Control Systems - 7th Semester - Week 5

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Contents that we have covered till now

We studied the following topics till now:

- ❑ Converting state-space to transfer function using formula
- ❑ Converting transfer functions to state-space models using canonical forms
- ❑ Analyzing step responses of first order systems (time constant and dc-gain)

We will study the following topics before mid term exam

- ❑ Block reduction of complex systems (today lecture)
- ❑ Analyzing step responses of second order systems (underdamped, undamped, over damped, critically damped)

Block reduction algebra

First we analyze a simple transfer function block

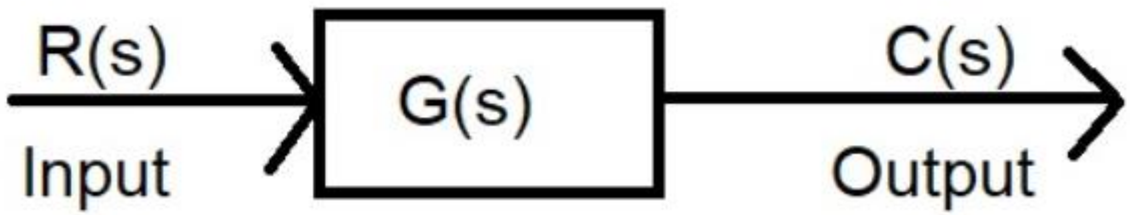


Figure: Transfer function block

The input signal is denoted by $\boldsymbol{R}(\boldsymbol{s})$ and output signal by $\boldsymbol{C}(\boldsymbol{s})$. We can write the following:

$$\boldsymbol{C}(\boldsymbol{s}) = \boldsymbol{G}(\boldsymbol{s})\boldsymbol{R}(\boldsymbol{s})$$

Sometimes, we skip the term (s) and write the following abusive notation:

$$\boldsymbol{C} = \boldsymbol{G}\boldsymbol{R}$$

Block reduction algebra

There are 3 types of interconnections in control systems:

- ❑ Series Interconnection
- ❑ Parallel Interconnection
- ❑ Feedback Interconnection

Besides, there are 4 operations which are as follows:

- ❑ Moving summing junction after transfer function
- ❑ Moving summing junction before transfer function
- ❑ Moving before pickoff point
- ❑ Moving after pickoff point

Let us introduce a summing junction or summer first, and then pick-off point

Block reduction algebra - Summer or Summing Junction

A summer or summing junction adds (or subtracts) two or more signals. The default sign is + in a summer or summing junction

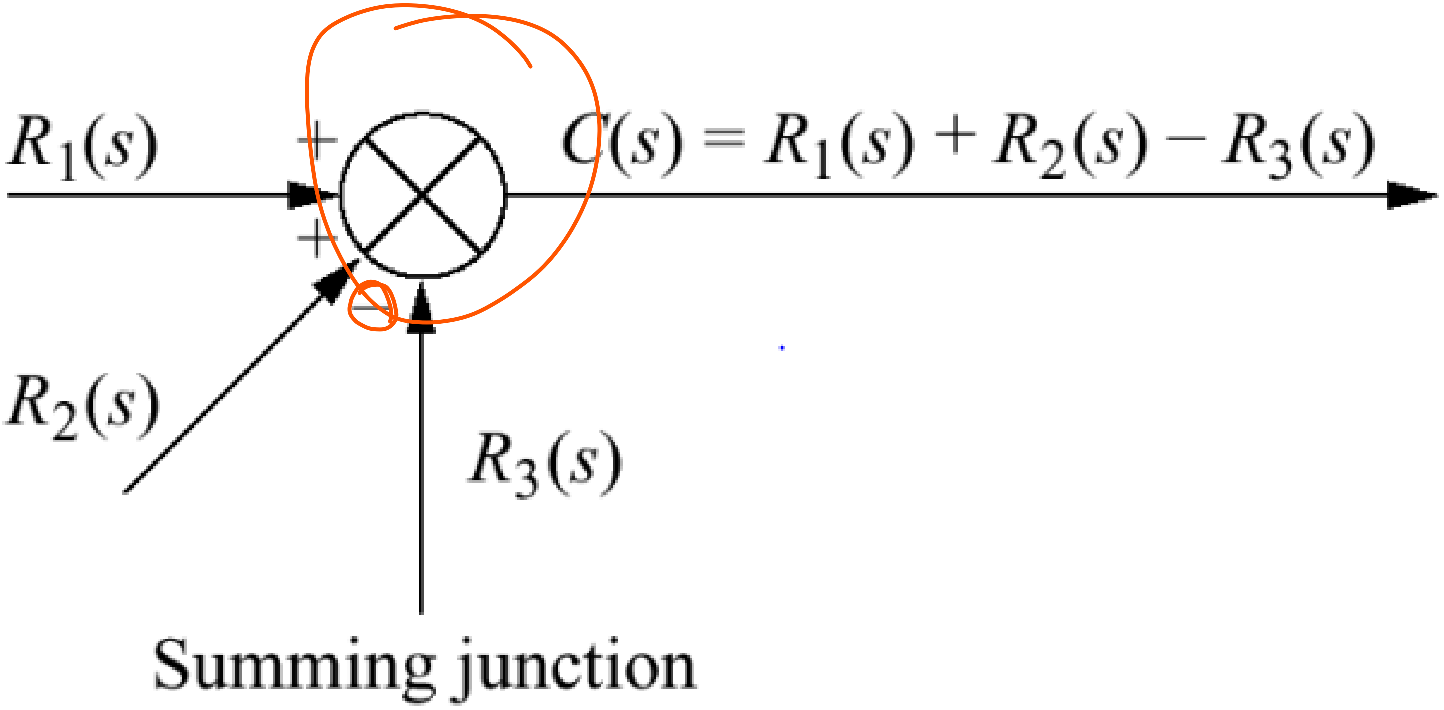


Figure: Summing Junction Symbol

Block reduction algebra - Pick off point

Pick off point: A point where the same signal has to propagate through more than one paths

