~~we~~

1) The likelihood function:

$$L(\theta) = \prod_i f(y_i|\theta) = \prod_i \theta^{y_i} \ast (1-\theta)^{1-y_i}$$

$$= \theta^{\Sigma y_i} \ast (1-\theta)^{(n-\Sigma y_i)}$$

$$= \theta^t \ast (1-\theta)^{n-t},$$

here $t = \Sigma y_i$

We need to test $H_0: \theta = 0.52$ versus $H_1: \theta = 0.48$. Let.

$$L(\theta) = \begin{cases} 0.52^t \ast (1-0.52)^{n-t} & \text{if } \theta = 0.52 \\ 0.48^t \ast (1-0.48)^{n-t} & \text{if } \theta = 0.48 \end{cases}$$

$$L_1 = L(0.52) = 0.52^t + (1-0.52)^{n-t}$$

and

$$L_2 = L(0.48) = 0.48^t \ast (1-0.48)^{n-t}$$

Thus we have:

$$\frac{L_1}{L_2} = \frac{0.52^t}{0.48^t} \ast \frac{(1-0.52)^{n-t}}{(1-0.48)^{n-t}}$$

$$= \left(\frac{13}{12}\right)^t \ast \left(\frac{12}{13}\right)^{n-t}$$

The likelihood ratio test ratio is:

$$\lambda = \frac{L_1}{\max(L_1, L_2)}$$

Note that, if $\max(L_1, L_2) = L_1$, then $\lambda = 1$. Because we want to reject for small values of λ, $\max(L_1, L_2) = L_2$, and we reject H_0 if $(L_1/L_2) \leq k$ or $(L_2/k_1) > k$.

(note that $\frac{L_2}{L_1} = \left(\frac{12}{13}\right)^t * \left(\frac{13}{12}\right)^{n-t}$)

That is reject H_0 if,

$$\left(\frac{12}{13}\right)^t \left(\frac{13}{12}\right)^{n-t} > k$$

$$\Leftrightarrow \left(\frac{12/13}{13/12}\right)^t > k_1$$

$$\Leftrightarrow \left(\frac{144}{169}\right)^t > k_1.$$

Hence reject H_0 if $T > C$; $P(T > C \mid H_0 : \theta = 0.52) \leq 0.52$

Here, C = Critical Region.

* If we want α to be exactly 0.52, we have to use randomized test.

The actual rejection region depends on the specific value of α.

2)

	H_0 True H_1 False	H_0 False H_1 True
Reject H_0	Type I error $P = \alpha$	Correct Rejection $P = 1 - \beta$
Retain H_0	Correct Retention $P = 1 - \alpha$	Type II error $P = \beta$

we have to determine $P[H_0 \text{ rejected} \mid H_1 \text{ true}]$ or $1 - \beta$.

In this problem we don't have n.

So, First we find the rejection region of the form $\{X \leq K\}$ so that $\alpha = 0.1$.

To find K such that.

$$\alpha = P\{X \leq K \mid P = 0.52\} = 0.1.$$

from here we will get a value of K, that will reject H_0.

Now,

$$\beta = P\{\text{accept } H_0 \mid H_1 \text{ true}\}$$
$$= P\{X > K \mid P = 0.48\}$$
$$= 1 - P\{X \leq K \mid P = 0.48\}$$

from this we also get a value. suppose we take this value b.

then, $P[H_0 \text{ rejected} \mid H_1 \text{ true}] = 1 - b$