Qusetion:

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}$$

1) find λ_1, λ_2 of $A^T A$ and their eigenvectors.

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B = A^T A = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\det(B - \lambda I) = \begin{vmatrix} 5-\lambda & 2 \\ 2 & 2-\lambda \end{vmatrix}$$

$$= 10 - 2\lambda - 5\lambda + \lambda^2 - 4$$

$$= \lambda^2 - 7\lambda + 6$$

$$= (\lambda - 6)(\lambda - 1)$$

$$\lambda_1 = 6 \ \& \ \lambda_2 = 1$$

for $\lambda_1 = 6$

$$\begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} \overset{R_1 \times 2}{=} \begin{bmatrix} -2 & 4 \\ 2 & -4 \end{bmatrix} \overset{R_2 \to R_1 - R_2}{\simeq} \begin{bmatrix} -2 & 4 \\ 0 & 0 \end{bmatrix}$$

$$\overset{R_1 \to R_1/2}{\simeq} \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$-1x_1 + 2x_2 = 0$$

$$\Rightarrow x_1 = 2x_2$$

$$x_2 = 1 \ \& \ x_1 = 2 \qquad \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

For $\lambda_1 = 1$

$$\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \underset{R_1 \to R_1/2}{\overset{R_2 \to R_1 - 2R_2}{\approx}} \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$$

$2x_1 + x_2 = 0$ $\qquad x_2 = 1$

$\Rightarrow 2x_1 = -x_2$ $\qquad x_1 = -\frac{1}{2}$

$$\begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix}$$

2) Find S, U, VT of SVD of $A = USV^T$. last

column $= \frac{1}{\sqrt{6}} \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$

$$v_1 = \frac{1}{\sqrt{2^2+1^2}} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$v_2 = \frac{1}{\sqrt{(-\frac{1}{2})^2 + 1^2}} \begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix} = \frac{2}{\sqrt{5}} \begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix}$$

$$\boxed{U = \frac{1}{\sqrt{\lambda_1}} A v_1} \qquad V = \begin{bmatrix} 2/\sqrt{5} & -\frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$$

$$U_1^* = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix} \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{30}} \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$$

$$U_2^* = \frac{1}{\sqrt{1}} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix} \frac{2}{\sqrt{5}} \begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix} = \frac{2}{\sqrt{5}} \begin{pmatrix} -0.5 \\ 0 \\ 1 \end{pmatrix}$$

$$U_3^* = \frac{1}{\sqrt{6}} \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$$

$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$U_3^* = e_1 - \langle e_1 \cdot U_1 \rangle U_1 - \langle e_1 \cdot U_2 \rangle U_2$

$= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \left(\frac{2}{\sqrt{30}}\right) \frac{1}{\sqrt{30}} \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} + \left(\frac{1}{\sqrt{5}}\right) \frac{2}{\sqrt{5}} \begin{pmatrix} -0.5 \\ 0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2/15 \\ 1/3 \\ 1/15 \end{pmatrix} + \begin{pmatrix} -1/5 \\ 0 \\ 2/5 \end{pmatrix}$

$$\Sigma = \begin{bmatrix} \sqrt{6} & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{pmatrix} 2/3 \\ -1/3 \\ 1/3 \end{pmatrix} \qquad v_3 = \begin{pmatrix} \frac{2/3}{\sqrt{6}/3} \\ \frac{-1/3}{\sqrt{6}/3} \\ \frac{1/3}{\sqrt{6}/3} \end{pmatrix} = \begin{pmatrix} 2/\sqrt{6} \\ -1/\sqrt{6} \\ 1/\sqrt{6} \end{pmatrix}$$

$$\sqrt{\left(\tfrac{2}{3}\right)^2 + \left(-\tfrac{1}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2} = \sqrt{6}/3$$

