Exercise 1: Let (x_1, …., x_n) € R_n be a set of samples. Show that for all a € R,

$$\sum_{i=1}^{n}(xi-a)^\wedge 2 = \sum_{i=1}^{n}(xi-\bar{x}n)^\wedge 2 + n * (\bar{x}n - a)^\wedge 2$$

Solution:

$$\sum_{i=1}^{n}(xi-a)^\wedge 2$$

= $\sum_{i=1}^{n}(xi-\bar{x}n+\ \bar{x}n-\ a)^\wedge 2$ [Add and subtract $\bar{x}n$ from the expression]

= $\sum_{i=1}^{n}(xi-\ \bar{x}n)$ ^ 2 + 2 * $\sum_{i=1}^{n}(xi-\ \bar{x}n)*(\bar{x}n-a)$ + $\sum_{i=1}^{n}(\bar{x}n-a)^\wedge 2$
[Take (xi - $\bar{x}n$) = a and ($\bar{x}n$ – a) = b then apply (a + b) ^ 2 = a^2 + 2ab + b^2]

……………………………… (1)

Now,

2 * $\sum_{i=1}^{n}(xi-\ \bar{x}n)*(\bar{x}n-a)$

= 2 *[[$\bar{x}n*\ \sum_{i=1}^{n}xi$] – [$a*\ \sum_{i=1}^{n}xi$] – [$\bar{x}n*\ \sum_{i=1}^{n}\bar{x}n$] + [$a*\ \sum_{i=1}^{n}\bar{x}n$]]

= 2 *[[n * $\bar{x}n$ ^ 2] – [a * n * $\bar{x}n$] – [n * $\bar{x}n$ ^ 2] + [a * n * $\bar{x}n$]]

= 2 * 0

= 0

Now (1) will be

$\sum_{i=1}^{n}(xi-\ \bar{x}n)$ ^ 2 + $\sum_{i=1}^{n}(\bar{x}n-a)^\wedge 2$

= $\sum_{i=1}^{n}(xi-\ \bar{x}n)$ ^ 2 + n * ($\bar{x}n$ – a) ^ 2

So,

$$\sum_{i=1}^{n}(xi-a)^\wedge 2 = \sum_{i=1}^{n}(xi-\bar{x}n)^\wedge 2 + n * (\bar{x}n - a)^\wedge 2$$