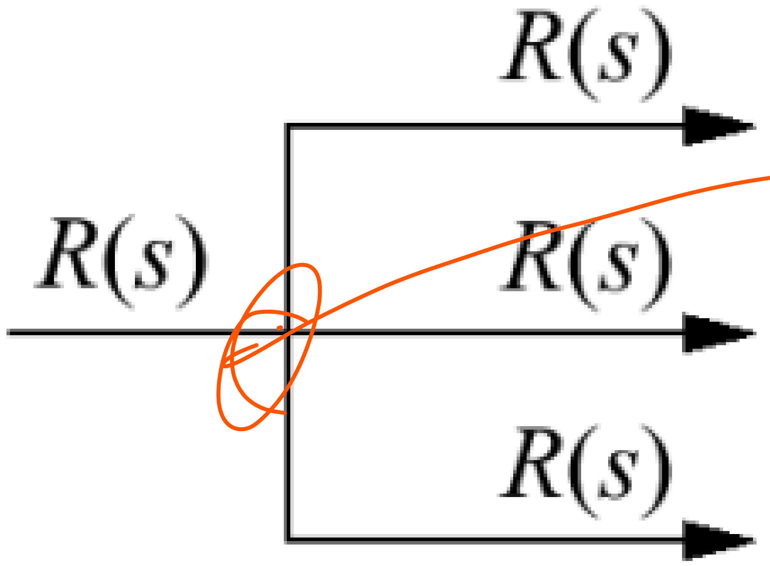


Figure: Pick Off point

First Interconnection - Series Interconnection

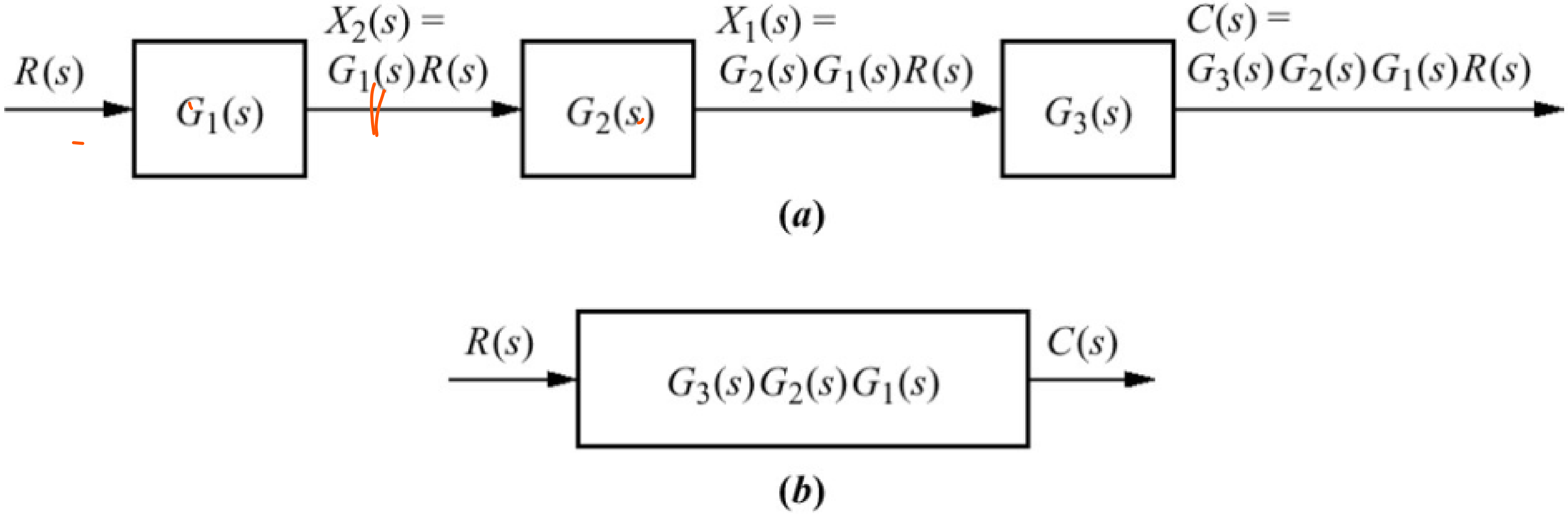


Figure: Series Interconnection of transfer functions

We can write $\boldsymbol{G_e = G_3 G_2 G_1}$

Second Interconnection - Parallel Interconnection

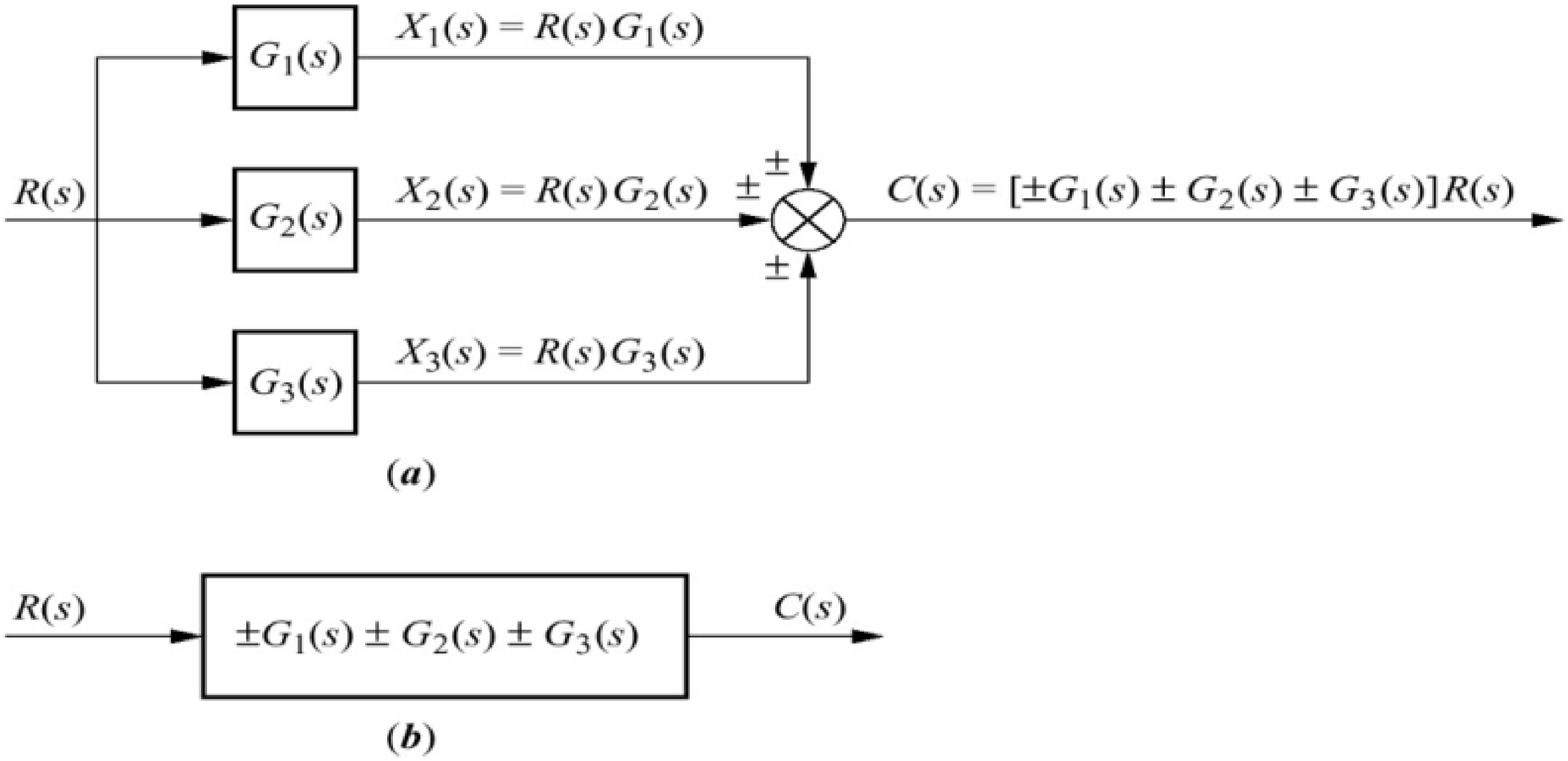


Figure: Parallel Interconnection of transfer functions

We can write $\boldsymbol{G_e} = \pm \boldsymbol{G_3} \pm \boldsymbol{G_2} \pm \boldsymbol{G_1}$

