VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
Fakulta informačních technologií

Elektronika pro informační technologie

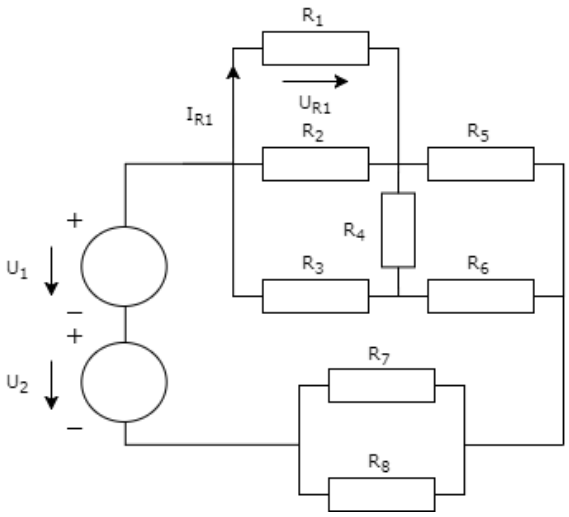
2017/2018

Semestrálny projekt

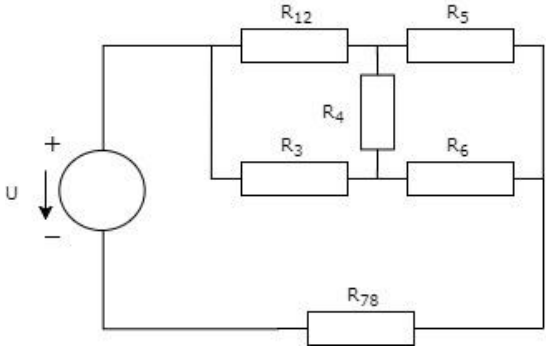
Sabína Gregušová (xgregu02) Brno, 20. decembra 2017

1B) Stanovte **napätie U_{R1}** a **prúd I_{R1}**. Použite metódu postupného zjednodušovania obvodu.

U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]	R_6 [Ω]	R_7 [Ω]	R_8 [Ω]
95	115	650	730	340	330	410	830	340	220



Postupne budeme zjednodušovať obvod:



$U = U_1 + U_2$

$U = 95\ V + 115\ V$

$U = 210\ V$

$$R_{12} = \frac{R_1 * R_2}{R_1 + R_2}$$

$$R_{12} = \frac{650\ \Omega * 730\ \Omega}{650\ \Omega + 730\ \Omega}$$

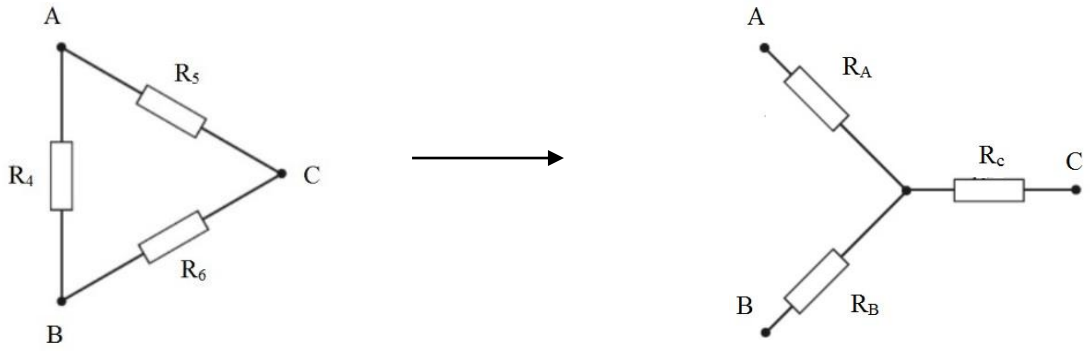
$R_{12} = 343{,}8406\ \Omega$

$$R_{78} = \frac{R_7 * R_8}{R_7 + R_8}$$

$$R_{78} = \frac{340\ \Omega * 220\ \Omega}{340\ \Omega + 220\ \Omega}$$

$R_{78} = 133{,}5714\ \Omega$

Transformácia trojuholník –> hviezda



$$R_A = \frac{R_4 * R_5}{R_4 + R_5 + R_6}$$

$$R_A = \frac{330\ \Omega * 410\ \Omega}{330\ \Omega + 410\ \Omega + 830\ \Omega}$$

