Institut für Mathematik
Abteilung Numerik
Jana de Wiljes

7. Problem sheet for **Statistical Data Analysis**

Exercise 1 (8 Points)
Consider a simple linear regression problem where $\mathcal{H} = \{h(x;\beta) = x\beta | \beta \in \mathbb{R}$ with samples $x_i, y_i \in \mathbb{R}$ iid and $y_i = h(x_i;\beta) + \epsilon_i$ where $\epsilon_i \sim \mathcal{N}(0,1)$ the associated

$$R_N(h) = \frac{1}{N}\sum_{i=1}^{N}(y_i - x_i\beta)^2 \quad \text{where } \beta_N^* = (\sum_{i=1}^{N} x_i^2)^{-1}(\sum_{i=1}^{N} x_i y_i)$$

A new data point (x_{N+1}, y_{N+1}) leads to a new parameter

$$\beta_{N+1}^* = (\sum_{i=1}^{N} x_i^2 + x_{N+1}x_{N+1})^{-1}(\sum_{i=1}^{N} x_i y_i + x_{N+1}y_{N+1}) \tag{1}$$

Derive a sequential update formulare for β_{N+1}^* that uses β_N^* and the new data point (x_{N+1}, y_{N+1}) and not require to repeat the calculations to determine β_N^*.

Exercise 2 (9 points)
Determine $\frac{\partial E}{\partial w_{ji}^H}$ of loss function

$$E(\mathbf{w}, \mathbf{b}) = \frac{1}{2}\sum_{k \in N_O}(\mathcal{O}_k - t_k)^2 \tag{2}$$

for a network with one input layer (with N_I neurons), output layer (with N_O neurons) and hidden layer (with N_H neurons). Note that every neuron is assumed to be connected to every neuron of the next layer, i.e., a Multi Layer Perceptron is considered. Further the sigmoid function is the considered action function for every neuron in the hidden and output layer.

 Due on the 12.12.2022 before midnight