Few Important Points

Series interconnection involves product of transfer functions

In parallel interconnection, be careful to identify the transfer functions correctly

Two blocks are in parallel if they have same input signal and the output goes towards same summing junction

Parallel interconnection involves sum or different of transfer functions

Third Interconnection - Feedback Interconnection

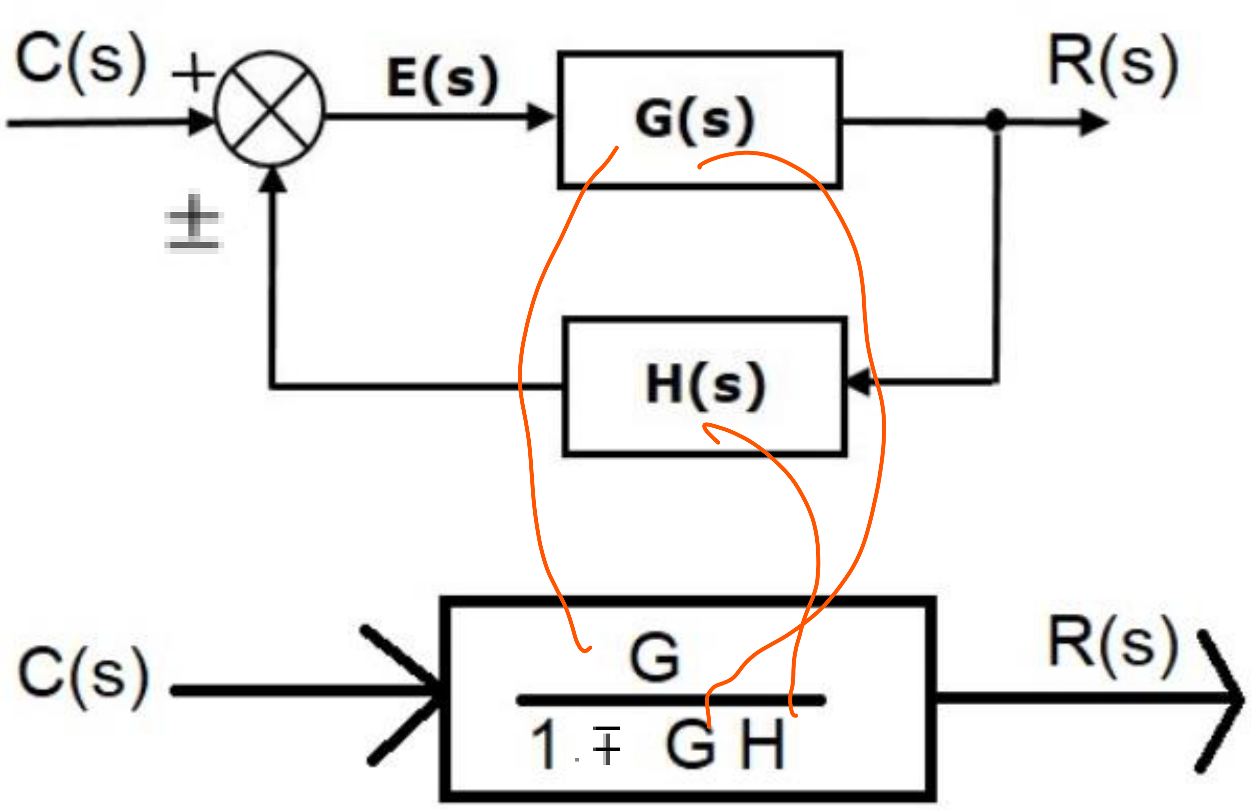
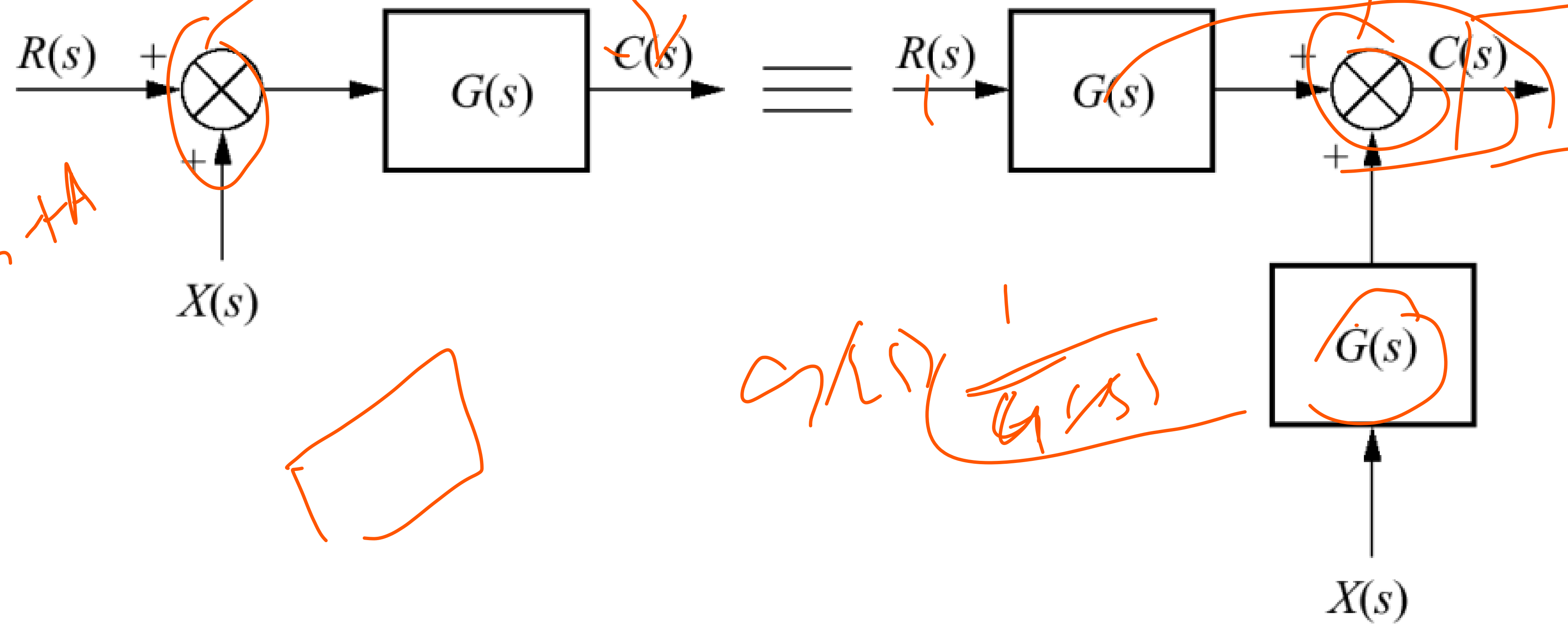


