Computer Security

Lecture 09: Public Key Cryptography

Prof. Dr. Sadeeq Jan

Department of Computer Systems Engineering
University of Engineering and Technology Peshawar

Lecture Outline

- Public Key Cryptography
- RSA

Public Key Cryptography

Private-Key Cryptography

- traditional **private/secret/single key** cryptography uses **one** key
- shared by both sender and receiver
- if this key is disclosed communications are compromised
- also is **symmetric**, parties are equal
- hence does not protect sender from receiver forging a message & claiming is sent by sender

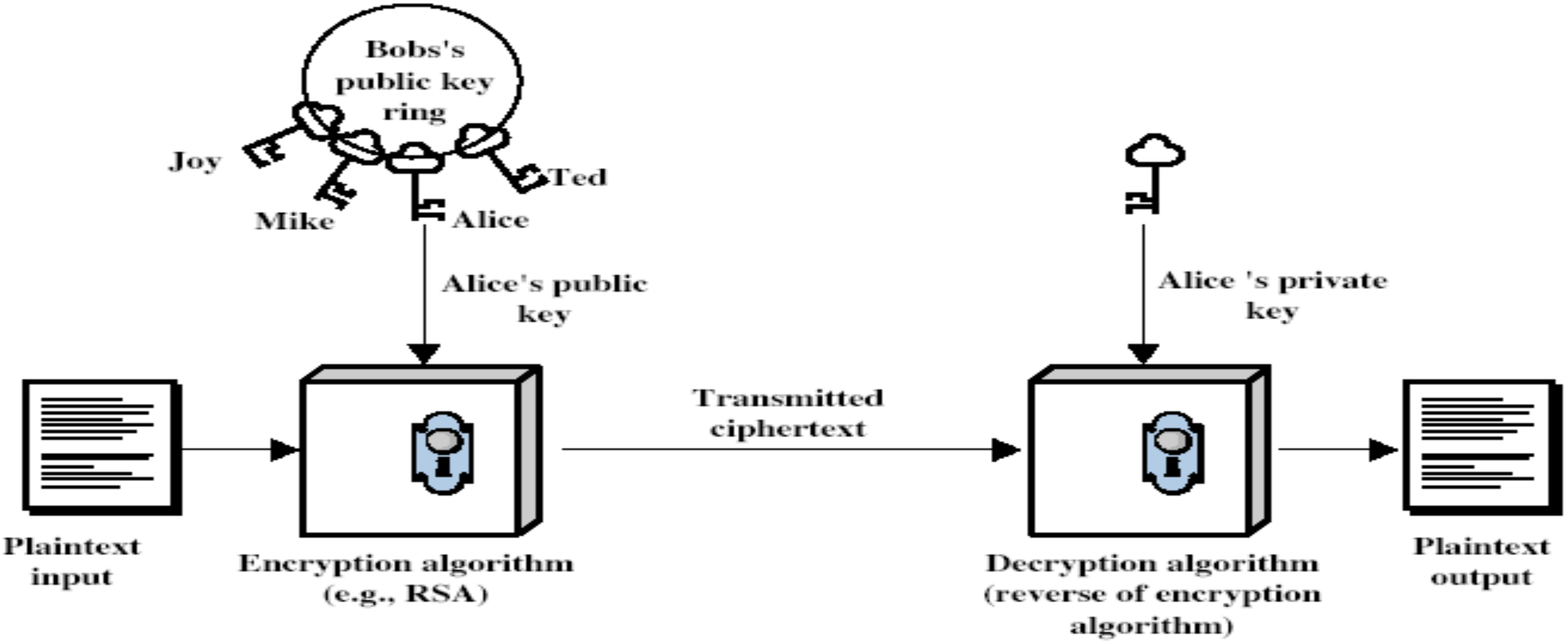
Public-Key Cryptography

- probably most significant advance in the 3000 year history of cryptography
- uses **two** keys – a public & a private key
- **asymmetric** since parties are **not** equal
- uses clever application of number theoretic concepts to function
- complements **rather than** replaces private key crypto

Public-Key Cryptography

- **public-key/two-key/asymmetric** cryptography involves the use of **two** keys:
  - a **public-key**, which may be known by anybody, and can be used to **encrypt messages**, and **verify signatures**
  - a **private-key**, known only to the recipient, used to **decrypt messages**, and **sign** (create) **signatures**
- is **asymmetric** because
  - those who encrypt messages or verify signatures **cannot** decrypt messages or create signatures

Public-Key Cryptography



Why Public-Key Cryptography?

