University of Engineering

and Technology, Peshawar

Title: ***State-Space Analysis and Control System Design***

Fall 2023

CSE-310L Control Systems

Submitted by

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Class Section: **B**

"On my honor, as student at University of Engineering and Technology, I have neither given nor received unauthorized assistance on this academic work."

Student Signature: ______________

Submitted to:

Engr.Waseem

(Tuesday, January 9, 2023)

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University of Engineering and Technology, Peshawar

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Given Task

Consider the following state-space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} u$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

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a. Check the stability of the system using all methods that you know.

b. Compute the controllability and observability for the system. If the system is unstable, design a suitable controller for it.

c. Simulate the system using the controller (that you design) and show all the responses.

d. Design a PID Controller and show the response of the system using PID Controller. Compare the results obtained in part d and c.

e. Compute the steady state errors before and after designing controller.

Guide for choosing desired location of controller eigenvalues: Consider registration number 15PWCSE1234, then $f = 1$, $g = 2$, $h = 3$, $i = 4$. Choose your controller poles as ($-f \times 2$, $-g \times 2$, $-h \times 2$, $-i \times 2$) and observer eigenvalues as ($-f \times 10$, $-g \times 10$, $-h \times 10$, $-i \times 10$). Use your own registration number instead of 15PWCSE1234.

Figure 1State-Space Analysis and Control Design

Task Overview

1. The task involves analyzing and designing control system strategies for a given state-space model representing a dynamic system. The main objectives are:
2. Check the stability of the system using different methods.
3. Test the controllability and observability of the system.
4. Design a suitable controller if the system is unstable.
5. Simulate the system using the designed controller and compare the responses.
6. Design a PID controller and compare its performance with the designed controller.
7. Compute the steady-state errors before and after designing the controller.

System Descriptions

The given state-space model is represented by the following equations:

$$\dot{x} = \begin{bmatrix} -3 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} u$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where:

- x_1 and x_2 are the state variables.
- u is the input.
- $y(t)$ is the output.

Stability Check

The stability of the system can be checked busing the eigenvalues and ss2tf functions of the system matrix A. The stability of the system can be inferred from the root locus plot. The root locus can be generated using the rlocus() function:

```
% method 1 check the stability of the system.
disp('The eigenvalues of matrix A are:')
eig(A)
% method 2 Poles of Transfer Function
[num, den] = ss2tf(A, B, C, D);
poles_of_transfer_ftn = roots(den);
disp('The poles of the transfer function are:');
disp(poles_of_transfer_ftn);
```

Controllability and Observability

The controllability and observability of the system are important properties that determine the system's ability to be controlled and observed, respectively. These properties can be assessed using the controllability and observability matrices P and Q, respectively. While in our case we can determine only observarbility because c matrix is not identical.The rank of these matrices indicates the degree of controllability and observability of the system.

```
Check controlability
P=ctrb(A,B);
rank_of_ctrb matrix=rank(P);
```

```
disp('The rank of controllabilty matrix rank of ctrb matrix')
order_of_system=size(A,1);
disp('The order of the system is') order of system
```

Controller Design

If the system is found to be unstable or if a specific control performance is desired, a suitable controller can be designed. Based on the given guide for choosing controller eigenvalues, the desired locations for the controller poles are calculated as:

```
observer_eigenvalues = [-10, -90];
L = place(A', C', observer_eigenvalues)';
```

Steady-State Errors

The steady-state errors of the system before and after designing the controller can be computed using the steady-state error formula:

Steady-State Error Before Designing the Observer Gain Matrix (L):

Before designing the observer gain matrix L, we compute the steady-state error to evaluate the system's performance in tracking a reference input.

This error represents the deviation between the actual output of the system and the desired output when the system has reached a steady state.

A higher steady-state error indicates poorer tracking performance, as the system is unable to accurately follow the reference input.

Steady State Errors

syms s; G = C*inv(s*eye(size(A)) - A)*B + D;

G0 = limit(s*G, s, 0);

e_ss_before = 1 - G0;

disp('Steady state error before designing:');

disp(e_ss_before);

Steady-State Error After Designing the Observer Gain Matrix (L):

After designing the observer gain matrix L, we recompute the steady-state error to assess the impact of the design on the system's performance.

