

12. Problem sheet for <b>Statistical Data Analysis</b>
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**Exercise 1** (8 Points)

Let  $A = U\Sigma V$  be the singular value decomposition of  $A \in \mathbb{R}^{m \times n}$  with  $m < n$  and singular values  $\sigma_1 \geq \dots \geq \sigma_p$  für  $p = \min(m, n)$ . Let  $u_1, \dots, u_m$  and  $v_1, \dots, v_n$  denote the columns of  $U$  and  $V$  respectively. Show that

$$\mathbf{Ker}(A) = \langle v_{r+1}, \dots, v_n \rangle \text{ and } \mathbf{Im}(A) = \langle u_1, \dots, u_r \rangle.$$

**Exercise 2** (8 Points)

Determine the SVD of the matrix

$$A = \begin{bmatrix} -1 & -1 \\ 2 & 2 \\ -1 & 1 \end{bmatrix} \tag{1}$$

without the help of computer and write down all the steps of the calculation.