Exercise 1

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Let $(x_1, ... x_n) \in \mathbb{R}^n$ be a set of sample. Show that for all $a \in \mathbb{R}$:

$$\sum_{i=1}^{n} (x_i - a)^2 = \sum_{i=1}^{n} (x_i - \bar{x}_n)^2 + n(\bar{x}_n - a)^2$$

Solution

$$\begin{aligned}
\sum_{i=1}^{n} (x_i - a)^2 &= \sum_{i=1}^{n} ((x_i - \bar{x}_n) + (\bar{x}_n - a))^2 \\
&= \sum_{i=1}^{n} (x_i - \bar{x}_n)^2 + 2\sum_{i=1}^{n} (x_i - \bar{x}_n)(\bar{x}_n - a) + \sum_{i=1}^{n}(\bar{x}_n - a)^2 \\
&= \sum_{i=1}^{n} (x_i - \bar{x}_n)^2 + 2(\bar{x}_n - a)\sum_{i=1}^{n} (x_i - \bar{x}_n) + n(\bar{x}_n - a)^2 \\
&= \sum_{i=1}^{n} (x_i - \bar{x}_n)^2 + 2(\bar{x}_n - a)\cancel{\sum_{i=1}^{n} (x_i - \bar{x}_n)} + n(\bar{x}_n - a)^2 \\
&= \sum_{i=1}^{n} (x_i - \bar{x}_n)^2 + n(\bar{x}_n - a)^2
\end{aligned}$$

References

https://www.math.arizona.edu/~jwatkins/N_unbiased.pdf