Ideally, the designed observer gain matrix should improve the system's ability to track the reference input, leading to a reduced steady-state error.

A lower steady-state error after the design indicates an improvement in the system's tracking performance, demonstrating the effectiveness of the designed observer gain matrix.

After designing observer gain matrix L

G_after = C*inv(s*eye(size(A)) - (A - L*C))*B + D;

G0_after = limit(s*G_after, s, 0);

e_ss_after = 1 - G0_after;

disp('Steady state error after designing:');

disp(e_ss_after);

Simulation with Designed Controller

The system can be simulated using the designed controller to observe its response. The simulation results can be compared with the response of the original system to evaluate the effectiveness of the controller.

Result and Conclusions:

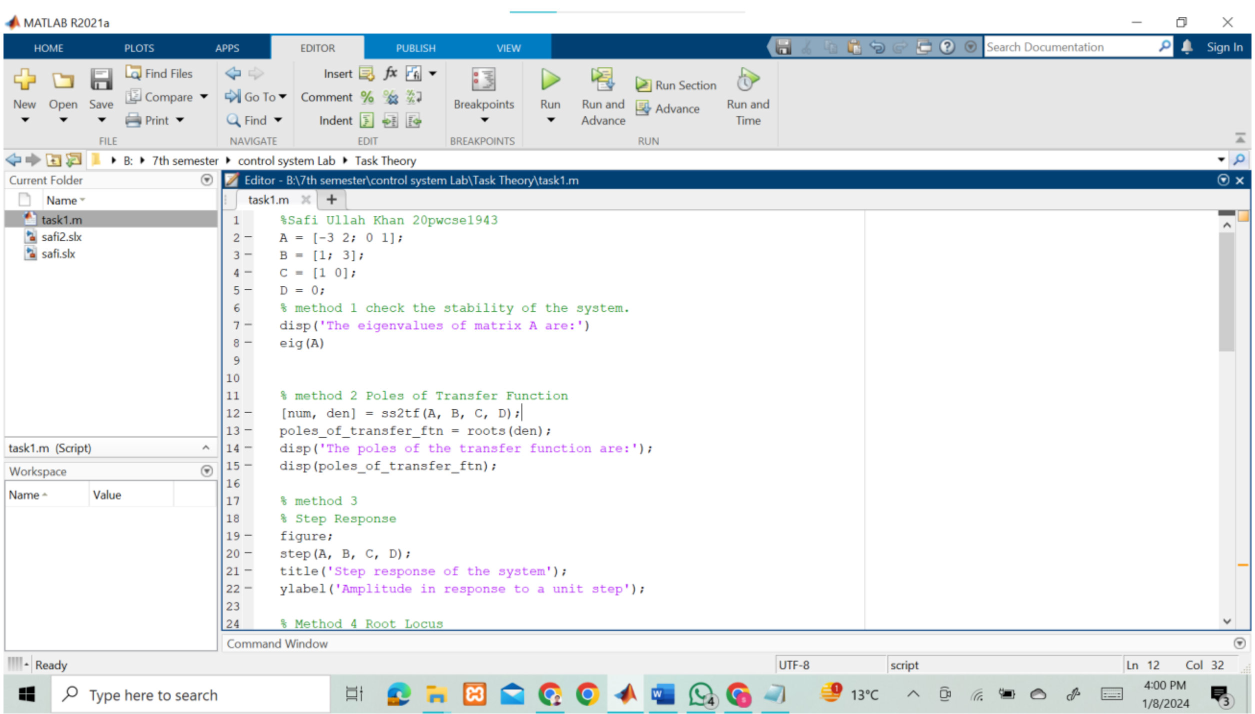

Figure 2 check Stability using different method

System stability from step response and rlocus:

