1. EXERCISE SHEET *Bayesian inference and data assimilation*

Please refer to **Assignment Submission Guidelines** on Moodle.

As for the plotting tasks on this sheet, plot the time interval wider than $0 \leqslant t \leqslant 5$.

Exercise 1. Assume you have a particle attached to an elastic spring. The equations of motion are

$$\dot{q}(t) = p(t), \quad (1)$$
$$\dot{p}(t) = -q(t) \quad (2)$$

for the *position* variables q and *momentum* variables p. Solutions to this system have the form of

$$q(t) = A\cos(t) + B\sin(t)$$

a) You observe $q_{\text{obs}}(1) = -2$, $q_{\text{obs}}(2) = 0$. Which values do A and B have? Plot the solution curve $t \rightarrow q(t)$ and the observations into one plot.

b) You now get another observation $q_{\text{obs}}(3) = -\frac{1}{2}$, so you have three observations overall. Find the least squares solution for A and B and again plot the solution curve and the observations.

c) How do the curves from Exercise 1a and 1b fit the observations? Can you explain the difference?

d) Plot $p(t)$ for the solutions you got in Exercise 1a and 1b. What are the initial values $q(0), p(0)$?

Exercise 2. In this task we make the spring constant γ variable. The equations of motion are now given as

$$\dot{q}(t) = p(t), \quad (3)$$
$$\dot{p}(t) = -\gamma q(t). \quad (4)$$

The general solutions have the form

$$q(t) = A\cos\left(\sqrt{\gamma}t\right) + B\sin\left(\sqrt{\gamma}t\right).$$

a) Prove that $q(t)$ given above actually solves the system of equations (3)-(4),

b) In Exercise 1a) we already found a solution for this system when $\gamma = 1$. Find the solution for $\gamma = 3$ when making the same observations as in Exercise 1a.

c) Now assume you have the same observations as in Exercise 1b. Describe how you could find values for A, B and γ to fit the curve through the observations.

Exercise 3. In this exercise you will numerically approximate the solutions from Exercise 1. Implement the forward Euler scheme for these equations.

a) Overlay the analytical solutions you got in Exercise 1a with its numerical approximation when you start at the same initial conditions $q(0)$ and $p(0)$, that you found in Exercise 1d. Do the experiment for $\Delta t = 0.2$ and $\Delta t = 0.01$.

b) How do the approximations in Exercise 3a differ for different time step sizes and why?