Exercise 2:

A statistics $T = T(X)$ is sufficient statistics for θ if the conditional distribution of X given T is free of θ; i.e. if the ratio

$$f_{X|T}(x|t) = \frac{f_X(x|\theta)}{f_T(t|\theta)}$$

is free of θ. for all $x \in X$. In other words, after conditioning on T, we have removed all information about θ from sample X.

The joint distribution of the n-order Statistics is

$$f_{X_1, X_2 \cdots X_n}(x_1 x_2 \cdots x_n|\theta)$$

$$= n!\, f_X(x_1|\theta)\, f_X(x_2|\theta) \cdots f_X(x_n|\theta)$$

$$= n!\, f_X(x|\theta)$$

for $-\infty < x_1 < x_2 < \cdots < x_n < \infty$. Therefore the ratio

$$\frac{f_X(x|\theta)}{f_T(x|\theta)} = \frac{f_X(x|\theta)}{n!\, f_X(x|\theta)} = \frac{1}{n!}$$

which is free of θ.

So, according to the defination of sufficiency: $T = T(X) = (X_1, X_2, \cdots X_n)$ is a sufficient statistic.

$\blacksquare$