

Exercise: 1

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1. Let X and Y be random variables. Shows that:

a) $E[a + bX] = a + bE[X]$, $a, b \in \mathbb{R}$

Solution:

We know that [1]

$$E[X] = \sum X_i p_i$$

Now,

$$E[a + bX] = \sum (a + bX_i) * p_i$$

$$\Rightarrow E[a + bX] = \sum a * p_i + \sum bX_i p_i$$

$$\Rightarrow E[a + bX] = a * \sum p_i + b * \sum X_i p_i$$

$$\Rightarrow E[a + bX] = a + bE[X] \text{ [Since } \sum p_i \text{ is sum of total probability 1]}$$

Reference:

[1] <https://www.quora.com/Prove-that-E-aX-b-aE-X-b>

b) $\text{Var}(X) = E[X^2] - (E[X])^2$

Solution:

$$\text{Var}(X) = E[(X - E[X])^2]$$

$$= E[X^2 - 2.X.E[X] + E(X)^2]$$

$$= E[X^2] + E[-2.X.E[X]] + E[E(X)^2]$$

$$= E[X^2] - 2.E[X].E[X] + E[X]^2.E[1]$$

$$= E[X^2] - 2.E[X]^2 + E[X]^2$$

$$= E[X^2] - E[X]^2$$

c) $\text{Var}(a + bX) = b^2 \cdot \text{Var}(X)$; $a, b \in \mathbb{R}$

Solution:

$$\begin{aligned}\text{Var}(a + bX) &= E [((a + bX) - E [(a + bX)])^2] \\&= E [(a + bX - (E(a) + E(bX)))^2] \\&= E [(a + bX - (a + b \cdot E(X)))^2] \\&= E [(a + bX - a - b \cdot E(X))^2] \\&= E [(bX - b \cdot E(X))^2] \\&= E [b^2 \cdot (X - E(X))^2] \\&= b^2 \cdot \text{Var}(X)\end{aligned}$$

d) $\text{Var}(a) = 0$; $a \in \mathbb{R}$

Solution:

We know $E(a) = a$.

$$\begin{aligned}\text{Now, } \text{Var}(a) &= E [(a - E(a))^2] \\&= E [(a - a)^2] \\&= E (0) \\&= 0 [E (0) = \sum_{x=0} x \cdot f_X x = 0 \cdot 1 = 0]\end{aligned}$$