Probability Methods in Engineering

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Lecture 1

Resource Material

- Course Book
  - A. Leon-Garcia, "Probability and Random Processes for Electrical Engineering", 3rd Edition, Pearson Prentice Hall, 2008
- Reference Books
  - D. Bertsekas and J. N. Tsitsiklis, "Introduction to Probability", 2nd Edition, Athena Scientific, 2008
  - Hossein Pishro-Nik, "Introduction to Probability, Statistics, and Random Processes", Kappa Research, 2014

Course Group

- Class code
  - yu2iscp
- Class link
  - https://classroom.google.com/c/NDgwMzE0MjY4ODQ3?cjc=yu2iscp
  - Teaching method
  - Combination of slides and white board
  - Interaction about concepts encouraged
  - Interruption to ask questions during lectures allowed

Tentative Grading Criteria

- Exams
  - Final exam: **50%**
  - Mid-term exam: **25%**
- Sessional
  - Attendance: **10%**
  - Assignments: **7.5%**
  - Quizzes: **7.5%**
- All lectures interrelated
  - Each lecture provides base for next lecture
  - Missing any lecture would result in problems in understanding subsequent lectures

- No mobile phone usage during class

Course Outline

- Introduction to Probability
  - Axioms
  - Probabilities using Counting methods
  - Conditional Probability
  - Law on total Probability
  - Bayes' Rule
  - …
- Random Variables (RVs)
  - Cumulative Distribution Function (CDF)
  - Probability Density Function (PDF)
  - Mean and variance
  - …
- Random Processes
  - Stationary processes
  - Integrals
  - Power spectral density
  - …

How is probability used in everyday life?

- Weather forecasting
- Sports Strategies
- Insurance
- Calculation of batting average in cricket.
- How likely one can win a lottery ticket.
- Playing cards
- Voting strategy in politics
- Rolling a dice.
- Pulling black socks from a drawer of white socks.
- Online shopping
- Online games

Course Significance

- Basis for numerous advanced technologies
  - Wave propagation
  - Wireless communication
  - Communication theory
  - Information theory
  - Pattern recognition
  - Radar and sonar signal processing
  - Network design and optimization

Course Significance (cont.)

- Disadvantages of weak probability concepts
  - No scope in research fields
  - Poor analytical skills
  - Fear of interview questions
  - Inability to conceptualize techniques
  - No major role possible in engineering problem solving
  - Minimum contribution towards nation building
  - Incapability to carry out feasibility studies for mega projects

Weekly Course Outline

Week	Contents
Week 1	Introduction to Mathematical Models, Deterministic Models, Probabilistic Models
Week 2	Basic Concepts of Probability, Axioms of Probability
Week 3	Computing Probabilities using Counting Methods, Conditional Probability
Week 4	Law on Total Probability, Bayes' Rule
Week 5	Independence of Events, Sequential Experiments
Week 6	Binomial Probability Law, Geometric Probability Law
Week 7	Sequences of Dependent Experiments, Random Variables, Notation of a Random Variable
Week 8	Types of Random Variable, Probability Mass Function
	Midterm Examination

Weekly Course Outline

Week	Contents
Week 9	Discrete Random Variables
Week 10	Expected Value, Variance, Standard Deviation
Week 11	Functions of a Random Variable, Expected Value of Function of Random Variables
Week 12	Entropy, Continuous Random Variables
Week 13	CDF, PDF, Memoryless Property
Week 14	Multiple Random Variables, Joint CDF and PDF, Conditional CDF and PDF, CCDF
Week 15	MATLAB for Generation of Pseudo Random Values
Week 16	Course Revision
	Final Term Examination

MAPPING OF CLOS WITH PLOS:

CLO #	Course Learning Outcomes (CLOs)	Level of Learning (Bloom's Taxonomy)	Program Learning Outcomes (PLOs)
1	Use essential concepts of probability and apply analytical methods for solving engineering problems.	Cog-3 (Application)	PLO1 (Engineering Knowledge)
2	Use the concepts of random variables and solve mathematical problems related to stochastic systems.	Cog-3 (Application)	PLO3 (Design/Development of Solutions)
3	Apply mathematical skills and demonstrate the use of software tools for implementation of probabilistic models.	Cog-3 (Application)	PLO5 (Modern Tool Usage)