$R_A = 86{,}1783\ \Omega$

$$R_B = \frac{R_4 * R_6}{R_4 + R_5 + R_6}$$

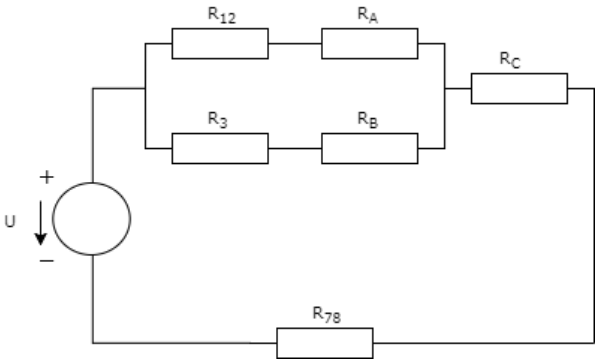
$$R_B = \frac{330\ \Omega * 830\ \Omega}{330\ \Omega + 410\ \Omega + 830\ \Omega}$$

$R_B = 174{,}4586\ \Omega$

$$R_C = \frac{R_4 * R_6}{R_4 + R_5 + R_6}$$

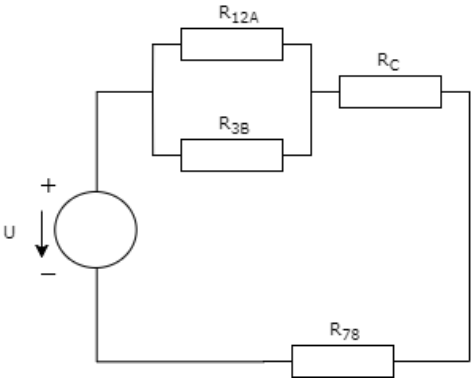
$$R_C = \frac{410\ \Omega * 830\ \Omega}{343{,}8406\ \Omega + 340\ \Omega + 330\ \Omega}$$

$R_C = 216{,}7516\ \Omega$

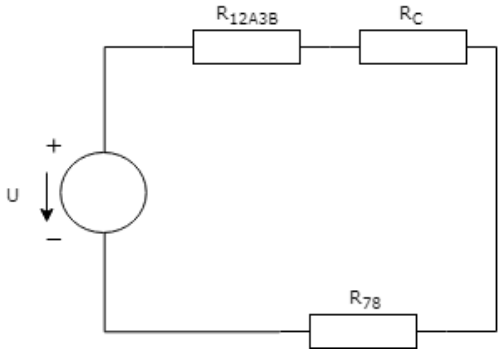


$R_{12A} = R_{12} + R_A$
$R_{12A} = 343{,}8406\ \Omega + 86{,}1783\ \Omega$
$R_{12A} = 430{,}0189\ \Omega$

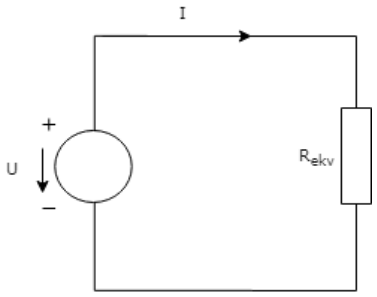
$R_{3B} = R_3 + R_B$
$R_{3B} = 340\ \Omega + 174{,}4586\ \Omega$
$R_{3B} = 514{,}4586\ \Omega$



$$R_{12A3B} = \frac{R_{12A} * R_{3B}}{R_{12A} + R_{3B}}$$
$$R_{12A3B} = \frac{430{,}0189\ \Omega * 514{,}4586\ \Omega}{430{,}0189\ \Omega + 514{,}4586\ \Omega}$$
$R_{12A3B} = 234{,}2321\ \Omega$

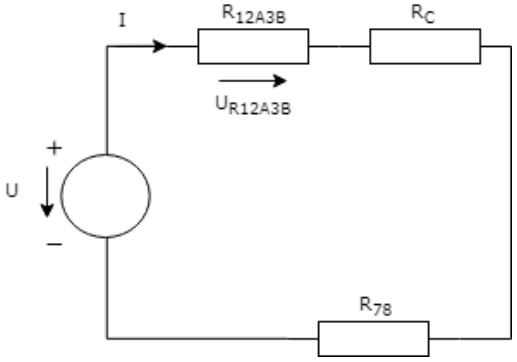


$R_{EKV} = R_{12A3B} + R_C + R_{78}$
$R_{EKV} = 234{,}2321\ \Omega + 216{,}7516\ \Omega + 133{,}5714\ \Omega$
$R_{EKV} = 584{,}5551\ \Omega$

