Investigating the Dynamics of the Stochastic Predator–Prey Population Model

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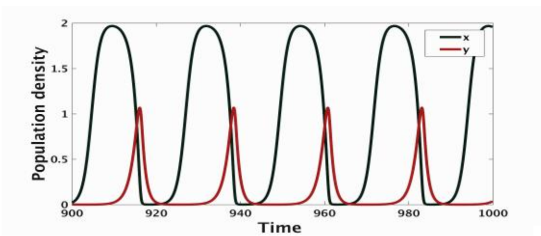
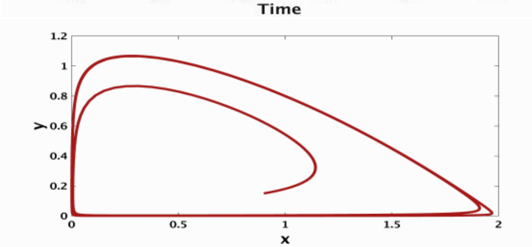
Rosenweigh-MacArthur Model

Dynamics of a biological system which contains two species that one acts like predator or parasite and the other is prey or host is oscillating in nature and described by **Lotka-Volterra** equations. Rosenweigh-MacArthur is one of the simple models based on Lotka-Voltera.

$$\dot{x} = ax - \frac{1}{2}x^2 - \frac{\sigma xy}{1+\sigma\tau x}$$

$$\dot{y} = -y + \frac{\sigma xy}{1+\sigma\tau x}$$

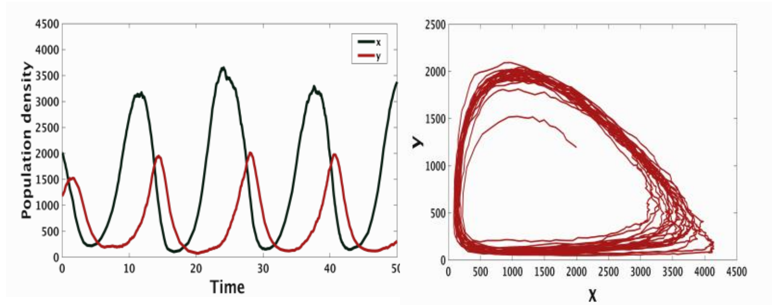




Population density over time and limit cycle in phase space of Rosenweigh-MacArthur model.

Stochastic Rosenweigh-MacArthur Model

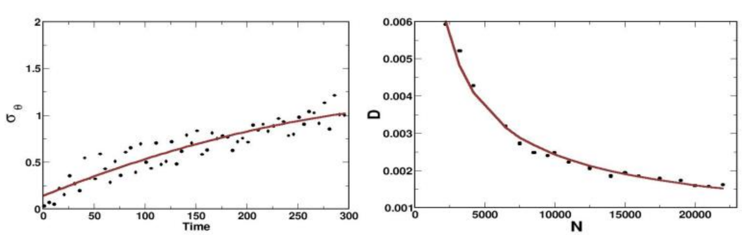
A stochastic model is better for describing nature than a deterministic one. The dynamics of the model is stochastic using Gillspie algorithm.



Stochastic population density over time and stochastic limit cycle in phase space of Rosenweigh-MacArthur model.

Phase is similar to **angular Brownian motion**.

$$\theta = \arctan\left(\frac{y}{x}\right) \qquad \dot{\theta} = \omega + \sqrt{D}\xi(t)$$



Standard deviation of phase over time and noise strength over system size.

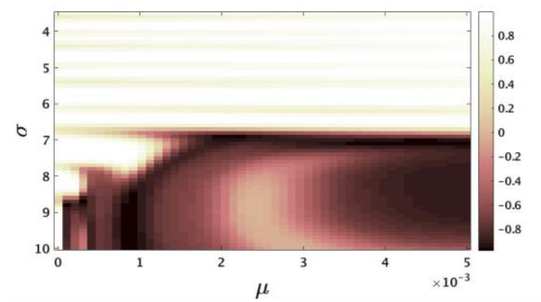
Coupled Stochastic Rosenweigh-MacArthur Model

Synchronization of population is seen between spatially separated patches in nature. Patch coupled by migration constant.

$$\dot{x}_i = ax_i - \frac{1}{2}x_i^2 - \frac{\sigma x_i y_i}{1+\sigma\tau x_i} + \mu(x_j - x_i)$$

$$\dot{y}_i = -y_i + \frac{\sigma x_i y_i}{1+\sigma\tau x_i} + \mu(y_j - y_i)$$

$$\varphi = \theta_j - \theta_i \qquad x = \cos(\varphi)$$



$< cos\varphi >$ for different σ and μ values.

References

[1] Lotka, Alfred J. "Contribution to the Theory of Periodic Reaction", Journal of Physical Chemistry 14: 271–274, 1910.
[2] Rosenzweig, Michael L., and Robert H. MacArthur. "Graphical representation and stability conditions of predator-prey interactions", The American Naturalist 97.895: 209-223, 1963.
[3] Gillespie, Daniel T. "A General Method for Numerically Simulating the Stochastic Time Evolution of Coupled Chemical Reactions", Journal of Computational Physics 22: 403–434, 1976.

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