

5. Problem sheet for Statistical Data Analysis

Exercise 1 (5 Points)

Let X_1, \dots, X_n be independent random variables with densities

$$f_{X_i}(x|\theta) = \begin{cases} \exp(i\theta - x) & x \geq i\theta \\ 0 & x < i\theta \end{cases} \quad (1)$$

Proof that $T((X_1, \dots, X_n)) = \min_i(X_i/i)$ is a sufficient statistic for θ .

Exercise 2 (5 Points)

Let X_1, \dots, X_n be a random sample from a population with pdf or pmf $f(x|\theta)$. Show that the order statistic $T(X_1, \dots, X_n) = X_{(1)}, \dots, X_{(n)}$ are a sufficient statistic for θ .

Exercise 3 (7 Points)

Let X_1, \dots, X_n be independent random variables distributed according to

$$f(x|\theta) = \begin{cases} \exp(-(x - \theta)) & x \geq \theta \\ 0 & x < \theta \end{cases} \quad (2)$$

where $-\infty < \theta < \infty$. Consider testing $H_0 : \theta \leq \theta_0$ versus $H_1 : \theta > \theta_0$. Derive and plot the corresponding LRT statistic $\lambda(x_1, \dots, x_n)$ for $\theta_0 \in \{-2, 0, 0.5, 1\}$.