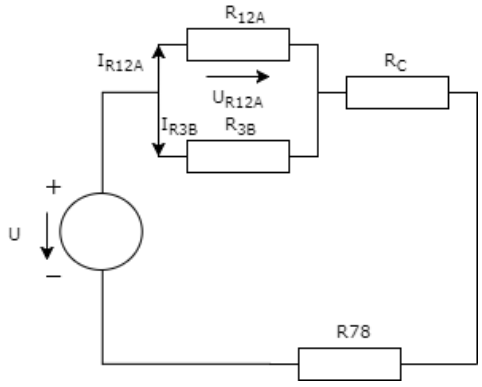


$$I = \frac{U}{R_{EKV}}$$
$$I = \frac{210\ V}{584{,}5550\ \Omega}$$
$I = 0{,}3592\ A$

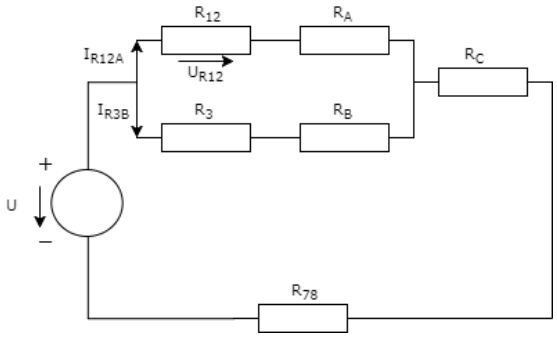
Spätne porozkladáme obvod na pôvodný aby sme dopočítali požadované hodnoty:



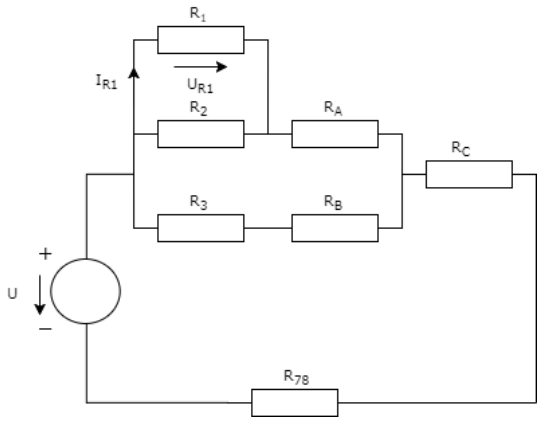
$U_{R12A3B} = R_{12A3B} * I$
$U_{R12A3B} = 216{,}7516\ \Omega * 0{,}3592\ A$
$U_{R12A3B} = 41{,}4193\ V$



$I_{R12A} = \frac{U_{R12A3B}}{R_{12A}}$
$I_{R12A} = \frac{84{,}1362\ V}{430{,}0189\ \Omega}$
$I_{R12A} = 0{,}1957\ A$



$U_{R12} = R_{12} * I_{R12A}$
$U_{R12} = 343{,}8406\ \Omega * 0{,}1957\ A$
$U_{R12} = 67{,}2896\ V$

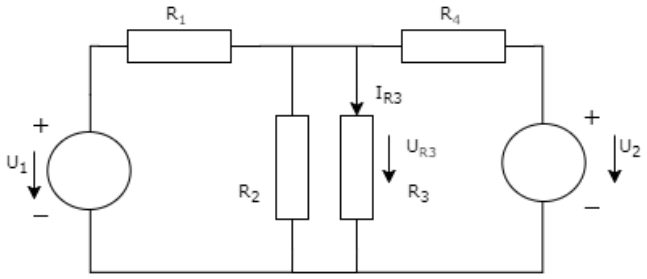


$I_{R1} = \frac{U_{R12}}{R_1}$
$I_{R1} = \frac{67{,}2896\ V}{650\ \Omega}$
$I_{R1} = \underline{0{,}1035\ A}$

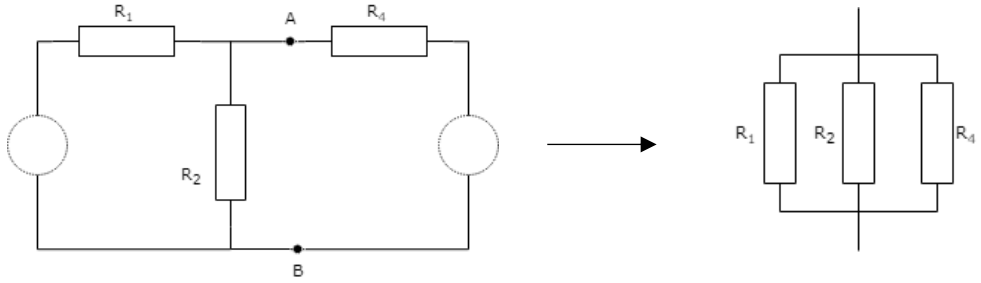
$U_{R1} = R_1 * I_{R1}$
$U_{R1} = 650\ \Omega * 0{,}1035\ A$
$U_{R1} = \underline{67{,}2896\ V}$

2C) Stanovte **napätie U_{R3}** a **prúd I_{R3}**. Použite metódu Théveninovej vety.

