Exercise 2:

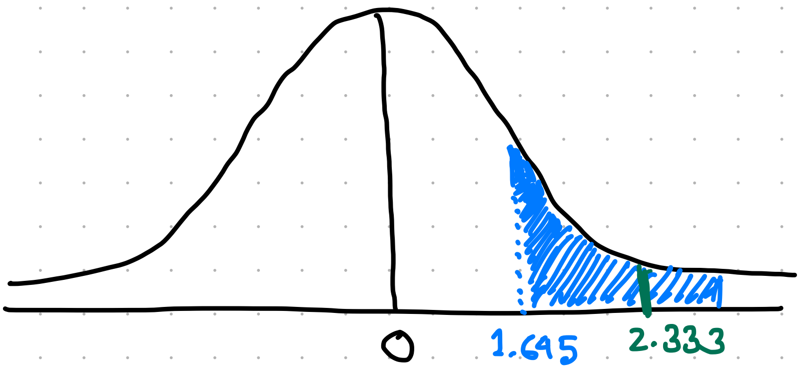
Part 1:

Here,

$\theta = 25,\ \alpha = 0.05,\ \bar{x} = 26,\ n = 49$

$\sigma^2 = 9$ or, $\sigma = 3$

$$\text{so, } z = \frac{\bar{x} - \theta}{\sigma/\sqrt{n}} = \frac{26 - 25}{3/\sqrt{49}} = 2.333$$



using z table for $\alpha = 0.05$ z value is 1.645

Here $1.645 < 2.333$

So, we can reject the null hypothesis.

The results of the sample data are statistically significant. There is sufficient evidence to conclude that Ho is an incorrect belief and the alternative hypothesis H_1 is true.

We can conclude that population mean(θ) is greater than 25.

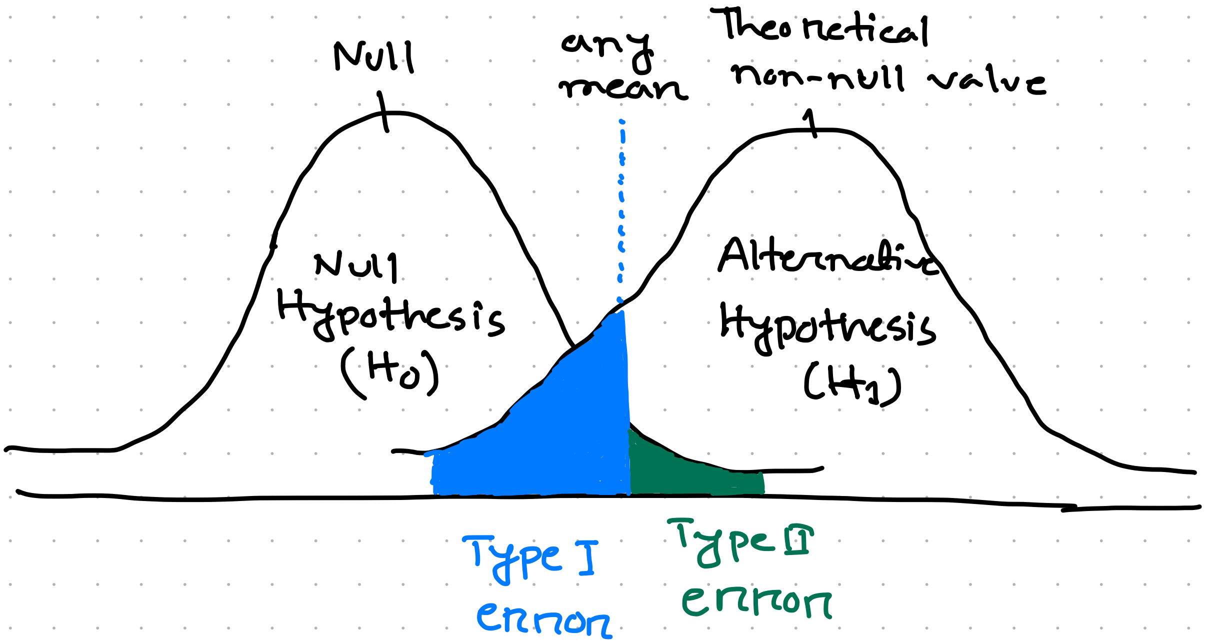
Part 2:

Type of Error Types

		Null Hypothesis (H_0) is	
		True	False
Decision about null Hypothesis (H_0)	Don't Reject	Correct inference (Power = $1 - \beta$)	Type II error (α)
	Reject	Type I error (β)	correct inference ($1-\alpha$)

When doing hypothesis testing, one endsup incorrectly rejecting the null hypothesis, when inreality it holds true. The probability of rejecting a null hypothesis when it actually holds goods is called Type I error. The probability of Type I error is α.

