiener})F - H_{Wiener}N))^2] \\
&= (1-HH_{Wiener})\overline{(1-HH_{Wiener})}E[F^2] - (1-HH_{Wiener})\overline{H_{Wiener}}E[F\overline{N}] \\
&\quad - \overline{(1-HH_{Wiener})}H_{Wiener}E[\overline{F}N] + H_{Wiener}\overline{H_{Wiener}}E[N^2]
\end{aligned}
$$

Image Restoration

- **Wiener Filter**
  - **Noise is independent to signal, and its mean is zero**

$$E[\overline{F}N] = 0 = E[F\overline{N}]$$

  - **Define power spectral densities (PSD) as follows**

$$D_F(u,v) = E[F(u,v)^2]$$

$$D_N(u,v) = E[N(u,v)^2]$$

  - **Hence**

$$\varepsilon = (1 - HH_{Wiener})\overline{(1 - HH_{Wiener})}D_F + H_{Wiener}\overline{H_{Wiener}}D_N$$

Image Restoration

- **Wiener Filter**
  - **To minimize the error, let the derivative to be zero**

$$\frac{d\varepsilon}{dH_{Wiener}} = 0 = -H\overline{(1 - HH_{Wiener})}D_F + \overline{H_{Wiener}}D_N$$

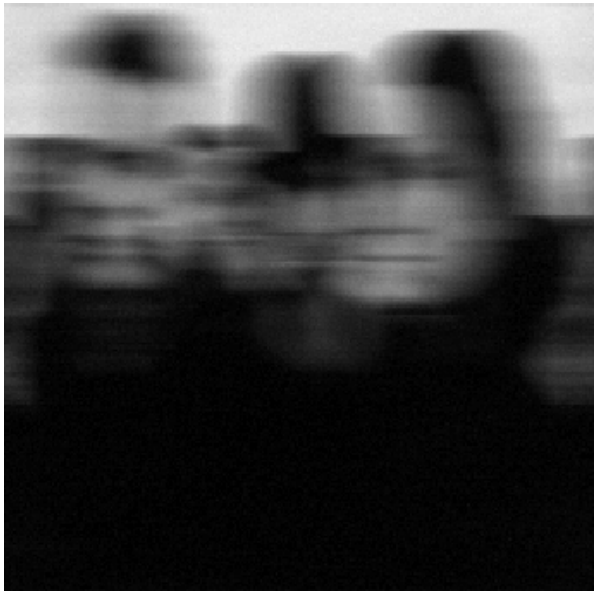
  - **Then**

$$H_{Wiener}(u,v) = \frac{\overline{H}(u,v)D_F(u,v)}{\left|H(u,v)\right|^2 D_F(u,v) + D_N(u,v)}$$

Image Restoration

- Examples

Original image



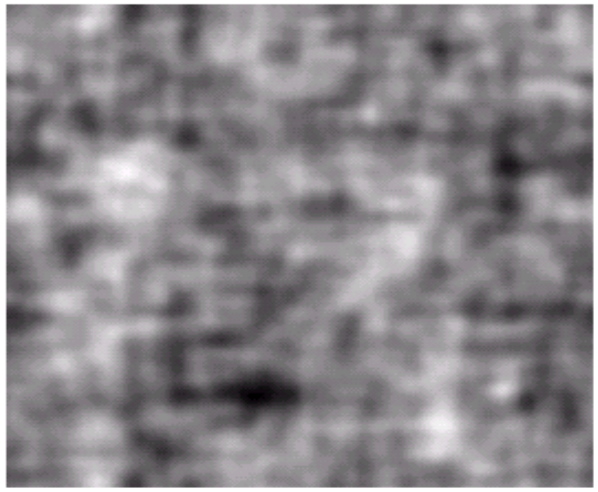
Add motion blur
and Gaussian noise

Restored by
Wiener filter

Image Restoration

- Other examples

degraded image



full inverse filter

limited inverse filter

Wiener filter

Image Restoration

- Other examples

with motion blur & additive noise

inverse filtering

Wiener filter