Exercise 2 (i)

$$P = \begin{pmatrix} 2/3 & 2/5 & 0 \\ 1/3 & 3/5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Intial State $x_0 = 1$

So; $x_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Probability after 1'st step

$$x_1 = Px = \begin{pmatrix} 0.67 \\ 0.33 \\ 0 \end{pmatrix}$$

2nd Step : $x_2 = Px_1 = \begin{pmatrix} 0.5786 \\ 0.4213 \\ 0.0 \end{pmatrix}$

3rd step : $x_3 = Px_2 = \begin{pmatrix} 0.5542 \\ 0.4456 \\ 0 \end{pmatrix}$

4th step : $x_4 = Px_3 = \begin{pmatrix} 0.5477 \\ 0.4521 \\ 0.0 \end{pmatrix}$

5th step : $x_5 = Px_4 = \begin{pmatrix} 0.5459 \\ 0.4538 \\ 0.0 \end{pmatrix}$

6th Step : $x_6 = Px_5 = \begin{pmatrix} 0.5454 \\ 0.4542 \\ 0.0 \end{pmatrix}$

7th step : $x_7 = Px_6 = \begin{pmatrix} 0.5452 \\ 0.4543 \\ 0.0 \end{pmatrix}$

8th step : $x_8 = Px_8 = \begin{pmatrix} 0.5452 \\ 0.4543 \\ 0.0 \end{pmatrix}$

Now, πP

$$= \begin{pmatrix} 0.5432 \\ 0.4543 \\ 0.0 \end{pmatrix} \begin{pmatrix} 2/3 & 2/5 & 0 \\ 1/3 & 3/5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.5452 \\ 0.4543 \\ 0.0 \end{pmatrix}$$

$$= \pi$$

$\therefore$ when $X_0 = 1$, the ~~distribution~~ X_t will tend towards invariant distribution, because it satisfy the rules $\pi = \pi P$.

Exercise 2 (ii)

$$P = \begin{pmatrix} 2/3 & 2/5 & 0 \\ 1/3 & 3/5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Intial state, $X_0 = 3$ so, $X = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Probability after 1'st step,

$$X_1 = PX = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

2nd step: $X_2 = P X_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Now, πP

$$= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 2/3 & 2/5 & 0 \\ 1/3 & 3/5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \pi$$

So, when $X_0 = 3$, the X_t will tend towards invariant distribution, because it satifies the rule $\pi = \pi P$.

Exercise 3 - 2 (iii)

$$P = \begin{pmatrix} 2/3 & 2/5 & 0 \\ 1/3 & 3/5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Now,

$$\frac{2}{3}\pi_1 + \frac{1}{3}\pi_2 = \pi_1 \quad \text{— (i)}$$

$$\frac{2}{5}\pi_1 + \frac{3}{5}\pi_2 = \pi_2 \quad \text{— (ii)}$$

$$\pi_3 = \pi_3$$

we also know, $\pi_1 + \pi_2 + \pi_3 = 1$. — (iii)

From (i) $\Rightarrow \frac{2}{3}\pi_1 = \frac{2}{3}\pi_2 \Rightarrow \pi_1 = \pi_2$

From (ii) $\Rightarrow \frac{2}{5}\pi_1 = \frac{2}{5}\pi_2 \Rightarrow \pi_1 = \pi_2$

if we take, $\pi_1 + \pi_2 = \alpha$

~~then~~, $\Rightarrow \pi_1 + \pi_1 = \alpha$

$\Rightarrow \pi_1 = \alpha/2$

So, $\pi_1 = \alpha/2 = \pi_2$

From (iii)

$$\pi_3 = 1 - \alpha/2$$

So, $\pi = [\alpha/2, \alpha/2, 1 - \alpha/2]$ where $0 < \alpha \leq 1$.

So, we can say that the markov chain have an invariant distribution.

As the markov chain is not irreducible ~~and~~

So, we can say that the markov chain

will converge whatever the α value is.