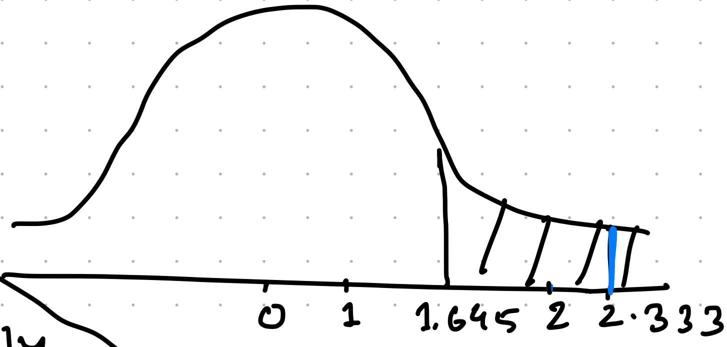
$\theta_0 = 25$, $\alpha = 0.05 \to 1.645$

$\boxed{\sigma^2 = 9. \quad \sigma = 3}$

$\bar{x} = 26$

$n = 49.$

$$z = \frac{26-25}{3/\sqrt{49}} = 2.333$$



$1.645 < 2.333$

Reject Ho,

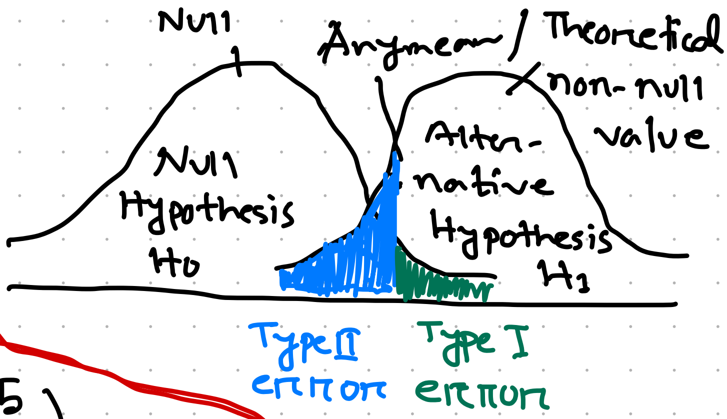
The results of the sample data are statistically significant. There is sufficient evidence to conclude that Ho is an incorrect belief and the alternative hypothesis H_1 is true.

We can conclude that population mean (θ) is greater than 25.

Table of error types		Null hypothesis (Ho) is: True	False
Decision about null hypothesis (Ho)	Don't Reject	Correct inference	Type II error
	Reject	Type I error	Correct Inference.

When doing hypothesis testing, one ends up incorrectly rejecting the null hypothesis, when in reality it holds true. The probability of rejecting a null hypothesis when it actually holds goods is called Type I error. The probability of Type I error is α.

Here the significant level, $\alpha = 0.05$ or 5%. This means that there is a 5% probability that the test will reject the null hypothesis when it is actually true. So, there are still 5% of the population mean are greater than 25 but the true population mean does not cross 25.



The critical value, c

$$P\left(Z \geq \frac{c-25}{3}\right) = 0.05$$

$$\Rightarrow \frac{c-25}{3} = 1.645$$

$$\therefore c = 29.935$$

25.705

True age, $\theta = 27$

$$P = (T < 29.935 \mid \theta = 27)$$

$$= P\left(\frac{T - 29.935}{3} < \frac{27 - 29.935}{3}\right)$$

$$= \phi(-0.9783)$$

$$= 0.1635$$

Which means, if the true age is 27, the Probability of making type 2 error is 0.1635 or 16.35%

From the previous exercise we know,

$$P\left(-1.96 < \frac{\bar{x} - \theta}{\sigma/\sqrt{n}} < 1.96\right) = 0.95$$

Here, $\bar{x} = 26$, $\sigma^2 = 9 \Rightarrow \sigma = 3$, $n = 49$

$$P\left(\bar{x} - \frac{1.96\,\sigma}{\sqrt{n}} < \theta < \bar{x} + \frac{1.96\,\sigma}{\sqrt{n}}\right) = 0.95$$