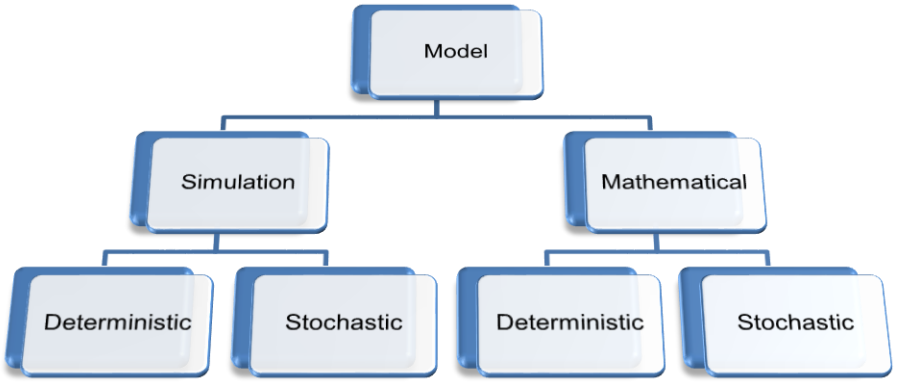
Randomness

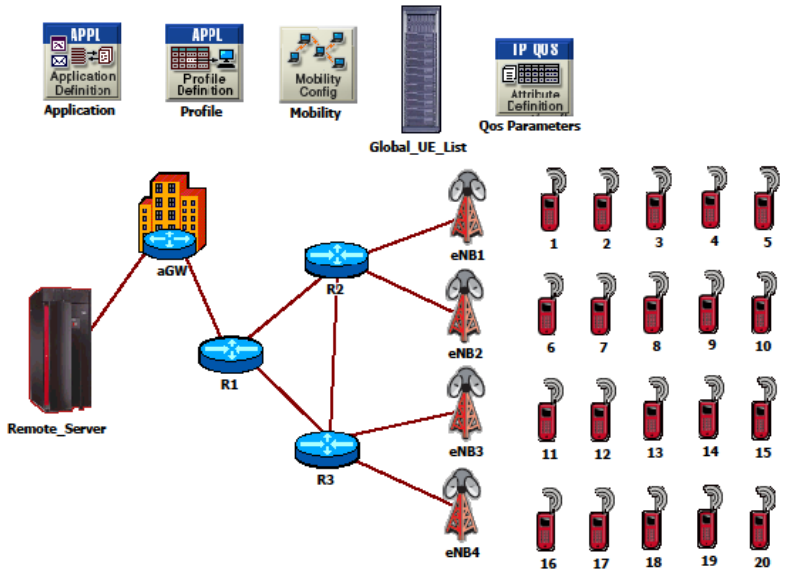
- What is Randomness?
  - Chaos
  - Uncertainty
  - Doubt
- Humans desire some level of 'certainty'
- Examples
  - Solar system
  - Weather forecast at Chitral Airport
  - Traffic situation on University road
- Engineers **quantify** 'certainty'

Source: Computer Vision Research Group, *CIIT Lahore*, Pakistan

Model of a Physical System

- **Model**: Approximate representation of physical situation
  - **Mathematical model**: Set of assumptions about how system works
    - **Deterministic model**: Offers repeatability of results, (e.g. Ohm's Laws)
    - **Stochastic model**: Characterizes randomness and uncertainty
  - **Simulation model**: Imitation of real system
    - **Deterministic model**: No random component involved, (e.g. chemical reaction)
    - **Stochastic model**: Must have random input component





Source: S. N. K. Marwat, PhD Thesis, *University of Bremen, Germany*

Random Experiment

- **Random Experiment**: The result varies in random manner
- **Sample Space**: Set of all possible experiment results
- **Outcome**: A single element of sample space
- **Event**: A subset of sample space

- Example: An urn containing three balls, one is drawn
  - How probable it is that a ball withdrawn at random is labeled '1'?
  - Can you quantify this 'chance'?
  - Everyone of you should be able to write the sample space for this experiment!