Figure: Feedback Interconnection of transfer functions

We can write $\boldsymbol{G_e} = \dfrac{\boldsymbol{G}}{\boldsymbol{1 \mp GH}}$

Operation 1: Moving summing junction after transfer function

$$C(s) = \big(R(s) + X(s)\big)G(s) = R(s)G(s) + X(s)G(s)$$



A+B=B+A

Figure: Moving a summing junction after transfer function

Operation 2: Moving summing junction before transfer function

$$C(s) = R(s)G(s) + X(s) = \left(R(s) + \frac{X(s)}{G(s)}\right)G(s)$$

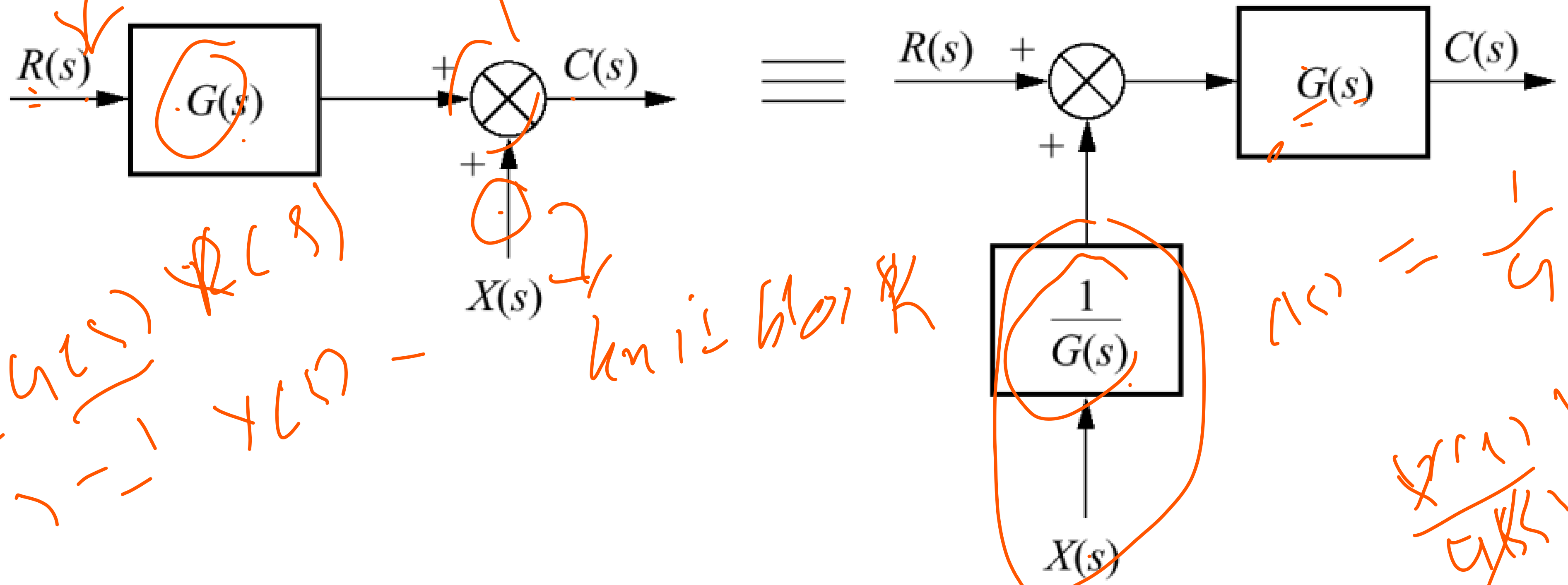


Figure: Moving a summing junction before transfer function

Operation 3: Moving before pickoff point

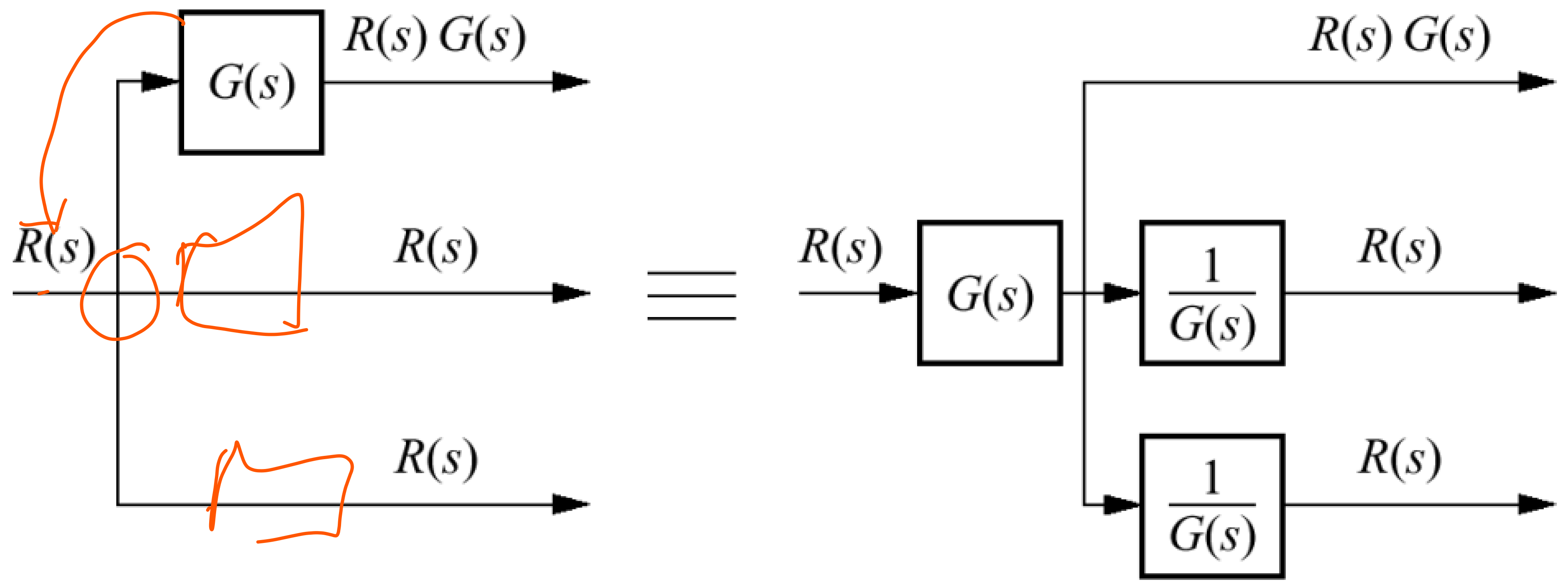


Figure: Moving before a pick-off point

Operation 4: Moving after pickoff point

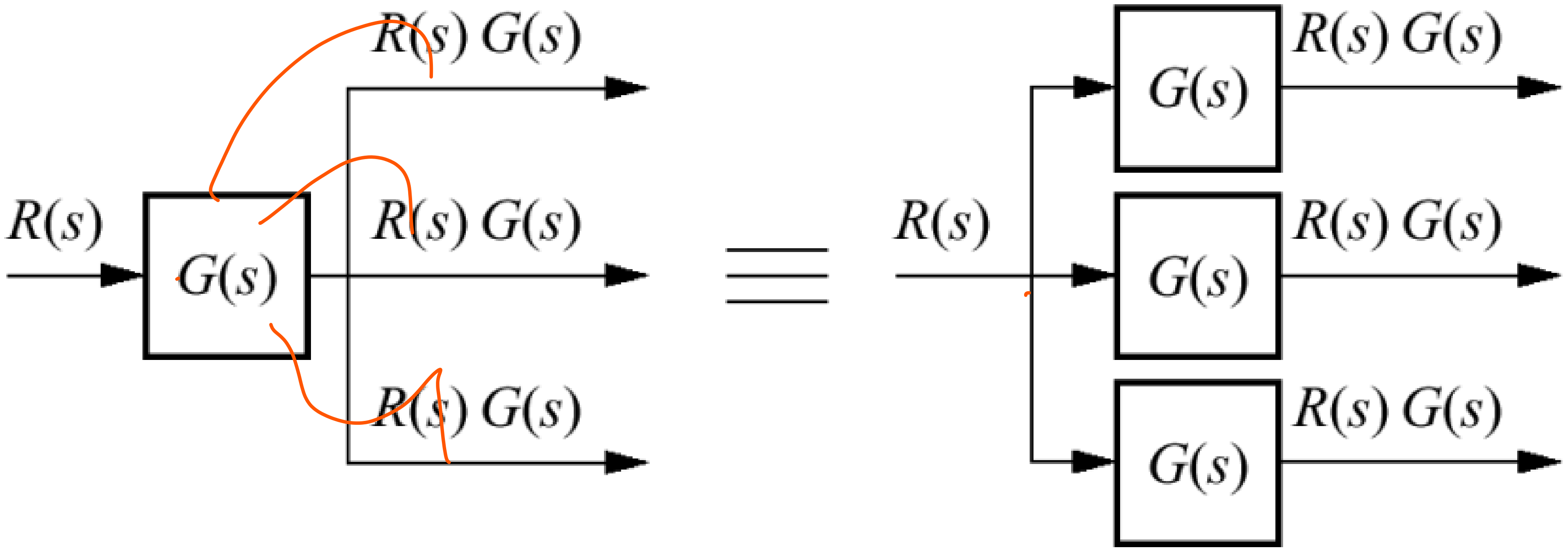


Figure: Moving after a pick-off point

Summary of block reduction rules

We will use the knowledge about these 3 interconnections, and 4 operations to reduce complex systems

You will be given a complex interconnection schematic, plus input and output, and will be asked to apply this knowledge to reduce or simplify complex systems

Example 1 - Problem to solve

$C(s) = R(s)\,G(s)$

$G(s) = \dfrac{C(s)}{R(s)}$

Can you obtain the transfer function, $\frac{C(s)}{R(s)}$?

