A mathematician applies for a job at a bank. She/he has to complete an aptitude test consisting of $n = 8$ multiple-choice questions. For each the questions the mathematician has to choose between *4* possible answers and only one is correct.

- The bank wants to strongly avoid the unfavorable scenario of hiring an incompetent person while they can live with not hiring a competent person. Write up a pair of suitable hypotheses.
- HR has decided that a mathematician is going to be hired, if she/he answers at least *6* questions correctly. Determine the error of type *1* i.e., H_0 is rejected despite it being true.
- Construct a test with sensitivity $\alpha = 0.05$. The mathematician answered *4* questions correctly. Is he going to be hired under this set-up?
- If the mathematician answers questions correctly with a probability of *0.85*. How large is the probability of the above test for an error of type *2* (H_0 is not rejected but is not true)?

Solution 1:

H0: Person is competent.

H1: Person is not competent.

Here, H0 is the null hypothesis, and H1 is the alternative hypothesis. The rejection of the null hypothesis will show that if person is not competent, the bank doesn't hire that person. This implies that some other hypothesis is accepted. This other hypothesis is called the alternative hypothesis.

Solution 2:

Action	**State of nature**	
	w1, Answers at least 6 questions correctly	**w2, Not answers at least 6 questions correctly**
H1, Person is competent	Correct decision	Incorrect decision, Type II (**β**) error
H2, Person is not competent	Incorrect decision, Type I (**α**) error	Correct decision

Type 1 error for this case, the bank failed to hired the correct mathematician. The person answers at least 6 questions correctly but because of error it shows that the person is not competent.

Solution 3:

Total question, n = 8.

Correct question = 4.

Sensitivity α = 0.05

Probability, $p = \frac{correct\ answer}{total\ answer} = \frac{4}{8} = 0.5$

Now, $X \sim Bin(n, p) \sim Bin(8, 0.5)$

H0 rejected if $\mathbb{P}(X \geq 6) < 0.05$

$$\mathbb{P}(X \geq 6) = \mathbb{P}(X = 6) + \mathbb{P}(X = 7) + \mathbb{P}(X = 8)$$

$$= \binom{8}{6} * 0.5^6 * 0.5^2 + \binom{8}{7} * 0.5^7 * 0.5^1 + \binom{8}{8} * 0.5^8 * 0.5^0$$

$$= 0.1094 + 0.03125 + 0.0039 = 0.14455$$

The P value of 0.14455 is statistically significant at an alpha level of 0.05. So, he is going to be hired under the given set-up.

Solution 4:

Take, Z is the number of questions a mathematician answer correctly.

Total question, n = 8.

Sensitivity α = 0.05

Probability, $p = 0.85$

Now, $X \sim Bin(n, p) \sim Bin(8, 0.85)$

$$\mathbb{P}(X = Z) = \mathbb{P}(X = 1) = \binom{8}{1} * 0.85^1 * 0.15^7 = 1.162 * 10^{-5}$$

$$\mathbb{P}(X = Z) = \mathbb{P}(X = 2) = \binom{8}{2} * 0.85^2 * 0.15^6 = 2.304 * 10^{-4}$$

$$\mathbb{P}(X = Z) = \mathbb{P}(X = 3) = \binom{8}{3} * 0.85^3 * 0.15^5 = 2.612 * 10^{-3}$$

$$\mathbb{P}(X = Z) = \mathbb{P}(X = 4) = \binom{8}{4} * 0.85^4 * 0.15^4 = 0.0185$$

$$\mathbb{P}(X = Z) = \mathbb{P}(X = 5) = \binom{8}{5} * 0.85^5 * 0.15^3 = 0.0839$$

$$\mathbb{P}(X = Z) = \mathbb{P}(X = 6) = \binom{8}{6} * 0.85^6 * 0.15^2 = 0.2376$$

$$\mathbb{P}(X = Z) = \mathbb{P}(X = 7) = \binom{8}{7} * 0.85^7 * 0.15^1 = 0.3847$$

$$\mathbb{P}(X = Z) = \mathbb{P}(X = 8) = \binom{8}{8} * 0.85^8 * 0.15^0 = 0.2725$$

Now,

$$\mathbb{P}(X \geq 1) = 1.0000$$

$$\mathbb{P}(X \geq 2) = 0.9999$$

$$\mathbb{P}(X \geq 3) = 0.9998$$

$$\mathbb{P}(X \geq 4) = 0.9972$$

$$\mathbb{P}(X \geq 5) = 0.9787$$

$$\mathbb{P}(X \geq 6) = 0.8948$$

$$\mathbb{P}(X \geq 7) = 0.6572$$

$$\mathbb{P}(X \geq 8) = 0.2725$$

We search for the Z value where $\mathbb{P}(X \geq Z \mid 8, 0.85) \geq 0.95$. and the value of Z is 5. Because, when $X \leq 4$, H_0 is rejected and when $X \geq 5$, H_0 is not rejected.

And the large probability of the test for an error of type 2 is:

$$\mathbb{P}(X = 5| \, 8, 0.85) = \binom{8}{5} * 0.85^5 * 0.15^3 = 0.0839$$