1(i) We can see that measure $\pi = \delta_0$ on the Point 0. It also said that

if

$u(x) > 0$ which means measure $u(x)$ is greater than 0 or positive.

and $v(x) = 0$. the measure $v(x) = 0$

the divergence between u and v is infinitive (∞).

We know that

$$KL(\pi | u_n) = \sum_{x \in supp(\pi)} \pi(x) \log\left(\frac{\pi(x)}{u_n(x)}\right)$$

$$= \sum_{x \in \{0\}} \delta_0(x) \log\left(\frac{\delta_0(x)}{u_n(x)}\right)$$

$$= \delta_0(0) \log\left(\frac{\delta_0(0)}{u_n(0)}\right)$$

$$= 1 \cdot \log\left(\frac{1}{0}\right)$$

$= \log(\infty)$

$= \infty$

Here, If x is 0 Dirac measure $\delta_0(x) = 1$. likewise, if $x = \frac{1}{n}$ then measure $u_n = 1$, otherwise 0

So, the KL divergence $KL(\pi | u_n)$ does not converge to π i.e. does not converge to zero.

Again,

$$KL(\pi | v_n) = \sum_{x \in supp(\pi)} \pi(x) \log\left(\frac{\pi(x)}{v(x)}\right)$$

$$= \sum_{x \in \{0\}} \delta_0(x) \log\left(\frac{\delta_0(x)}{v_n(x)}\right)$$

$$= \delta_0(0) \log\left(\frac{\delta_0(0)}{v_n(0)}\right)$$

$$= 1 \cdot \log\left(\frac{1}{(1-\frac{1}{n})}\right)$$

$\because v_n(0) = 1 - \frac{1}{n}$

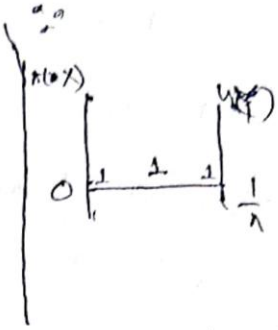
$$= \log\left(\frac{n}{n-1}\right)$$

If the values of n goes to then $\log\left(\frac{n}{n-1}\right)$ goes to zero. So, we can say. If n goes to infinity v_n converge to π.

1(ii) Now we have to prove the same things like questions (1) for Wasserstien-2 distance.

To find the coupling between two measures π and U_n —

$\pi(x)$ \ $U_n(y)$	$\frac{1}{n}$
0	1



There is one coupling between two measures.

∴ Wasserstien-2 distance

$$W(T) = \sqrt{1\left(\frac{1}{n}-0\right)^2}$$

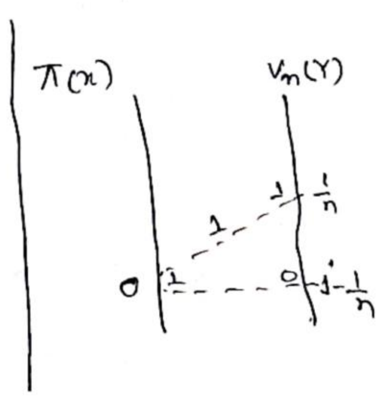
$$= \sqrt{\tfrac{1}{n}}$$

$= 0$ If value of n goes to infinitive (∞)

So, we can say, U_n converges to π.

Likewise,

$\pi(x)$ \ $V_n(Y)$	$1-\frac{1}{n}$	$\frac{1}{n}$
0	0	1



$X = \{0\}$

$Y = \{1-\frac{1}{n}, \frac{1}{n}\}$

$$\therefore W(T) = \sqrt{\left(1-\frac{1}{n}\right)(0-0)^2 + \frac{1}{n}(1-0)^2}$$

$$= \sqrt{\frac{1}{n}}$$

$= 0$ If n goes to infinity (∞)

So, we can say v_n converge to π.