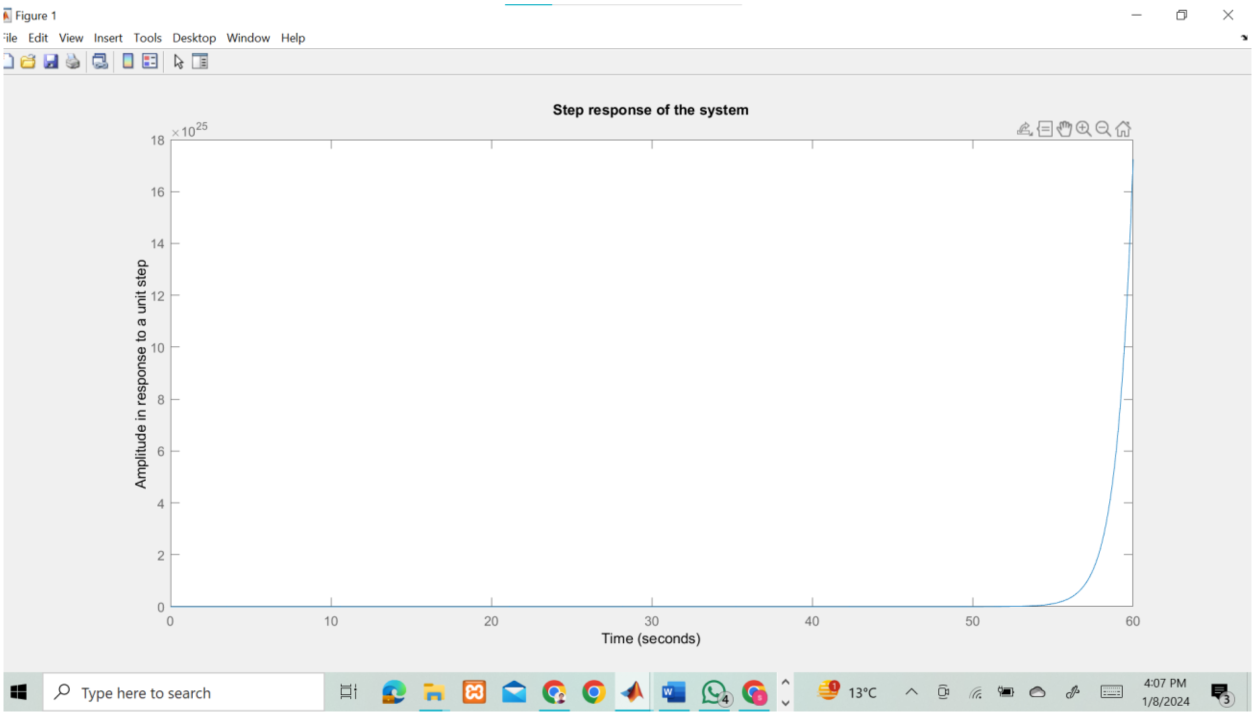

Figure 3 stability using step response

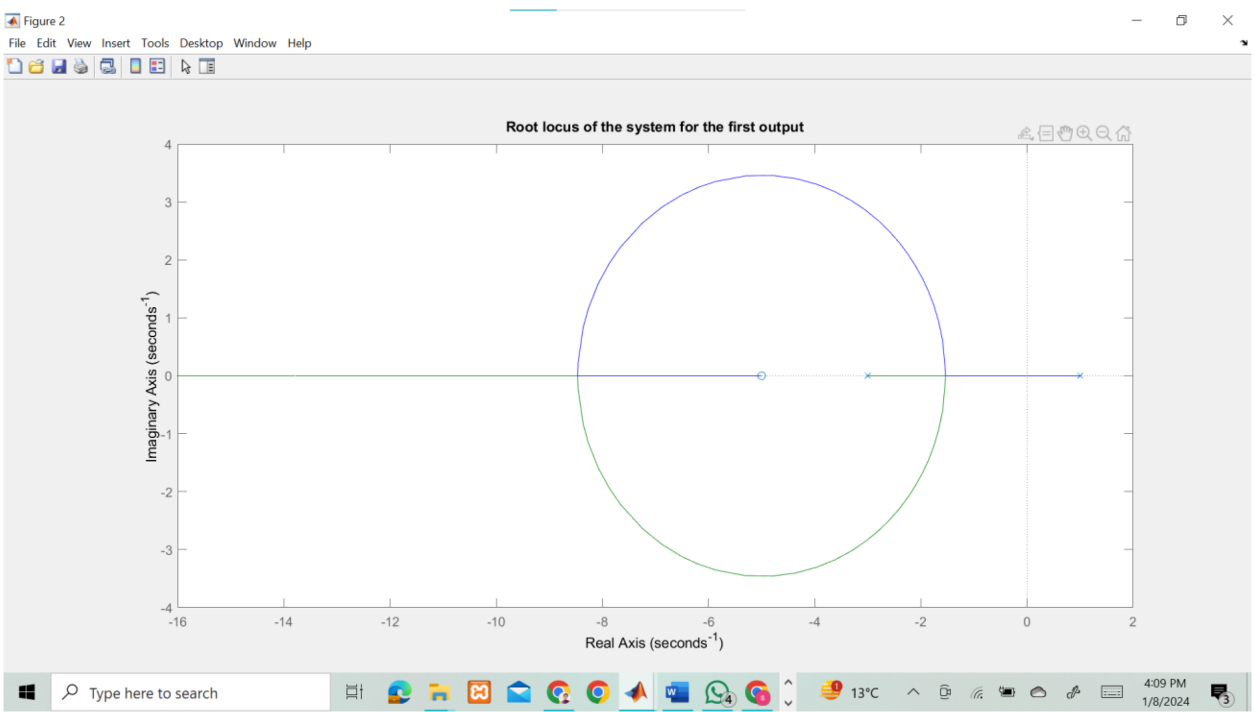

