Exercise 2:

Here, $M=2$,

$P_1 = [\frac{1}{4}, \frac{3}{4}]$ [As they are already in decending order we don't need to order them]

$P_2 = [1/2, 1/2]$

$t = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$i = 1$ and $j = 1$.

$a1 = [0,1]$

$a2 = [0,1]$

1st step:

$P_1[1] = \frac{1}{4}$ and $P_2[1] = \frac{1}{2}$

As, $P_1[1] < P_2[1]$

So, $\boxed{t[1][1] = \frac{1}{4}}$

$P_2[1] = \frac{1}{2} - \frac{1}{4}$

$= \frac{1}{4}$

$i = i+1 = 1+1 = 2$

2nd step:

$P_1[2] = \frac{3}{4}$ and $P_2[1] = \frac{1}{4}$

As, $P_1[2] > P_2[1]$

So, $\boxed{t[2][1] = \frac{1}{4}}$

$P_1[2] = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$j = j+1 = 1+1$

$= 2$

Algorithm:

```
While i<M and j<M:
    If P1[i] < P2[i]:
        t[i][j] = P1[i]
        P2[j] = P2[j] - P1[i]
        i = i+1.
    ~~End If~~.
    Else:
        t[i][j] = P2[j]
        P[i] = P[i] - P2[j]
        j = j+1.
    End If.
End While.
Sum:=0
For i=1 to 2
    For j = 1 to 2
        Sum:= sum + t[i][j]
                  + (a1[i] - a2[j])**2
    End For
End For.
Output: Sum
```

3rd step:

$$P_1[2] = \frac{3}{4} \text{ and } P_2[2] = \frac{1}{2}$$

As, $P_1[2] > P_2[2]$

So, $\boxed{t[2][2] = \frac{1}{2}}$

$$P_1[2] = \frac{1}{2} - \frac{1}{2} = 0$$

$$j = 2 + 1 = 3$$

Now, $j = 3 > M = 2$ so, we terminate the while loop.

So, the coupling matrix

$$\boxed{t = \begin{bmatrix} 1/4 & 0 \\ 1/4 & 1/2 \end{bmatrix}}$$

The transport cost,

$$J(T) = \frac{1}{4} \times (0-0)^2 + 0 \times (0-1)^2 + \frac{1}{4} \times (1-0)^2 + \frac{1}{2} \times (1-1)^2$$

$$\therefore \boxed{J(T) = \frac{1}{4}}$$