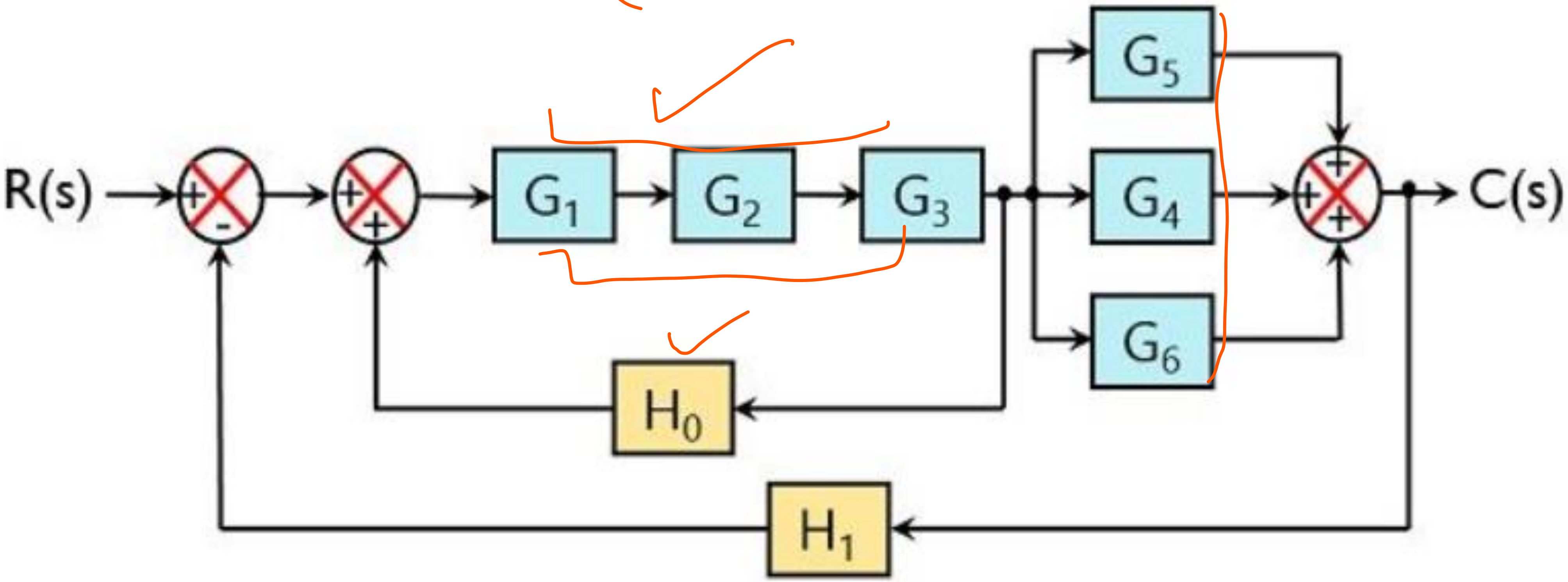


Figure: Example 1

Example 1 - Solution part a - Simplify series interconnection

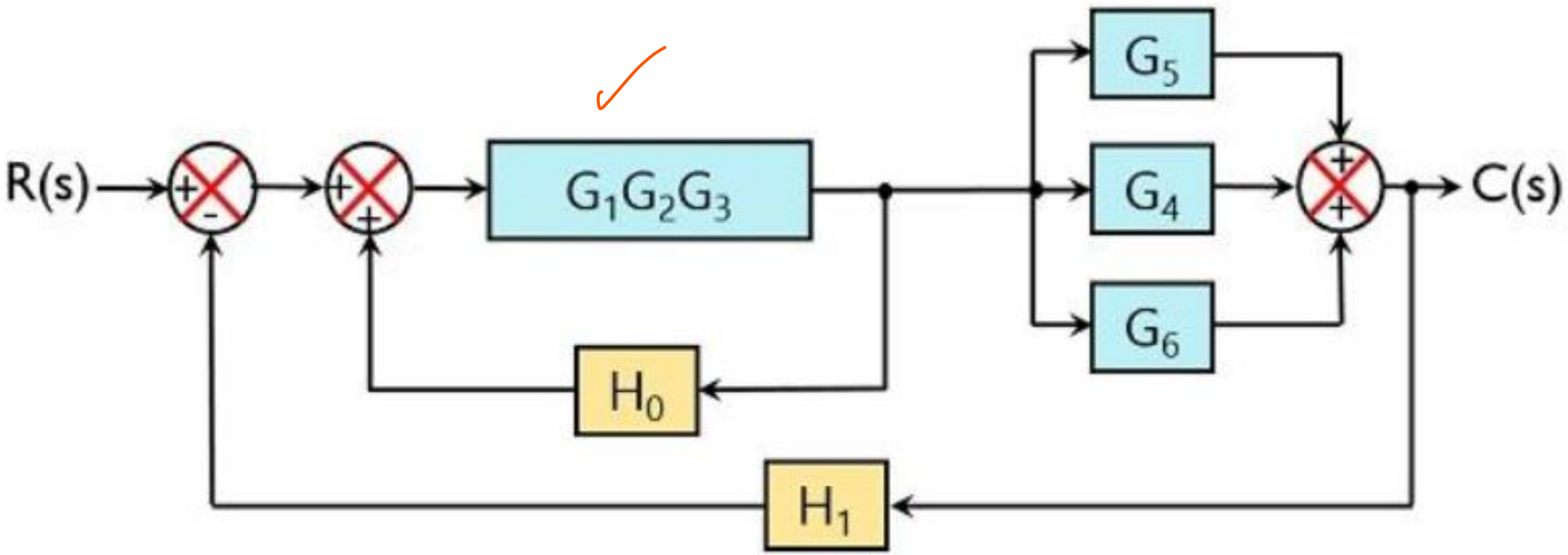


Figure: Example 1 – Solution part a

Example 1 - Solution part b - Simplify parallel interconnection

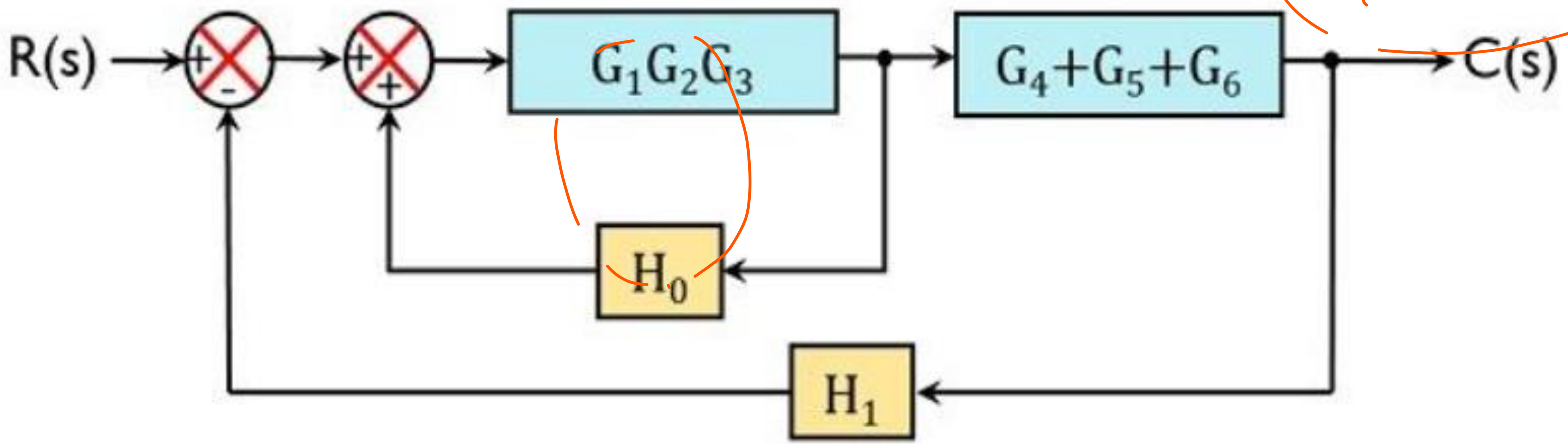


Figure: Example 1 – Solution part b

Example 1 - Solution part c - Simplify inner feedback interconnection