Now, $U\Sigma V^T = \begin{bmatrix} \frac{2}{\sqrt{30}} & \frac{-1}{\sqrt{5}} & \frac{-2}{\sqrt{6}} \\ \frac{5}{\sqrt{30}} & 0 & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{30}} & \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \sqrt{6} & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix} = A.$$

3. Pseudo code of A using SVD of A.

$$V\Sigma^{+}U^{*}$$

$$\begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \sqrt{6} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{30}} & \frac{5}{\sqrt{30}} & \frac{1}{\sqrt{30}} \\ -\frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} = A^T.$$

*Extra: Find Singular value of

$$A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$$

$$B = A^T A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 25 & 20 \\ 20 & 25 \end{bmatrix}$$

$$\det(B - \lambda I) = \begin{vmatrix} 25-\lambda & 20 \\ 20 & 25-\lambda \end{vmatrix}$$

$$= (25-\lambda)^2 - 400$$

$$= 625 - 50\lambda + \lambda^2 - 400$$

$$= \lambda^2 - 50\lambda + 225$$

$$= (\lambda - 5)(\lambda - 45)$$

So, $\lambda_1 = 5$ & $\lambda_2 = 45$

$\therefore \sigma_1 = \sqrt{5}$ and $\sigma_2 = \sqrt{45}$

$$\boxed{\begin{array}{l} tr(B) = \lambda_1 + \lambda_2 \\ \text{and } \det(B) = \lambda_1 \lambda_2 \end{array}}$$

Question:

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\det(C - \lambda I) = \begin{bmatrix} 0-\lambda & 1 & 0 \\ 1 & -1-\lambda & 1 \\ 0 & 1 & 0-\lambda \end{bmatrix}$$

$$= -\lambda\left((-1-\lambda)(-\lambda) - 1\right) - 1(-\lambda - 0) + 0$$

$$= -\lambda^3 - \lambda^2 + 2\lambda$$

$$= -\lambda(\lambda^2 - \lambda + 2)$$

$$= -\lambda(\lambda+2)(\lambda-1)$$

So, $\lambda_1 = 0$ $\lambda_2 = -2$ & $\lambda_3 = 1$

For, $\lambda_1 = 0$,

$$\cdots \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_1 + x_3 = 0 \Rightarrow x_1 = -x_3$$
$$x_2 = 0$$

$$x_3 = 1 \quad x_1 = -1$$

$$\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$\lambda_2 = -2$

$$\cdots \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_3 = 0 \Rightarrow x_1 = x_3$$
$$x_2 + 2x_3 = 0$$
$$\Rightarrow x_2 = -2x_3$$

$x_3 = 1, \quad x_1 = 1, \quad x_2 = -2$

$$\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$\lambda_3 = 1,$

$$\cdots \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_3 = 0 \Rightarrow x_1 = x_3$$

$$x_2 - x_3 = 0 \Rightarrow x_2 = x_3$$

$x_3 = 1, \quad x_1 = 1, \quad x_2 = 1$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\lambda_1 = 0, \quad \lambda_2 = -2, \quad \lambda_3 = 1.$

$$\frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} = 0, \quad \frac{-2}{0 - 2 + 1} = 2, \quad \frac{1}{0 - 2 + 1} = -1$$

* দুইটা eigen vector এর dot always 0 হলে। তারা orthogonal হবে।

$$v_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \quad v_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$v_1 \cdot v_2 = -1 \times 1 + 0 \times 1 + 1 \times 1 = 0$$

