ESERCIZIO 1 (Foglio A)

(A) Si enuncino il Teorema Master e il suo Corollario.

(B) Si definiscano le notazioni asintotiche $\Theta(f(n))$, $o(f(n))$, $\Omega(f(n))$ per una data funzione $f\colon \mathbb{N} \to \mathbb{N}$.

(C) Si risolva l'equazione di ricorrenza $T(n) = a \cdot T\left(\frac{n}{3}\right) + \Theta\left(n^3 \log^2 n\right)$ al variare del parametro reale $a > 1$.

(D) Sia $T(n)$ la funzione di cui al punto precedente. Per quali valori di a si ha:

(i) $T(n) = \Theta(n^4)$; (ii) $T(n) = \Omega(n^3 \log^4 n)$; (iii) $T(n) = o(n^3 \log^3 n)$?

(B)

$$\Theta(f(n)) = \{ g(n) : (\exists c_1, c_2 > 0)(\exists n_0 \in \mathbb{N})(\forall n \geq n_0)\ 0 \leq c_1 f(n) \leq g(n) \leq c_2 f(n) \}$$

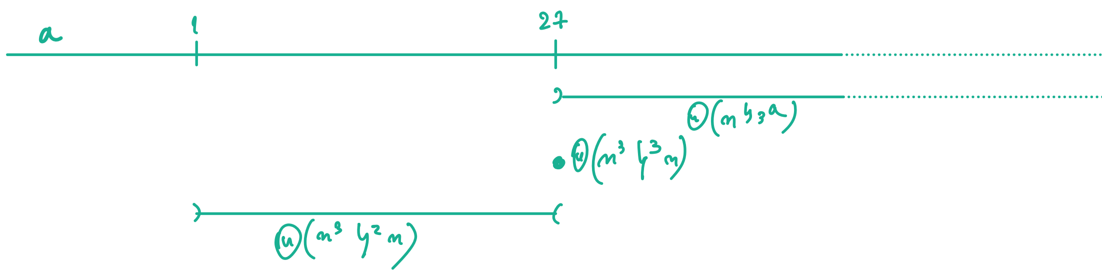
$$o(f(n)) = \{ g(n) : (\forall c > 0)(\exists n_0 \in \mathbb{N})(\forall n \geq n_0)\ 0 \leq g(n) < c f(n) \}$$

$$\Omega(f(n)) = \{ g(n) : (\exists c > 0)(\exists n_0 \in \mathbb{N})(\forall n \geq n_0)\ 0 \leq c f(n) \leq g(n) \}$$

(C) Si risolva l'equazione di ricorrenza $T(n) = a \cdot T\left(\frac{n}{3}\right) + \Theta\left(n^3 \log^2 n\right)$ al variare del parametro reale $a > 1$.

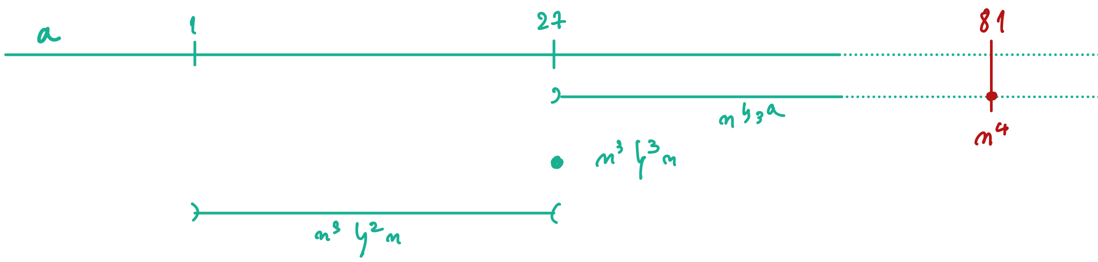
$$T(n) = \begin{cases} \Theta(n^{\log_3 a}) & \log_3 a > 3 \quad (\Leftrightarrow a > 3^3 = 27) \\ \Theta(n^3 \log^3 n) & \log_3 a = 3 \quad (\Leftrightarrow a = 3^3 = 27) \\ \Theta(n^3 \log^2 n) & 0 < \log_3 a < 3 \quad (\Leftrightarrow 1 < a < 27) \end{cases}$$

$$= \begin{cases} \Theta(n^{\log_3 a}) & a > 27 \\ \Theta(n^3 \log^3 n) & a = 27 \\ \Theta(n^3 \log^2 n) & 1 < a < 27 \end{cases}$$



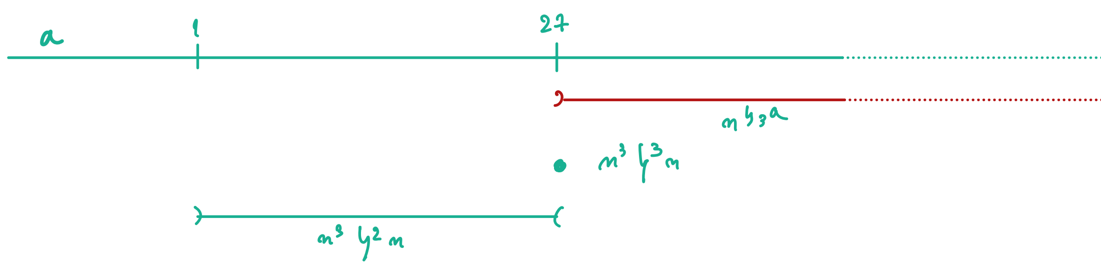
(D) (i) $T(n) = \Theta(n^4)$; (ii) $T(n) = \Omega(n^3 \log^4 n)$; (iii) $T(n) = o(n^3 \log^3 n)$?

(i) $T(n) = \Theta(n^4)$



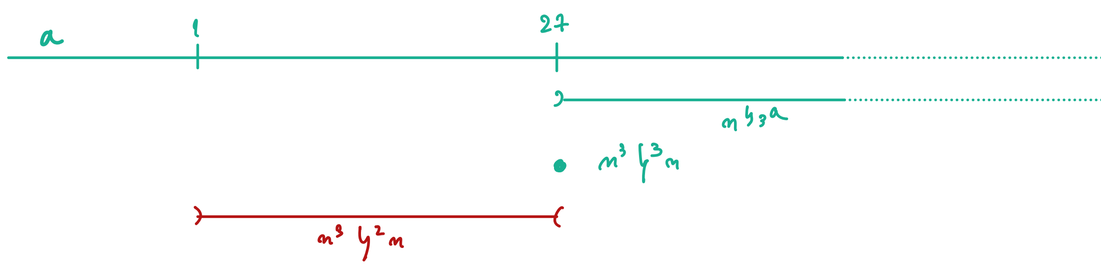
$\Leftrightarrow \log_3 a = 4 \Leftrightarrow a = 3^4 = 81$

(ii) $T(n) = \Omega(n^3 \log^4 n)$



$\Leftrightarrow a > 27$

(iii) $T(n) = o(n^3 \log^3 n)$



$\Leftrightarrow 1 < a < 27$