- developed to address two key issues:
  - **key distribution** – how to have secure communications in general without having to trust a KDC with your key
  - **digital signatures** – how to verify a message comes intact from the claimed sender
- public invention due to Whitfield Diffie & Martin Hellman at Stanford Uni in 1976
  - known earlier in classified community

Public-Key Characteristics

- Public-Key algorithms rely on two keys with the characteristics that it is:
  - computationally infeasible to find decryption key knowing only algorithm & encryption key
  - computationally easy to en/decrypt messages when the relevant (en/decrypt) key is known
  - either of the two related keys can be used for encryption, with the other used for decryption (in some schemes)

Public-Key Cryptosystems

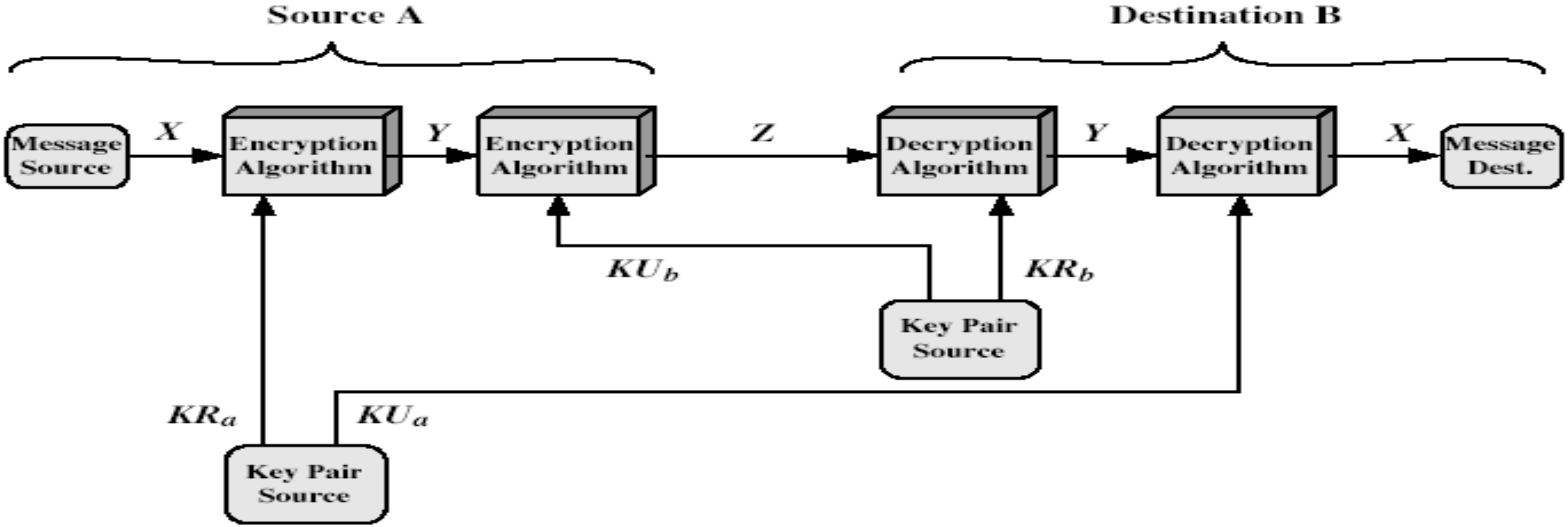


Figure 9.4 Public-Key Cryptosystem: Secrecy and Authentication

Public-Key Applications

- can classify uses into 3 categories:
  - **encryption/decryption** (provide secrecy)
  - **digital signatures** (provide authentication)
  - **key exchange** (of session keys)
- some algorithms are suitable for all uses, others are specific to one

Security of Public Key Schemes

- like private key schemes brute force **exhaustive search** attack is always theoretically possible
- but keys used are too large (>512bits)
- security relies on a **large enough** difference in difficulty between **easy** (en/decrypt) and **hard** (cryptanalyse) problems
- more generally the **hard** problem is known, its just made too hard to do in practise
- requires the use of **very large numbers**
- hence is **slow** compared to private key schemes

RSA

RSA

- by Rivest, Shamir & Adleman of MIT in 1977
- best known & widely used public-key scheme
- based on exponentiation in a finite (Galois) field over integers modulo a prime
- uses large integers (eg. 1024 bits)
- security due to cost of factoring large numbers