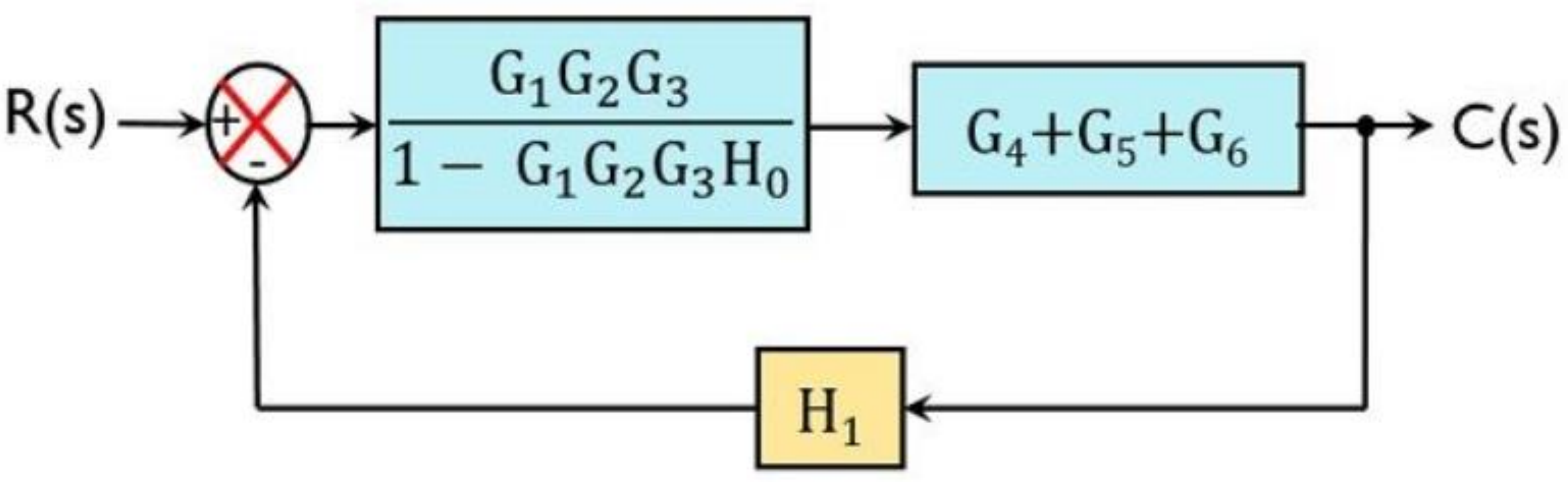


Figure: Example 1 – Solution part c

Example 1 - Solution part d

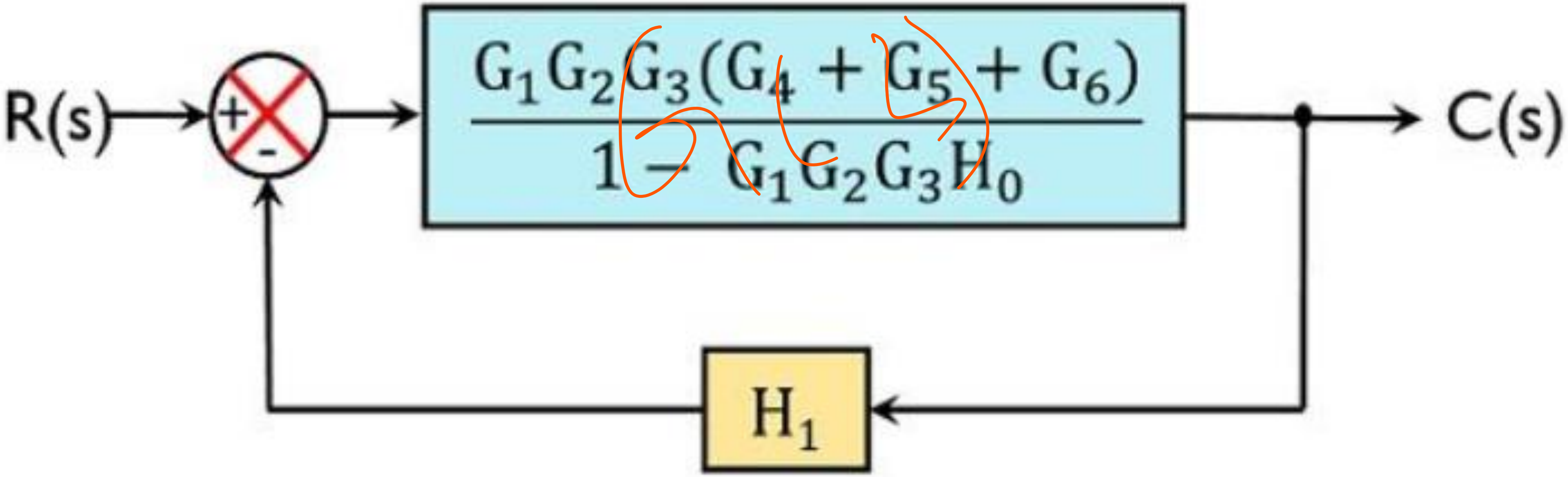


Figure: Example 1 – Solution part d

Example 1 - Final Solution

Let us introduce $G(s)$ as follows:

$$G(s) = \frac{G_1G_2G_3(G_4 + G_5 + G_6)}{1 - G_1G_2G_3H_0}$$

$G(s) = C(s) / R(s)$

Then, we can write the following:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H_1(s)}$$

