Exercise 2:

(i) We know,

$$d_{ij} = \delta_{ij} - \frac{1}{M-1}\left(z_i^f - \bar{z}_M^f\right)^T H^T \left(H P_M^f H^T + R\right)^{-1} \left(H z_j^f + \xi_j - y_{obs}\right)$$

$$= \delta_{ij} - \frac{1}{M-1}\left(z_i^f - \bar{z}_M^f\right)^T \left(P_M^f + R\right)^{-1} \left(z_j^f + \xi_j - y_{obs}\right)$$

when $H = 1$.

Here ξ_j is a random number so, d_{ij} will be random whatever value y_{obs} have.

(ii)

$$\sum_{i=1}^{M} d_{ij} = \sum_{i=1}^{M}\left[\delta_{ij} - \frac{1}{M-1}\left(z_i^f - z_M^f\right)^T \left(P_M^f + R\right)^{-1} \left(z_j^f + \xi_j - y_{obs}\right)\right]$$

$$= \sum_{i=1}^{M} \delta_{ij} - \frac{1}{M-1}\sum_{i=1}^{M}\left(z_i^f - \bar{z}_M^f\right)^T \left(P_M^f + R\right)^{-1}\left(z_j^f + \xi_j - y_{obs}\right)]$$

$$= 1 - \frac{1}{M-1}\left(M\bar{z}_M^f - M\bar{z}_M^f\right)^T \left(P_M^f + R\right)^{-1}\left(z_j^f + \xi_j - y_{obs}\right)\}$$

$$\left[\because \sum_{i=1}^{M} \delta_{ij} = 1 \text{ and } \sum_{i=1}^{M} z_i^f = M\bar{z}_M^f\right]$$

$$= 1 - \frac{1}{M-1}\cdot 0 \cdot \left(P_M^f + R\right)^{-1}\left(z_j^f + \xi_j - y_{obs}\right)$$

$$= 1 - 0$$

$$= 1.$$

$\square$

(ii) We know,

$$d_{ij} = \delta_{ij} - \frac{1}{M-1}(z_i^f - \bar{z}_M^f)^T H^T (H P_M^f H^T + R)^{-1} (H z_j^f + \xi_j - y_{obs})$$

Here δ_{ij} denotes the kronecker delta. i.e.
$\delta_{ij} = 0$ for $i \neq j$

if $\delta_{ij} = 0$ then

$$d_{ij} = 0 - \frac{1}{M-1}(z_i^f - \bar{z}_M^f)^T H^T (H P_M^f H^T + R)^{-1} (H z_j^f + \xi_j - y_{obs})$$

$$= - \frac{1}{M-1}(z_i^f - \bar{z}_M^f)^T H^T (H P_M^f H^T + R)^{-1} (H z_j^f + \xi_j - y_{obs})$$

So, when $i \neq j$ then d_{ij} will be negative.