RSA Key Setup

- each user generates a public/private key pair by:
- selecting two large primes at random - p, q
- computing their system modulus N=p.q
  - note ø(N)=(p-1)(q-1)
- selecting at random the encryption key e
  - where 1<e<ø(N), gcd(e,ø(N))=1
- solve following equation to find decryption key d
  - e.d=1 mod ø(N) and 0≤d≤N
- publish their public encryption key: KU={e,N}
- keep secret private decryption key: KR={d,N}

7 . d = 1 mod ø(N)

7 . 23 mod ø(N)

161 mod ø(N) 160

= 1

RSA Use

- to encrypt a message M the sender:
  - obtains **public key** of recipient KU={e,N}
  - computes: $C=M^e \bmod N$, where $0 \le M < N$
- to decrypt the ciphertext C the owner:
  - uses their private key KR={d,p,q}
  - computes: $M=C^d \bmod N$
- note that the message M must be smaller than the modulus N (block if needed)
  - $C = M^e \bmod n$
  - $M = C^d \bmod n = (M^e)^d \bmod n = M^{ed} \bmod n$

Key Generation

Select p, q	p and q both prime, $p \neq q$
Calculate $n = p \times q$	
Calculate $\phi(n) = (p-1)(q-1)$	
Select integer e	$\gcd(\phi(n), e) = 1;\ 1 < e < \phi(n)$
Calculate d	$d \equiv e^{-1} \pmod{\phi(n)}$
Public key	$PU = \{e, n\}$
Private key	$PR = \{d, n\}$

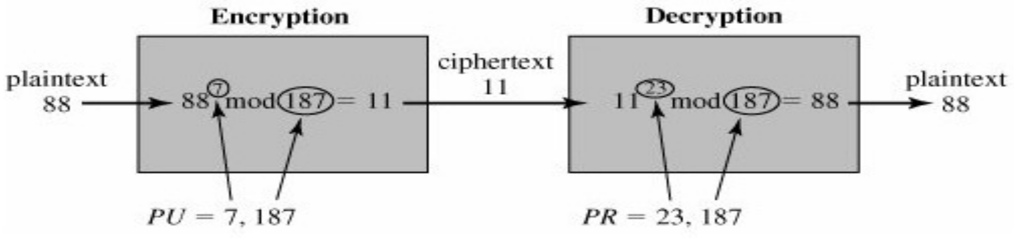
Encryption

Plaintext:	$M < n$
Ciphertext:	$C = M^e \bmod n$

Decryption

Ciphertext:	C
Plaintext:	$M = C^d \bmod n$

Figure 9.6. Example of RSA Algorithm



RSA Example

- Select two primes: $p=17$ & $q=11$
- Compute $n = pq = 17\times 11 = 187$
- Compute $\varnothing(n) = (p-1)(q-1) = 16\times 10 = 160$
- Select e : $\gcd(e,160)=1$; choose $e=7$

(Select e such that e is relatively prime to ø(n) = 160 and less than ø(n) we choose e=7)

- Determine d: $de=1 \mod 160$ and $d < 160$ Value is $d=23$ since $23\times 7=161= 10\times 160+1$
- Publish public key $KU=\{7,187\}$
- Keep secret private key $KR=\{23,187\}$

RSA Example cont

- sample RSA encryption/decryption is:
- given message M = 88 (nb. 88<187)
- encryption:
  - $C = 88^7 \bmod 187 = 11$
- decryption:
  - $M = 11^{23} \bmod 187 = 88$

$M^e \bmod N$

$C = M^e \bmod N$

$= 88^7 \bmod 187 =$

Exponentiation

- can use the Square and Multiply Algorithm
- a fast, efficient algorithm for exponentiation
- concept is based on repeatedly squaring base
- and multiplying in the ones that are needed to compute the result
- look at binary representation of exponent
  - eg. $7^5 = 7^4.7^1 = 3.7 = 10 \bmod 11$
  - eg. $3^{129} = 3^{128}.3^1 = 5.3 = 4 \bmod 11$

RSA Key Generation

- users of RSA must:
  - determine two primes at random - p, q
  - select either e or d and compute the other
- primes p,q must not be easily derived from modulus N=p.q
  - means must be sufficiently large
- exponents e, d are inverses, so use Inverse algorithm to compute the other

RSA Security

- Three approaches to attacking RSA:
  - brute force key search (infeasible given size of numbers)
  - mathematical attacks (based on difficulty of computing ø(N), by factoring modulus N)
  - timing attacks (on running of decryption)

8

41
187) 7744
748
264
187
77

77 × 77 × 77 + 88
231 + 88 = 319
319 mod 187
1

1
187) 319
187
132

Summary

- have considered:
  - principles of public-key cryptography
  - RSA algorithm, implementation, security

END