Be careful: please donot introduce G_1 as follows (its wrong):

$$G_1(s) = \frac{G_1G_2G_3(G_4 + G_5 + G_6)}{1 - G_1G_2G_3H_0}$$

Example 2 - Problem to solve

Can you obtain the transfer function, $\frac{\boldsymbol{C(s)}}{\boldsymbol{R(s)}}$?

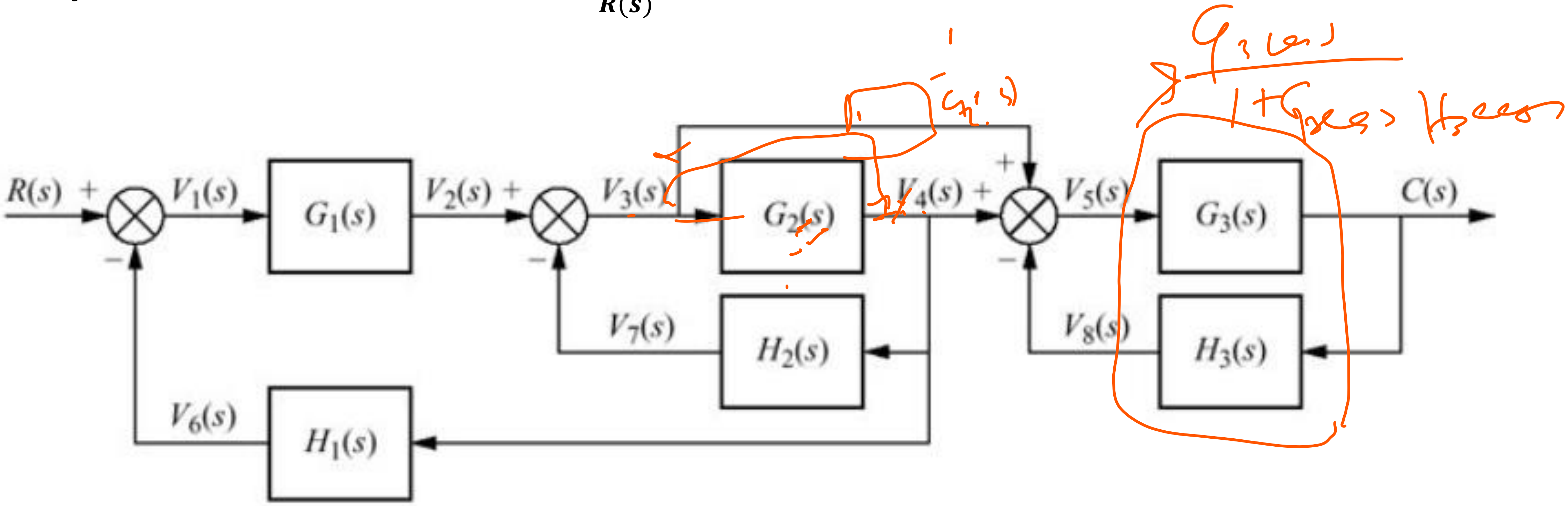
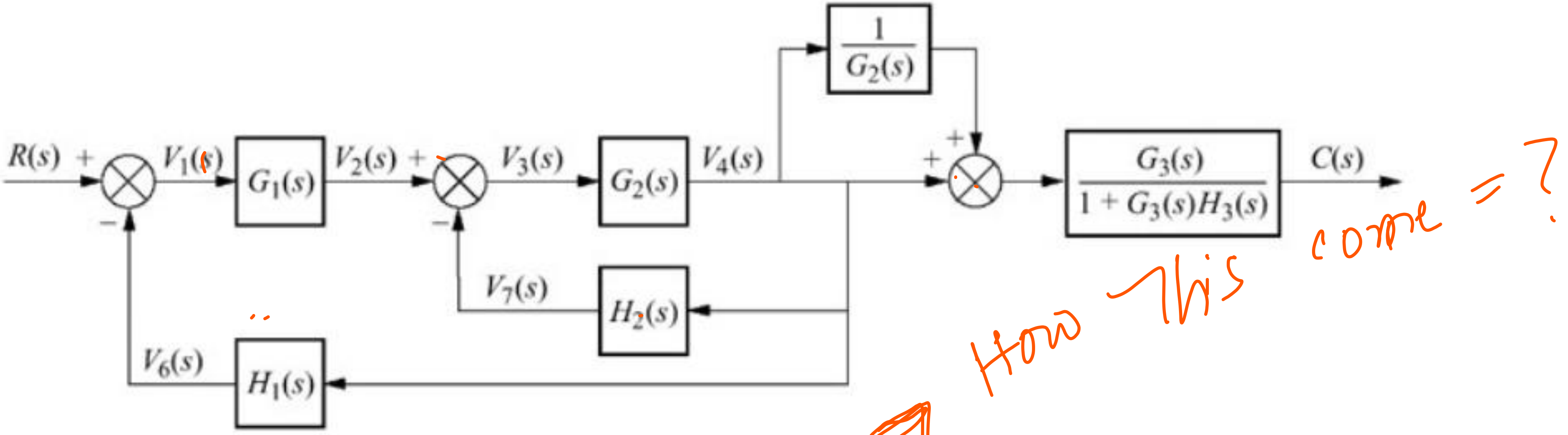
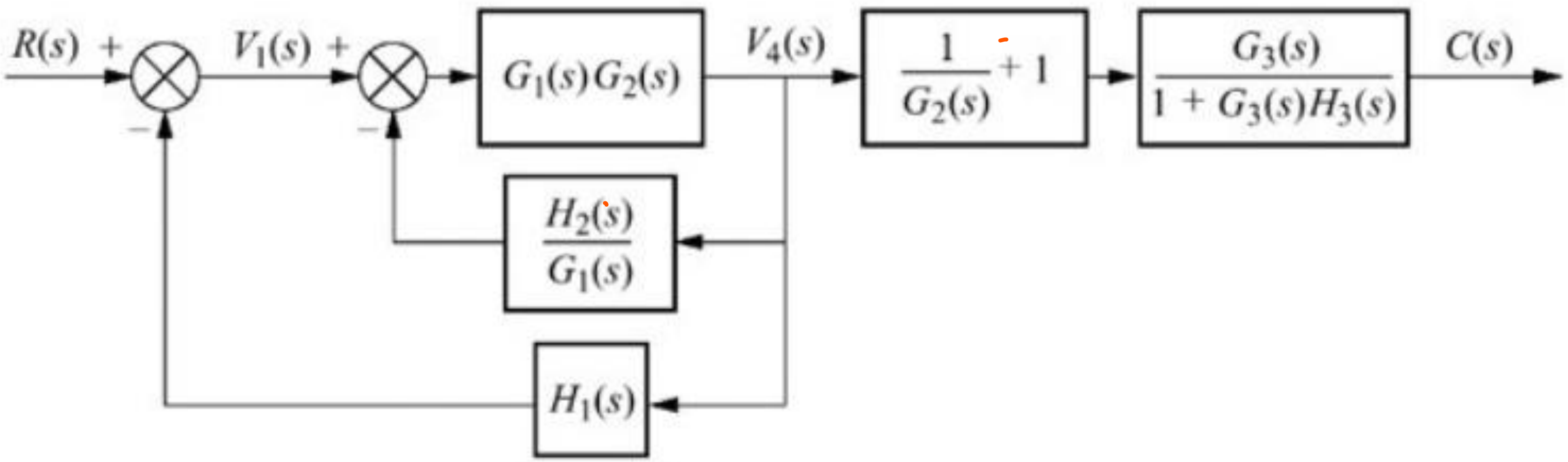