U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]
200	0,85	220	630	240	450



Nahradíme napäťové zdroje skratom a spočítame R_i



$$\frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$R_1*R_2*R_4 = R_i*R_2*R_4 + R_i*R_1*R_4 + R_i*R_1*R_2$

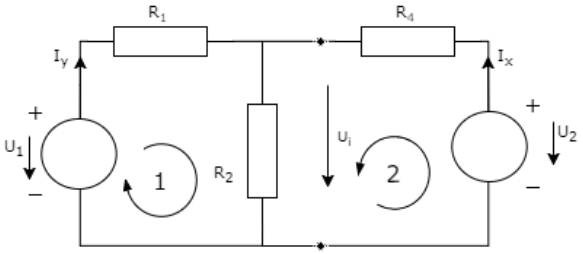
$R_1*R_2*R_4 = R_i*(R_2*R_4 + R_1*R_4 + R_1*R_2)$

$$R_i = \frac{R1*R2*R4}{R2*R4 + R1*R4 + R1*R2}$$

$$R_i = \frac{220*630*450}{630*450 + 220*450 + 220*630}$$

$R_i = 119{,}6891\ \Omega$

Vypočítame si prúdy v náhradnom obvode pomocou riešenia dvoch rovníc s dvomi neznámymi



1) $U_{R1} + U_{R2} - U_1 = 0$

$R_1*I_y + R_2*(I_y + I_x) = U_1$

$220I_y + 630I_y + 630I_x = 200$

$850I_y + 630I_x = 200$

$85I_y + 63I_x = 20$

Vyjadríme si I_y a dosadíme do prvej rovnice

2) $U_{R4} + U_{R2} - U_2 = 0$

$R_4*I_x + R_2*(I_x + I_y) = U_2$

$450I_x + 630I_x + 630I_y = 200$

$1080I_x + 630I_y = 70$

$108I_x + 63I_y = 7$

$108I_x + 63I_y = 7$

$63I_y = 7 - 108I_x$

$$I_y = \frac{7 - 108Ix}{63}$$

$$85*\left(\frac{7-108I_x}{63}\right) + 63I_x = 20$$

$$\frac{595-9180I_x}{63} + 63I_x = 20$$

$$595 - 9180I_x + 3969I_x = 1260$$

$$-5211I_x = 665$$

$$I_x = -0{,}1276 \text{ A}$$

Dopočítame si I_x ⟶

$$I_y = \frac{7-108*(-0{,}1276)}{63}$$

$$I_y = 0{,}3299 \text{ A}$$

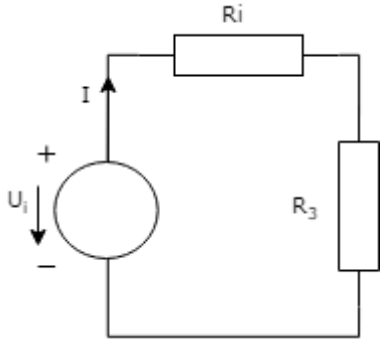
Keďže poznáme prúd I_x, môžeme si dopočítať U_i

$$U_{R4} + U_i - U_2 = 0$$

$$U_i = U_2 - R_4 * I_x$$

$$U_i = 70 - 450*(-0{,}1276)$$

$$U_i = 127{,}42 \text{ V}$$



$$I = \frac{Ui}{Ri + R3}$$

$$I = \frac{127{,}42}{119{,}6891 + 240}$$

$$I = \underline{0{,}3542 \text{ A}}$$

$$I_{R3} = \underline{0{,}3542 \text{ A}}$$

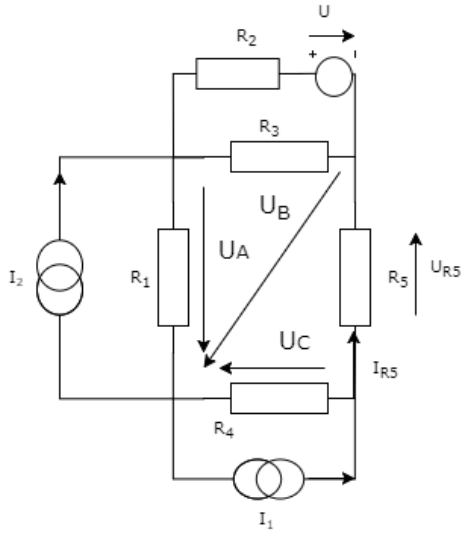
$$U_{R3} = R_3 * I$$

$$U_{R3} = 240 * 0{,}3542$$

$$U_{R3} = \underline{85{,}008 \text{ V}}$$

3C) Stanovte napätie U_{R5} a prúd I_{R5}. Použite metódu uzlových napätí (U_A, U_B, U_C)

