Exercise: 1

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1. Let X and Y be random variables. Shows that:

a) E [a + bX] = a + bE[X], a, b € R

Solution:

We know that [1]

E[X] = $\sum Xi$þi

Now,

X: R.V.

E [a + bX] = ∑ (a + bXi) * þi

you need a different notation for realization of R.V.

=> E [a + bX] = ∑a*þi + ∑ bXiþi

=> E [a + bX] = a * þi + b * ∑Xiþi

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=> E [a + bX] = a + bE[X] [Since ∑þi is sum of total probability 1]

Reference:

[1] https://www.quora.com/Prove-that-E-aX-b-aE-X-b

b) Var(X) – E[X^2] – (E[X])^2

Solution:

Var(X) = E [(X – E[X])^2]

= E [X^2 – 2.X.E[X] + E(X)^2]

= E[X^2] + E [-2. X. E[X]] + E[E(X)^2]

= E[X^2] – 2. E[X]. E[X] + E[X]^2. E [1]

= E[X^2] – 2. E[X]^2 + E[X]^2

= E[X^2] – E[X]^2

c) Var (a + bX) = b^2. Var(X); a, b € R

Solution:

Var (a + bX) = E [((a + bX) - E [((a + bX)])^2]

= E [(a + bX – (E(a) + E(bX)))^2]

= E [(a + bX – (a + b. E(X))^2]

= E [(a + bX – a – b. E(X))^2]

= E [(bX – b. E(X))^2]

= E [b^2. (X – E(X))^2]

= b^2. Var(X)

d) Var(a) = 0; a € R

Solution:

We know E(a) = a.

Now, Var(a) = E [(a – E(a))^2]

= E [(a – a)^2]

= E (0)

= 0 [E (0) = $\sum_{x=0} x. f_X x = \underline{0.1} = 0$]

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