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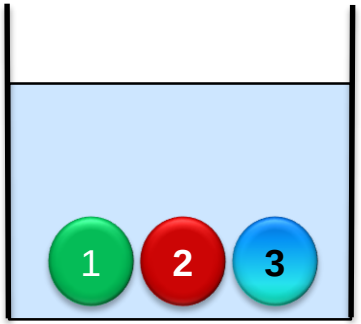


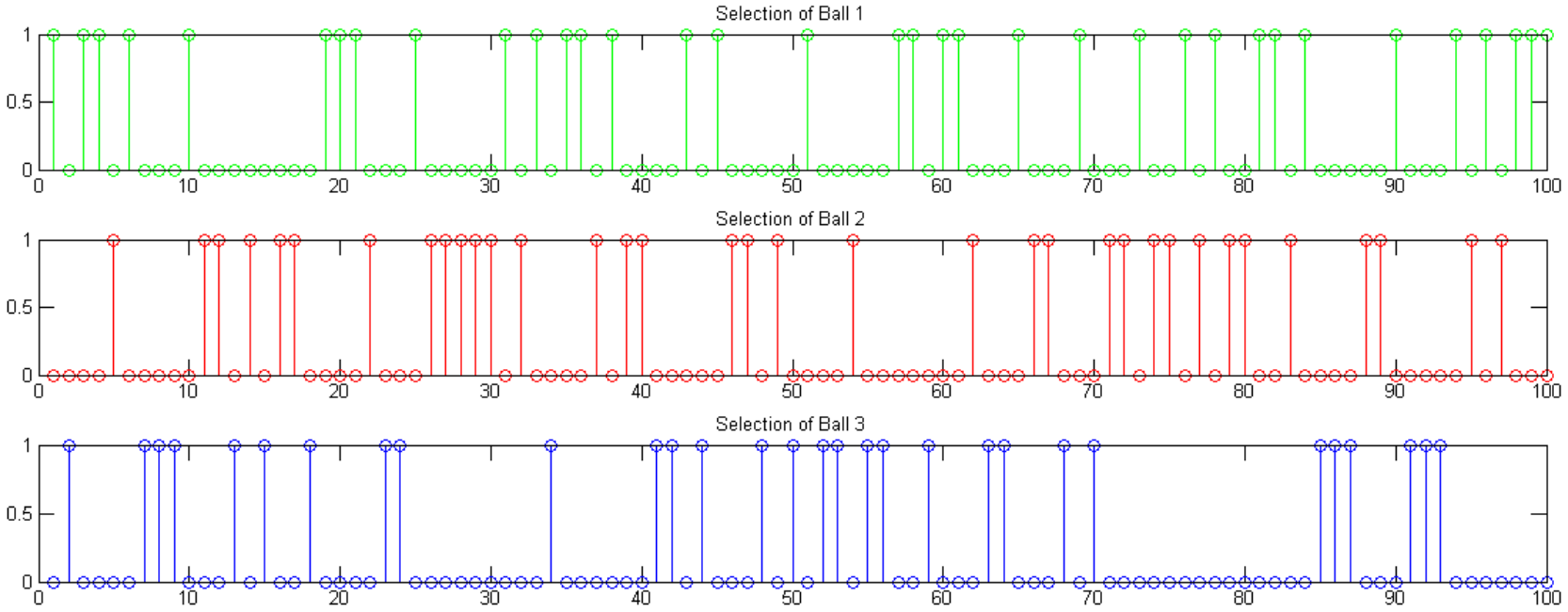
Random Experiment (cont.)

- Some more questions to answer
  - Is withdrawing all the three balls equi-probable (or is any ball more likely to be drawn)?
  - If '1' means 'sure occurrence' and '0' means 'no chance of occurrence', what number can be given to the chance of getting 'ball 1'?
  - What is the chance of withdrawing an odd-numbered (or even-numbered) ball?

Let the nature answer this

Random Experiment (cont.)

- Take a ball from the urn
- Record the outcome
- Put it back in the urn
- Do the experiment 'n' times



Random Experiment (cont.)

- Number of times k^{th} outcome occurred (or **frequency** of k) in a total of n trails