U [V]	I_1 [A]	I_2 [A]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]
110	0,85	0,75	44	31	56	20	30



Zdroj U si transformujeme na prúdový zdroj I_3 pomocou Ohmovho zákona:

$$I_3 = \frac{U}{R_3}$$

$$I_3 = \frac{110}{31}$$

$$I_3 = 3{,}5484 \text{ A}$$

Vyjadríme si prúdy na základe napätí:

$$I_{R1} = \frac{U_A}{R_1}$$

$$I_{R2} = \frac{U + U_B - U_A}{R_2}$$

$$I_{R3} = \frac{U_A - U_B}{R_3}$$

$$I_{R4} = \frac{U_C}{R_4}$$

$$I_{R5} = \frac{U_B - U_C}{R_5}$$

Na základe vzťahu $G = \frac{1}{R}$ upravíme rovnice prúdu

$$I_{R1} = G_1 U_A$$

$$I_{R2} = G_2(U + U_B - U_A)$$

$$I_{R3} = G_3(U_A - U_B)$$

$$I_{R4} = G_4 U_C$$

$$I_{R5} = G_5(U_B - U_C)$$

Zostavíme si všeobecnú maticu pre výpočet napätia. Na diagonálu zapisujeme vodivosť odporov, ktoré sú priamo spojené s daným uzlom.

Do ostatných polí matice zapisujeme vodivosti odporov so znamienkom mínus, ktoré sa nachádzajú medzi dvoma danými uzlami.

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} G_1 + G_2 + G_3 & -G_2 - G_3 & 0 \\ -G_2 - G_3 & G_2 + G_3 + G_5 & -G_5 \\ 0 & -G_5 & G_4 + G_5 \end{pmatrix} \end{array}\begin{pmatrix} I_1 + I_3 \\ -I_3 \\ I_2 \end{pmatrix}$$

Dosadíme a vypočítame hodnoty:

$$\begin{pmatrix} 0{,}0728 & -0{,}0501 & 0 \\ -0{,}0501 & 0{,}0834 & -0{,}0333 \\ 0 & -0{,}0333 & 0{,}0833 \end{pmatrix}\begin{pmatrix} 4{,}3984 \\ -3{,}5484 \\ 0{,}75 \end{pmatrix}$$

Vyjadríme determinant matice:

$$\text{Det} = \begin{vmatrix} 0{,}0728 & -0{,}0501 & 0 \\ -0{,}0501 & 0{,}0834 & -0{,}0333 \\ 0 & -0{,}0333 & 0{,}0833 \end{vmatrix} = 2{,}1595 * 10^{-4}$$

Do druhého stĺpca si dosadíme hodnoty prúdu a vypočítame hodnotu U_B:

$$U_B = \frac{\begin{vmatrix} 0{,}0728 & 4{,}3984 & 0 \\ -0{,}0501 & -3{,}5484 & -0{,}0333 \\ 0 & 0{,}75 & 0{,}0833 \end{vmatrix}}{\text{Det}} = \frac{-1{,}3442 * 10^{-3}}{2{,}1595 * 10^{-4}} = \text{-6,2246 V}$$

Do tretieho stĺpca si dosadíme hodnoty prúdu a vypočítame hodnotu U_C:

$$U_C = \frac{\begin{vmatrix} 0{,}0728 & -0{,}0501 & 4{,}3984 \\ -0{,}0501 & 0{,}0834 & -3{,}5484 \\ 0 & -0{,}0333 & 0{,}75 \end{vmatrix}}{\text{Det}} = \frac{1{,}4069 * 10^{-3}}{2{,}1595 * 10^{-4}} = 6{,}5149 \text{ V}$$

Dosadíme hodnoty do počiatočného vzťahu a vyjadríme si prúd I_{R5}:

$$I_{R5} = \frac{U_B - U_C}{R_5}$$

$$I_{R5} = \frac{-6{,}2246 - 6{,}5149}{30}$$

$$I_{R5} = \text{-0,4247 A}$$

Vezmeme absolútnu hodnotu z prúdu I_{R5}

$$I_{R5} = \underline{0{,}4247 \text{ A}}$$

Dopočítame si U_{R5}:

