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3. Problem sheet for **Statistical Data Analysis**

Exercise 1 (3+3 Points)
Let $X_1, \dots, X_n \sim N(\mu, \sigma^2)$ independent, where

- σ^2 is known and $\mu \in \mathbb{R}$ is unknown
- $\mu \in \mathbb{R}$ known and $\sigma^2 > 0$ unknown

Estimate the respective unknown parameters via the Maximum Likelihood Method.

Exercise 2 (3+3 Points)
Given is a realisation $(x_1, \dots, x_n)$ of samples of $n = 100$ coin flips. The values $x_i = 1$ represent heads and $x_i = 0$ tails. We are interested in estimate the unknown parameter θ that is associated with the probability of heads. In order to approximate θ consider two different statistical models:

- it is assumed for the statistical model that the underlying $X_1, \dots, X_n$ are independent and identical distributed random variables following the Bernoulli distribution
- it is assumed for the statistical model that the underlying $X_1, \dots, X_n$ are independent and identical distributed random variables following the Binomial distribution.

Estimate θ using the Maximum Likelihood Method for the two statistical models. Compare the resulting values and comment on the difference.

Exercise 3 (6 Points)
Every human is a carrier of one of the three genotypes AA, Aa, or aa. The genotypes are occurring with the probabilities $(1-p)^2$, $2p(1-p)$ and p^2 whereas $0 < p < 1$ and testing of n persons yielded

- x persons had the genotype AA
- y persons had the genotype Aa
- z persons had the genotype aa

Describe the corresponding statistical model and determine the Maximum Likelihood Estimator for p.