Digital Signal Processing

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Part I

Digital filters

Filters

- **Digital filter** is an algorithm, which transforms *input discrete signal* into another *output discrete signal*.
- The process may include low-pass filtering (smoothing), band-pass filtering, interpolation, generation of derivatives etc.
- Filter is thus just another name given to a *discrete system*, when it is used in the context of signal processing.
- Filters thus have mathematical properties, which we defined earlier for discrete systems … (linearity, causality, stability)
- … and also other properties, which stem from the frequency response and concern signal processing operations.

Filters II

- **Allpass filter:**
- The magnitude of an *allpass filter* is constant and independent on frequency

$$\left|H(e^{j\omega})\right| = c, c \in \mathcal{R} \tag{1}$$

- **Frequency selective filters:**
- *Low-pass, high-pass, band-pass, band-stop*
- Presented responses are ideal and unachievable. In practice, these need to be closely approximated.
- *Stop band* - $\left|H(e^{j\omega})\right| = 0$
- *Pass band* - $\left|H(e^{j\omega})\right| = 1$
- *Cutoff frequency* - Frequency separating pass and stop bands

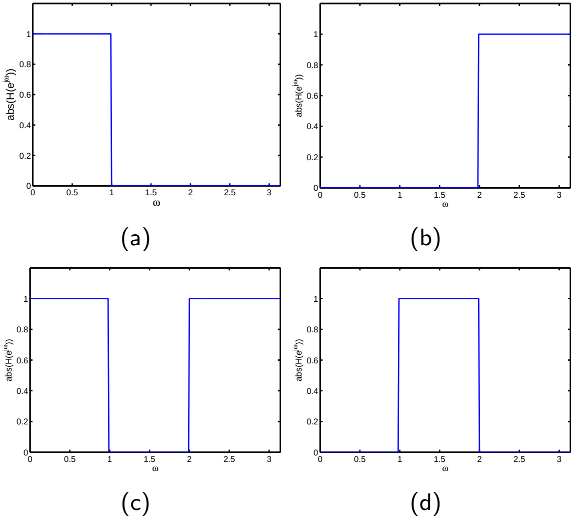


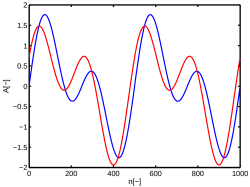
Figure: (a) Low-pass (b) High-pass (c) Band-stop (d) Band-pass

Part II

Filters with linear phase

Filters with linear phase I

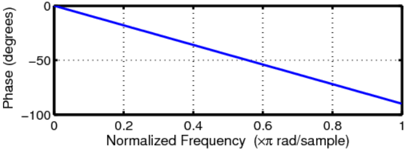
- For many filtering applications, the *magnitude response* is of prime interest
- In some cases, it is important to consider the influence of filtering on the phase spectrum as well
- **Phase response** gives the change of phase of the harmonic function at specific frequency when it passes through the filter
- In the case, when various frequency components are delayed differently, the *phase distortion* arises
- This distortion modifies the shape of the signal in the time-domain, even when all frequency components should pass the filter
- This behavior is undesirable, for example when the signal should be analyzed in the time-domain (ECG/EEG)
- Systems/filters, which do not deform the phase spectrum are denoted as filters with *linear phase*



- Digital filter has the **linear phase** when

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\alpha\omega}, \quad \alpha \in \mathcal{R} \tag{2}$$

$A(e^{j\omega}) \in \mathcal{R}$ - amplitude (can be positive and negative)



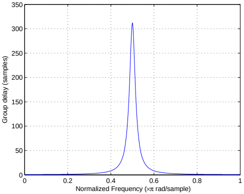
- Systems with linear phase have constant phase/group delay

$$\tau_g(\omega) = \tau_p(\omega) = \alpha. \tag{3}$$

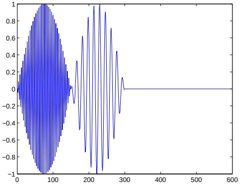
- These filters only delay the processed signal in the time-domain and (almost) do not distort it (if the spectrum of the signal is contained in the pass-band of the filter).

Systems with non-linear phase: phase distortion

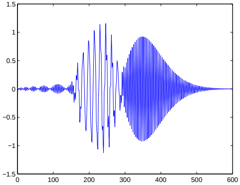
All pass filter: $H(e^{j\omega}) = \left(\frac{e^{-2j\omega}+0.95^2}{1+0.95^2 e^{-2j\omega}}\right)^8$



(a)



(b)



(c)

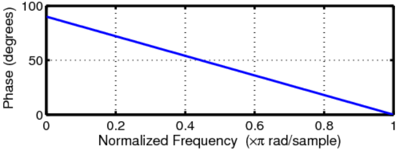
(a) Group delay $\tau(\omega)$, (b) Modulated input signal $x[n]$ -Carrier frequencies $\pi/2$ a $\pi/8$.
(c) Output signal $y[n] = \text{IDTFT}(H(e^{j\omega})X(e^{j}\omega))$

This example is inspired by a lecture given by professor Barry Van Veen (University of Wisconsin) "Characterizing Filter Phase Response", available on Youtube.

