| Distribution | Parameters | Possible Description | $\mathbf{Range}\ \Omega_X$ | $\mathbb{E}[\mathbf{X}]$ | $\mathbf{Var}(\mathbf{X})$ | $\mathrm{PDF}/\mathrm{PMF}$ | $\begin{array}{ c c } \mathbf{CDF} \\ (\mathbb{P}(X \le x)) \end{array}$ |
|-------------------------------------|--|--|---|--|---|---|--|
| Uniform (disc) | $X \sim Unif(a, b)$ for $a, b \in \mathbb{Z}$ and $a \le b$ | Equally likely to be any $integer$ in $[a, b]$ | $\{a,\ldots,b\}$ | $\frac{a+b}{2}$ | $\frac{(b-a)(b-a+2)}{12}$ | $\frac{1}{b-a+1}$ | |
| Bernoulli | $X \sim Ber(p)$ for $p \in [0, 1]$ | Takes value 1 with prob p and 0 with prob $1-p$ | {0,1} | p | p(1-p) | $p^k \left(1 - p\right)^{1 - k}$ | |
| Binomial | $X \sim Bin(n, p)$ for $n \in \mathbb{N}$, $p \in [0, 1]$ | Sum of n independent Bernoulli trials, each with parameter p | $\{0,1,\ldots,n\}$ | np | np(1-p) | $\binom{n}{k} p^k \left(1 - p\right)^{n - k}$ | |
| Poisson | $X \sim Poi(\lambda)$ for $\lambda > 0$ | # of events that occur in a unit of time independently with rate λ per unit time | $\{0,1,\ldots\}$ | λ | λ | $e^{-\lambda} \frac{\lambda^k}{k!}$ | |
| Geometric | $Geo(p)$ for $p \in [0, 1]$ | # of independent Bernoulli trials with parameter p up to and including first success | $\{1,2,\ldots\}$ | $\frac{1}{p}$ | $\frac{1-p}{p^2}$ | $(1-p)^{k-1} p$ | $1-\left(1-p\right)^x$ |
| Hypergeometric | $HypGeo(N, K, n)$ for $n, K \leq N$ and $n, K, N \in \mathbb{N}$ | # of successes in n draws (w/out replacement) from N items that contain K successes in total | $\{\max(0, n + K - N), \dots, \min(n, K)\}$ | $n\frac{K}{N}$ | $n\frac{K(N-K)(N-n)}{N^2(N-1)}$ | $\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$ | |
| Negative Binomial | $NegBin(r,p)$ for $r \in \mathbb{N}, p \in [0,1]$ | # of trials until r^{th} success in Bernoulli process | $\{r,r+1,\ldots\}$ | $\frac{r}{p}$ | $\frac{r\left(1-p\right)}{p^2}$ | $\binom{k-1}{r-1}p^r\left(1-p\right)^{k-r}$ | |
| Multinomial | $\mathbf{X} \sim Mult_r(n, \mathbf{p})$ for $r, n \in \mathbb{N}$ and $\mathbf{p} = (p_1, p_2,, p_r),$ $\sum_{i=1}^r p_i = 1$ | generalization of the binomial distribution, n trials with r categories each with probability p_i | $k_i \in \{0, \dots, n\},$ $i \in \{1, \dots, r\}$ and $\Sigma k_i = n$ | $\mathbb{E}[\mathbf{X}] = n\mathbf{p} = \begin{bmatrix} np_1 \\ \vdots \\ np_r \end{bmatrix}$ | $Var(X_i) = np_i(1 - p_i)$ $Cov(X_i, X_j) =$ $-np_i p_j, i \neq j$ | $\binom{n}{k_1,\dots,k_r}\prod_{i=1}^r p_i^{k_i}$ | |
| Multivariate Hypergeomet- ric | $\mathbf{X} \sim MVHG_r(N, \mathbf{K}, n)$ for $r, n \in \mathbb{N}$, $\mathbf{K} \in \mathbb{N}^r$ and $N = \sum_{i=1}^r K_i$ | generalization of the hypergeometric distribution, n draws from r categories each with K_i successes (w/out replacement) | $k_i \in \{0, \dots, K_i\},$ $i \in \{1, \dots, r\}$ and $\Sigma k_i = n$ | $\mathbb{E}[\mathbf{X}] = n \frac{\mathbf{K}}{N} = \begin{bmatrix} n \frac{K_1}{N} \\ \vdots \\ n \frac{K_r}{N} \end{bmatrix}$ | $Var(X_i) = n\frac{K_i}{N} \cdot \frac{N - K_i}{N} \cdot \frac{N - n}{N - 1}$ $Cov(X_i, X_j) = -n\frac{K_i}{N} \frac{K_j}{N} \cdot \frac{N - n}{N - 1}, i \neq j$ | $\frac{\prod_{i=1}^{r} \binom{K_i}{k_i}}{\binom{N}{n}}$ | |