$$\Rightarrow P(25.16 < \theta < 26.84) \approx 0.95.$$

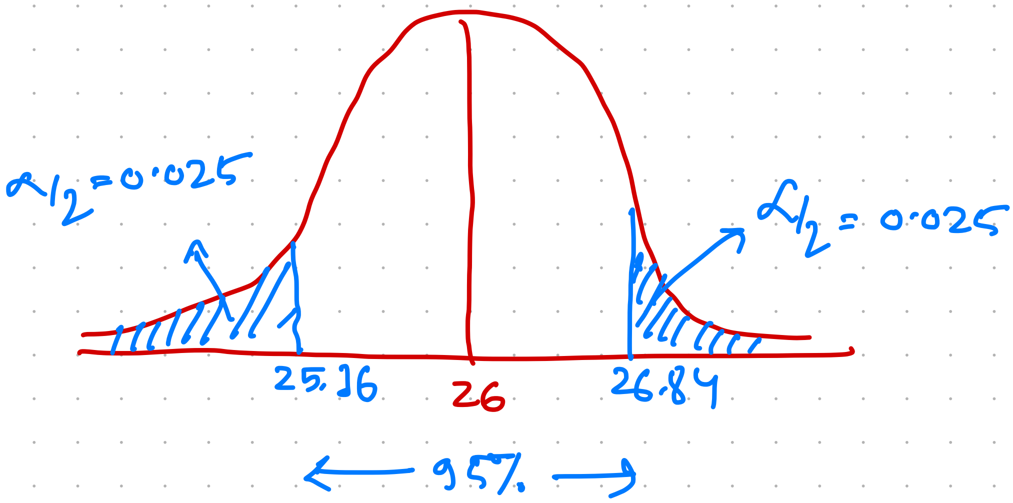
So the confidence interval is $(25.16, 26.84)$.

So if we have 100 Samples mean of 95 samples will be between 25.16 to 26.84.

⊛ We can reduce the risk of committing a type I error by using a lower value for α. For example a α-value of 0.01 would mean there is a 1% chance of committing a Type I error.

However, using a lower value for alpha means that it will be less likely to detect a true difference if one really exists.

⊛ The assumation (age>25) is supported by this interval.



Marginof error (moe)

$Z_\alpha \frac{\sigma}{\sqrt{n}}$

To decrease marginal of error
1. decrease Confidence interval.
2. Increase sample size.

* Confidence Interval ↑ moe↑

* Sample Size (n) ↑ moe↓

Part 3:

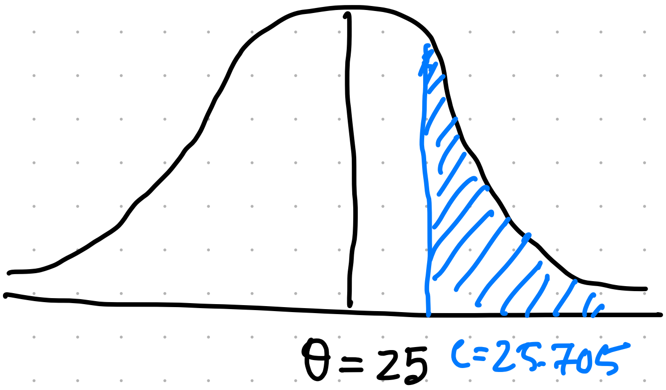
$$C = \mu + z_c\left(\frac{\sigma}{\sqrt{n}}\right)$$

$\mu = 25, \ \sigma^2 = 9 \Rightarrow \sigma = 3, \quad n = 49.$

for $\alpha = 0.05$ $\quad z_c = 1.645$

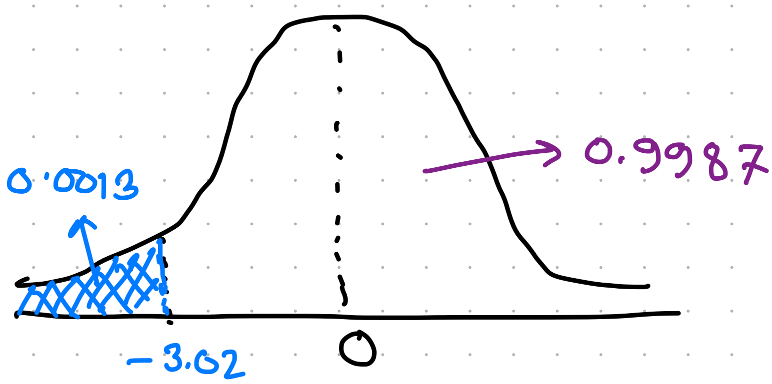
So, $\quad C = 25 + 1.645 \times \dfrac{3}{\sqrt{49}}$

$\quad = 25.705.$



Now, $\quad z = \dfrac{\bar{x} - \theta}{\sigma/\sqrt{n}} = \dfrac{25.705 - 27}{3/\sqrt{49}}$

