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$$f(t) = \begin{cases} 1/2\theta\sqrt{t}\, \exp(-\sqrt{t}/\theta) & \text{for } t>0 \\ 0, & \text{for } t\leq 0 \end{cases}$$

$$L(t;\theta) = \prod_{i=1}^{n} f(t_i;\theta)$$

$$= \prod_{i=1}^{n} \frac{1}{2\theta\sqrt{t_i}} \exp\left(-\frac{\sqrt{t_i}}{\theta}\right)$$

Taking log

$$L(t;\theta) = \text{Log}\left(\prod_{i=1}^{n} \frac{1}{2\theta\sqrt{t_i}} \exp -\frac{\sqrt{t_i}}{\theta}\right)$$

$$= \text{Log}\left(\sum_{i=1}^{n} \frac{1}{2\theta\sqrt{t_i}}\right) - \frac{\sum_{i=1}^{n}\sqrt{t_i}}{\theta}$$

$$= \sum_{i=1}^{n} \left(\log(1) - \log(2\theta\sqrt{t_i})\right) - \frac{\sum_{i=1}^{n}\sqrt{t_i}}{\theta}$$

$$= -\sum_{i=1}^{n} \text{Log}\, 2\theta\sqrt{t_i} - \frac{\sum_{i=1}^{n}\sqrt{t_i}}{\theta}$$

$$= -\sum_{i=1}^{n} \text{Log}\, 2\theta - \sum_{i=1}^{n}\sqrt{t_i} - \frac{\sum_{i=1}^{n}\sqrt{t_i}}{\theta}.$$

Differentiating w. r. to. θ.

$$\frac{\partial L(t;\theta)}{\partial \theta} = \sum_{i=1} \frac{1}{2\theta} - \sum_{i=1}^{n} \frac{1}{2\theta}\cdot 2 - 0 + \frac{\sum_{i=1}^{n}\sqrt{t_i}}{\theta^2}$$

$$= -\sum_{i=1}^{n} \frac{1}{\theta} + \frac{\sum_{i=1}^{n}\sqrt{t_i}}{\theta^2}$$

$$= -\frac{n}{\theta} + \frac{\sum_{i=1}^{n}\sqrt{t_i}}{\theta^2}$$

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To find the MLE of θ, $\hat{\theta}$, we solve

$$-\frac{n}{\theta} + \frac{\sum_{i=1}^{n} \sqrt{t_i}}{\theta^2} = 0$$

since $\log x$ is continuous increasing, the maximum stays at the same point

why can we get ML estimator from $\log L$? (-0.5)

$$\Rightarrow -n\theta + \sum_{i=1}^{n} \sqrt{t_i} = 0$$

$$\Rightarrow \hat{\theta}_{MLE} = \frac{1}{n} \sum_{i=1}^{n} \sqrt{t_i}$$

confirm it is a maximum by using second derivative (-0.5)

For sample, t_1, t_2, t_3, t_4 & t_5.

careful! $\sqrt{a}+\sqrt{b} \neq \sqrt{a+b}$

$$\hat{\theta}_{MLE} = \frac{\sqrt{11300 + 5000 + 4300 + 8500 + 7900}}{5}$$

$$= 2\sqrt{\cancel{370}}. \quad 84.73 \quad (-0.5)$$

2.

Given, $\mathbb{E}(T) = \int_0^{\infty} t \, \frac{1}{2\theta\sqrt{t}} e^{-\sqrt{t}/\theta} \, dt$

$$= 2\theta^2$$

Using method of moments,

$$2\theta^2 = \bar{t}$$

$$\Rightarrow \hat{\theta}_{MOM} = \sqrt{\bar{t}/2}$$

here, $\bar{t} = \frac{11300 + 3000 + 4300 + 8500 + 7900}{5}$

$$= 7400$$

so, $\hat{\theta}_{MOM} = \sqrt{\frac{7400}{2}} = 10\sqrt{37}$.

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