Exercise 2:

Given, $$E(w,b) = \frac{1}{2} \sum_{k \in N_0} (O_k - t_k)^2$$

We take,

output of input layer $= O_i^I$

output of hidden layer $= O_j^H$

Bias of hidden layer $= b_i^H$

Input of hidden layer

$$z_j^H = \sum_{i=1}^{N^I} w_{ji}^H O_i^I + b_i^H$$

output of output layer $= O_k^O$

Bias of output layer $= b_k^O$

Input of output layer

$$z_k^O = \sum_{j=1}^{N^H} w_{kj}^O O_j^H + b_k^O$$

Activation function $= Sig(t)$

and $Sig'(t) = Sig(t)\,(1 - Sig(t))$

So, $O_j^H = Sig(z_j^H)$

and $O_k^O = Sig(z_k^O)$

$$\frac{\partial E}{\partial o_k^o} = \frac{1}{2} \cdot 2 \sum_{k=1}^{N^o} (o_k^o - t_k) = \sum_{k=1}^{N^o} (o_k^o - t_k)$$

$$\frac{\partial o_k^o}{\partial z_k^o} = sig'(z_k^o)$$

$$= sig(z_k^o)\,(1 - sig(z_k^o))$$

$$\frac{\partial z_k^o}{\partial o_j^H} = \cancel{w_{kj}^O} \quad w_{iu}^o$$

$$\frac{\partial o_j^H}{\partial z_j^H} = sig'(z_j^H)$$

$$= sig(z_j^H)\,(1 - sig(z_j^H))$$

$$\frac{\partial z_j^H}{\partial w_{ji}^H} = \cancel{w_{ji}^H} \quad o_j^I$$

Now,

$$\frac{\partial E}{\partial w_{ji}^H} = \frac{\partial E}{\partial o_k^o} \cdot \frac{\partial o_k^o}{\partial z_k^o} \cdot \frac{\partial z_k^o}{\partial o_j^H} \cdot \frac{\partial o_j^H}{\partial z_j^H} \cdot \frac{\partial z_j^H}{\partial w_{ji}^H}$$

$$= \sum_{k=1}^{N^o} (o_k^o - t_k) \cdot sig(z_k^o)\,(1 - sig(z_k^o))$$

$$\cdot \cancel{w_{kj}^O} \cdot sig(z_j^H)\,(1 - sig(z_j^H))$$

$$\cdot \cancel{w_{ji}^H}$$

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