Question,

x_i :	1	2	3	4	5
y_i :	-4.2947	-2.3880	-1.0445	-1.1596	-0.8999

$$\hat{\beta} = \underbrace{(X^T X)^{-1}}\, X^T y$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 15 \\ 15 & 55 \end{bmatrix}$$

$$(X^T X)^{-1} = \frac{1}{50} \begin{bmatrix} 55 & -15 \\ -15 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1.1 & -0.3 \\ -0.3 & 0.1 \end{bmatrix}$$

$$\hat{\beta} = \begin{bmatrix} 1.1 & -0.3 \\ -0.3 & 0.1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} -4.2947 \\ -2.3880 \\ -1.0445 \\ -1.1596 \\ -0.8999 \end{bmatrix}$$

$$= \begin{bmatrix} -4.36274 \\ 0.8018 \end{bmatrix}$$

$$\beta_0 = -4.36274 \;\&\; \beta_1 = 0.8018$$

$$\boxed{\hat{\beta}_{ridge} = (X^T X + \lambda I_p)^{-1} X^T y}$$

$$X^T X + \lambda I_p = \begin{pmatrix} 5 & 15 \\ 15 & 55 \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 15 \\ 15 & 56 \end{bmatrix}$$

$$(X^T X + \lambda I_p)^{-1} = \begin{bmatrix} 0.5045 & -0.135 \\ -0.135 & 0.054 \end{bmatrix}$$

$$(X^TX + \lambda I_p)^{-1} X^T y = \begin{bmatrix} -2.05336667 \\ 0.1689 \end{bmatrix}$$

$$\sigma^2_{ML} = \frac{\hat{\varepsilon}\hat{\varepsilon}}{n} ; \quad \hat{\varepsilon} = y - x\hat{\beta}$$

$$\hat{\varepsilon} = \begin{bmatrix} -0.73376 \\ 0.37114 \\ 0.91284 \\ -0.00406 \\ -0.54616 \end{bmatrix}$$

$$\sigma^2_{ML} = 0.3615465$$

Sample $\underbrace{\text{standard errors}}_{\text{error of } y \text{ and } \hat{y}.}$ of residuals.

High error $\rightarrow$ sample mean widely spread around the population mean.

Low error $\rightarrow$ Sample mean closely distributed around the population mean.

$$\sigma^2_{ad} = \frac{\hat{\varepsilon}\hat{\varepsilon}}{n-p-1}$$

$n = 5$ (length of x or y)

$P = 1$ (original X এর column Number)

$$\sigma^2_{ad} = 0.602577577$$

$$R^2 = \frac{\sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} = \frac{\hat{\beta}^T X^T y - n\bar{y}^2}{y^T y - n\bar{y}^2}$$

$\hat{y}_i \rightarrow x\hat{\beta}$

$$R^2 = \frac{6.4288324}{8.236565} = 0.78052347$$

R^2 range is $[0, 1]$. Low when 0 and High when 1.

Mean square error:

$$\hat{y} = x\hat{\beta}$$

$$mse = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2$$

Emperical Bias, mean residual:

$$emp\_bias = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)$$

Emperical variance;

$$emp\_variance = \frac{1}{n-1}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2$$

Sample variance:

$$sam\_variance = \frac{1}{n-1}\sum_{i=1}^{n}\left(\hat{y}_i - \left[\frac{1}{n}\sum_{i=1}^{n}\hat{y}_i\right]\right)^2$$

l_1-norm — $\|x\|_1 = |x_1| + |x_2| + \cdots + |x_n|$

l_2-norm — $\|x\|_2 = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2} = \sqrt{x^T x}$

$$\|x\|_2^2 = x_1^2 + x_2^2 + \cdots + x_n^2 = x^T x$$

$$var(x) = \mathbb{E}[x^2] - \mathbb{E}^2[x]$$

$$\mathbb{E}[xy] = \mathbb{E}[x]\mathbb{E}[y] + cov(x,y)$$

$$var(x+y) = var(x) + var(y) + 2cov(x,y)$$

$$var(x-y) = var(x) + var(y) - 2cov(x,y)$$

$cov(x,y) = 0$, if x and y are independent.

