Given data points:

$$x^{(1)} = (2,8), \quad x^{(2)} = (2,5), \quad x^{(3)} = (1,2),$$
$$x^{(4)} = (5,8), \quad x^{(5)} = (7,3), \quad x^{(6)} = (6,4)$$
$$x^{(7)} = (8,4), \quad x^{(8)} = (4,7)$$

Distance Matrix:

	$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	$x^{(4)}$	$x^{(5)}$	$x^{(6)}$	$x^{(7)}$	$x^{(8)}$
$x^{(1)}$	0	3.0000	6.0828	3.0000	7.0711	5.6569	7.2111	2.2361
$x^{(2)}$	3.0000	0	3.1623	4.2426	5.3852	4.1231	6.0828	2.8284
$x^{(3)}$	6.0828	3.1623	0	7.2111	6.0828	5.3852	7.2801	5.8310
$x^{(4)}$	3.0000	4.2426	7.2111	0	5.3852	4.1231	5.0000	1.4142
$x^{(5)}$	7.0711	5.3852	6.0828	5.3852	0	1.4142	1.4142	5.0000
$x^{(6)}$	5.6569	4.1231	5.3852	4.1231	1.4142	0	2.0000	3.6056
$x^{(7)}$	7.2111	6.0828	7.2801	5.0000	1.4142	2.0000	0	5.0000
$x^{(8)}$	2.2361	2.8284	5.8310	1.4142	5.0000	3.6056	5.0000	0

Initial point:

$$\mu^{(1)} = x^{(3)}$$
$$\mu^{(2)} = x^{(4)}$$
$$\mu^{(3)} = x^{(6)}$$

Compute closest centroids:

$$c^{(1)} = c^{(4)} = c^{(8)} = 2$$

$$c^{(2)} = c^{(8)} = 1$$

$$c^{(5)} = c^{(6)} = c^{(7)} = 3$$

Move centroids:

$$\mu(1) = \frac{1}{2}\left(x^{(2)} + x^{(3)}\right) = \begin{pmatrix} 1.5 \\ 3.5 \end{pmatrix}$$

$$\mu(2) = \frac{1}{3}\left(x^{(1)} + x^{(4)} + x^{(8)}\right) = \begin{pmatrix} 3.67 \\ 7.67 \end{pmatrix}$$

$$\mu(3) = \frac{1}{3}\left(x^{(5)} + x^{(6)} + x^{(7)}\right)$$

$$= \begin{pmatrix} 7 \\ 3.67 \end{pmatrix}$$

Loss function,

$$J(c^{(1)}, \ldots, c^{(m)}, \mu_1 \ldots, \mu_k) = \sum_{i=1}^{m} \| x^{(i)} - \mu_{c^{(i)}} \|^2$$

For, $x^{(2)}$ and $x^{(3)}$, $\mu_c = (1.5, 3.5)$

So, $\| x^{(2)} - \mu_c \|^2 + \| x^{(3)} - \mu_c \|^2$

$= 2.5 + 2.5$

$= 5$

For $x^{(1)}, x^{(4)}$ and $x^{(8)}$, $M_c = (3.67, 7.67)$

So, $\|x^{(1)} - M_c\|^2 + \|x^{(4)} - M_c\|^2 + \|x^{(8)} - M_c\|^2$

$= 2.8978 + 1.8778 + 0.5578$

$= 5.3334$

For, $x^{(5)}, x^{(6)}$ and $x^{(7)}$, $M_c = (7, 3.67)$

So, $\|x^{(5)} - M_c\|^2 + \|x^{(6)} - M_c\|^2 + \|x^{(7)} - M_c\|^2$

$= 0.4489 + 1.1089 + 1.1089$

$= 2.6667.$

$J_1 = 5 + 5.3334 + 2.6667$

$= 13.0001$

2nd Iteration:

Now,

$M^{(1)} = (1.5, 3.5)$

$M^{(2)} = (3.67, 7.67)$

$M^{(3)} = (7, 3.67)$

Distance

	$\mu^{(1)}$	$\mu^{(2)}$	$\mu^{(3)}$
$x^{(1)}$	4.52	1.70	6.61
$x^{(2)}$	1.58	3.15	5.17
$x^{(3)}$	1.58	6.27	6.23
$x^{(4)}$	5.70	1.37	4.77
$x^{(5)}$	5.52	5.74	0.67
$x^{(6)}$	4.52	4.35	1.05
$x^{(7)}$	6.52	5.68	1.05
$x^{(8)}$	4.30	0.74	4.48

Closest centroids:

$$c^{(2)} = c^{(3)} = 1$$
$$c^{(1)} = c^{(4)} = c^{(8)} = 2$$
$$c^{(5)} = c^{(6)} = c^{(7)} = 3$$

✓

Centroid points are unchanged.

So, loss is also unchanged. $J_2 = 13.0001$.

6/6