Exercise 2:

$$\text{Given. } \beta^*_N = \left(\sum_{i=1}^{N} x_i^2 \right)^{-1} \left(\sum_{i=1}^{N} x_i y_i \right)$$

$$= \frac{\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2}$$

If a new data point (x_{N+1}, y_{N+1}) will add then.

$$\beta^*_{N+1} = \frac{\sum_{i=1}^{N} x_i y_i + x_{N+1} y_{N+1}}{\sum_{i=1}^{N} x_i^2 + x_{N+1}^2} \quad \left[\text{add } (N+1)\text{th term} \right]$$

$$= \frac{\dfrac{\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2} \sum_{i=1}^{N} x_i^2 + x_{N+1} y_{N+1}}{\sum_{i=1}^{N} x_i^2 + x_{N+1}^2}$$

$$= \frac{\beta^*_N \sum_{i=1}^{N} x_i^2 + x_{N+1} y_{N+1}}{\sum_{i=1}^{N} x_i^2 + x_{N+1}^2}$$