Question:

$x^1 = (1,1)$, $x^2 = (3,1)$, $x^3 = (3,3)$, $x^4 = (4,2)$

$x^5 = (6,5)$, $x^6 = (5,6)$, $x^7 = (6,7)$, $x^8 = (4,7)$

Initial x^3, x^4, and x^6

	θ_1 $x^3 = (3,3)$	θ_2 $x^4 = (4,2)$	θ_3 $x^6 = (5,6)$	
$x^1 = (1,1)$	8	10	41	θ_1
$x^2 = (3,1)$	4	2	29	θ_2
$x^5 = (6,5)$	13	13	2	θ_3
$x^7 = (6,7)$	25	29	2	θ_3
$x^8 = (4,7)$	17	25	2	θ_3

$M_1 = \{x^1, x^3\}$

$M_2 = \{x^2, x^4\}$

$M_3 = \{x^5, x^6, x^7, x^8\}$

$$\theta_1 = \frac{1}{2}\left((1,1) + (3,3)\right) = (2,2)$$

$$\theta_2 = \frac{1}{2}\left((3,1) + (4,2)\right) = (3.5, 1.5)$$

$$\theta_3 = \frac{1}{4}\left((6,5) + (5,6) + (6,7) + (4,7)\right)$$

$$= (5.25, 6.25)$$

$$\text{Loss} = \sum_{k=1}^{n} \sum_{x^i \in M_k} d(x^{(i)}, \theta_k)$$

$$\begin{cases} d(x^1, \theta_1) = (1-2)^2 + (1-2)^2 = 2 \\ d(x^3, \theta_1) = (3-2)^2 + (3-2)^2 = 2 \end{cases}$$

$$\begin{cases} d(x^2, \theta_2) = (3-3.5)^2 + (1-1.5)^2 = 0.5 \\ d(x^4, \theta_2) = (4-3.5)^2 + (2-1.5)^2 = 0.5 \end{cases}$$

$$\begin{cases} d(x^5, \theta_3) = (6-5.25)^2 + (5-6.25)^2 = 2.125 \\ d(x^6, \theta_3) = (5-5.25)^2 + (6-6.25)^2 = 0.125 \\ d(x^7, \theta_3) = (6-5.25)^2 + (7-6.25)^2 = 1.125 \\ d(x^8, \theta_3) = (4-5.25)^2 + (7-6.25)^2 = 2.125 \end{cases}$$

$L = 10.5.$

	$\theta_1 = (2,2)$	$\theta_2 = (3.5, 1.5)$	$\theta_3 = (5.25, 6.25)$	
$x^1 = (1,1)$	2	6.5	45.625	θ_1
$x^2 = (3,1)$	2	0.5	32.625	θ_2
$x^3 = (3,3)$	2	2.5	15.625	θ_1
$x^4 = (4,2)$	4	0.5	19.625	θ_2
$x^5 = (6,5)$	25	18.5	2.125	θ_3
$x^6 = (5,6)$	25	22.5	0.125	θ_3
$x^7 = (6,7)$	41	36.5	1.125	θ_3
$x^8 = (4,7)$	29	30.5	2.125	θ_3

$M_1 = \{ x^1, x^3 \}$

$M_2 = \{ x^2, x^4 \}$

$M_3 = \{ x^5, x^6, x^7, x^8 \}$

remains unchanged

$L = 10.5$ unchanged

Question:

Geometric Distribution.

Samples:	1	2	3	4	5	6	7	8	9	10
trials success :	2	5	8	4	12	2	2	7	2	18

$$f(x, p) = (1-p)^{x-1} p$$

$$\boxed{L(p) = \prod_{i=1}^{n} (1-p)^{x_i - 1} p}$$

Take the log likelihood.