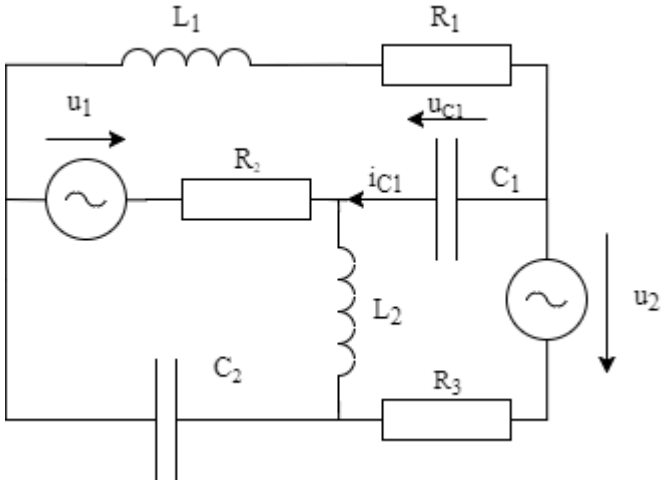
$$U_{R5} = R_5 * I_{R5}$$

$$U_{R5} = 30 * 0{,}4247$$

$$U_{R5} = \underline{12{,}7410 \text{ V}}$$

4B) Pre napájacie napätie platí: $u_1 = U_1\sin(2\pi ft)$, $u_2 = U_2\sin(2\pi ft)$. Vo vzťahu pre napätie $u_{C1} = U_{C1}\sin(2\pi ft + \varphi c_1)$. Určte $|U_{c1}|$ a φc_1. Použite metódu smyčkových prúdov. Pomocné „smery šipiek napájacích zdrojov platí pre špeciálny časový okamih ($t = \frac{\pi}{2\omega}$).

U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	L_1 [mH]	L_2 [mH]	C_1 [μF]	C_2 [μF]	f [Hz]
25	40	11	15	12	100	85	220	95	80



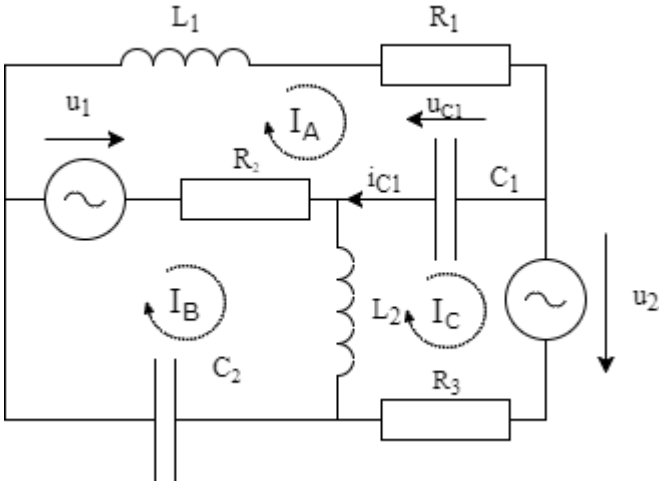
Vypočítame si hodnotu ω:

$$\omega = 2\pi f = 2\pi * 80 = 502{,}6548 \text{ rad/s}$$

Vypočítame si impedanciu cievok a kondenzátorov:

$$Z_{L1} = j\omega L_1 = 502{,}6548 * 100 * 10^{-3} = 50{,}2655j$$
$$Z_{L2} = j\omega L_2 = 502{,}6548 * 85 * 10^{-3} = 42{,}7257j$$
$$Z_{C1} = -\frac{1}{\omega C_1}j = -\frac{1}{502{,}6548 * 220 * 10^{-6}}j = -9{,}0429j$$
$$Z_{C2} = -\frac{1}{\omega C_2}j = -\frac{1}{502{,}6548 * 95 * 10^{-6}}j = -20{,}9414j$$

Určíme si uzlové smyčky a zostavíme rovnice:



I_A: $(Z_{L1} + R_1)I_A + Z_{C1}(I_A - I_C) + R_2(I_A - I_B) - U_1 = 0$
I_B: $U_1 + R_2(I_B - I_A) + Z_{L2}(I_B - I_C) + Z_{C2}\, I_B = 0$
I_C: $R_3 I_C + Z_{L2}(I_C - I_B) + Z_{C1}(I_C - I_A) + U_2 = 0$

$$\begin{pmatrix} Z_{L1} + R_1 + Z_{C1} + R_2 & -R_2 & -Z_{C1} \\ -R_2 & R_2 + Z_{L2} + Z_{C2} & -Z_{L2} \\ -Z_{C1} & -Z_{L2} & R_3 + Z_{L2} + Z_{C1} \end{pmatrix} \begin{pmatrix} U_1 \\ -U_1 \\ -U_2 \end{pmatrix}$$

