A Probability - Refresher Training I

Exercise A.1 (easy)
In a communication channel, a zero or a one is transmitted. The probability that a zero is transmitted is 0.1. Due to the noise in the channel, a zero can be received as one with probabbilty 0.01 and a one can be received as a zero with probability 0.05. If you receive a zero, what is the probability that a zero was transmitted ? If you receive a one, what is the probability that a one was transmitted ?

Exercise A.2 (easy)
Four suppliers provide 10%, 20%, 30% and 40% of the bolts sold by a hardware shop and the rate of defects in their products are 1%, 1.5%, 2% and 3% respectively. Calculate the probability of a given defective bolt coming from supplier 1.

Exercise A.3 (easy)
The two events A and B have $\Pr(A) = 1/2$, $\Pr(B) = 1/3$, $\Pr(A \cup B) = 2/3$. Are the events A and B independent ? Are they mutually exclusive ?

Exercise A.4 (easy)
What is the probability P of having at least six heads when tossing a coin ten times ?

Exercise A.5 (easy)
A die is thrown six times. What is the probability of having at two 4s, two 5s and two 6s ?
Hints : think about the multinomial distribution.

Exercise A.6 (easy)
Two boxes containing marbles are placed on a table. The boxes are labeled B_1 and B_2. Box B_1 contains 7 green marbles and 4 white marbles. Box B_2 contains 3 green marbles and 10 yellow marbles. The boxes are arranged so that the probability of selecting box B_1 is $1/3$ and the probability of selecting box B_2 is $2/3$. Kathy is blindfolded and asked to select a marble. She will win a color TV if she selects a green marble.

1. What is the probability that Kathy will win the TV (that is, she will select a green marble) ?
2. If Kathy wins the color TV, what is the probability that the green marble was selected from the first box ?

Exercise A.7 (easy)
Let the random variable X have the density function

$$f(x) = \begin{cases} k\,x & \text{for } 0 \le x \le \sqrt{\frac{2}{k}}, \\ 0 & \text{elsewhere}, \end{cases}$$

where k is a positive real value. If the mode is at $x = \frac{\sqrt{2}}{4}$, then what is the median of X ?

Exercise A.8 (easy)
What is the probability density function of the random variable whose cumulative distribution function is

$$F(x) = \frac{1}{1 + e^{-x}}, \quad -\infty < x < \infty?$$

Exercise A.9 (easy)
The length of time required by students to complete a 1-hour exam is a random variable with a pdf given by

$$f(x) = \begin{cases} c\,x^2 + x & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

What the probability a student finishes in less than a half hour ?

Exercise A.10 (easy)
Show that the variance of an exponential random variable with parameter λ is $\frac{1}{\lambda^2}$.

Exercise A.11 (easy)
Let X be a random variable with the cumulative distribution function

$$F(x) = \begin{cases} 1 - e^{-x} & \text{for } x > 0, \\ 0 & \text{elsewhere.} \end{cases}$$

What is $\Pr(0 \leq e^X \leq 4)$?

Exercise A.12 (easy)
Let X be a random variable with the probability density function

$$f(x) = \begin{cases} 1 - |x| & \text{for } |x| < 1, \\ 0 & \text{elsewhere.} \end{cases}$$

Calculate $\mathbb{E}[X]$ and var$[X]$.

Exercise A.13 (medium)
Let $W = e^X$ where $X \sim \mathcal{N}(\mu, \sigma^2)$. What is the probability density function of W for $w \in (0, +\infty)$?

Exercise A.14 (medium)
If the random variable X is normal with mean 1 and standard deviation 2, then what is

$$\Pr(X^2 - 2X \leq 8)?$$

B Probability - Refresher Training II

Exercise B.1 (medium)
Let X an univariate normal random variable with mean 0 and variance σ^2. Let $Y = X^2$.

1. Calculate the cumulative distribution function $F_Y(y)$ of Y.
2. Calculate the probability density function $f_Y(y)$ of Y.
3. Calculate $\mathbb{E}[Y]$.

Exercise B.2 (easy)
Let us suppose that X is a continuous random variable with pdf $f_X(x)$. Let $Y = aX + b$ where $a \neq 0$ and b are two real numbers. Show that

$$f_Y(y) = \frac{1}{a} f_X\left(\frac{y-b}{a}\right).$$

Exercise B.3 (easy)
Let X be normally distributed with mean μ and standard deviation σ. Determine $\Pr(|X-\mu| \geq 2\sigma)$. Compare with Chebyshev's inequality.

Exercise B.4 (medium)
Let X and Y be any two random variables and let a and b be any two real numbers. Then show that

$$\text{var}[aX + bY] = a^2\text{var}[X] + b^2\text{var}[Y] + 2ab\,\text{cov}(X, Y).$$

Exercise B.5 (difficult)
If the random variable $X \sim Exp(\theta)$ with $\theta > 0$, then what is the probability density function of the random variable $Y = X\sqrt{X}$?

Exercise B.6 (easy)
Let X and Y be discrete random variables with joint probability density function

$$f(x,y) = \begin{cases} \frac{1}{21}(x+y) & \text{for } x = 1, 2, 3; y = 1, 2, \\ 0 & \text{otherwise.} \end{cases}$$

What are the marginals of X and Y ?

Exercise B.7 (difficult)
Let X and Y have the joint probability density function

$$f(x,y) = \begin{cases} \exp^{-(x+y)} & \text{for } 0 \leq x, y < +\infty, \\ 0 & \text{otherwise.} \end{cases}$$

What is $\Pr(X \geq Y \geq 2)$?

Exercise B.8 (easy)
Suppose the random variables X and Y are independent and identically distributed. Let $Z = aX + Y$. If the correlation coefficient between X and Z is $\frac{1}{3}$, then what is the value of the constant a ?

Exercise B.9 (difficult)
Let X and Y be two independent random variables. If $X \sim BIN(n, p)$ and $Y \sim BIN(m, p)$, then what is the distribution of $X + Y$?

Exercise B.10 (difficult)
Let X and Y be two independent random variables. If X and Y are both standard normal, then what is the distribution of the random variable $Z = \frac{1}{2}(X^2 + Y^2)$?

Exercise B.11 (easy)
Let X_1, X_2, $\ldots$, X_n be a random sample of size n from a Bernoulli distribution with probability of success $p = \frac{1}{2}$. What is the limiting distribution of the sample mean $\bar{X}$?