Figure: Example 2

Example 2 - Condensed Solution Part 1



(a)



Example 2 - Condensed Solution Part 2

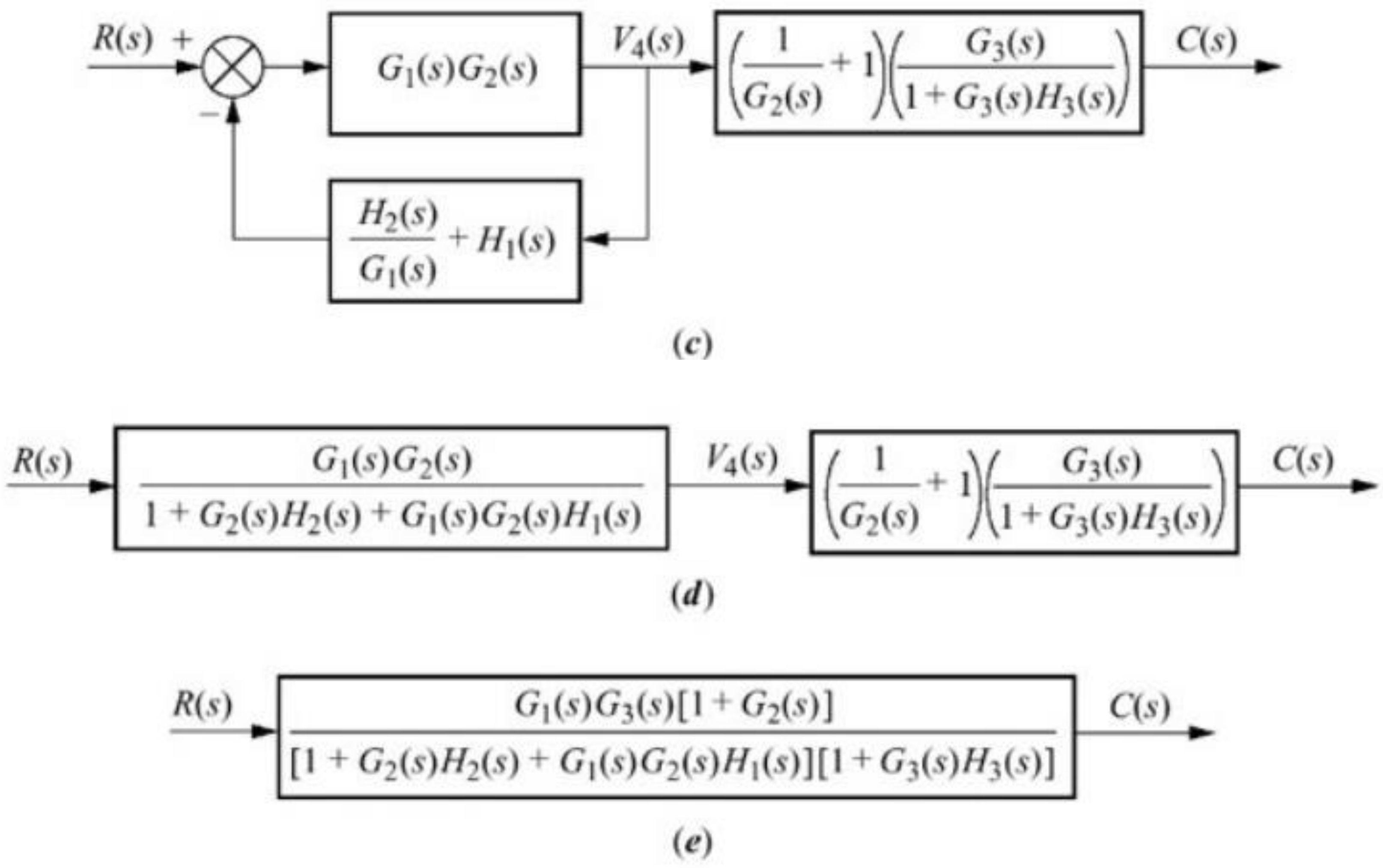


Figure: Example 2 - Final Solution

Try it yourself!