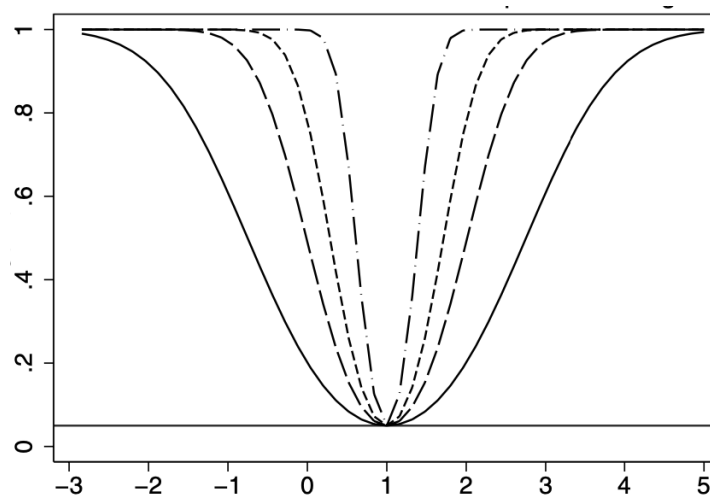


6. Problem sheet for **Statistical Data Analysis**

**Exercise 1** (8 Points)

Let  $X_i \sim \mathcal{N}(\theta, \sigma)$  be iid random variable where  $\sigma^2 = 4$ . Consider the problem  $H_0 : \theta = 1$  versus  $H_1 : \mu \neq 1$ .

1. Test if  $H_0$  is accepted or rejected for  $\alpha = 0.05$  for observed  $\bar{x} = 1.5$  with  $n = 15$ .
2. In the figure below the associated power functions for different number of samples are given. Determine which power function is associated with which sample sizes:  $n_1 = 100$ ,  $n_2 = 15$ ,  $n_3 = 5$  and  $n_4 = 30$ . Provide reasoning for each of your assignments.
3. Determine a confidence interval for  $\theta$  for the error probability  $\alpha = 0.05$  if the empirical mean  $\bar{x} = 1.5$  was calculated for a sample of size  $n = 15$ .
4. What effect does an increase in the sample size have on the confidence interval calculated in (c), assuming that the empirical mean  $\bar{x}$  and the variance  $\sigma^2$  remain unchanged?



**Exercise 2** (8 Points)

As part of a large-scale study on *Women and Pregnancy*, the age of women at the birth of their first child is of interest. It is assumed that the average age of first-time mothers is over 25 years. To test this hypothesis, 49 mothers are randomly selected and asked about their age at the birth of their first child. The mean age was  $\bar{x} = 26$ .

- check the statistical null hypothesis  $H_0 : \theta \leq 25$  against the alternative  $H_1 : \theta > 25$  at level  $\alpha = 0.05$ . Assume that the age of primiparous women is normally distributed. The variance with  $\sigma^2 = 9$  is known from experience. Interpret your result.
- How is the first type of error defined and what does it mean here?
- Determine the probability of making a type 2 error under the Assumption that  $\theta = 27$  is the true age of primiparae.
- Determine a 95% confidence interval for the age of the primiparae.