Probability Methods in Engineering

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Lecture 16

Expected Value of Functions of RV

- The expected value of a random variable is denoted by **E[X]**.
- The expected value can be thought of as the "**average**" value attained by the random variable; in fact, the expected value of a random variable is also called its **mean** or **first moment**)
- Let X be a random variable and *g* be any function.

1. If X is discrete, then the expectation of *g*(X) is defined as, then

   $$E[g(X)] = \sum_{x \in X} g(x)f(x)$$

   where f is the probability mass function of X and X is the support of X

2. If X is continuous, then the expectation of *g*(X) is defined as,

   $$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)\, dx$$

   where f is the probability density function of X.

Expected Value of Functions of RV

- Function $g(X)$ of RV X can be denoted by Z
- Expected value of Z would be

$$E[Z] = E[g(X)] = \sum_k g(x_k)p_X(x_k)$$

- Or simply multiply each value of Z with its probability and add the products for each k
- For more than one values of X are mapped to one value of Z

$$E[Z] = \sum_k g(x_k)p_X(x_k) = \sum_j z_j p_Z(z_j)$$

- Property (see others in book):

$$E[ag(X)+c] = aE[g(X)]+c$$

Examples

- Let X be a noise voltage that is uniformly distributed in S_X = {-3, -1, 1, 3} with $p_X(k) = 1/4$ for k in S_X. Find $E[Z]$ where $Z = X^2$.

Examples (cont.)

- Let X be a noise voltage that is uniformly distributed in S_X = {-3, -1, 1, 3} with $p_X(k) = 1/4$ for k in S_X. Find $E[Z]$ where $Z = (2X+10)^2$.

Examples (cont.)

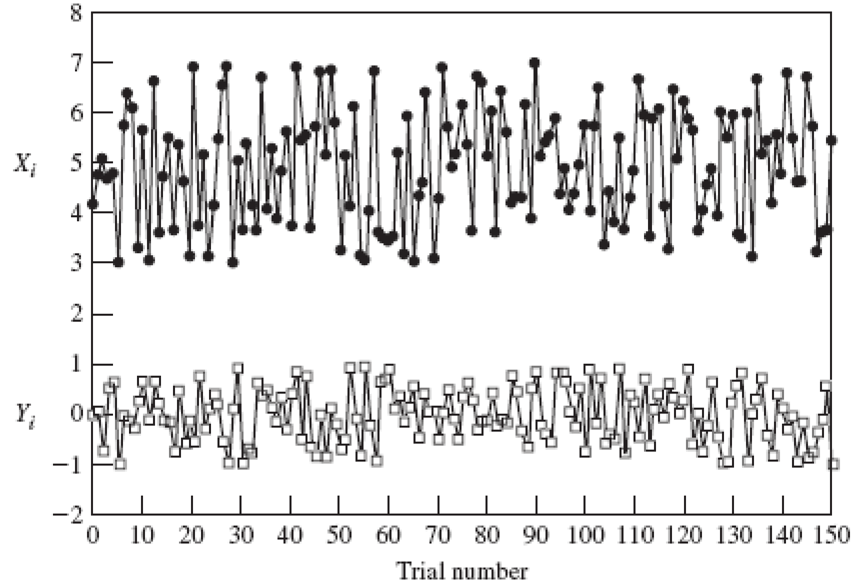
- A fair coin is tossed three times and the sequence of heads and tails is noted. Let X be the number of heads in each outcome. Find $E[X^2] = E[Z]$.

Examples (cont.)

- Let V be the voltage of a signal in S_V having possible values 1, 2 and 3 with $p_V(k)$ as 1/4, 1/2 and 1/4 respectively. Find the mean power $E[P]$ of the signal where $P = V^2$ (considering $R = 1$). Find $E[Z]$ where $Z = (V+1)^3$.

Variance of Discrete RV

- Expected value provides limited information
- Interest also in the variation about expected value $X - E[X]$
- Squaring the variations gives positive values $(X - E[X])^2$
- **Variance** defined as the expected value of this square



$$\sigma^2{}_X = VAR\left[X\right] = E[(X - E[X])^2]$$

Variance of Discrete RV (cont.)

$$\sigma^2_X = \sum_{x \in S_X} (x - E[X])^2 p_X(x) = \sum_{k=1}^{\infty} (x_k - E[X])^2 p_X(x_k)$$

- The square root of variance is **standard deviation**

$$\sigma_X = STD[X] = \sqrt{VAR[X]}$$

- Variance also expressed as

$$E[(X - E[X])^2] = E[X^2 - 2E[X]X + E^2[X]]$$
$$= E[X^2] - 2E[X]E[X] + E^2[X]] = E[X^2] - E^2[X]]$$

- $E[X^2]$ is 2nd moment of X, similarly $E[X^n]$ the **nth moment**

Examples (cont.)

- Let X be the number of heads in three tosses of a fair coin. Find $\mathrm{VAR}[X]$.

Examples (cont.)

- Find the variance of the Bernoulli random variable X having success probability p. The value for success is 1 and failure is 0.