Let's also assign the points to each exercise. I just remember that there were only 2 with 8 points.(40 points) [15 mins for reading and 90 mins for writing]

1. optimal coupling like sheet3, ex2 (6 points)
   a) Find T*
   b) find the Differential

2. Markov chain/MCMC (6 points)
   a) like sheet5 ex2
   b) Given a p*(¼,¾) prob vector, describe the MCMC
   c) Find an expression(?) for the Markov transition kernel (?) and prove that the thing is stationary like Exercise Sheet 7.2. Show it is invariant.

3. Score like sheet8 ex3 (6 points)
   a) Prove that the score RME is proper, but not strictly proper[E[Scrps] =...., is RME](solution in 4.2 slide)
   b) Given two models, with PDFS: 1) Std-Normal 2) Gaussian Mixture and wi(0.2,0.8) 0.8xN(0,1)+0.3*N(1?,3). Which model is better, given an observation yobs=1.6 for a logarithmic scoring rule?
   c) Explain the 3 cases when the histogram is: Flat, U-Shaped, Inverted-U-shaped curves

4. Kalman Filter like sheet9, ex2 (observation had variance gamma square. Zn+1=lambda * Z)(8 points)
   a) Prove that K = P^ak * (gamma)^-2
   b) Another thing that involved the gamma^2…[Pa(k) <gamma^2, k=1,2...]
   c) proving that something (ek=Z^a(k)-Z^ref(k)) going to infinity gives us something [ek=Z^a(k)-Z^ref(k) obtain recursive formula of this equation]

5. Ensemble Kalman Filter with perturbed observations(6 points)
   a) … Hint: Use the kalman filter update formulas?...
   b) show the mean and the variance of the resulting posterior if Yobs =2

6. !(8 points)

Xf and Xa are both random variable

Pfi=1/M, Pai= Wi

   a) determine mean and variance of both random variables in terms of outcome X1→ probability vectors of Pfi and Pai …
   b) computing the matrix sum(tij)=1/M, sum(tij)=Wi derive the transform matrix
   c) show that the expectation of Xa and X(hat)a agree while the variances of Xa and X(hat)a is less or equal to the variance