



Probability Methods in Engineering

Lecture 2

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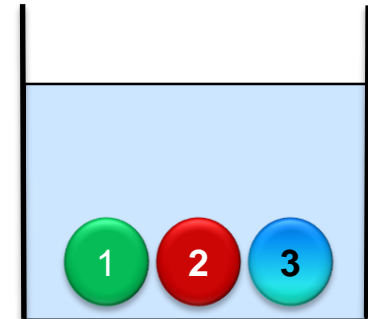
Properties of Relative Frequency

- Number of occurrences of an outcome in n trials
 - ❑ A number between zero and n
 - ❑ Also called frequency

$$0 \leq N_k(n) \leq n$$

- Relative frequencies are
 - ❑ A number between zero and one
 - ❑ Divide the above equation by n to get

$$0 \leq f_k(n) \leq 1$$





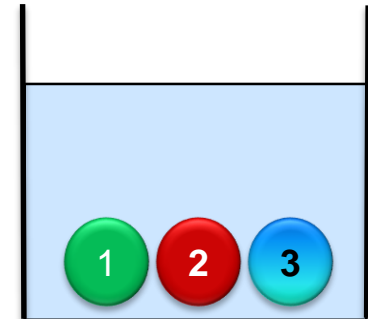
Properties of Relative Frequency (cont.)

- Sum of number of occurrences of all possible outcomes
 - ❑ Must be n
 - ❑ n , sum all frequencies

$$\sum_{k=1}^K N_k(n) = n$$

- Sum of all relative frequencies
 - ❑ Must be 1

$$\sum_{k=1}^K f_k(n) = 1$$

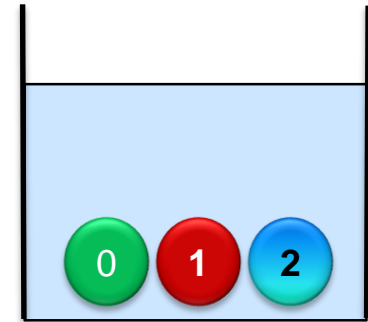




Properties of Relative Frequency (cont.)

- E is an event that the outcome is even

$$f_E(n) = f_0(n) + f_2(n)$$



- If C is an event such that, "either A or B occurs" but simultaneous occurrence not possible

$$f_C(n) = f_A(n) + f_B(n)$$



Axioms of Probability

- Axiom
 - Universally accepted principle or rule
- If A and B are two events from a sample space S
- In modern theory of probability, probability of event A

$$0 \leq P[A] \leq 1$$

- Probability of S

$$P[S] = 1$$

- If A and B cannot occur simultaneously

$$P[A \text{ or } B] = P[A] + P[B]$$



Problems

- Consider the following three random experiments:
 - ❑ Experiment 1: Toss a coin.
 - ❑ Experiment 2: Roll a die.
 - ❑ Experiment 3: Select a ball at random from an urn containing balls numbered 0 to 9.

- a) Specify the sample space of each experiment.
- b) Find the relative frequency of each outcome in each of the above experiments in a large number of repetitions of the experiment. Explain your answer.



Problems (cont.)

- Explain how the following experiments are equivalent to random urn experiments:
 - a) Flip a fair coin twice.
 - b) Toss a pair of fair dice.
 - c) Draw two cards from a deck of 52 distinct cards, with replacement after the first draw; without replacement after the first draw.



Problems (cont.)

- Explain under what conditions the following experiments are equivalent to a random coin toss. What is the probability of heads in the experiment?
- a) Observe a pixel (dot) in a scanned black-and-white document.
 - b) Receive a binary signal in a communication system.
 - c) Test whether a device is working.
 - d) Determine whether your friend Joe is online.
 - e) Determine whether a bit error has occurred in a transmission over a noisy communication channel.



Problems (cont.)

- An urn contains three electronically labelled balls with labels 00, 01, 10. Lisa, Homer, and Bart are asked to characterize the random experiment that involves selecting a ball at random and reading the label. Lisa's label reader works fine; Homer's label reader has the most significant digit stuck at 1; Bart's label reader's least significant digit is stuck at 0.
 - a) What is the sample space determined by Lisa, Homer, and Bart?
 - b) What are the relative frequencies observed by Lisa, Homer, and Bart in a large number of repetitions of the experiment?



Problems (cont.)

- A random experiment has sample space $S = \{1, 2, 3, 4\}$ with probabilities $p_1 = 1/2$, $p_2 = 1/4$, $p_3 = 1/8$ and $p_4 = 1/8$.
- a) Describe how this random experiment can be simulated using tosses of a fair coin.
 - b) Describe how this random experiment can be simulated using an urn experiment.
 - c) Describe how this experiment can be simulated using a deck of 52 distinct cards.



Problems (cont.)

- A random experiment consists of selecting two balls in succession from an urn containing two black balls and one white ball.
- a) Specify the sample space for this experiment.
 - b) Suppose that the experiment is modified so that the ball is immediately put back into the urn after the first selection. What is the sample space now?
 - c) What is the relative frequency of the outcome (white, white) in a large number of repetitions of the experiment in part a? In part b?
 - d) Does the outcome of the second draw from the urn depend in any way on the outcome of the first draw in either of these experiments?



Problems (cont.)

- Let A be an event associated with outcomes of a random experiment, and let the event B be defined as "event A does not occur." Show that

$$f_B(n) = 1 - f_A(n)$$



Problems (cont.)

- Let A , B , and C be events that cannot occur simultaneously as pairs or triplets, and let D be the event "A or B or C occurs." Show that

$$f_D(n) = f_A(n) + f_B(n) + f_C(n)$$



Problems (cont.)

Write the sample space S for the following random experiments.

- a. We toss a coin until we see two consecutive tails. We record the total number of coin tosses.
- b. A bag contains 4 balls: one is red, one is blue, one is white, and one is green. We choose two distinct balls and record their color in order.
- c. A customer arrives at a bank and waits in the line. We observe T , which is the total time (in hours) that the customer waits in the line. The bank has a strict policy that no customer waits more than 20 minutes under any circumstances.