# 1.2 Integrals as General and Particular Solutions

In this section we consider diff. eq'ns of the form:

$$\boxed{\frac{dy}{dx} = f(x)}$$

ie the rate of change of y does not depend on y, it only depends on x.

Note: the gen. sol'n is given by

$$\boxed{y(x) = \int f(x)\,dx + C.}$$

## Examples

(#6) $\frac{dy}{dx} = x\sqrt{x^2+9}, \quad y(-4) = 0$

$$y(x) = \int x\sqrt{x^2+9}\,dx + C$$

is the general sol'n of the diff. eq'n.

Substitution: $\left[\begin{array}{c} u = x^2+9 \\ du = 2x\,dx \end{array}\right]$

$$y(x) = \int \frac{1}{2}\sqrt{u}\,du + C$$

$$= \int \frac{1}{2} u^{1/2} du + C$$

$$= \frac{1}{2}\cdot\frac{2}{3} u^{3/2} + C$$

$$= \frac{1}{3}(x^2+9)^{3/2} + C$$

$\therefore$ $\boxed{y(x) = \frac{1}{3}(x^2+9)^{3/2} + C}$ is the gen. sol'n of the diff. eq'n.

$y(-4) = 0 \implies \frac{1}{3}\left((-4)^2+9\right)^{3/2} + C = 0$

$$\implies \frac{1}{3}(25)^{3/2} + C = 0$$

$$\implies \frac{1}{3}(5)^3 + C = 0$$

$$\implies C = \frac{-125}{3}$$

$\therefore$ the <u>particular sol'n</u> of the diff. eq'n satisfying the initial condition $y(-4)=0$ is

$$\boxed{y(x) = \frac{1}{3}\left[(x^2+9)^{3/2} - 125\right].}$$

(#10) $\frac{dy}{dx} = xe^{-x}$, $y(0)=1$

$$y(x) = \int xe^{-x}\,dx + C$$

Integration by parts:

$$(fg)' = f'g + fg'$$

$$fg = \int f'g + \int fg'$$

$$\int fg' = fg - \int f'g$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$\left[\begin{array}{ll} u = x & dv = e^{-x}dx \\ du = dx & v = -e^{-x} \end{array}\right]$$

$$y(x) = \int xe^{-x}dx + C$$

$$= x\cdot(-e^{-x}) - \int -e^{-x}dx + C$$

$$= -xe^{-x} + \int e^{-x}dx + C$$

$$= -xe^{-x} - e^{-x} + C$$

$$= -(x+1)e^{-x} + C$$

gen. sol'n

$$y(0)=1 \implies -(0+1)e^{-0}+C = 1$$

$$\implies -1 + C = 1$$

$$\implies C = 2.$$

$$\therefore \boxed{y(x) = -(x+1)e^{-x} + 2}$$

is the particular sol'n of the diff. eq'n that satisfies the initial condition.

# Velocity and Acceleration

Gravitational acceleration $g$ at the surface of the Earth is

$$\boxed{g \approx 32\ \mathrm{ft/s^2} \approx 9.8\ \mathrm{m/s^2}.}$$

Let $y(t)$ = height of object at time $t$
$v(t)$ = velocity of object at time $t$
$a(t)$ = acceleration of object at time $t$.

Then $y'(t) = v(t)$ and $v'(t) = a(t)$.

Thus $a(t) = -g$ (assuming no air resistance)

$v(t) = -gt + v_0$ ← initial velocity

$y(t) = -\frac{1}{2}gt^2 + v_0 t + y_0$ ← initial height

# Examples

(#27) A ball is thrown straight downward from the top of a tall building. The initial speed of the ball is 10 m/s. It strikes the ground with a speed of 60 m/s. How tall is the building?

$$a(t) = -9.8 \ m/s^2$$

$$\left[V_0 = -10 \ m/s\right]$$

$$v(t) = -9.8t + v_0$$

$$Y_0 = ?$$

$$= -9.8t - 10 \ m/s$$

$$y(t) = -\frac{9.8}{2}t^2 - 10t + y_0$$

$\hat{t}$ = time ball hits ground.

$$v(\hat{t}) = -60 \ m/s$$

$$-9.8\hat{t} - 10 = -60$$

$$\hat{t} = \frac{-50}{-9.8} = \frac{500}{98} = \frac{250}{49} \approx 5.1 \ s$$

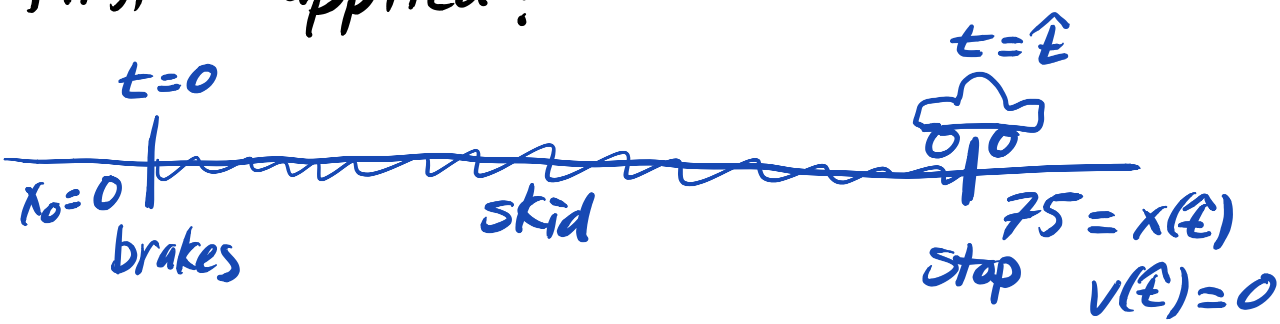
$$y(\hat{t}) = 0 \Rightarrow -4.9\hat{t}^2 - 10\hat{t} + y_0 = 0$$

$$\Rightarrow y_0 = 4.9\left(\frac{250}{49}\right)^2 + 10\left(\frac{250}{49}\right)$$

$$\therefore\ y_0 = \frac{1250}{7} \approx 178.57 \text{ m}$$

$\therefore$ The building is approx. $\boxed{178.57 \text{ m}}$ tall.

(#31) The skid marks made by an automobile indicated that its brakes were fully applied for a distance of 75 m before it came to a stop. The car in question is known to have a constant deceleration of 20 m/s² under these conditions. How fast — in km/h — was the car traveling when the brakes were first applied?




$$a(t) = -20 \text{ m/s}^2$$

$$\boxed{v_0 = ?}$$

$$v(t) = -20t + v_0$$

$$x(t) = -10t^2 + v_0 t + \cancel{x_0}^{\,0}$$

$$v(\hat{t}) = 0 \implies -20\hat{t} + v_0 = 0$$

$$\implies \boxed{v_0 = 20\hat{t}}$$

$$x(\hat{t}) = 75 \implies -10\hat{t}^2 + 20\hat{t}^2 = 75$$

$$\implies 10\hat{t}^2 = 75$$

$$\implies \hat{t}^2 = \frac{75}{10} = \frac{15}{2}$$

$$\implies \hat{t} = \sqrt{\frac{15}{2}} \approx 2.74\, s$$

$$\therefore v_0 = 20\hat{t} = 20\sqrt{\frac{15}{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 10\sqrt{30}$$

$$\implies v_0 \approx 54.77 \ m/s$$

$$\implies v_0 = 10\sqrt{30} \cdot \frac{3600}{1000} \ km/h$$

$$\approx \boxed{197.18 \ km/h.}$$