Taif University
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TITLE OF THESIS

A thesis submitted by

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in partial fulfillment of the requirements
for the degree of Master of Science in Pure Mathematics

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2017

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Word count: 39048

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Abstract

Describe briefly your thesis in a single page.

We study in this thesis the influence of prescribed flows on three distinct types of flame, namely flame balls, premixed flames and triple flames. The interaction between flame propagation and fluid flow is examined using asymptotic analyses and numerical simulations of thermo-diffusive models. We consider first the effect of a flow of hot inert gas, either a source or a sink, located at the origin of flame balls. It is shown that the flow gives rise to new kinds of flame balls characterised by having nonzero burning speeds, which we refer to as *generalised flame balls*.

The second part of the thesis is concerned with premixed flame propagation in the presence of parallel and vortical flow fields. Special attention is devoted to examining the effect of high-intensity flow on flame propagation. In this limit, the study identifies several behaviours of the effective flame speed depending on the flow intensity, the flow scale and the Reynolds number.

Finally, we study the response of triple flames to the presence of parallel flows in the direction of flame propagation. The effect of flow on flame propagation is found to be determined mainly by the scale of the flow compared to the radius of curvature of the flame: large scale flows are shown to have no remarkable influence on the flame structure, whereas flows whose scale is of the same order of magnitude or smaller than the radius of curvature can affect the flame in different ways, such as wrinkling its premixed branches or shifting its leading edge from the stoichiometric line.

Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

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Acknowledgements

Above all, I must thank Allah, most Gracious, Most Merciful, for helping me finishing this work.

I would like to express my deep and sincere gratitude to all those who provide me any kind of help. Thank you for anyone who encouraged me to discover new and exciting aspects of the life.

Dedication

To my parents, my wife, etc.

Chapter 1

Introduction

Flame propagation in a flow field is a fundamental problem in combustion theory with a wide range of practical applications. The problem is characterised by the presence of different kinds of interlinked inhomogeneities that are inherited not only from the combustion process itself but also from the flow field. These inhomogeneities include the spatial and temporal non-uniformities of the flow field and the spatial non-uniformities in the enthalpy of the reactants. These inhomogeneities have significant implications for the structure, propagation and stability of flames.

Chapter 2

Literature review

This chapter provides a brief overview of combustion theory. The discussion is limited to some of the concepts used in the following chapters. The aim is to provide the necessary ingredients to model combustion phenomena mathematically. We begin by introducing some basic terms such as premixed flames, diffusion flames and partially premixed flames in section 2.1. Subsequently, we present in section 2.2 the mathematical equations that describe flame propagation in a flow field.

2.1 Review

In addition to the significance of flame balls as a possible mode of combustion in weakly flammable mixtures, an important related aspect, of particular relevance to the present work, is their significance in ignition problems involving heat addition by an external source such as an electric spark. In this context, they may indeed serve to estimate the minimum energy to be deposited by the source for successful ignition whereby an initially formed hot kernel generates an outwardly propagating flame front [1, p. 331]. For successful ignition to occur, however, both the power and the duration of the source need be taken into account [2, 3, 4]. In the particular case of a point source of constant power, two branches of stationary solutions are obtained depending on the power of the source [2]; the lower branch representing small flame balls being stable, the upper branch representing large flame balls, including Zeldovich flame balls, being unstable.

In this work, we extend these studies, by considering a model for flame balls in the presence of a flow of hot inert gas, either a source or a sink, at their origin. Depending on the direction and magnitude of the flow, these flames can have positive, zero or negative burning speeds, with zero speeds characterising Zeldovich flame balls. We shall refer to these stationary solutions of the advection-diffusion-reaction heat and mass transport equations as *generalised flame balls*.

One motivation for studying these solutions is that they provide a simple framework for analysing the important problem of flame initiation, intentional or accidental, by a hot gas stream [5, p. 265]-[6]. They also provide valuable analytical information on the effect of convection, albeit for a specific radial flow, on the existence and stability of flame balls. Finally, the dependence of their burning velocity, positive or negative, on their curvature, small or large, may provide some insight into such dependence when studying the local behaviour of more complex premixed flames such as edge-flames in strained mixing layers or flamelets in turbulent flow fields.