$$L(p) = \log(1-p) \sum_{i=1}^{n} (x_i - 1) + \sum_{i=1}^{n} \log p$$

Diff. w.r.to p

$$\frac{\partial L(p)}{\partial p} = -\frac{1}{1-p} \sum_{i=1}^{n} (x_i - 1) + \frac{n}{p} \stackrel{!}{=} 0$$

$$\Rightarrow -\frac{1}{1-p} \sum_{i=1}^{n} x_i + \frac{n}{1-p} + \frac{n}{p} = 0$$

$$\Rightarrow \frac{-p\sum_{i=1}^{n} x_i + \cancel{np} + n - \cancel{np}}{p(1-p)} = 0$$

$$\Rightarrow \boxed{\hat{p}_{MLE} = \frac{n}{\sum_{i=1}^{n} x_i}}$$

Here, $n = 10$

$$\sum_{i=1}^{n} x_i = 2+5+6+4+12+2+2+7+2+18$$
$$= 62$$

$$\hat{p}_{MLE} = \frac{10}{62} = 0.16129$$

$$\mathbb{E}[\hat{p}] = \mathbb{E}\left[\frac{n}{\sum_{i=1}^{n} x_i}\right]$$

$$= n \; \mathbb{E}\left[\frac{1}{\sum_{i=1}^{n} x_i}\right]$$

$$= n \cdot \frac{1}{\sum_{i=1}^{n} \mathbb{E}[x_i]}$$

$$= n \cdot \frac{1}{\sum_{i=1}^{n} 1/p} \quad \left[\because \text{ mean of Geometric distribution} = \frac{1}{p}\right]$$

$$= n \cdot \frac{1}{\frac{n}{p}}$$

$$= n \cdot \frac{p}{n}$$

$$= p$$

$$\therefore \mathbb{E}[\hat{p}] = p \quad \text{(Unbiased)}$$

Question.

When the measurement uncertainty is large and the estimate uncertainty is low, the Kalman gain is close to zero. Hence we give significant weight to the estimate and a small weight to the measurement.

When the measurement uncertainty is low and the estimate uncertainty is large, the Kalman Gain is close to one. Hence we give a low weight to the estimate and a significant weight to the measurement.

If the measurement uncertainty equals the estimate uncertainty, then the Kalman gain equals 0.5.

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \quad y = [0.5] \quad H = [0 \quad 1] \quad R = [0.1]$$

$$B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \quad m_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad P_0 = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$$

Forecast: $\hat{m}_{n+1} = A m_n$

$\hat{C}_{n+1} = A C_n A^T + B$

Kalman gain:

$$K_{n+1} = \hat{C}_{n+1} H^T (R + H \hat{C}_{n+1} H^T)^{-1}$$

Analysis:

$$m_{n+1} = \hat{m}_{n+1} - K_{n+1}(H\hat{m}_{n+1} - y_{n+1})$$

$$C_{n+1} = \hat{C}_{n+1} - K_{n+1} H \hat{C}_{n+1}.$$

$$m_1 = A m_0 = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$P_1 = A P_0 A^T + B$$

$$= \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 3.5 & 4 \\ 4 & 22.5 \end{bmatrix}$$

$$K = P_1 H^T (R + H P_1 H^T)^{-1}$$

$$= \begin{bmatrix} 3.5 & 4 \\ 4 & 22.5 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(0.1 + \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 3.5 & 4 \\ 4 & 22.5 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)^{-1}$$

$$= \begin{bmatrix} 0.17699115 \\ 0.99557522 \end{bmatrix}$$

$$m\_new = m_1 - K(Hm_1 - y)$$

$$= \begin{pmatrix} 0 \\ 5 \end{pmatrix} - \begin{bmatrix} 0.17699115 \\ 0.9955752 \end{bmatrix} \left(\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} - 0.5 \right)$$

$$= \begin{bmatrix} -0.79646018 \\ 0.5199115 \end{bmatrix}$$

$$P\_new = P_1 - KHP_1$$

$$= \begin{bmatrix} 3.5 & 4 \\ 4 & 22.5 \end{bmatrix} - \begin{bmatrix} 0.1769915 \\ 0.99557522 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 3.5 & 4 \\ 4 & 22.5 \end{bmatrix}$$

$$= \begin{bmatrix} 2.7920354 & 0.01769912 \\ 0.01769912 & 0.0995752 \end{bmatrix}$$