

Exercise 2:

Given, $E(w, b) = \frac{1}{2} \sum_{k \in N_0} (O_k - t_k)^2$

We take,

Output of input layer = O_i^I

Output of hidden layer = O_j^H

Bias of hidden layer = b_i^H

Input of hidden layer

$$Z_j^H = \sum_{i=1}^{N^I} w_{ji}^H O_i^I + b_i^H$$

Output of output layer = O_k^O

Bias of output layer = b_k^O

Input of output layer

$$Z_k^O = \sum_{j=1}^{N^H} w_{kj}^O O_j^H + b_k^O$$

Activation function = $\text{Sig}(t)$

and $\text{Sig}'(t) = \text{Sig}(t) (1 - \text{Sig}(t))$

So, $O_j^H = \text{Sig}(Z_j^H)$

and $O_k^O = \text{Sig}(Z_k^O)$

$$\frac{\partial E}{\partial o_k^o} = \frac{1}{2} \cdot 2 \sum_{k=1}^{N^o} (o_k^o - t_k) = \sum_{k=1}^{N^o} (o_k^o - t_k)$$

$$\begin{aligned} \frac{\partial o_k^o}{\partial z_k^o} &= \text{sig}'(z_k^o) \\ &= \text{sig}(z_k^o) (1 - \text{sig}(z_k^o)) \end{aligned}$$

$$\frac{\partial z_k^o}{\partial o_j^H} = \omega_{kj}^o$$

$$\begin{aligned} \frac{\partial o_j^H}{\partial z_j^H} &= \text{sig}'(z_j^H) \\ &= \text{sig}(z_j^H) (1 - \text{sig}(z_j^H)) \end{aligned}$$

$$\frac{\partial z_j^H}{\partial \omega_{ji}^H} = \omega_{ji}^H$$

Now,

$$\begin{aligned} \frac{\partial E}{\partial \omega_{ji}^H} &= \frac{\partial E}{\partial o_k^o} \cdot \frac{\partial o_k^o}{\partial z_k^o} \cdot \frac{\partial z_k^o}{\partial o_j^H} \cdot \frac{\partial o_j^H}{\partial z_j^H} \cdot \frac{\partial z_j^H}{\partial \omega_{ji}^H} \\ &= \sum_{k=1}^{N^o} (o_k^o - t_k) \cdot \text{sig}(z_k^o) (1 - \text{sig}(z_k^o)) \\ &\quad \cdot \omega_{kj}^o \cdot \text{sig}(z_j^H) (1 - \text{sig}(z_j^H)) \\ &\quad \cdot \omega_{ji}^H \end{aligned}$$

Εξάσκηση:

$$\begin{aligned}\frac{\partial E}{\partial w_{kj}^0} &= \frac{\partial E}{\partial o_k^0} \cdot \frac{\partial o_k^0}{\partial z_k^0} \cdot \frac{\partial z_k^0}{\partial w_{kj}^0} \\ &= (o_k^0 - t_k) \cdot \text{sig}(z_k^0) (1 - \text{sig}(z_k^0)) \\ &\quad \cdot o_{jh}\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial o_{jh}} &= \frac{\partial E}{\partial o_k^0} \cdot \frac{\partial o_k^0}{\partial z_k^0} \cdot \frac{\partial z_k^0}{\partial o_{jh}} \\ &= \sum_{k=1}^{N^0} (o_k^0 - t_k) \cdot \text{sig}(z_k^0) (1 - \text{sig}(z_k^0)) \\ &\quad \cdot w_{kj}^0.\end{aligned}$$

$$E = \frac{1}{2} (y(x_n, w) - t_n)^2$$

$$y_k = b_k$$

Hidden layer

$$\left\{ \begin{array}{l} \text{output: } z_j = h(a_j) \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{Input: } a_j = \sum_i w_{ji} x_i \end{array} \right. \xrightarrow{\text{Input}}$$

$$\sum_j w'_{kj} z_j$$

$$\begin{aligned} \frac{\partial E}{\partial w_{ji}} &= \frac{\partial E}{\partial z_j} \cdot \frac{\partial z_j}{\partial a_j} \cdot \frac{\partial a_j}{\partial w_{ji}} \\ &= (y_k - t_n) \cdot \frac{\partial}{\partial z_j} (y_k) \cdot h'(a_j) \cdot x_i \end{aligned}$$

$$= \sum_k \underbrace{(y_k - t_n)}_{= \delta_k \text{ (given)}} w'_{kj} h'(a_j) x_i$$

$$= \left(h'(a_j) \sum_k \delta_k w'_{kj} \right) x_i$$