$$\begin{pmatrix} 50{,}2655\text{j} + 11 - 9{,}0429\text{j} + 15 & -15 & 9{,}0429\text{j} \\ -15 & 15 + 42{,}7257\text{j} - 20{,}9414\text{j} & -42{,}7257\text{j} \\ 9{,}0429\text{j} & -42{,}7257\text{j} & 12 + 42{,}7257\text{j} - 9{,}0429\text{j} \end{pmatrix} \begin{pmatrix} 25 \\ -25 \\ -40 \end{pmatrix}$$

$$\begin{pmatrix} 16 + 41{,}2226\text{j} & -15 & 9{,}0429\text{j} \\ -15 & 15 + 21{,}7843\text{j} & -42{,}7257\text{j} \\ 9{,}0429\text{j} & -42{,}7257\text{j} & 12 + 33{,}6828\text{j} \end{pmatrix} \begin{pmatrix} 25 \\ -25 \\ -40 \end{pmatrix}$$

Vyjadríme determinant matice:

$$\text{Det} = \begin{vmatrix} 16 + 41{,}2226\text{j} & -15 & 9{,}0429\text{j} \\ -15 & 15 + 21{,}7843\text{j} & -42{,}7257\text{j} \\ 9{,}0429\text{j} & -42{,}7257\text{j} & 12 + 33{,}6828\text{j} \end{vmatrix} = -11602{,}8113 + 66559{,}7388\text{j}$$

Do tretieho stĺpca si dosadíme hodnoty napätia a vypočítame hodnotu I_C:

$$I_C = \frac{\begin{vmatrix} 16+41{,}2226\text{j} & -15 & 25 \\ -15 & 15+21{,}7843\text{j} & -25 \\ 9{,}0429\text{j} & -42{,}7257\text{j} & -40 \end{vmatrix}}{\text{Det}} = \frac{78276{,}6616 - 59138{,}7995\text{j}}{-11602{,}8113 + 66559{,}7388\text{j}} = -1{,}0613 - 0{,}9910\text{j}$$

Do prvého stĺpca si dosadíme hodnoty napätia a vypočítame hodnotu I_A:

$$I_A = \frac{\begin{vmatrix} 25 & -15 & 9{,}0429\text{j} \\ -25 & 15+21{,}7843\text{j} & -42{,}7257\text{j} \\ -40 & -42{,}7257\text{j} & 12+33{,}6828\text{j} \end{vmatrix}}{\text{Det}} = \frac{9754{,}3948 - 13674{,}3900j}{-11602{,}8113 + 66559{,}7388\text{j}} = -0{,}2242 - 0{,}1075\text{j}$$

Vypočítame si prúd I_{C1}:

$$I_{C1} = I_C - I_A$$
$$I_{C1} = -1{,}0613 - 0{,}9910\text{j} - (-0{,}2242 - 0{,}1075\text{j})$$
$$I_{C1} = -1{,}0613 - 0{,}9910\text{j} + 0{,}2242 + 0{,}1075\text{j}$$
$$I_{C1} = -0{,}8371 - 0{,}8835\text{j}$$

Vypočítame si napätie U_{C1}:

$$U_{C1} = I_{C1} * Z_{C1}$$
$$U_{C1} = (-0{,}8371 - 0{,}8835\text{j}) * -9{,}0429\text{j}$$
$$U_{C1} = 7{,}5698\text{j} - 7.9894$$
$$U_{C1} = -7.9894 + 7{,}5698\text{j}$$

Vypočítame amplitúdu U_{C1} (A je reálna časť, B je imaginárna časť):

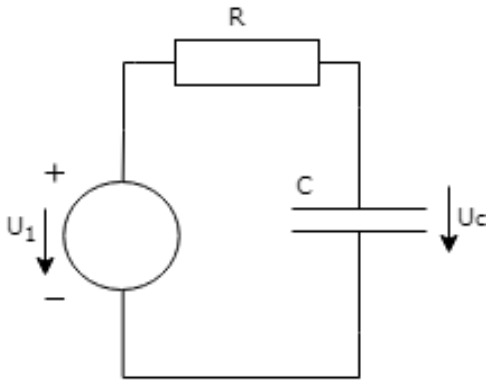
$$|U_{C1}| = \sqrt{A^2 + B^2}$$
$$|U_{C1}| = \sqrt{(-7.9894)^2 + (7{,}5698)^2}$$
$$|U_{C1}| = \sqrt{121{,}1324}$$
$$|U_{C1}| = \underline{11{,}0060\ V}$$

Dopočítame fázový posun φc_1. Keďže sa komplexné číslo nachádza v II. kvadrante, k výpočtu arkus tangensu pripočítame π.

$$\varphi c_1 = \pi + \text{arctg}(\frac{B}{A})$$
$$\varphi c_1 = \pi + \text{arctg}(\frac{7{,}5698}{-7.9894}) * \frac{\pi}{180}$$
$$\varphi c_1 = \underline{2{,}3832\ rad}$$

5C) Zostavte diferenciálnu rovnicu popisujúcu správanie obvodu na obrázku, ďalej ju upravte dosadením hodnôt parametrov. Vypočítajte analytické riešenie $u_c = f(t)$. Urobte kontrolu výpočtu dosadením do zostavenej diferenciálnej rovnice.

