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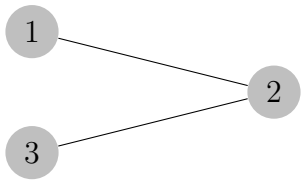
Extra problem sheet for **Statistical Data Analysis**

Exercise 1 (6 Points)

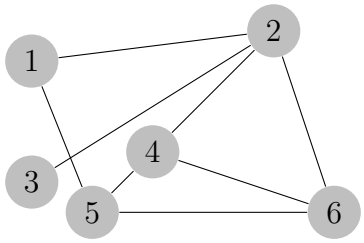
For the following graph compute the Laplacian matrix and its eigenvalues and eigenvectors.[1]



Find (by hand) a bisection of this graph such that RatioCut is minimal and one such that NCut is minimal.

Exercise 2 (12 Points)

Given is the following graph



(i) Construct its corresponding Laplacian matrix.

(ii) Perform spectral clustering for $K = 2$ and determining the cluster assignment The K-means step can be determined with a routine on the computer and should use a random initialization).

(iii) Construct the corresponding vector f for the determine partitioning of the vertex set.

[1] Use Sarrus' scheme to compute the determinant.

- Verify the equation

$$f^\top Lf = |V| \cdot RatioCut(A, \bar{A}) \tag{1}$$

  for this particular choice of f.

- Show that f is orthogonal to the all-one-vector and that $\|f\|^2 = n$ holds.