1 Coclustering by Block Value Decomposition

Seja X uma matriz de dados que representam dados em algum domÃŋnio de aplicaÃğÃčo, sendo $X \in \mathbb{R}_+^{\times}$, contendo nÃžmeros reais positivos com n linhas e m colunas. Esta matriz Ãľ formada por um conjunto de linhas $\mathscr{X} = \{x_{1\cdot}, \dots, x_{n\cdot}\}$ e um conjunto de colunas $\mathscr{Y} = \{y_{\cdot 1}, \dots, y_{\cdot m}\}$.

Um dos primeiros algoritmos propostos na literatura para a resoluÃğÃčo do problema de *co-clustering* baseado em NMF, Ãľ o DecomposiÃğÃčo de Valores em Blocos (*Block Value Decomposition* - DVB). Este algoritmo tem o objetivo de clusterizar a matriz de dados X simultÃćneamente em k clusters de $\mathscr{X}$ exclusivos e l clusters de $\mathscr{Y}$.

A idÃľia do algoritmo Ãľ reconstruir a matriz de dados X atravÃľs de combinaÃğÃţes lineares dos centros dos blocos de X, similar Ãă alguns tipos de estratÃľgias de *clustering* tradicional, como *k-means.*

Definition 1 (A resoluÃğÃčo do problema de *coclustering* em X Ãľ dada pela otimizaÃğÃčo do problema)**.**

$$\begin{array}{ll} \underset{U,S,V}{minimizar} & \|X - USV^T\|_F^2 \\ sujeito\ a & U \geq 0, S \geq 0, V \geq 0 \end{array}$$

onde $U \in \mathbb{R}_+^{n \times k}$, $S \in \mathbb{R}_+^{k \times l}$ $\|\cdot\|$, $V \in \mathbb{R}^{m \times l}$ e $\|\dots\|$ denota a norma de Frobenius para matrizes.

This is a co-clustering algorithm called Block Value Decomposition (BVD) based on Nonnegative Matrix Factorization (NMF) technique. The goal is to find a factorization for the data matrix $X \in \mathbb{R}_+^{N \times M}$, where N is the number of objects, M is the number of features of these objects and the factorization takes the form

$$X \approx USV^T$$

, where $U \in \mathbb{R}_+^{N \times L}$ is a matrix of rows factors representing features clusters, $S \in \mathbb{R}_+^{L \times K}$ is a block matrix representing how blocks are related, and $V \in \mathbb{R}_+^{M \times K}$ is a matrix of columns factors representing rows clusters.

This algorithm solves the following optimization problem:

$$min\, ||X - USV^T||^2 \ s.t.\ U \geq 0, S \geq 0, V \geq 0$$

The optimization problem can be solved using Lagrange multipliers (λ), optimizing the following Lagrange function:

$$\mathscr{L} = |X - USV^T||^2 - tr(\lambda_1 U^T) - tr(\lambda_2 S^T) - tr(\lambda_3 V^T)$$

Then $\mathscr{L}$ must satisfy the K.K.T. conditions:

$$\frac{\partial \mathscr{L}}{\partial U} = 0$$

$$\frac{\partial \mathscr{L}}{\partial S} = 0$$

$$\frac{\partial \mathscr{L}}{\partial V} = 0$$

$$\lambda_1 \odot U = 0$$

$$\lambda_2 \odot S = 0$$

$$\lambda_3 \odot V = 0$$

Solving the derivatives and equal them to 0, is possible to solve the optimization problem by applying gradient ascending on $\mathscr{L}$ with the following update rules:

$$U \leftarrow U \odot \frac{XVS^T}{USV^TVS^T}$$

$$V \leftarrow V \odot \frac{U^TXV}{U^TUSV^TV}$$

$$S \leftarrow S \odot \frac{S^TU^TX}{S^TU^TUSV^T}$$