Problem 1:

Part 1:

$$Z \text{ statistic} = \frac{\bar{x} - \theta}{\sigma/\sqrt{n}}$$

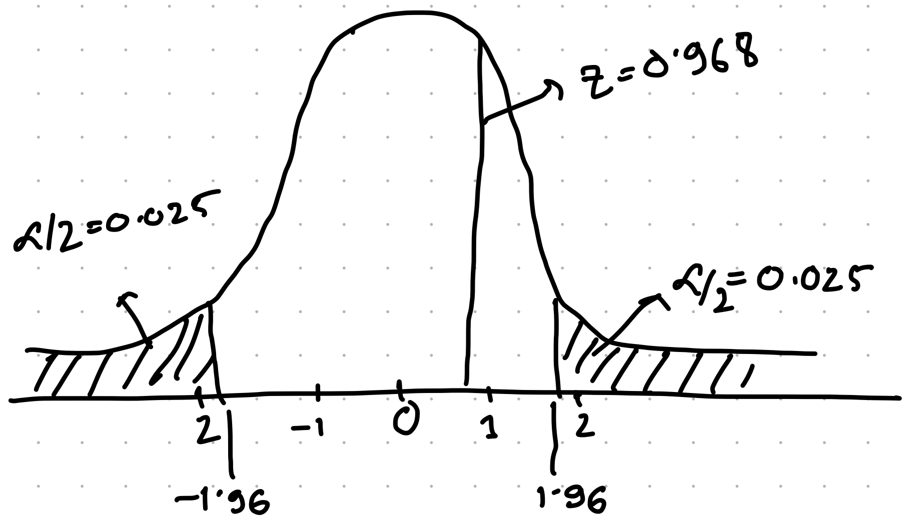
here $\bar{x} = 1.5$, $\theta = 1$, $n = 15$, and $\sigma^2 = 4$
$\Rightarrow \sigma = 2$

so,

$$z = \frac{1.5 - 1}{2/\sqrt{15}}$$

$$= 0.968$$

Here, $\alpha = 0.05$, two tailed test, $\alpha/2 = 0.025$
using Z table the z value of Z for the two tailed test is 1.96

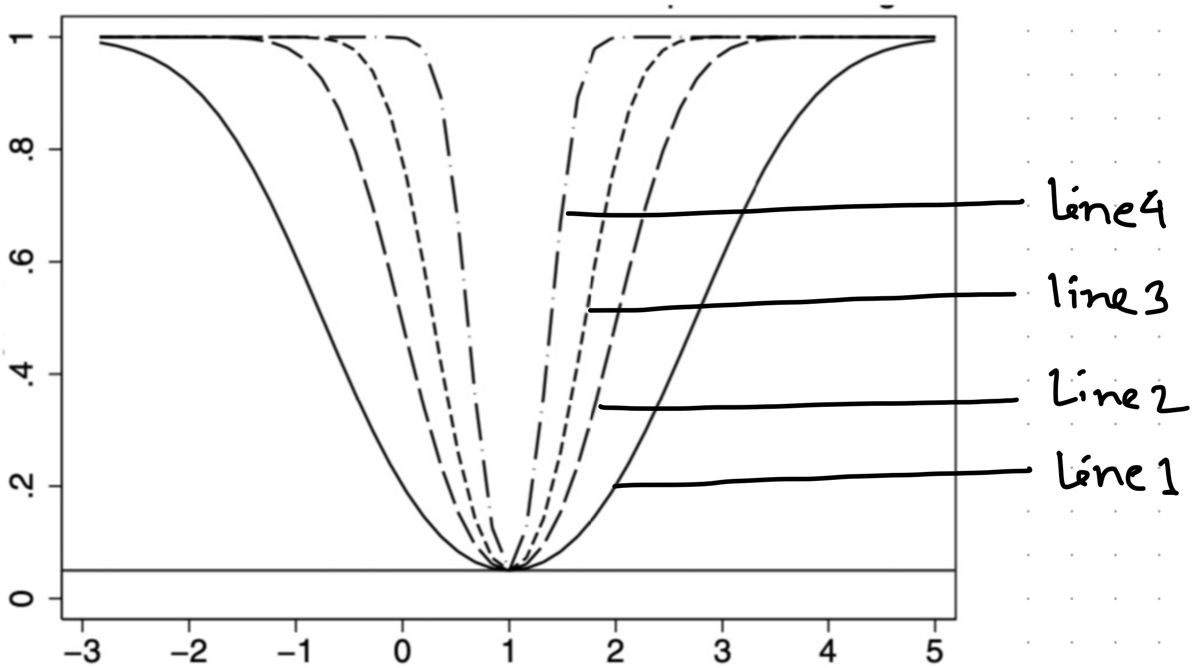


we can reject H_0 if $Z \leq -1.96$ or if $Z \geq 1.960$

Here $0.968 < 1.960$

So, we cannot reject H_0, on the other hand H_0 is accepted.

part 2 :



For understanding I rename the line with Line 1, Line 2, Line 3 and Line 4.

we know. $Z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

here when the sample size gets Larger, the z value increases therefore we will more likely to reject the null hypothesis; less likely to fail to reject the null hypothesis, thus the power of test increase.

Also when increase the sample size, the sampling distributions are getting narrow.

and when they are getting narrow then

difficult to reject the null hypothesis.

Sample size	Line
$n_1 = 100$	Line 4
$n_2 = 15$	Line 2
$n_3 = 5$	Line 1
$n_4 = 30$	Line 3.

Part 3:

From Part 1 we found that for $\alpha = 0.05$ two tailed test the value is ± 1.96

So,

$$P(-1.96 < Z < 1.96) = 0.95$$

$$\Rightarrow P\left(-1.96 < \frac{\bar{x}-\theta}{\sigma/\sqrt{n}} < 1.96\right) = 0.95$$

$$\Rightarrow P\left(-1.96\frac{\sigma}{\sqrt{n}} < \bar{x}-\theta < 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

$$\Rightarrow P\left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < \theta < \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

$$\Rightarrow P(1.5 - 0.51\sigma < \theta < 1.5 + 0.51\sigma) = 0.95$$

$[\because \bar{x} = 1.5$ and $n = 15]$

$$\Rightarrow P(0.48 < \theta < 2.52) = 0.95$$

$[\because \sigma^2 = 4. \Rightarrow \sigma = 2]$

So, when error probability $\alpha = 0.05$ then the confidence interval of θ is $(0.48, 2.52)$

Part 4:

From Part 3 (*)

$$P\left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < \theta < \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right) \approx 0.95$$

$$\Rightarrow P\left(1.5 - 1.96\frac{2}{\sqrt{n}} < \theta < 1.5 + 1.96\frac{2}{\sqrt{n}}\right) \approx 0.95$$

$$\Rightarrow P\left(1.5 - \frac{3.92}{\sqrt{n}} < \theta < 1.5 + \frac{3.92}{\sqrt{n}}\right) \approx 0.95$$

Now, we take some sample size and calculate the corresponding confidence interval.

n	confidence interval
5	(−0.25, 3.25)
10	(0.26, 2.74)
15	(0.49, 2.51)
20	(0.62, 2.38)
50	(0.95, 2.05)
100	(1.11, 1.892)

So, when sample size is increasing then the lower bound of the interval is increasing and the upper bound is decreasing. So, the sampling distribution are getting narrow, and we have less area to reject the null hypothesis. At some big sample size the upper and lower bound will be same.