Institut für Mathematik
Abteilung Numerik
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1. Problem sheet for **Statistical Data Analysis**

Exercise 1 (2+2+2+2 Points)

Let X and Y be random variables. Show that

1. $\mathbb{E}[a + bX] = a + b\mathbb{E}[X], \quad a, b \in \mathbb{R}$,
2. $\mathrm{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X]))^2$,
3. $\mathrm{Var}(a + bX) = b^2 \mathrm{Var}(X), \quad a, b \in \mathbb{R}$,
4. $\mathrm{Var}(a) = 0, \quad a \in \mathbb{R}$.

Exercise 2 (2+2 Points)
Let $X_1, \ldots, X_n$ be independent and identical random variables with $\mathbb{E}[X_i] = \mu$ and $Var[X_i] = \sigma^2$ and define the empirical variance

$$S_n^2 := \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X}_n)^2 \tag{1}$$

Show

- that for estimator S_n^2 the following equivalence holds true

$$S_n^2 = \frac{1}{n-1} (\sum_{i=1}^{n} X_i^2 - n\bar{X}_n^2) \tag{2}$$

- that estimator S_n^2 is an unbiased estimator of the variance

$$\mathbb{E}[S_n^2] = \sigma^2. \tag{3}$$

Exercise 3 (4+5+3 Points)
Plot

1. the probability of a random variable that follows the Binomial distribution Bin(n,p) for different $p \in \{0.3, 0.5, 0.8\}$ and $n \in \{10, 50\}$
2. the probability of a random variable that follows the Geometric distribution Geom(p) and the corresponding cumulative distribution function F for different $p \in \{0.3, 0.5, 0.8\}$ for all $x \leq 11$
3. the probability of a random variable that follows the poisson distribution for different $\lambda \in \{0.3, 2, 6\}$ for $x \leq 16$

in python.

Due on the 30.10.2022 23:59