

1. solve each of the following sets of simultaneous congruences.

(a) $x \equiv 1 \pmod{3}$, $x \equiv 2 \pmod{5}$, $x \equiv 3 \pmod{7}$

Ans: System $x \equiv 1 \pmod{3}$,

$$x \equiv 2 \pmod{5},$$

$$x \equiv 3 \pmod{7}.$$

Compute distributions:

$$N_1 = 105/3 = 35, 35 \equiv 2 \pmod{3}, 2^{-1} \pmod{3} = 2.$$

Term: $1.35.2 = 70$.

$$N_2 = 105/5 = 21, 21 \equiv 1 \pmod{5}, 1^{-1} = 1.$$

Term: $2.21.1 = 42$.

$$N_3 = 105/7 = 15, 15 \equiv 1 \pmod{7}, 1^{-1} = 1.$$

Term: $3.15.1 = 45$.

Sum = $70 + 42 + 45 = 157$. Reduce modulo ~~105:157~~

$$105:157 \equiv 52 \pmod{105}$$

So $x \equiv 52 \pmod{105}$

(check $52 \equiv 1 \pmod{3}$, $2 \pmod{5}$, $3 \pmod{7}$.)

(b) $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$.

Ans:

System $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$,
 $x \equiv 15 \pmod{31}$.

Product $N = 11 \cdot 29 \cdot 31 = 9889$

compute contributions:

$$N_1 = 9889/11 = 899, 899 \equiv 8 \pmod{11},$$

$$8^{-1} \pmod{11} = 7.$$

Term: $5 \cdot 899 \cdot 7 = 31465$

$$N_2 = 9889/29 = 341, 341 \equiv 22 \pmod{29},$$

$$22^{-1} \pmod{29} = 4$$

Term: $14 \cdot 341 \cdot 4 = 19096$.

$$N_3 = 9889/31 = 319, 319 \equiv 9 \pmod{31}, 9^{-1} \pmod{31} = 7$$

Term: $15 \cdot 319 \cdot 7 = 33495$

$$\text{Sum} = 31465 + 19096 + 33495 = 84056.$$

Reduce modulo 9889:

$$84056 - 8 \cdot 9889 = 84056 - 79112 = 4944$$

So, $x = 4944 \pmod{9889}$

(Check: $4944 \equiv 5 \pmod{11}$, $14 \pmod{29}$, $15 \pmod{31}$.)

(c) $x \equiv 5 \pmod{6}$, $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$

Ans:

System: $x \equiv 5 \pmod{6}$, $x \equiv 4 \pmod{11}$,
 $x \equiv 3 \pmod{17}$.

product $N = 6 \cdot 11 \cdot 17 = 1122$,

compte contributions:

$N_1 = 1122/6 = 187$, $187 \equiv 1 \pmod{6}$, $1^{-1} = 1$

Term: $5 \cdot 187 \cdot 1 = 935$.

$N_2 = 1122/11 = 102$, $102 \equiv 3 \pmod{11}$, $3^{-1} \pmod{11} = 4$

Term: $4 \cdot 102 \cdot 4 = 1632$

$N_3 = 1122/17 = 66$, $66 \equiv 15 \pmod{17}$, $15^{-1} \pmod{17} = 8$

Term: $3 \cdot 66 \cdot 8 = 1584$

sum = $3 \cdot 66 \cdot 8 = 1584$

sum = $935 + 1632 + 1584 = 4151$.

Reduce modulo 1122:

$4151 - 3 \cdot (1122) = 4151 - 3366 = 785$

so, $x \equiv 785 \pmod{1122}$.

Check: $785 \equiv 5 \pmod{6}$, $785 \equiv 4 \pmod{11}$, $785 \equiv 3 \pmod{17}$,