Polar Curves ($r = f(\theta)$)

$$A = \int_\alpha^\beta \frac{1}{2} r^2 \, d\theta \qquad A = \int_\alpha^\beta \frac{1}{2} r_1^2 - \frac{1}{2} r_2^2 \, d\theta$$

"outside" - "inside"

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2} \, d\theta$$

Trig Identities

$$\cos^2\theta = \frac{1}{2}[1+\cos 2\theta]$$

$$\sin^2\theta = \frac{1}{2}[1-\cos 2\theta]$$

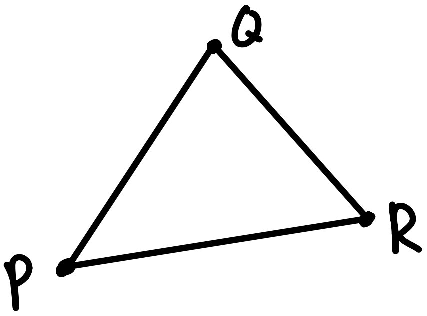
Vectors

unit vector: $\frac{\vec{a}}{|\vec{a}|}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \cdots$$



Area of ΔPQR: $\frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$

Lines (point: (x_0, y_0, z_0), dir. vector: $\vec{v} = \langle a, b, c \rangle$)

i) $\vec{r}(t) = \vec{v}t + \vec{r}_0$, where $\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$

ii) $x = at + x_0 \quad y = bt + y_0 \quad z = ct + z_0$

iii) $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Planes (point: (x_0, y_0, z_0), normal vec: $\vec{n} = \langle a, b, c \rangle$)

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Vector Functions ($\vec{r}(t) = \langle \underset{x}{f(t)}, \underset{y}{g(t)}, \underset{z}{h(t)} \rangle$)

Tangent vector: $\vec{r}\,'(t)$

Unit tangent vector: $\dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$

$$L = \int_a^b \sqrt{\underset{\frac{dx}{dt}}{[f'(t)]^2} + \underset{\frac{dy}{dt}}{[g'(t)]^2} + \underset{\frac{dz}{dt}}{[h'(t)]^2}}\, dt$$

Limits

Suppose $f(x,y) \to L_1$ as $(x,y) \to (a,b)$ along C_1

and $f(x,y) \to L_2$ as $(x,y) \to (a,b)$ along C_2

If $L_1 \neq L_2$ then the limit does not exist.

Tangent Planes

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Gradients / Directional Derivatives

Gradient of $f(x,y)$:

$$\nabla f(x,y) = \langle f_x(x,y), f_y(x,y) \rangle$$

Let $u = \langle a, b \rangle$ be a unit vector:

$$D_{\vec{u}} f(x,y) = f_x(x,y)\, a + f_y(x,y)\, b$$

$$= \nabla f(x,y) \cdot \vec{u}$$

$D_{\vec{u}}f$ is maximized when $\vec{u}$ is in the direction of ∇f. (i.e. $\vec{u} = \frac{\nabla f}{|\nabla f|}$)