Here the significant Level, $\alpha = 0.05$ or 5%. . This means that there is a 5% probability that the test will reject the null hypothesis when it is actually true. So, there are still 5% of the population mean are greater than 25 but the true population mean does not cross 25.



We can reduce the risk of committing a Type 1 error by using a lower value for α. For example a α value of 0.01 would mean there is a 1% chance of committing Type I error.

However, using a lower value for α means that it will be less likely to detect a true difference if one really exists.

Part 3:

We know, $z_c = \dfrac{c-\mu}{\sigma/\sqrt{n}}$

$$\Rightarrow c-\mu = z_c \frac{\sigma}{\sqrt{n}}$$

$$\therefore c = \mu + z_c \frac{\sigma}{\sqrt{n}}$$

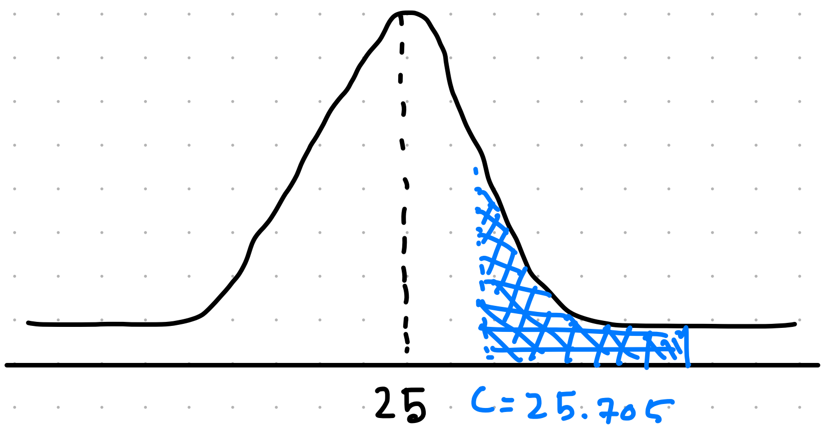
Here, $\mu = 25$, $\sigma^2 = 9$ or $\sigma = 3$

$n = 49$,

for, $\alpha = 0.05$. $Z_c = 1.645$

So, $c = 25 + 1.645 \times \frac{3}{\sqrt{49}}$

$= 25.705$



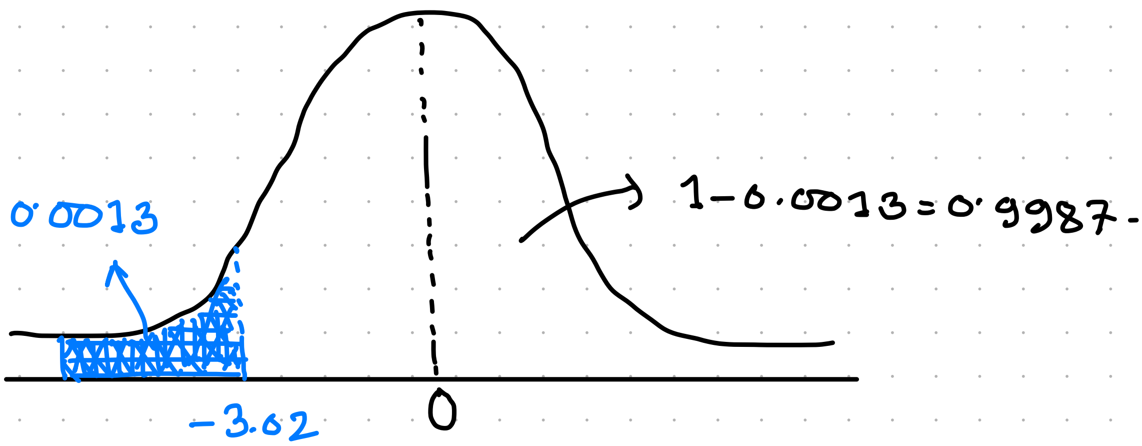
Given, true age = 27

Now, $z = \frac{\bar{x} - \theta}{\sigma/\sqrt{n}}$

$= \frac{25.705 - 27}{3/\sqrt{49}}$

$= -3.02$

for, $z = -3.02$ using table $p = 0.0013$



So, probability of Type 2 error is 99.87%

Part 4:

From the previous exercise we know,

$$P\left(-1.96 < \frac{\bar{x}-\theta}{\sigma/\sqrt{n}} < 1.96\right) = 0.95$$

Here, $\bar{x}=26$, $\sigma^2=9$ or $\sigma=3$, $n=49$

$$P\left(\bar{x} - \frac{1.96\sigma}{\sqrt{n}} < \theta < \bar{x} + \frac{1.96\sigma}{\sqrt{n}}\right) = 0.95$$

$$\Rightarrow P(25.16 < \theta < 26.84) = 0.95$$

So, the confidence interval is $(25.16, 26.84)$

So, if we have 100 samples, mean of 95 samples will be between 25.16 to 26.84.

The assumation (age > 25) is supported by this interval.