$$N_k(n)$$

- The **relative frequency** of k^{th} outcome

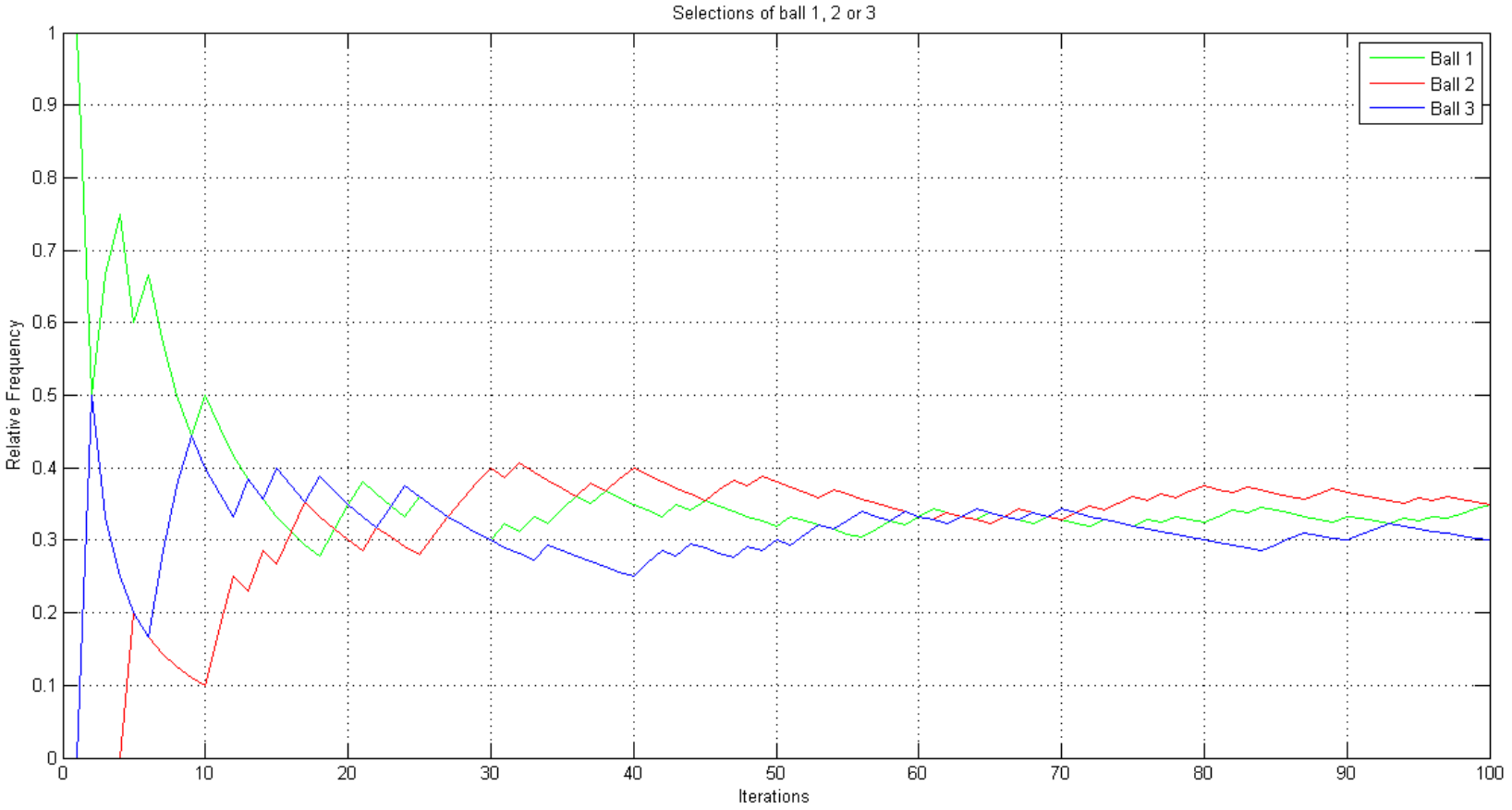
$$f_k(n) = \frac{N_k(n)}{n}$$

- **Relative frequency** can be defined as the number of times an event occurs divided by the total number of events occurring in a given scenario.
- The relative frequency formula is given as:

  Relative Frequency = Subgroup frequency/ Total frequency.

Random Experiment (cont.)

