

$$m\ddot{x} + kx = 0$$

$$mr^2 + k = 0$$

$$r = \pm i \sqrt{\frac{k}{m}}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$r^2 + 2\beta r + \beta^2 = 0$$

$$(r + \beta)^2 = 0 \quad r = -\beta, -\beta$$

$$x = Ae^{-\beta t} + Bte^{-\beta t} = (A + Bt)e^{-\beta t}$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$r = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

$$c^2 - 4mk = 0$$

critical
damping

$$r = -\frac{c}{2m}, -\frac{c}{2m}$$

$$mr^2 + cr + k = 0$$

$$\beta = \frac{c}{2m}$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

$$k = \frac{c^2}{4m}$$

$$\ddot{x} + 2\beta\dot{x} + \beta^2x = 0$$

$$\frac{k}{m} = \frac{c^2}{4m^2} = \beta^2$$