

Exercise 2

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Determine whether S_n is an unbiased estimator of σ . In case it is not an unbiased estimator, which one is larger $E[S_n]$ or σ ?

Solution

The standard deviation of a sample is defined as :

$$S_n = \sqrt{\sum_{i=1}^n \frac{(X_i - \bar{X}_n)^2}{n-1}},$$

Considering $\text{Var}[S_n] \neq 0$, we take:

$$0 < \text{Var}[S_n]$$

We can rewrite the variance as:

$$E[S_n^2] - E^2[S_n]$$

So:

$$E^2[S_n] < E[S_n^2]$$

Taking the square root in both sides:

$$E[S_n] < \sqrt{E[S_n^2]} = \sigma$$

Therefore is a biased estimator

References

https://www.math.arizona.edu/~jwatkins/N_unbiased.pdf