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"# Initial value problem for ordinary differential equations"

]

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"%config InlineBackend.figure\_format = 'svg'\n",

"import numpy as np\n",

"from matplotlib import pyplot as plt"

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"def f(t,y):\n",

" return y"

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"def yexact(t):\n",

" return np.exp(t)"

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"def euler(t0,T,y0,h):\n",

" N = int((T-t0)/h)\n",

" y = np.zeros(N)\n",

" t = np.zeros(N)\n",

" y[0] = y0\n",

" t[0] = t0\n",

" for n in range(1,N):\n",

" y[n] = y[n-1] + h\*f(t[n-1],y[n-1])\n",

" t[n] = t[n-1] + h\n",

" return t, y"

]

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"Q 24.171875 6.390625 31.78125 6.390625 \r\n",

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"Q 47.125 21.390625 47.125 36.375 \r\n",

"Q 47.125 51.421875 43.28125 58.90625 \r\n",

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"Q 56.984375 17.96875 50.515625 8.265625 \r\n",

"Q 44.046875 -1.421875 31.78125 -1.421875 \r\n",

"Q 19.53125 -1.421875 13.0625 8.265625 \r\n",

"Q 6.59375 17.96875 6.59375 36.375 \r\n",

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"Q 43.3125 58.59375 39.234375 62.25 \r\n",

"Q 35.15625 65.921875 28.609375 65.921875 \r\n",

"Q 23.96875 65.921875 18.8125 64.3125 \r\n",

"Q 13.671875 62.703125 7.8125 59.421875 \r\n",

"L 7.8125 69.390625 \r\n",

"Q 13.765625 71.78125 18.9375 73 \r\n",

"Q 24.125 74.21875 28.421875 74.21875 \r\n",

"Q 39.75 74.21875 46.484375 68.546875 \r\n",

"Q 53.21875 62.890625 53.21875 53.421875 \r\n",

"Q 53.21875 48.921875 51.53125 44.890625 \r\n",

"Q 49.859375 40.875 45.40625 35.40625 \r\n",

"Q 44.1875 33.984375 37.640625 27.21875 \r\n",

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"Q 22.515625 -1.421875 17.65625 -0.515625 \r\n",

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"L 29.109375 43.015625 \r\n",

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"Q 44.09375 59.90625 40.109375 62.90625 \r\n",

"Q 36.140625 65.921875 28.71875 65.921875 \r\n",

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"L 9.8125 71.09375 \r\n",

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"Q 40.828125 74.21875 47.359375 69.109375 \r\n",

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"z\r\n",

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"L 47.609375 25.390625 \r\n",

"L 58.015625 25.390625 \r\n",

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"Q 40.625 48.1875 47.75 41.5 \r\n",

"Q 54.890625 34.8125 54.890625 23.390625 \r\n",

"Q 54.890625 11.625 47.5625 5.09375 \r\n",

"Q 40.234375 -1.421875 26.90625 -1.421875 \r\n",

"Q 22.3125 -1.421875 17.546875 -0.640625 \r\n",

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"L 7.71875 11.625 \r\n",

"Q 12.109375 9.234375 16.796875 8.0625 \r\n",

"Q 21.484375 6.890625 26.703125 6.890625 \r\n",

"Q 35.15625 6.890625 40.078125 11.328125 \r\n",

"Q 45.015625 15.765625 45.015625 23.390625 \r\n",

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"L 36.8125 0 \r\n",

"L 27.59375 0 \r\n",

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"L 2.6875 47.703125 \r\n",

"L 2.6875 54.6875 \r\n",

"L 9.28125 54.6875 \r\n",

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"z\r\n",

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"Q 47.40625 15.53125 47.40625 23.390625 \r\n",

"Q 47.40625 31.296875 43.53125 35.828125 \r\n",

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"Q 27.828125 65.921875 22.671875 59.328125 \r\n",

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"Q 19.671875 43.65625 24.015625 45.921875 \r\n",

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"Q 38.8125 6.390625 42.859375 10.171875 \r\n",

"Q 46.921875 13.96875 46.921875 20.515625 \r\n",

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"z\r\n",

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"Q 45.3125 48.734375 45.3125 54.390625 \r\n",

"Q 45.3125 60.0625 41.71875 63.234375 \r\n",

"Q 38.140625 66.40625 31.78125 66.40625 \r\n",

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" <use x=\"127.246094\" xlink:href=\"#DejaVuSans-48\"/>\r\n",

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"L 12.5 54.6875 \r\n",

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"L 115.75 14.2 \r\n",

"Q 115.75 12.2 113.75 12.2 \r\n",

"L 53.965625 12.2 \r\n",

"Q 51.965625 12.2 51.965625 14.2 \r\n",

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"Q 14.984375 11.71875 18.140625 8.90625 \r\n",

"Q 21.296875 6.109375 26.703125 6.109375 \r\n",

"Q 34.1875 6.109375 38.703125 11.40625 \r\n",

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"Q 6 8.015625 6 15.921875 \r\n",

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"z\r\n",

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"ye = yexact(te)\n",

"plt.plot(te,ye,'--')\n",

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"H = [0.2,0.1,0.05]\n",

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"for h in H:\n",

" t,y = euler(t0,T,y0,h)\n",

" plt.plot(t,y)\n",

"\n",

"plt.legend(('Exact','0.2','0.1','0.05'),loc=2)\n",

"plt.xlabel('t')\n",

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"plt.grid(True);"

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"from numpy import zeros,exp\n",

"from matplotlib.pyplot import plot,xlabel,ylabel,legend"

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" h = step size\n",

" N = number of steps to take\n",

" \"\"\"\n",

" y = zeros(N)\n",

" t = zeros(N)\n",

" t[0], y[0] = 0, 1\n",

" t[1], y[1] = h, exp(alpha\*h)\n",

" a = 2.0\*alpha\*h\n",

" for i in range(1,N-1):\n",

" y[i+1] = a\*y[i] + y[i-1]\n",

" t[i+1] = t[i] + h\n",

" ye = exp(alpha\*t)\n",

" plot(t,y,'o-',t,ye,'r-')\n",

" xlabel('t')\n",

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" legend(('Numerical','Exact'))"

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