Ex-1

data points R^2

$$d_1 = \begin{pmatrix} 5 \\ 10 \end{pmatrix}, \quad d_2 = \begin{pmatrix} 10 \\ -5 \end{pmatrix}, \quad d_3 = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$$

via dimension reduction techniques identified that the subspace $U = \left\langle \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\rangle$ is sufficient to represent the data

a. Define orthogonal projection that can be used to project data points onto U

b. project 3 data points onto U using orthogonal projection

c. Consider first components of projected data points to be the x values and the second component of data points to be values in linear regression. Estimate associated regression parameter β_1 (note β_0 is assumed to be 0) corrosponding to the three projected data points.

Ex-4. Based on theory about the inheritance of intelligence one expects that the intelligence quotient (IQ) of a randomly chosen person is larger than 100 (on average). This leads to the following statistical test problem.

$$H_0 : \mu \leq 100 \text{ versus } H_1 : \mu > 100$$

the IQ is assumed to be normally distributed with standard deviation $\sigma = 15$. Significance level is set $\alpha = 0.1$ (Hint: $z_{0.9} \approx 1.28$). You observe an average IQ of 104 in your sample set. on the basis of this sample set, would you reject H_0?

Ex-3

consider Stochastic multi-armed bandits problem with $k=4$ Bernoulli arms and with potential reward in each epoch for each arm with mean probabilities $\mu_1 = 0.3, \mu_2 = 0.15, \mu_3 = 0.55, \mu_4 = 0.7$

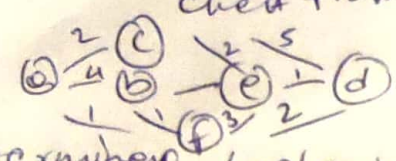
epoch i	chosen arm in epoch i	reward arm 1	reward arm 2	" 3	" 4
1	1	1	0	1	1
2	2	1	0	0	1
3	3	0	1	1	1
4	4	0	0	1	1
5	1	0	0	0	1
6	1	0	1	1	1
7	4	0	0	1	0
8	3	1	0	0	1
9	3	0	0	1	0
10	2	0	0	1	1

a. Compute the regret if every arm is chosen exactly T/k times (assume T is multiple of k) for the given Bernoulli arms (see μ_i values)

b. using collected info from chosen arm up till epoch 10 compute the current empirical estimate $\hat{\mu}_i$.

c. considering UCB1 algorithm, which arm would be selected next on basis of given info. (Hint: work with logarithm to basis 10 and use following approx: $\sqrt{1/2} \approx 0.7071$, $\sqrt{1/3} \approx 0.5774$, $\sqrt{1/4} \approx 0.5$, what do you expect happen when algorithm is used for longer time horizon.

Ex-5



a. determine Laplacian matrix (L_G) for given graph G

b. let $A_1 = \{a, b\}$, $A_2 = \{c, d\}$, $A_3 = \{e, f\}$
Calculate $Ncut(A_1, A_2, A_3)$ and $Ratiocut(A_1, A_2, A_3)$

c. comment on the choice of A_1, A_2, A_3 with respect to graph cutting considering the associate $Ncut(A_1, A_2, A_3)$ and $Ratiocut(A_1, A_2, A_3)$ values

Ex-6 Emma collects figures from suprise eggs, yet there is not a figure in every egg. Her grandma buys her n = 10 surprise eggs every week. She writes down how many of n eggs had a figure in it

week	1	2	3	4	5	6	7	8	9	10
number of figures found in 10 eggs	4	4	3	6	5	7	6	6	6	8

Assume that the samples generated by drawing from iid random variables $x_i \sim Binomial(n, \theta)$, i.e. samples are distributed according to the binomial distribution with parameter θ

a. what's the likelihood with respect to the underlying model assumption for parameter θ.

b. Derive the associated MLE.

c. Calculate MLE of θ with respect to sample set given in the table

d. Show MLE of θ is unbiased

Consider the Grid world game with state space $S = \{(1,1) \dots (4,3)\} \setminus \{(2,2)\}$ and action space $A = \{\uparrow, \rightarrow, \downarrow, \leftarrow\} =$

$\{(0,1), (1,0), (0,-1), (-1,0)\}$ while the reward is defined via

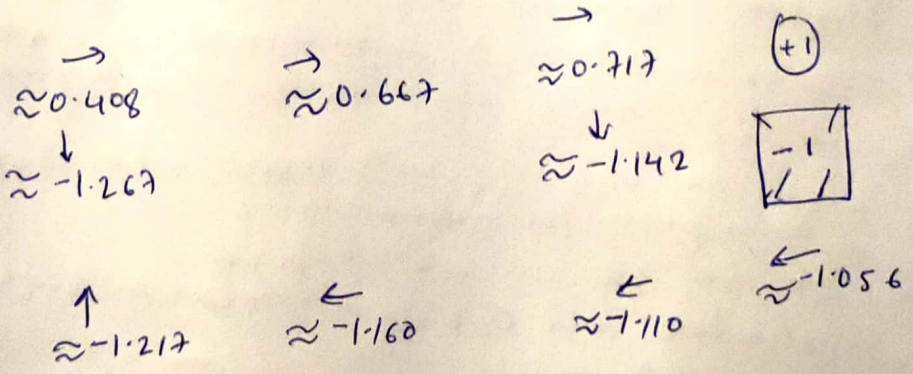
$$R(s,a) = \begin{cases} -0.04 & \text{each step} \\ -1 & \text{on } \times \\ +1 & \text{on } \circ \end{cases}$$

Here we want to evaluate the strategy μ (given by arrows in figure below) by approximating the associated value function $V(s)$ iteratively. The values of current iteration are shown in the grid in the figure below.

a. compute $(j+1)$-st iteration $\hat{V}_{j+1}(s)$ for $s = (3,2)$ and $s = (1,2)$

b. How can the values be interpreted and what do they tell about the given policy μ?

c. Does this iterative approach converge to the true value and if yes why?



Ex-2 Hint

$$A(a,s) = \begin{cases} s, & s'_* \in S \\ s & s' \in S \end{cases}$$

$$P^a_{ss'} = \begin{cases} 0.8 \\ 0.1 \\ 0.1 \end{cases} \qquad s'_* = s + a \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$s'_* = s + a$$

$$s' = s + a \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$