Lec 1

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Course Policy:

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Assignments: 25 (5 or 6 take home)

Quiz: 20 (2 × 10, no requiz)

Minor: 25 (re-minor on medical ground @ last week of course)

Major: 30

Books: E. Kreyszig - Introductory functional Analysis
B. V. Limaye - Linear Functional Analysis for Scientists and Engineers
M.T. Nair - Functional Analysis

Linear space / vector space

X (non empty set), $\mathbb{K}$ (Field)

$+ : X \times X \to X$

$\cdot : X \times \mathbb{K} \to X$

satisfy distributive and additive property

denoted as $X(\mathbb{K})$

we mostly work with $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$

ex: ① $\mathbb{R}^2$, $\mathbb{R}^3$

Field should have atleast 2 elements,
0, 1
additive identity — multiplicative identity

② $\mathbb{K}^n(\mathbb{K})$ is also vector space

$(x_1, x_2, \dots, x_n)$ $x_i \in \mathbb{K}$

③ $P_n(\mathbb{K})$ = set of all polynomial of degree $\leq n$ $(n \in \mathbb{N})$ and coef from $\mathbb{K}$
is a vector space over $\mathbb{K}$

$\mathcal{P} = \bigcup_{n=0}^{\infty} P_n$ is also vector space over $\mathbb{K}$

$x \in \mathcal{P} \iff x \in P_n$ for some $n \in \mathbb{N}$

Metric Space

$(X \neq \phi)$, (X, d) is metric space:

$d : X \times X \to \mathbb{R}$

$d(x, y) = 0 \iff x = y$ —— ①

$d(x, y) = d(y, x)$ —— ②

$d(x, z) \leq d(x, y) + d(y, z)$ —— ③

Every set can be made a metric space with discrete metric.

$\mathbb{R}$ $d(x,y) = |x - y|$

$\mathbb{R}^2$ $d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$ $[x = (x_1, x_2), y = (y_1, y_2)]$

$$d_1(x,y) = |x_1 - y_1| + |x_2 - y_2|$$

$\mathbb{R}^n$ $$d(x,y) = \sqrt{\sum (x_i - y_i)^2}$$

$$d_\infty(x,y) = \max_{1 \le i \le n} |x_i - y_i|$$

$$d_1(x,y) = \sum_{i=1}^{n} |x_i - y_i|$$

$$d_2(x,y) = \sqrt{\sum_{i=1}^{n} |x_i - y_i|^2}$$

$$d_p(x,y) = \left(\sum_{i=1}^{n} |x_i - y_i|^p\right)^{1/p}$$

$$d_{dis}(x,y) = \begin{cases} 0 & x = y \\ 1 & x \neq y \end{cases}$$

$C[a,b]$ = set of continous func^n in $[a,b]$

$$d_\infty(f,g) = \sup_{x \in [a,b]} |f(x) - g(x)| \qquad f : [a,b] \to \mathbb{R}$$

sup exists for bounded subset of $\mathbb{R}$

continous func^n over bounded interval is bounded

$$d_1(f,g) = \int_a^b |f(x) - g(x)| \, dx$$

In $[a,b]$ there should be a metric, as in the δ,ε-def of continuity

for any $\varepsilon > 0$ $\exists$ $\delta > 0$, $|f(x) - f(x_0)| < \varepsilon$

$\forall$ $|x - x_0| < \delta$

$d(x, x_0)$

$C(\Omega)$ = the set of $x, y \in C(\Omega)$ collection of all continuous functions defined on a domain Omega

$$(x+y)(s) = x(s) + y(s)$$
$$(\alpha x)(s) = \alpha x(s)$$

$\alpha \in \mathbb{K}$

$(C(\Omega), \mathbb{K})$

Ex $k \in \mathbb{N}$

$C^k[a,b]$ = set of all $\mathbb{K}$-valued functions defined on $[a,b]$ such that $j \in \{1, 2, \dots, k\}$ j^{th} derivative of x exists.

$S \neq \phi$

$F(S, \mathbb{K})$ = set of all func^n from $S \mapsto \mathbb{K}$ General Function Spaces

$x, y \in F(S, \mathbb{K})$ collection of all maps from S into the field K

$$(x+y)(s) = x(s) + y(s)$$

$+$ in $F(S, \mathbb{K})$ $\qquad$ $+$ in field $\mathbb{K}$

$\alpha \in \mathbb{K}$, $(\alpha x)(s) = \alpha \cdot x(s)$

Consider $S = \{1, 2, 3 \dots n\}$

$f : \{1, 2, \dots, n\} \to \mathbb{K}$

$$f \longmapsto \{f(1), f(2), \ldots, f(n)\}$$

$$J : F(S, \mathbb{K}) \longrightarrow \mathbb{K}^n$$

$$J(x) = \{x(1), x(2), \ldots x(n))$$

J is a bijection

$$J(x+y) = J(x) + J(y)$$

$$J(\alpha x) = \alpha J(x)$$

$$F(S, \mathbb{K}) \cong \mathbb{K}^n$$

$$x = (x(1), \ldots, x(n))$$
$$X(x_1, x_2, \ldots, x_n)$$

$$(x_n)_{n \in \mathbb{N}} \in \mathbb{K}^n$$

$$x_1 = (x_1(1), \ldots, x_1(n))$$

$$S = \mathbb{N}$$

$F(S, \mathbb{K})$ = set of all seq

$$c_0 = \{ x \in F(\mathbb{N}, \mathbb{K})$$
$$x(n) \to 0 \text{ as } n \to \infty$$

$$c = \{ x \in F(\mathbb{N}, \mathbb{K}), \ x(n) \text{ converges} \}$$

$$x_1 = (x_1(1), \ldots, x_1(n)\}$$

$$x \in c_{00} = \bigcup_{u=1}^{\infty} \{ (x(1), x(2), \ldots x(j), \ldots) \}$$
$$x(j) = 0 \ \forall j \geq u$$

$$e_i = (0, 0, \ldots 0, \underset{i\text{th}}{1}, 0, 0, \ldots)$$

$$c_{00} = \operatorname{span}_{i \in \mathbb{N}} \{e_i\}$$

$x \in c_{00}$ then $x = \sum_{i=1}^{n} a_i e_i$ $\boxed{\text{finite sum}}$ ☆☆

$\boxed{c_{00} \text{ is seperable set}}$

what is dense set in l_2?

next class on Saturd