Probability Methods in Engineering

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Lecture 11

Geometric Probability Law

- Independent Bernoulli trials till occurrence of first success
- Success probability p
- $m-1$ trials result in failure
- m^{th} trial results in success
- Probability of such event is

$$p(m) = P\left[A_1^c A_2^c \ldots A_{m-1}^c A_m\right] = (1-p)^{m-1} p$$

Geometric Probability Law (cont.)

- If more than K trials required before a success
  - Probability of performing at least K trials before a success

$$P[\{m > K\}] = q^K$$

Examples

- What is the probability that the coin has to be flipped i) 4 times ii) more than 4 times, for getting heads for the first time? The probability of heads is 0.6 and the probability of tails is 0.4.

Examples (cont.)

- Computer A sends a message to computer B over an unreliable radio link. The message is encoded so that B can detect when errors have been introduced into the message during transmission. If B detects an error, it requests A to retransmit it. If the probability of a message transmission error is 0.1, what is the probability that a message needs to be transmitted i) twice ii) more than two times?

Sequences of Dependent Experiments

- Sequence or "chain" of subexperiments
- Outcome of a given subexperiment determines
  - Which subexperiment is performed next

$$P[\{s_0\} \cap \{s_1\} \cap \{s_2\}] = P[\{s_2\} | \{s_0\} \cap \{s_1\}].P[\{s_0\} \cap \{s_1\}]$$

$$= P[\{s_2\} | \{s_0\} \cap \{s_1\}].P[\{s_1\} | \{s_0\}].P[\{s_0\}]$$

- If next subexperiment depends only on last outcome
  - The sequence is called "Markov chain"

$$P[\{s_0\} \cap \{s_1\} \cap \{s_2\}] = P[\{s_2\} | \{s_1\}].P[\{s_1\} | \{s_0\}].P[\{s_0\}]$$

- Simplify notation by removing braces

Examples (cont.)

- A sequential experiment involves repeatedly drawing a ball from one of two urns, noting the number on the ball, and replacing the ball in its urn. Urn 0 contains a ball with the number 1 and two balls with the number 0, and urn 1 contains five balls with the number 1 and one ball with the number 0.The urn from which the first draw is made is selected at random by flipping a fair coin. Urn 0 is used if the outcome is heads and urn 1 if the outcome is tails. Thereafter the urn used in a subexperiment corresponds to the number on the ball selected in the previous subexperiment. Draw the corresponding trellis diagram.

Examples (cont.)

- Find the probability of the sequence 0011 for the urn experiment