

Given data points:

$$x^{(1)} = (2, 8), \quad x^{(2)} = (2, 5), \quad x^{(3)} = (1, 2),$$

$$x^{(4)} = (5, 8), \quad x^{(5)} = (7, 3), \quad x^{(6)} = (6, 4)$$

$$x^{(7)} = (8, 4), \quad x^{(8)} = (4, 7)$$

Distance Matrix:

	$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	$x^{(4)}$	$x^{(5)}$	$x^{(6)}$	$x^{(7)}$	$x^{(8)}$
$x^{(1)}$	0	3.0000	6.0828	3.0000	7.0711	5.6569	7.2111	2.2361
$x^{(2)}$	3.0000	0	3.1623	4.2426	5.3852	4.1231	6.0828	2.8284
$x^{(3)}$	6.0828	3.1623	0	7.2111	6.0828	5.3852	7.2801	5.8310
$x^{(4)}$	3.0000	4.2426	7.2111	0	5.3852	4.1231	5.0000	1.4142
$x^{(5)}$	7.0711	5.3852	6.0828	5.3852	0	1.4142	1.4142	5.0000
$x^{(6)}$	5.6569	4.1231	5.3852	4.1231	1.4142	0	2.0000	3.6056
$x^{(7)}$	7.2111	6.0828	7.2801	5.0000	1.4142	2.0000	0	5.0000
$x^{(8)}$	2.2361	2.8284	5.8310	1.4142	5.0000	3.6056	5.0000	0

Initial point:

$$\mu^{(1)} = x^{(3)}$$

$$\mu^{(2)} = x^{(4)}$$

$$\mu^{(3)} = x^{(6)}$$

Compute closest centroids:

$$c^{(1)} = c^{(4)} = c^{(8)} = 2$$

$$c^{(2)} = c^{(6)} = 1$$

$$c^{(5)} = c^{(6)} = c^{(7)} = 3$$

Move centroids:

$$\mu^{(1)} = \frac{1}{2} (x^{(2)} + x^{(3)}) = \begin{pmatrix} 1.5 \\ 3.5 \end{pmatrix}$$

$$\mu^{(2)} = \frac{1}{3} (x^{(1)} + x^{(4)} + x^{(8)}) = \begin{pmatrix} 3.67 \\ 7.67 \end{pmatrix}$$

$$\begin{aligned} \mu^{(3)} &= \frac{1}{3} (x^{(5)} + x^{(6)} + x^{(7)}) \\ &= \begin{pmatrix} 7 \\ 3.67 \end{pmatrix} \end{aligned}$$

Loss function,

$$J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_k) = \sum_{i=1}^m \|x^{(i)} - \mu_{c^{(i)}}\|^2$$

For,  $x^{(2)}$  and  $x^{(3)}$ ,  $\mu_c = (1.5, 3.5)$

$$\text{So, } \|x^{(2)} - \mu_c\|^2 + \|x^{(3)} - \mu_c\|^2$$

$$= 2.5 + 2.5$$

$$= 5$$

For  $x^{(1)}, x^{(4)}$  and  $x^{(8)}$ ,  $M_c = (3.67, 7.67)$

$$\begin{aligned}\text{So, } \|x^{(1)} - M_c\|^2 + \|x^{(4)} - M_c\|^2 + \|x^{(8)} - M_c\|^2 \\ = 2.8978 + 1.8778 + 0.5578 \\ = 5.3334\end{aligned}$$

For,  $x^{(5)}, x^{(6)}$  and  $x^{(7)}$ ,  $M_c = (7, 3.67)$

$$\begin{aligned}\text{So, } \|x^{(5)} - M_c\|^2 + \|x^{(6)} - M_c\|^2 + \|x^{(7)} - M_c\|^2 \\ = 0.4489 + 1.1089 + 1.1089 \\ = 2.6667.\end{aligned}$$

$$\begin{aligned}J_1 &= 5 + 5.3334 + 2.6667 \\ &= 13.0001\end{aligned}$$

## 2nd Iteration:

Now,

$$M^{(1)} = (1.5, 3.5)$$

$$M^{(2)} = (3.67, 7.67)$$

$$M^{(3)} = (7, 3.67)$$

## Distance

	$\mu(1)$	$\mu(2)$	$\mu(3)$
$x^{(1)}$	4.52	1.70	6.61
$x^{(2)}$	1.58	3.15	5.17
$x^{(3)}$	1.58	6.27	6.23
$x^{(4)}$	5.70	1.37	4.77
$x^{(5)}$	5.52	5.74	0.67
$x^{(6)}$	4.52	4.35	1.05
$x^{(7)}$	6.52	5.68	1.05
$x^{(8)}$	4.30	0.74	4.48

Closest centroids:

$$c^{(2)} = c^{(3)} = 1$$

$$c^{(1)} = c^{(4)} = c^{(8)} = 2$$

$$c^{(5)} = c^{(6)} = c^{(7)} = 3$$

Centroid points are unchanged.

So, loss is also unchanged.  $J_2 = 13.0001$ .