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10. Problem sheet for **Statistical Data Analysis**

Exercise 1 (8 Points)
Let $A = U\Sigma V^t$ be the singular value decomposition of $A \in \mathbb{R}^{m\times n}$ with $m < n$ and singular values $\sigma_1 \geq \cdots \geq \sigma_p$ für $p = \min(m, n)$. Let $u_1, \ldots, u_m$ and $v_1, \ldots, v_n$ denote the columns of U and V respectively. Show that

$$\mathbf{Ker}(A) = \langle v_{r+1}, \ldots, v_n \rangle \text{ and } \mathbf{Im}(A) = \langle u_1, \ldots, u_r \rangle.$$

Exercise 2 (8 Points)
Determine the SVD of the matrix

$$A = \begin{bmatrix} -1 & -1 \\ 2 & 2 \\ -1 & 1 \end{bmatrix} \tag{1}$$

without the help of a computer and write down all the steps of the calculation.