| Continuous Distributions | | | | | | | | | | | |
|--------------------------|---|---|--|--|--|--|--|--|--|--|--|
| Distribution | Parameters | Possible Description | Range Ω_X | $\mathbb{E}[\mathbf{X}]$ | $\mathbf{Var}(\mathbf{X})$ | PDF/PMF | $\begin{aligned} \mathbf{CDF} \\ (\mathbf{F_X}\left(\mathbf{x}\right) = \mathbb{P}(\mathbf{X} \leq \mathbf{x})) \end{aligned}$ | | | | |
| Uniform | Unif(a,b) for $a < b$ | Equally likely to be any real number in $[a, b]$ | [a,b] | $\frac{a+b}{2}$ | $\frac{(b-a)^2}{12}$ | $\frac{1}{b-a}$ | $\begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \le x < b \\ 1 & \text{if } x \ge b \end{cases}$ | | | | |
| Exponential | $Exp(\lambda)$ for $\lambda > 0$ | Time until next event in Poisson process | $[0,\infty)$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ | $\lambda e^{-\lambda x}$ | $\begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & \text{if } x \ge 0 \end{cases}$ | | | | |
| Normal | $ \mathcal{N}(\mu, \sigma^2) \text{for } \mu \in \mathbb{R}, \sigma^2 > 0 $ | Standard bell curve | $(-\infty,\infty)$ | μ | σ^2 | $\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ | $\Phi\left(\frac{x-\mu}{\sigma}\right)$ | | | | |
| Gamma | $Gam(r,\lambda)$ for $r,\lambda>0$ | Time to r^{th} event in Poisson process. Conjugate prior for Exp, Poi parameter | $(0,\infty)$ | $\frac{r}{\lambda}$ | $rac{r}{\lambda^2}$ | $\frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$ | Note: $\Gamma(r) = (r-1)!$ for integers r . | | | | |
| Beta | $Beta(\alpha,\beta)$ for $\alpha,\beta>0$ | Conjugate prior for Ber, Bin, Geo, NegBin parameter p . | (0, 1) | $\frac{\alpha}{\alpha + \beta}$ | $\frac{\alpha\beta}{\left(\alpha+\beta\right)^{2}\left(\alpha+\beta+1\right)}$ | $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$ | | | | | |
| Dirichlet | $\mathbf{X} \sim Dir(\alpha_1, \alpha_2, \dots, \alpha_r)$ for $\alpha_i, r > 0$ and $r \in \mathbb{N}, \alpha_i \in \mathbb{R}$ | Generalization of Beta distribution. Conjugate prior for Multinomial parameter p | $x_i \in (0,1);$ $\sum_{i=1}^{r} x_i = 1$ | $\mathbb{E}[X_i] = \frac{\alpha_i}{\sum_{j=1}^r \alpha_j}$ | | $\frac{\frac{1}{B(\alpha)} \prod_{i=1}^{r} x_i^{a_i - 1}}{x_i \in (0, 1), \sum_{i=1}^{r} x_i} = 1$ | | | | | |
| Multivariate Normal | $\mathbf{X} \sim \mathcal{N}_n(oldsymbol{\mu}, oldsymbol{\Sigma})$ for $oldsymbol{\mu} \in \mathbb{R}^n$ and $oldsymbol{\Sigma} \in \mathbb{R}^{n 	imes n}$ | Multivariate generalization of Normal distribution. | \mathbb{R}^n | μ | Σ | $\frac{\frac{1}{(2\pi)^{n/2} \Sigma ^{1/2}}\cdot}{\exp(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu))}$ | | | | | |
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