This chapter is structured as follows. We begin by formulating the model and

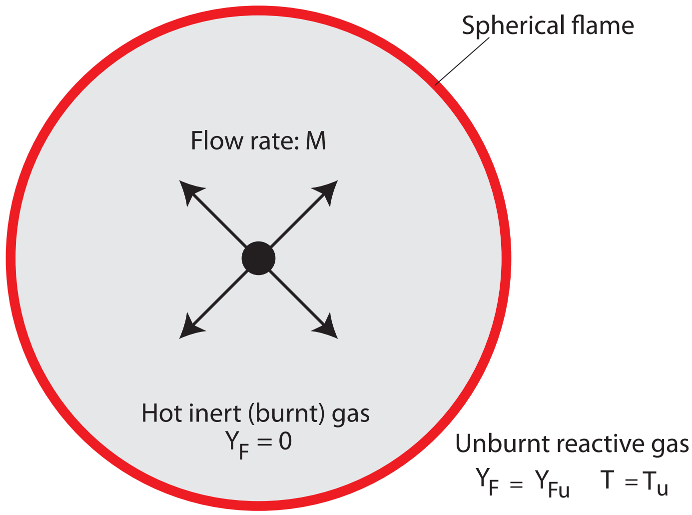

Figure 2.1: Spherical flame sustained by a point source of hot inert gas, with flow rate $M > 0$. Negative values of M correspond to a sink (not shown in this figure).

identifying its main non-dimensional parameters. An asymptotic analysis is then presented, where the stationary solutions, their multiplicity and their stability are fully described. This is followed by a numerical study which validates and illustrates the analytical results. Finally, a conclusion section where the main findings are summarised and additional extensions of the work suggested, closes the chapter.

2.2 Mathematical formulation

We consider a spherically symmetric flame around a point source of hot inert gas located at the origin, as shown in figure 2.1. The governing equations can be written in spherical coordinates using the conservation equations presented in Chapter 2.

Within the thermo-diffusive approximation of constant density and constant transport properties, a relevant non-dimensional model consists of the equations

$$\frac{\partial\theta}{\partial t} + \frac{M}{r^2}\frac{\partial\theta}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\theta}{\partial r}) + \omega, \qquad)(2.1$$

$$\frac{\partial y_F}{\partial t} + \frac{M}{r^2}\frac{\partial y_F}{\partial r} = \frac{1}{\mathrm{Le}}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial y_F}{\partial r}) - \omega, \qquad)(2.2$$

subject to the far-field boundary condition

$$\theta = 0, \qquad y_F = 1 \qquad \text{as} \quad r \to \infty, \qquad)(2.3$$

and the requirement inside the ball that

$$\begin{cases} (\theta, y_F) = (\theta_0, 0) & \text{for} \quad M > 0 \\ (\theta, y_F)\ \text{bounded are} & \text{for} \quad M \leq 0 \end{cases} \qquad \text{as} \quad r \to 0\,. \qquad)(2.4$$

Here θ and y_F are the non-dimensional temperature and mass fraction of the fuel, respectively, and Le is the Lewis number of the fuel. They are given by $\theta = (T - T_u)/(T_{ad} - T_u)$ and $y_F = Y_F/Y_{Fu}$, where T_u and Y_{Fu} are the temperature and the fuel mass fraction of the reactive mixture in the far-field, and T_{ad} is the adiabatic *planar flame* temperature. The fuel is assumed to limit the reaction rate ω, which is taken to follow the standard (non-dimensional) Arrhenius form

$$\omega = \frac{\beta^2}{2\ \mathrm{Le}} y_F \exp\left(\frac{\beta(\theta - 1)}{1 + \alpha(\theta - 1)}\right),$$

where β is the non-dimensional activation energy or Zeldovich number, and $\alpha = (T_{ad} - T_u)/T_{ad}$ the heat release parameter. A convection term of strength M is included in the equations to account for the presence of point-source radial flow at the origin if $M > 0$, where M represents the non-dimensional volumetric flow rate; when $M < 0$, we are in the presence of a sink which sucks the burnt gas. The units for speed and length chosen for the non-dimensionalisation correspond to the propagation speed S_L and the thickness ℓ_{Fl} of the *planar flame* (more precisely to the asymptotic values of these as $\beta \to \infty$, which are discussed in Chapter 2).

The far-field boundary condition (2.3) corresponds to a frozen mixture with prescribed temperature and composition. The boundary condition (2.4) specifies the temperature of the fuel-free (inert) stream at the origin when $M > 0$; this condition is redundant when $M \leq 0$ if θ and y_F are simply required to be bounded.

We have now completed the formulation of the problem. The main task is to find time-independent solutions to equations (2.1) to (2.4) which represent spherical flame balls whose radius R depend on the parameters Le, M and θ_0, in addition to β and α.

We begin by reformulating the problem in the asymptotic limit $\beta \to \infty$ in the next section. This is followed by an analytical treatment which allows R to be determined in terms of three parameters representing Le, M and θ_0, along with the multiplicity of the solutions. The linear stability of these to one-dimensional perturbations is then addressed analytically. Finally, a numerical treatment of the problem with a finite value of β is provided, which supports and illustrates the analytical findings. We also discuss the application of the results to the problem of ignition by a hot inert gas flow and to clarifying the dependence of the speed and the burning rate of the flames encountered on their curvatures.

