Max-Cut Offline Report

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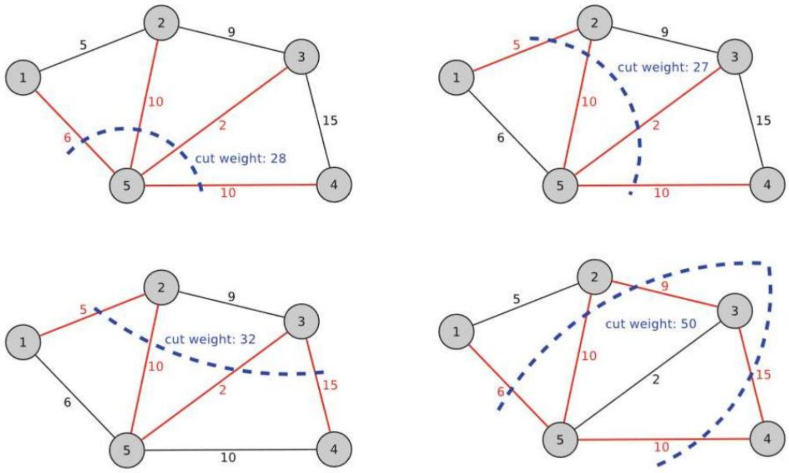
1. Introduction:

Given an undirected graph G = (V,E), where V is the set of vertices and E is the set of edges, and weights is associated with each edge, the maximum cut $(u,\ v) \in E$, the maximum cut (MAX-CUT) problem consists in finding a non-empty proper subset of vertices $S \subset V\ (S \neq \emptyset)$, such that the weight of the cut $(S,\ \overline{S})$ given by $w(S,\ \overline{S})\ = \sum_{u \in S,\, v \in \overline{S}} w_{uv}$, is maximized



2. GRASP:

The Greedy Randomized Adaptive Search Procedure (GRASP) is an algorithmic approach used to solve combinatorial optimization problems. It combines elements of greedy construction with randomization and iterative improvement.

The GRASP approach involves the following key steps:

- **Greedy Construction Phase:** During each iteration of GRASP, a greedy construction algorithm is applied to build a candidate solution. The construction algorithm makes locally optimal choices based on some heuristic criteria. This produces a feasible solution that might not be of the highest quality
- **Randomized Component:** A certain level of randomness is introduced. The algorithm randomly perturbs or diversifies the constructed solution to explore different areas of solution space. This randomization helps the algorithm avoid getting stuck in suboptimal solutions
- **Local Search and Iterative Improvement:** Following the greedy and randomization steps, a local search or iterative improvement procedure is executed. The local search explores the neighborhood of the current solution, making small adjustments to obtain the local optima.

The GRASP problem presents itself as a candidate for solving the maxcut problem. With each iteration of the GRASP algorithm, a better solution is crafted through the application of constructive algorithm. The addition of randomness into the solution by GRASP renders semi-greedy and randomized strategies effective. The inclusion of controlled randomness facilitates the exploration of diverse search domains through successive iterations, effectively introducing the concept of random restarts.

3. Approaches:

The following three construction methods are used:

- **Greedy:**

  This approach closely follows the problem specification. The procedure involves iterating over each vertex individually. For every vertex, the cumulative contribution it can make to the cut upon inclusion in set X or Y is calculated. The function value g(v) for a given vertex v is determined as the maximum potential contribution to either set, as expressed by: $g(v) = \max\{\sigma_X(v), \sigma_Y(v)\}$, where, $\sigma_X(v) = \sum_{u \in X} w_{vu}$ and $\sigma_Y(v) = \sum_{u \in Y} w_{vu}$

- **Semi-Greedy:** Instead of selecting the single optimal vertex (or edge, depending on the approach), a predefined number of top solutions are chosen to construct a Restricted Candidate List (RCL) with a threshold alpha of the best solution. Subsequently, a candidate is randomly selected from this list to augment the solution. The construction of the RCL is achieved through a value-based methodology. The threshold was determined as (max − min) · α + min, where α represents a randomly generated value. Candidates surpassing or equating this threshold were included in the RCL.
- **Randomized:** The Randomized-1 strategy is characterized by its complete randomness. For each vertex, it is randomly assigned to either set X or Y with equal probability, without any further consideration. It is equivalent to the Semi-Greedy approach with alpha = 0.
- **Local Search:** Local search operates as follows: for each vertex, we experiment with moving it from its current set to the opposite one. If this transition yields an enhanced cut, we adopt the change. Once we identify an improvement achieved by moving a vertex, we adopt the new configuration. We repeat this process iteratively, terminating when no further enhancements are attained. This technique effectively leads to the local optima. Therefore, local search is used to refine solutions from the construction algorithms.

4. Discussion:

- In the generated csv file, there are values for randomized algorithm, semi-greedy algorithm, greedy algorithm, local search algorithm and GRASP algorithm

- The values of the randomized, semi-greedy and greedy algorithms were calculated by running the algorithms for a specific iteration and then the average value was taken
- The local search algorithm value consists of number of iterations and the best value of the local search.
- The number of iterations was calculated by running local search for each iteration of the previous three algorithms and then taken the average value.
- The local search value was calculated in the same manner.
- The GRASP algorithm was run for a specific number of iterations and at each iteration, a semi-greedy value was determined, and then compared with the best known value so far, if improvement was found, it was accepted.
- From primary inspection of the results, we can see that randomized algorithm sometimes gives very poor result, even giving negative values for negative weight edge graphs.
- The greedy and semi-greedy algorithm provides moderately good results, and the results were further refined by subjecting them to local search
- The GRASP method has provided the best overall result for each dataset.
- So, in terms of result:

  randomized < semi-greedy < Greedy < Local Search < GRASP

  GRASP provides the best overall value, whilst randomized provides the worst value. Iteration count was consistent throughout the experiment, the value was 50.

- The best value from among the algorithms was picked, and labeled as the best possible value for that dataset.
- However, it's essential to note that, in most cases, the algorithms do not reach the exact known upper limit. This could be attributed, in part, to the algorithm being an approximation algorithm, and to the constraint of limiting iterations to just 50, a measure taken to conserve

computational resources, especially given the scale of the dataset, which comprises 54 distinct graphs.