Example Solution

According to the information given,

X	$P(X=x \mid \theta=\theta_1)$	$P(X=x \mid \theta=\theta_2)$
0	0.7	0.5
1	0.2	0.3
2	0.1	0.2

the prior probability :

$$\begin{cases} P(\theta=\theta_1) = 0.75 \\ P(\theta=\theta_2) = 0.25 \end{cases}$$

the likelihood :

$$P(X_1=0, X_2=1 \mid \theta=\theta_1) = (0.7)(0.2) = 0.14$$
$$P(X_1=0, X_2=1 \mid \theta=\theta_2) = (0.5)(0.3) = 0.15$$

the marginal likelihood :

$$\begin{aligned} P(X_1=0, X_2=1) &= P(X_1=0, X_2=1 \mid \theta=\theta_1)\, P(\theta=\theta_1) \\ &\quad + P(X_1=0, X_2=1 \mid \theta=\theta_2)\, P(\theta=\theta_2) \\ &= (0.7)(0.2)(0.75) + (0.5)(0.3)(0.25) \\ &= 0.1425 \end{aligned}$$

Therefore, the posterior probability of θ :

$$\begin{aligned} P(\theta=\theta_1 \mid X_1=0, X_2=1) &= \frac{P(X_1=0, X_2=1 \mid \theta=\theta_1)\, P(\theta=\theta_1)}{P(X_1=0, X_2=1)} \\ &= \frac{(0.7)(0.2)(0.75)}{0.1425} \\ &= 0.7368 \end{aligned}$$

$$\begin{aligned} P(\theta=\theta_2 \mid X_1=0, X_2=1) &= \frac{P(X_1=0, X_2=1 \mid \theta=\theta_2)\, P(\theta=\theta_2)}{P(X_1=0, X_2=1)} \\ &= \frac{(0.5)(0.3)(0.25)}{0.1425} \\ &= 0.2632 \end{aligned}$$

Not much change on Hani's belief on the proportion of the students being lost on her tutorial.