Chapter 3

First Chapter

3.1 Introduction

Premixed flame propagation in a flow field is a fundamental problem in combustion theory with applications in many practical combustion devices. The problem is characterised by the interaction of curved flame with a flow that can involve several length and velocity scales, as can be seen in turbulent combustion. This multiscale nature of the problem, coupled with the presence of several interlinked processes such as diffusion and reaction, has a significant impact on the flame characteristics such as its shape and propagation speed.

3.2 Equation

Premixed flame propagation in a flow field is a fundamental problem in combustion theory with applications in many practical combustion devices. The problem is characterised by the interaction of curved flame with a flow that can involve several length and velocity scales, as can be seen in turbulent combustion. This multiscale nature of the problem, coupled with the presence of several interlinked processes such as diffusion and reaction, has a significant impact on the flame characteristics such as its shape and propagation speed.

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Chapter 4

Second Chapter

4.1 Introduction

Flame propagation in a flow field is an important problem from both theoretical and practical points of view, characterised in general by the interaction of a curved flame with a flow that can involve a wide range of temporal and spatial scales. This complication is sometimes coupled by the presence of different kinds of inhomogeneity in the combustion mixture itself. These include the spatial non-uniformities in the compositions of the reactants and their temperature, as frequently encountered in non-premixed devices such as that in mixing layers of initially non-premixed reactants. Analysing such problems in real-life applications is a formidable task, but it is instructive to examine such interactions in mixing layers, at least for simple prototypical flows.

4.2 Mathematical Model

Flame propagation in a flow field is an important problem from both theoretical and practical points of view, characterised in general by the interaction of a curved flame with a flow that can involve a wide range of temporal and spatial scales. This complication is sometimes coupled by the presence of different kinds of inhomogeneity in the combustion mixture itself. These include the spatial non-uniformities in the compositions of the reactants and their temperature, as frequently encountered in non-premixed devices such as that in mixing layers of initially non-premixed reactants. Analysing such problems in real-life applications is a formidable task, but it is instructive to examine such interactions in mixing layers, at least for simple prototypical flows.

4.3 Methodology

This section contains the methodology used throughout the chapter.

4.4 Results

Describe your genius results here :)

Chapter 5

Conclusions

We have examined in this thesis the effect of prescribed flow fields on the propagation of three distinct types of flame namely flame balls, premixed flames and triple flames. In general, the main aim was to assess the influence of the flow on some of the characteristics of these flames such as flame structure, propagation speed and stability. The current investigation has been restricted to prescribed flows within the constant-density approximation, neglecting the effect of thermal expansion. The study of the flame-flow interaction has been based on asymptotic analyses and numerical simulations of the thermo-diffusive models. We summarise in the following the main findings for each of these flames along with recommendations for future work.

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Appendix A

Title

We now consider the classical problem with a source of reactive mixture at the origin sustaining a spherically symmetric flame, mentioned briefly at the end of section **??**. In the limit $\beta \to \infty$, the reaction is confined to an infinitely thin reaction sheet, located at $r = r_f$, say. We adopt the near-equidiffusion flame approximation for which $l \equiv \beta(\text{ Le} - 1)$ is $\mathcal{O}(1)$, supplemented by the assumption that the temperature at infinity θ_∞ may deviate from unity by an amount at most of $\mathcal{O}(\beta^{-1})$, and thus may be written as $\theta_\infty = 1 + h_\infty/\beta$, which defines h_∞. These assumptions insure that the leading order temperature θ^0 is unity in the burnt gas ($\theta^0(r \geq r_f) = 1$), and allow the problem to be reformulated in terms of θ^0 and the (excess) enthalpy $h \equiv \theta^1 + y_F^1 \sim \beta(\theta + y_F - 1)$. In terms of θ^0 and h we have to solve

الملخص العربي

سوف نستعرض في هذا الفصل بعضاً الطرق العددية المستخدمة لحل أنظمة المعادلات الخطية. تصنف هذه الطرق عادة إلى نوعين هما: الطرق المباشرة (direct methods) وهي الطرق التي يمكن أن تستخدم لإيجاد حل مضبوط (Exact solution)

للنظام الخطي و هي الطرق التي تتأثر فقط بأخطاء التدوير. أما النوع الأخر فهور الطرق التكرارية (Iterative methods) وهي الطرق التي تستخدم لإيجاد حلول تقريبية للنظام الخطي.

سوف نركز في هذا الفصل على محاولة حل نظام خطي يأخذ الشكل العام التالي

$$\left\{\begin{array}{ll} E_1: & a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1, \\ E_2: & a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2, \\ \vdots & \quad\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ E_n: & a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \ldots + a_{nn}x_n = b_n, \end{array}\right. \tag{1. }$$

حيث $x_1, x_2, \ldots, x_n$ هي عبارة عن المجاهيل أما a_{ij} و b_i لكل $i, j = 1, 2, .., n$ هي أعداد حقيقية. يشير الرمز E_i إلى المعادلة الخطية رقم i في النظام الخطي.

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