U[V]	C [F]	R [Ω]	$U_c(0)$ [V]
60	5	30	7



Základný vzťah je: $u_c' = \frac{1}{C} * i$

Vyjadríme si I pomocou Kirchhoffovho zákona

$$0 = U_R + u_c - U$$
$$0 = R*i + u_c - U$$
$$R*i = u_c - U$$
$$i = \frac{U - u_c}{R}$$
$$i = \frac{60 - u_c}{30}$$

Dosadíme I do základného vzorca:

$$u_c' = \frac{1}{5} * \frac{60 - u_c}{30}$$
$$u_c' = \frac{60 - u_c}{5*30}$$
$$u_c' = \frac{60 - u_c}{150}$$
$$u_c' = \frac{60}{150} - \frac{1}{150} u_c$$
$$u_c' + \frac{1}{150} u_c = \frac{2}{5}$$

Obecný tvar riešenia je:

$$u_c(t) = k(t)e^{\lambda t}$$

Vyjadríme si λ z charakteristickej rovnice:

$$\lambda + \frac{1}{150} = 0$$
$$\lambda = -\frac{1}{150}$$

Obecný tvar riešenia je:

$$u_c(t) = k(t)e^{-\frac{1}{150}t}$$
$$u_c'(t) = k'(t)e^{-\frac{1}{150}t} - \frac{1}{150}k(t)e^{-\frac{1}{150}t}$$

Známe hodnoty spätne dosadíme:

$$u_c' + \frac{1}{150}u_c = \frac{2}{5}$$
$$k'(t)e^{-\frac{1}{150}t} - \frac{1}{150}k(t)e^{-\frac{1}{150}t} + \frac{1}{150}k(t)e^{-\frac{1}{150}t} = \frac{2}{5}$$
$$k'(t)e^{-\frac{1}{150}t} = \frac{2}{5}$$

Vyjadríme k(t):

$$k'(t)e^{-\frac{1}{150}t} = \frac{2}{5}$$
$$k'(t) = \frac{2}{5}e^{\frac{1}{150}t}$$
$$\int k'(t)dt = \frac{2}{5}e^{\frac{1}{150}t}$$
$$k(t) = 60e^{\frac{1}{150}t} + c$$

Známe hodnoty dosadíme do obecnej rovnice:

$$u_c(t) = k(t)e^{-\frac{1}{150}t}$$
$$u_c(t) = (60e^{\frac{1}{150}t} + c)e^{-\frac{1}{150}t}$$
$$u_c(t) = 60 + ce^{-\frac{1}{150}t}$$

Dosadíme počiatočnú podmienku:

$$u_c(0) = 60 + ce^{-\frac{1}{150}0}$$
$$7 = 60 + c$$
$$c = -53$$

Riešenie je:

$$u_c(t) = 60 - 53e^{-\frac{1}{150}t}$$

Urobíme kontrolu dosadením do pôvodnej rovnice:

$$u_C' + \frac{1}{150} u_C = \frac{2}{5}$$

$$\frac{53e^{-\frac{1}{150}}}{150} + \frac{1}{150}\left(60 - 53e^{-\frac{1}{150}}\right) = \frac{2}{5}$$

$$\frac{53e^{-\frac{1}{150}}}{150} + \frac{60}{150} - \frac{53e^{-\frac{1}{150}}}{150} = \frac{2}{5}$$

$$\frac{60}{150} = \frac{2}{5}$$

$$\frac{2}{5} = \frac{2}{5}$$

$$0 = 0$$

Tabuľka riešení

Príklad	Verzia	Riešenie
1	B	I_{R1} = 0,1035 A, U_{R1} = 67,2896 V
2	C	I_{R3} = 0,3542 A, U_{R3} = 85,0080 V
3	C	I_{R5} = 0,4247 A, U_{R5} = 12,7410 V
4	B	$\lvert U_{C1} \rvert$ = 11,0060 V, φc_1 = 2,3832 rad
5	C	$u_c(t) = 60 - 53e^{-\frac{1}{150}t}$