

1. solve each of the following sets of simultaneous congruences.

(a) $x \equiv 1 \pmod{3}$, $x \equiv 2 \pmod{5}$, $x \equiv 3 \pmod{7}$

Ans: System $x \equiv 1 \pmod{3}$,

$$x \equiv 2 \pmod{5},$$

$$x \equiv 3 \pmod{7}.$$

Compute distributions:

$$N_1 = 105/3 = 35, 35 \equiv 2 \pmod{3}, 2^{-1} \pmod{3} = 2.$$

$$\text{Term: } 1 \cdot 35 \cdot 2 = 70.$$

$$N_2 = 105/5 = 21, 21 \equiv 1 \pmod{5}, 1^{-1} = 1.$$

$$\text{Term: } 2 \cdot 21 \cdot 1 = 42.$$

$$N_3 = 105/7 = 15, 15 \equiv 1 \pmod{7}, 1^{-1} = 1.$$

$$\text{Term: } 3 \cdot 15 \cdot 1 = 45.$$

$$\text{Sum} = 70 + 42 + 45 = 157. \text{ Reduce modulo } 105: 157$$

$$\equiv 52 \pmod{105}$$

$$105 : 157 \equiv 52 \pmod{105}$$

$$\text{So } \boxed{x \equiv 52 \pmod{105}}$$

(check $52 \equiv 1 \pmod{3}$, $2 \pmod{5}$, $3 \pmod{7}$.)

(b) $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$.

Ans:

system $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$,
 $x \equiv 15 \pmod{31}$.

product $N = 11 \cdot 29 \cdot 31 = 9889$

compute contributions:

$N_1 = 9889/11 = 899$, $899 \equiv 8 \pmod{11}$,
 $8^{-1} \pmod{11} = 7$.

Term: $5 \cdot 899 \cdot 7 = 31465$

$N_2 = 9889/29 = 341$, $341 \equiv 22 \pmod{29}$,
 $22^{-1} \pmod{29} = 4$

Term: $14 \cdot 341 \cdot 4 = 19096$.

$N_3 = 9889/31 = 319$, $319 \equiv 9 \pmod{31}$, $9^{-1} \pmod{31} = 7$

Term: $15 \cdot 319 \cdot 7 = 33495$

Sum $= 31465 + 19096 + 33495 = 84056$.

Reduce modulo 9889:

$84056 - 8 \cdot 9889 = 84056 - 79112 = 4944$

So, $x = 4944 \pmod{9889}$

(Check: $4944 \equiv 5 \pmod{11}$, $14 \pmod{29}$, $15 \pmod{31}$.)

$$(c) x \equiv 5 \pmod{6}, x \equiv 4 \pmod{11}, x \equiv 3 \pmod{17}$$

Ans:

$$\text{system: } x \equiv 5 \pmod{6}, x \equiv 4 \pmod{11}, \\ x \equiv 3 \pmod{17}.$$

$$\text{product } N = 6 \cdot 11 \cdot 17 = 1122,$$

compute contributions:

$$N_1 = 1122/6 = 187, 187 \equiv 1 \pmod{6}, 1^{-1} = 1$$

$$\text{Term: } 5 \cdot 187 \cdot 1 = 935.$$

$$N_2 = 1122/11 = 102, 102 \equiv 3 \pmod{11}, 3^{-1} \pmod{11} = 4$$

$$\text{Term: } 4 \cdot 102 \cdot 4 = 1632$$

$$N_3 = 1122/17 = 66, 66 \equiv 15 \pmod{17}, 15^{-1} \pmod{17} = 8$$

$$\text{Term: } 3 \cdot 66 \cdot 8 = 1584$$

$$\underline{\text{sum} = 3 \cdot 66 \cdot 8 = 1584}$$

$$\text{sum} = 935 + 1632 + 1584 = 4151.$$

Reduce modulo 1122:

$$4151 - 3 \cdot (1122) = 4151 - 3366 = 785$$

$$\text{So, } x \equiv 785 \pmod{1122}.$$

$$\text{Check: } 785 \equiv 5 \pmod{6}, 4 \pmod{11}, 3 \pmod{17}.$$