Exercise 2:

Given, $E(w,b) = \frac{1}{2} \sum_{k \in N_0} (O_k - t_k)^2$

We take,

output of input Layer $= O_i^I$

output of hidden Layer $= O_j^H$

Bias of hidden Layer $= b_i^H$

Input of hidden Layer

$$Z_j^H = \sum_{i=1}^{N^I} w_{ji}^H O_i^I + b_i^H$$

output of output layer $= O_k^O$

Bias of output Layer $= b_k^o$

Input of output Layer

$$Z_K^o = \sum_{j=1}^{N^H} w_{Kj}^O O_j^H + b_K^o$$

Activation function $= Sig(t)$

and $Sig'(t) = Sig(t)\,(1 - Sig(t))$

So, $O_j^H = Sig(Z_j^H)$

and $O_K^O = Sig(Z_K^O)$

$$\frac{\partial E}{\partial O_k^o} = \frac{1}{2} \cdot 2 \sum_{k=1}^{N^o} (O_k^o - t_k) = \sum_{k=1}^{N^o} (O_k^o - t_k)$$

$$\frac{\partial O_k^o}{\partial Z_k^o} = \text{sig}'(Z_k^o)$$

$$= \text{sig}(Z_k^o)\left(1 - \text{sig}(Z_k^o)\right)$$

$$\frac{\partial Z_k^o}{\partial O_j^H} = \omega_{kj}^O$$

$$\frac{\partial O_j^H}{\partial Z_j^H} = \text{sig}'(Z_j^H)$$

$$= \text{sig}(Z_j^H)\left(1 - \text{sig}(Z_j^H)\right)$$

$$\frac{\partial Z_j^H}{\partial \omega_{ji}^H} = \omega_{ji}^H$$

Now,

$$\frac{\partial E}{\partial \omega_{ji}^H} = \frac{\partial E}{\partial O_k^o} \cdot \frac{\partial O_k^o}{\partial Z_k^o} \cdot \frac{\partial Z_k^o}{\partial O_j^H} \cdot \frac{\partial O_j^H}{\partial Z_j^H} \cdot \frac{\partial Z_j^H}{\partial \omega_{ji}^H}$$

$$= \sum_{k=1}^{N^o} (O_k^o - t_k) \cdot \text{sig}(Z_k^o)\left(1 - \text{sig}(Z_k^o)\right)$$

$$\cdot\ \omega_{kj}^O \cdot \text{sig}(Z_j^H)\left(1 - \text{sig}(Z_j^H)\right)$$

$$\cdot\ \omega_{ji}^H$$