1. $a(t) = a_0 + a_1 t$ $\quad y_0 = 0, \; y_1 = \cancel{4}\, -1$ at $t = 0$ and $t = 1$

(i) minimize $l(\alpha_0, \alpha_1) = \frac{1}{2}\left((a(t) - y_0)^2 + (a(t) - y_1)^2\right)$

~~~~ find ~~~~ a_0, a_1

does the model fit the given values?

(ii) consider another point $y_2 = \cancel{0}\,2$ at $t = 2$

$l(\alpha_0, \alpha_1) = \frac{1}{2}\left((a(t) - y_0)^2 + (a(t) - y_1)^2 + (a(t) - y_2)^2\right)$

does the model fit the given values?

2. Given $P = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

(i) find the invariant distribution

(ii) given $p^* = \begin{pmatrix} 5/6 \\ 1/6 \end{pmatrix}$ find $\tilde{P}$ such that p^* is invariant of $\tilde{P}$ using MCMC.
p^* is invariant of $\tilde{P}$ & verify.

(iii) ~~Find the~~ Show that p^* is

(Importance sampling)

3. Given Bimodal distribution

$$\pi_x = \sum_{i=1}^{2} \omega^i \, n(\mu^i, \sigma^{i2})$$

	1	2
μ^i	4	-4
σ_i^2	1	1
ω^i	0.5	0.5

(i) show mean of π_x, $m = 0$, variance $v = 17$.

(ii) we want to estimate the expectation of $f(x)$ such that

$$E[f(x)] = \sum \alpha_i f(x_i)$$

How do we choose α_i?
~~~~

5. Given some $Z^0 \sim N(-,-)$ (Kalman Filter)

E and $\Sigma \sim N(0,1)$

$Z^1 = Z^0 + 1 + \sqrt{2}\,\xi^{1}$

$Y^1 = Z \cdots + \Sigma$ observation operator.

(i) Find m^f and v^f.

(ii) For $Y_{obs} = 3$ find m^a and v^a and distribution of $Z^a = Z^1 \mid Y_{obs} = 3$

(iii) Given a perturbed distribution, find value of α such that the mean and variance of perturbed dist^n $\hat{Z}^a$ is same as Z^a

$$\hat{Z}^a = Z^1 + \alpha(1 - Z^1) + \Sigma^1$$ ← something like this.

7. $P(X_1 = -1) = P(X_2 = 1) = P(X_1 = -1) = P(X_2 = 1) = \frac{1}{2}$ (coupling of measures)

(i) find all possible coupling between X_1 and X_2

(ii) Which coupling maximizes the correlation? Which one minimizes the correlation

(iii) In which case they are uncorrelated (independent)?