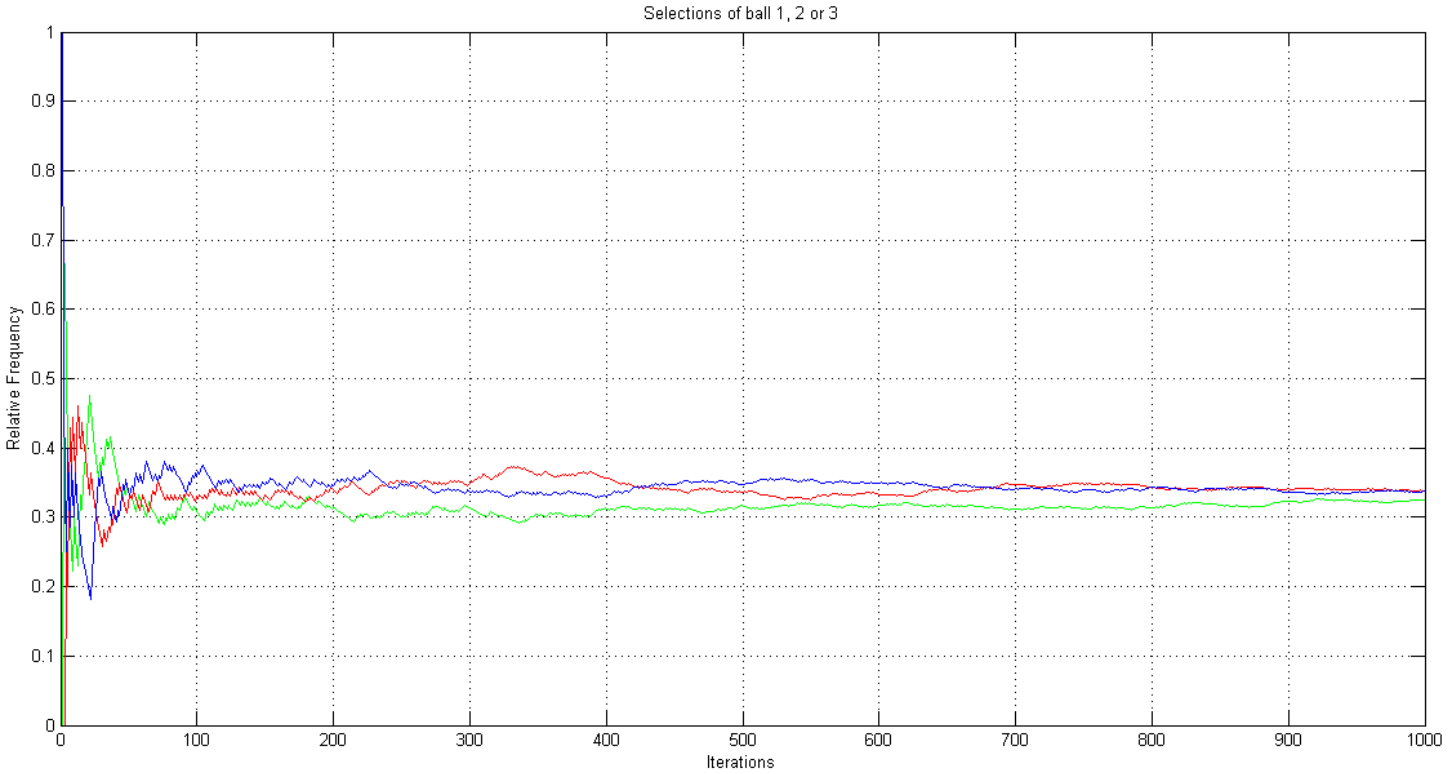
- Statistical Regularity
  - Averages obtained in long sequences yield same value



Random Experiment (cont.)

- **Probability** defined by *von Mises* as 'limiting case of relative frequency'

$$\lim_{n\to\infty} f_k(n) = \lim_{n\to\infty} \frac{N_k(n)}{n} = p_k$$



Properties of Relative Frequency

- Number of occurrences of an outcome in n trials
  - A number between zero and n

$$0 \leq N_k(n) \leq n$$

- Relative frequencies are
  - A number between zero and one
  - Divide the above equation by n to get

$$0 \leq f_k(n) \leq 1$$

Properties of Relative Frequency (cont.)

- Sum of number of occurrences of all possible outcomes
  - Must be n

$$\sum_{k=1}^{K} N_k(n) = n$$

- Sum of all relative frequencies
  - Must be 1

$$\sum_{k=1}^{K} f_k(n) = 1$$

Examples

- A die is tossed 40 times and lands 6 times on the number 4. What is the relative frequency of observing the die land on the number 4?

Examples

- A coin is tossed 20 times and lands 15 time on heads. What is the relative frequency of observing the coin land on heads?

Examples

- Example: Travel Survey
- 92 people were asked how they got to work:
  - 35 used a car
  - 42 took public transport
  - 8 rode a bicycle
  - 7 walked

- Find the Relative Frequencies (to 2 decimal places) for
- Car:
- Public Transport:
- Bicycle:
- Walking:

Examples

- Example 2: Anna has a packet containing 20 candies. Her favorites are the yellow ones and the red ones. The table below shows the frequency of each different candy selected as she picked all 20 sweets one by one and finished them all.

Candy color	Yellow	Red	Green	Brown
Frequency	6	6	3	5

- A) What is the relative frequency of the picked candy being one of her favorites?
- B) What is the relative frequency for the brown candy