Problem Sheet 01

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Exercise 1

Let X and Y be random variables. Show that

1. $E(a+bX) = a + bE(X)$

$$\begin{aligned}
E(a+bX) &= \Sigma_x(a+bx)p(x) && \text{by definition}\\
&= \Sigma_x(ap(x)+bxp(x)) &&\\
&= \Sigma_x(ap(x)) + \Sigma_x(bxp(x)) && \text{using properties of sums}\\
&= a\Sigma_x(p(x)) + b\Sigma_x(xp(x)) && \text{pulling constants out of the sum}\\
&= a\cdot 1 + bE(X) && \text{the probabilities have to sum up to 1}\\
& && \text{and } \Sigma_x(xp(x)) \text{ is } E(X) \text{ by definition}
\end{aligned}$$

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2. $\operatorname{Var}(X) = E(X^2) - E(X)^2$

$$\begin{aligned}
\operatorname{Var}(X) &= E[(X-E(X))^2] && \text{by definition}\\
&= E[(X^2 + E(X)^2 - 2XE(X)] && \text{binomial formula}\\
&= E(X^2) + E(E(X)^2) - E(2XE(X)) && \text{Linearity of } E\\
&= E(X^2) + E(X)^2 - 2E(X)E(E(X)) && E \text{ of const. is const. and Linearity of } E\\
&= E(X^2) + E(X)^2 - 2E(X)E(X) && E \text{ of const. is const.}\\
&= E(X^2) + E(X)^2 - 2E(X)^2 &&\\
&= E(X^2) - E(X)^2 &&
\end{aligned}$$

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3. $\operatorname{Var}(a+bX) = b^2 \operatorname{Var}(X)$

$$\begin{aligned}
\operatorname{Var}(a+bX) &= E((a+bX)^2) - (E(a+bX))^2 && \text{by definition}\\
&= E(a^2 + 2abX + b^2X^2) - (E(a) + bE(X))^2 && \text{Binomial and Linearity of } E\\
&= E(a^2) + 2abE(X) + b^2E(X^2) - (a+bE(X))^2 &&\\
&= \cancel{a^2} + \cancel{2abE(X)} + b^2E(X^2) - \cancel{a^2} - \cancel{2abE(X)} - b^2E(X)^2 && \text{Binomial}\\
&= b^2E(X^2) - b^2E(X)^2 &&\\
&= b^2(E(X^2) - E(X)^2) &&\\
&= b^2 \operatorname{Var}(X) && \text{proven above in Exercise 1.2}
\end{aligned}$$

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4. $\mathrm{Var}(a) = 0$

$$\begin{aligned}
\mathrm{Var}(a) &= E((a)^2) - (E(a))^2 && \text{proven above in Exercise 1.2} \\
&= a^2 - a^2 && \text{expected value of a constant is the constant itself} \\
&= 0
\end{aligned}$$

$\blacksquare$