IT-24667
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problem statement: Is set of odd numbers with binary operations (+), i.e., $\langle O, + \rangle$ an abelian group? If not explain the reasons with necessary notations.

Ans: No, the set of odd integers under addition, $(O, +)$, is not a group (and hence not abelian) because it's not closed under + and has no additive identity in O.

Set and operation:
Let $O = \{2k+1 \mid k \in \mathbb{Z}\}$ be the set of all odd integers, and consider the binary operation + restricted to O.
A group $(G, *)$ must satisfy closure, associativity, identity and inverse axioms, and it's abelian if in addition $*$ is commutative.

Failure of closure:
Take arbitary $a, b \in O$ with $a = 2m+1$ and $b = 2n+1$; then $a+b = (2m+1) + (2n+1)$
$= 2(m+n+1)$
Which is even and thus not in O, So, O is not closed under +.
Since, closure is a required group axiom, the failure closure alone shows $(O, +)$ is not a group.

P.T.O

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No identity element:

The additive identity for addition is 0, i.e., for all integers x, $x+0=0+x=x$.
But 0 is even, hence $0 \notin O$, so, there is no $e \in O$ with $a+e=a$ for all $a \in O$, and thus $(O,+)$ lacks an identity element.

Other axioms:

Addition on integers is associative and commutative, but these properties do not reuse group structure once closure and identity fail.
Therefore, even though $+$ is associative and commutative on Z, $(O,+)$ can't be a group and hence can't be an abelian group.