Problem Sheet 01

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Exercise 2

Show that for the cumulative distribution function F(x) of the geometric distribution the following equation holds:

$$\sum_{i=1}^{x} p(1-p)^{i-1} = 1-(1-p)^x$$

Solution: Show that it is true for $x = 1$:

$$\begin{aligned}\sum_{i=1}^{1} p(1-p)^{i-1} &= p(1-p)^0 \\ &= p \\ &= 1-(1-p)^1\end{aligned}$$

Show that from

$$\sum_{i=1}^{x} p(1-p)^{i-1} = 1-(1-p)^x \tag{1}$$

follows

$$\sum_{i=1}^{x+1} p(1-p)^{i-1} = 1-(1-p)^{x+1}: \tag{2}$$

$$\begin{aligned}\sum_{i=1}^{x+1} p(1-p)^{i-1} &= \sum_{i=1}^{x} p(1-p)^{i-1} + p(1-p)^x && \text{pulling the last element out of the sum} \\ &= 1-(1-p)^x + p(1-p)^x && \text{using the statement from equation 1} \\ &= 1-(-p(1-p)^x + (1-p)^x) && \text{rearranging} \\ &= 1-(1-p)(1-p)^x && \text{simplifying} \\ &= 1-(1-p)^{x+1}\end{aligned}$$

$\blacksquare$