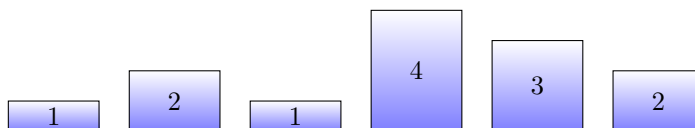


## Eksaminatorier DM573 Uge 48/49

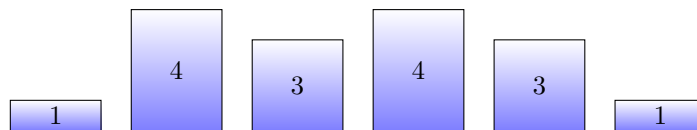
Husk at læse de relevante sider i slides før du/I forsøger at løse en opgave.

### I: Løses i løbet af øvelsestimerne i uge 48

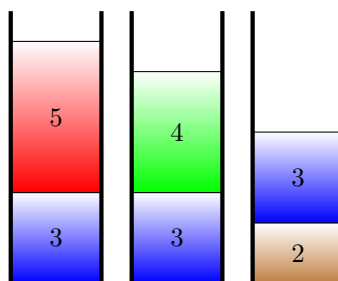
1. For  $m = 3$ , which schedule does the List Scheduling algorithm, Ls, produce on the following input sequence:



2. In the lecture, we proved that the machine scheduling algorithm, Ls, could not perform better than  $2 - \frac{1}{m}$ . We now consider only two machines. Thus,  $m = 2$ , and the ratio is then  $\frac{3}{2}$ . Just because Ls cannot perform better, it could be that some other algorithm could. Prove (for  $m = 2$ ) that this is not the case. You must design an input, where *no* algorithm, no matter what decisions it makes, can do better than  $\frac{3}{2}$  times OPT. You only need sequences with three jobs and a case analysis with only two cases, depending on what an algorithm does with the second job that is given.
3. Consider the first bin packing example given in the lecture (slide 19), where the First-Fit algorithm, FF, uses four bins. Show that OPT only needs three.
4. How does the First-Fit algorithm, FF, behave on the input sequence below? Item sizes are given in multiples of  $\frac{1}{6}$ .



5. Why can the following configuration *not* have been produced by the bin packing algorithm FF? Item size are given in multiples of  $\frac{1}{9}$ .



6. Prove that no matter which other algorithm than the one from the lecture slides we define for ski rental, the algorithm will perform worse, i.e., the competitive ratio will be strictly higher than  $\frac{19}{10}$ . (As explained in the lecture slides, any algorithm is of the form **Buy on day X**, where the choice of **X** determines the algorithm).

Start by analyzing the algorithms **Buy on day 5** and **Buy on day 15** to see what happens. The skis still cost 10 units to buy and 1 unit per day to rent.

## II: Løses hjemme inden øvelsestimerne i uge 49

1. For bin packing, one can prove the upper bound that FF is 1.7-competitive. However, this is a quite hard proof. In this exercise, we will try to improve (raise) the lower bound.

In the lecture, we saw an example demonstrating that FF can be as bad as  $\frac{3}{2} = 1.5$  times OPT.

Let that example inspire you, and try to use items of the following three sizes:

$$\frac{1}{7} + \frac{1}{1000}, \quad \frac{1}{3} + \frac{1}{1000}, \quad \frac{1}{2} + \frac{1}{1000}$$

Find a sequence where FF performs  $\frac{5}{3} = 1.666\dots$  times worse than OPT.

Now try using

$$\frac{1}{43} + \frac{1}{10000}, \quad \frac{1}{7} + \frac{1}{10000}, \quad \frac{1}{3} + \frac{1}{10000}, \quad \frac{1}{2} + \frac{1}{10000}$$

to get a lower bound even closer (than 1.666...) to the 1.7 upper bound.

2. It is very easy to implement FF in PYTHON, if there are no efficiency requirements: just use a list to hold the current level in the bins, and for each item, search for the first bin with enough space. If you make sure there are enough bins from the beginning, then there are no special cases. And you simply count the number of non-empty bins at the end to get the result.

Implement FF.

Try to define your own algorithm, from scratch or as a variant of FF.

Test your own algorithm up against FF and try to determine which one is best; for instance on uniformly distributed sequences, i.e., each item size is chosen like this:

```
from random import seed, random

seed(42)

.
.
item = random()
nextitem = random()
.
.
```

You don't have to use `seed`; the intention is simply to provide a *reproducible* random sequence, i.e., `seed` is called once and the seed value initializes the sequence.