

Here is what I jotted down while in the train headed home after the exam:

5 questions (so one less than in exam1), 8 points each (same amount as in exam1).

But all in all there were 15 sub-questions (so same amount as in exam1).

Again 15 min read-only time (no pen allowed), 90 min sharp for solving.

1. Distance proof
 - a. $0 \leq D_{\text{hell}} \leq 1$
 - b. $D_{\text{hell}}^2 \leq \frac{1}{2} D_{\text{KL}}$ almost like in sheet 2 ex4. Hint: $2(1-\sqrt{x}) \leq -\log(x)$?
2. Coupling, but different approach

X,Y with 3 states $\{-1, 0, 1\}$, X is equally distributed, and probability of Y-states unevenly ($0 \rightarrow \frac{1}{2}$, $\pm 1 \rightarrow \frac{1}{4}$)

 - a. find bi-stochastic coupling matrix T_0 . X,Y independent
 - b. X,Y dependent, minimise $E((X-Y)^2)$, find Optimal transport coupling of bi-stochastic matrix T^*
 $W(T^*) = E((X-Y)^2)$
 - c. show $W(T_0) > W(T^*)$
3. discrete time deterministic dynamical system
 - a. show invariant, π^* given (exactly like sheet 7 ex1a))
 $p^{n+1} = (1/(4*(\sqrt{1-z}))) (p^n(f(z)) + p^n(g(z)))$
 $f(z) = \frac{1}{2} + \frac{1}{2}(1-z)$
 $g(z) = \frac{1}{2} - \frac{1}{2}(1-z)$
 - b. in Chapter 4-2 slide, page 29 (Example (Continuous ranked probability score)) . Given the graph of Scrps from 3 models, Which one is the worst model?
 - c. Given 2 models, the first model is gaussian with $N(0,1)$, and the second model is a gaussian mixture (2 gaussian, kind of " $0.5xN(x;0.5,1.1) + 0.5*N(x;0.5,2.2)$ "), with given mean, variance and equal weight. With Yobs = 1. Determine using Slog scoring, which model will give better results.
4. Kalman Filter(Further development of Ex 9.2)
 - a. show $P_{k_a} \leq R$ and $P_{k_a} \leq P_{k_f}$. explain why this is desirable (for $P_{k_a} \leq R$ proving is in ex 9.2 b)
 - b. $k \rightarrow \infty$ define the fixed point for P_{k_a} , show that it is the quadratic solution for P_{k_a} . check Ex9.2c
 - c. We have 2 sensors. with variances $R_1 > R_2$, show that $K_1 < K_2$ (Kalman gain) and give statements about your interpretation regarding that fact. Hint: $K = P_a/R$
5. ensemble Kalman with perturbed observations

$\xi \sim N(0,1)$, $y = Hz + \xi$
 $\hat{Z}_a = Z_{k_f} + P_{k_f} H^*(K) (y_{\text{obs}} - Hz)$

 - a. Find the distribution of \hat{Z}_a by determining the mean and covariance
 - b. Define better z_{a_mean} using Kalman update formula to improve the posterior model
 - c. Compare covariance a and b. Show that b is better model than a (wasn't there a "+ 1" somewhere in \hat{Z}_a ?)

All in all I found it very hard, nothing looked exactly like in the assignments, all texts did differ. 4 pages of text for the questions.