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Abteilung Numerik
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4. Problem sheet for **Statistical Data Analysis**

Exercise 1 (6 Points)
Let $(x_1, \ldots, x_n) \in (0, \infty)^n$ be a realisation of independent on $[0, \theta]$ uniformly distributed random variables $X_1, \ldots, X_n$. What is Maximum Spacing Estimator in this case? Using the data set provide on Moodle computer the unknown parameter θ via the Maximum spacing estimator for the three different sets of samples (note that they are of different sizes).

Exercise 2 (10 Punkte)
Consider the following evolution equation

$$z_n = 0.99 z_{n-1} + \xi_{n-1} \tag{1}$$

with $\xi_{n-1} \sim \mathcal{N}(0, 0, 5)$ and initial value $z_0 \sim \mathcal{N}(0, 0, 5)$. The associated observations are linked to the signal via

$$y_n = z_n + \eta_n \tag{2}$$

where $\eta_{n-1} \sim \mathcal{N}(0, 0, 5)$. Implement the Kalman filter and the ensemble Kalman filter (with $M = 5$, $M = 10$, $M = 25$ and $M = 50$ ensemble members) for the data assimilation problem above and estimate the true signal via the Bayes estimator. Compare the estimates of the Ensemble Kalman Filter with the Kalman filter by computing the MSE for different ensemble sizes and comment on it the results. The corresponding observation data and the true signal are provided in the Moodle.

Due on the 20.11.2022 before midnight