(3) $$f(t) = \begin{cases} \frac{1}{2\theta\sqrt{t}} \exp\left(-\frac{\sqrt{t}}{\theta}\right) & t > 0 \\ 0 & t \leq 0 \end{cases}$$

(a) $$L(\theta, t) = \prod_{i=1}^{n} \frac{1}{2\theta\sqrt{t_i}} \exp\left(\frac{-1}{\theta}\sqrt{t_i}\right)$$

$$= \frac{1}{(2\theta)^n}\left(\prod_{i=1}^{n} \frac{1}{\sqrt{t_i}}\right) \exp -\frac{1}{\theta}\sum_{i=1}^{n}\sqrt{t_i}$$

$$\ln L(\theta, t) = -n\ln(2\theta) - \frac{1}{2}\sum \ln(t_i) - \frac{1}{\theta}\sum_{i=1}^{n}\sqrt{t_i}$$

$$\frac{\partial \ln L(\theta, t)}{\partial\theta} = \frac{-2n}{8\theta} + \frac{1}{\theta^2}\sum_{i=1}^{n}\sqrt{t_i} = 0$$

$$\Rightarrow \hat{\theta} = \frac{1}{n}\sum_{i=1}^{n}\sqrt{t_i}$$

$$\frac{\partial^2 \ln L(\theta, t)}{\partial\theta^2} = \frac{n}{\theta^2} - \frac{2}{\theta^3}\sum\sqrt{t_i}$$

easy to check $\frac{\partial^2 \ln L(\hat{\theta}, t)}{\partial\theta^2} < 0$ for $n > 0$

$$\hat{\theta}_{MLE} = 84.733$$

(b) $E(T) = 2\Theta^2$, hence

$$\Theta = \sqrt{\frac{1}{2} E(T)}$$

Using the method of moments, we obtain

$$\hat{\Theta}_{moment} = \sqrt{\frac{1}{2n} \sum_{i=1}^{n} t_i}$$

which, for this particular case is

$$\hat{\Theta}_{moment} = 60.828$$