$\quad = -3.02$



Probability of Type 2 error is 99.87%

$\mu = 60, \quad \bar{x} = 57, \quad n = 51$

$\sigma = 12 \quad \alpha = 0.01$

$\downarrow$

-2.33

-1.645

$$C = \mu + z_c \frac{\sigma}{\sqrt{n}}$$

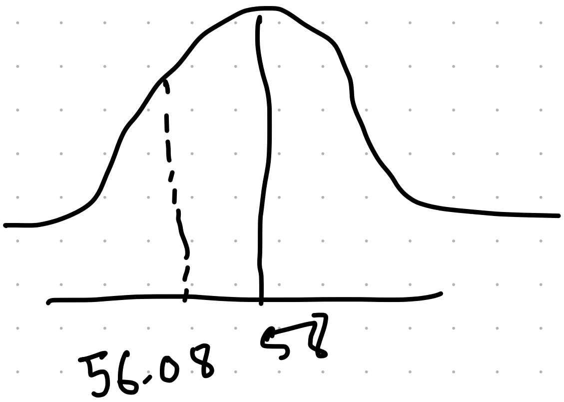
$$= 60 + (-2.33)\frac{12}{\sqrt{51}}$$

$$= 56.08$$

$\underline{24.295}$

$= \underline{25.705}$

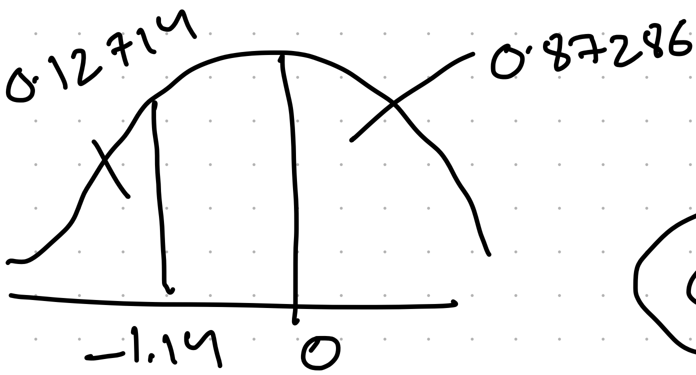
22.885



$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{56.08 - 58}{12/\sqrt{51}}$$

$$= -1.14$$



0.12714

$\underline{-6.31167}$

$= -3.02$

$= 0.0013$

Probability - 87.286%

0.9987

If

Sample average value (μ) < Assume Claim Average (hypothesis)

$$c = \mu - |z_c| \frac{\sigma}{\sqrt{n}}$$

otherwise

$$c = \mu + |z_c| \frac{\sigma}{\sqrt{n}} .$$ ✓

0.95

0.95

26 < 25 ✗

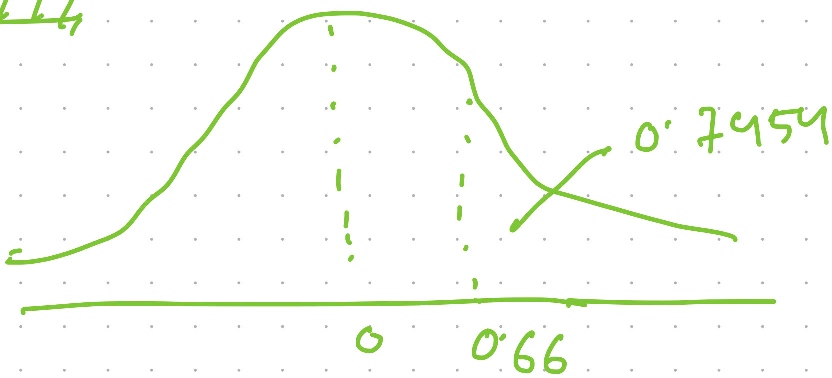
0.0013

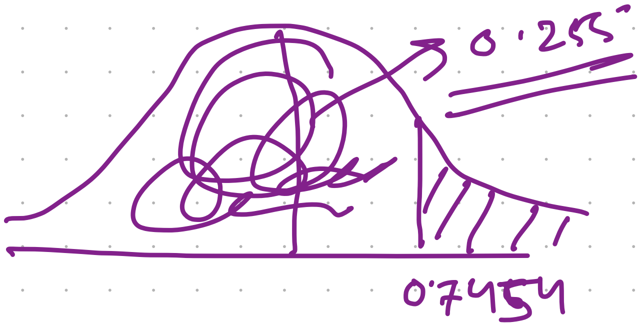
0·91 0·745

0·09 0·255

1·57 → 0·9345 0·66 → 0·7454

0·2546





1.51 → 0.9345