- Digital filter has the **generalized linear phase** when

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j(\alpha\omega-\beta)}, \quad \alpha, \beta \in \mathcal{R} \tag{4}$$

$A(e^{j\omega}) \in \mathcal{R}$ - amplitude (can be positive and negative)



- Systems with generalized linear phase have constant group delay (not the phase delay)

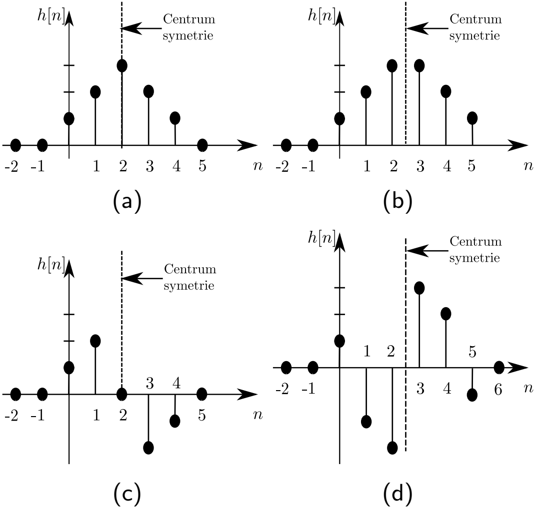
Filters with linear phase IV

- A filter has the **(generalized) linear phase**, is stable, causal and has real-valued impulse response if the following conditions are fulfilled.
  1. The impulse response $h[n]$ must be finite (FIR)
  2. The impulse response $h[n]$ must feature specific form of symmetry
- Based on these requirements, there are four types of FIR filters with linear phase. Let $N+1$ be the length of $h[n]$ then
  1. Type 1 - symmetric $h[n]$, N is even number, linear phase
  2. Type 2 - symmetric $h[n]$, N is odd number, linear phase
  3. Type 3 - antisymmetric $h[n]$, N is even number, gen. linear phase
  4. Type 4 - antisymmetric $h[n]$, N is odd number, gen. linear phase
- For symmetric impulse response $h[n] = h[N-n]$ holds, whereas for the antisymmetric ones $h[n] = -h[N-n]$.
- Phase/group delays of filters of Types 1 and 2 (with linear phase) fulfill

$$\tau_g(\omega) = \tau_p(\omega) = \alpha = \frac{N}{2} \tag{5}$$

- DETAILS: Boaz Porat, *A course in digital signal processing*, 256 / chapter 8.4.3

Filters with linear phase V



Impulse responses for various types of (generalized) linear phase filters
(a) Type 1, (b) Type 2 (c) Type 3, (d) Type 4

Part III

Filter interconnection

Filter interconnection I

- **Serial / cascade interconnection:**
- Overall impulse response is a convolution of the partial ones
- $h[n] = h_1[n] * h_2[n]$
- Overall frequency response is a multiplication of the partial ones
- $H(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega})$
- QUESTION: What holds for the magnitude and phase?

- **Parallel interconnection:**
- Overall impulse response is a sum of the partial ones
- $h[n] = h_1[n] + h_2[n]$
- Overall frequency response is a multiplication of the partial ones
- $H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})$
- QUESTION: What holds for the magnitude and phase?

- **Feedback loop:**
- Frequency response:

$$H(e^{j\omega}) = \frac{H_1(e^{j\omega})}{1 - H_1(e^{j\omega})H_2(e^{j\omega})} \tag{6}$$

- QUESTION: How is this formula derived?

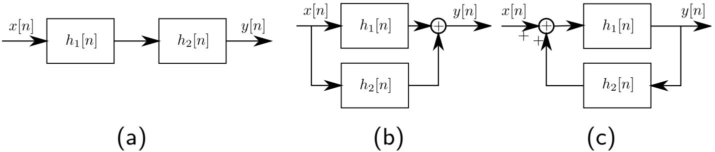


Figure: (a) Cascade (b) Parallel interconnection (c) Feedback loop

Part IV

Decibel, Signal-to-Noise Ratio

Decibel

Decibel [dB]:

- Logaritmic unit expressing ratio of two values of a physical quantity (often of energy).
- It can be used as a measure of an attenuation/amplification of a signal after filtering
- It is defined as a ratio of an investigated and a reference variable

$$L_{dB} = 10\log_{10}\left(\frac{E_{x[n]}}{E_{x_{ref}[n]}}\right) \tag{7}$$

- **Amplitude** of a periodic/harmonic signal is its maximal change (height of the peak) within a single period.
- For periodic signals, the SNR value is computed using the amplitude A as

$$L_{dB} = 20\log_{10}\left(\frac{A_{x[n]}}{A_{x_{ref}[n]}}\right) \tag{8}$$

SNR:

- Quantity measuring a ratio of energy of a desired signal $s[n]$ and the undesired background noise $v[n]$ in the mixture

$$x[n] = s[n] + v[n] \tag{9}$$

- Usually given in decibels as

$$\mathrm{SNR} = 10\log_{10}\left(\frac{E_{s[n]}}{E_{v[n]}}\right) \tag{10}$$

- In denoising applications another related quantity is stated, the **SNR improvement** defined by

$$\mathrm{SNR}_{\mathrm{imp}} = \mathrm{SNR}_{\mathrm{enh}} - \mathrm{SNR}_{\mathrm{orig}}, \tag{11}$$

  where $\mathrm{SNR}_{\mathrm{orig}}$ and $\mathrm{SNR}_{\mathrm{enh}}$ are SNRs prior/after the enhancement.

- Prior the computation, it is necessary to decompose the signal into the desired and the noise components, since these are usually unknown.

SNR in the context of acoustic signal denoising

Denoising:

- Removal / Suppression of undesired signal component (noise, interference) in the audio signal.
- Evaluation proceeds via *objective* / *subjective* criteria (SNR / listening tests)
- **Example** of interfering speech suppression in real acoustic conditions (beamforming, 4 microphones)

Mixture - two speakers (SNR = 0.7 dB),

Interference (SNR = -10.3 dB, interference amplification by 11dB)

Desired speaker (SNR = 8.3dB, SNR_{imp} = 7.6 dB),

Desired speaker (distortion, SNR = 18.2 dB, SNR_{imp} = 17.5 dB)

Thank you for attention!