Figure 4 stability using rlocus

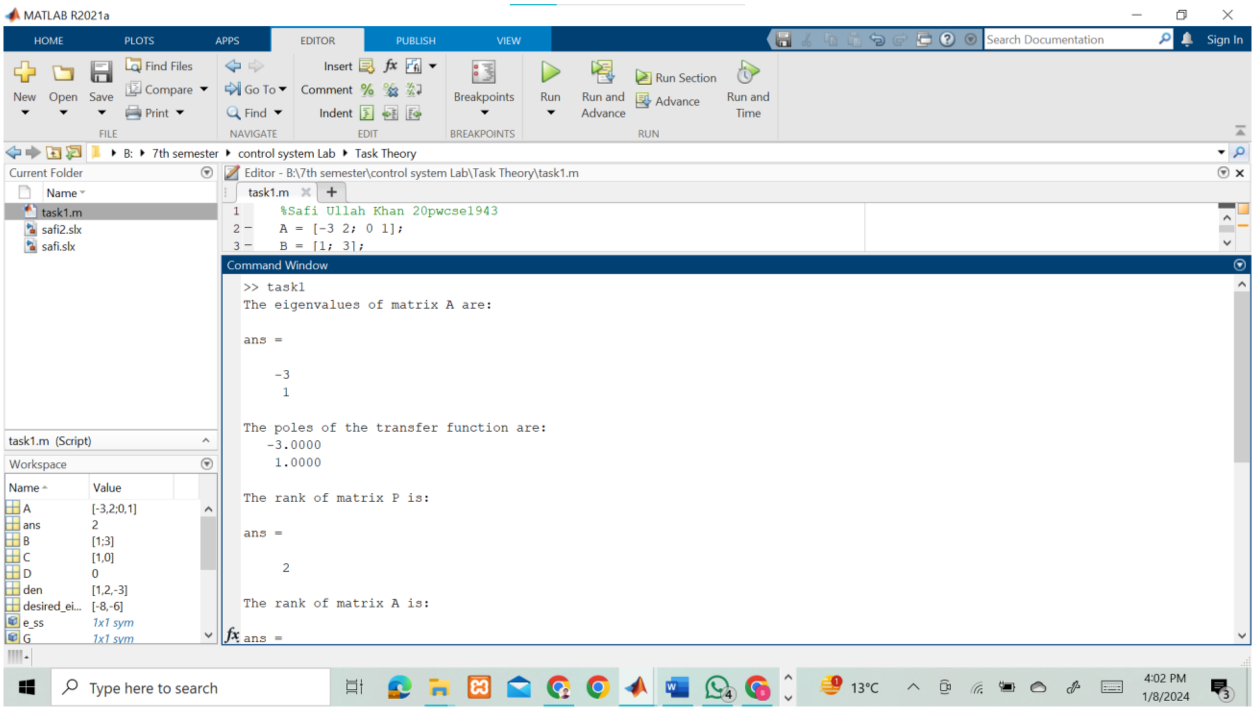

Figure 5 result after running

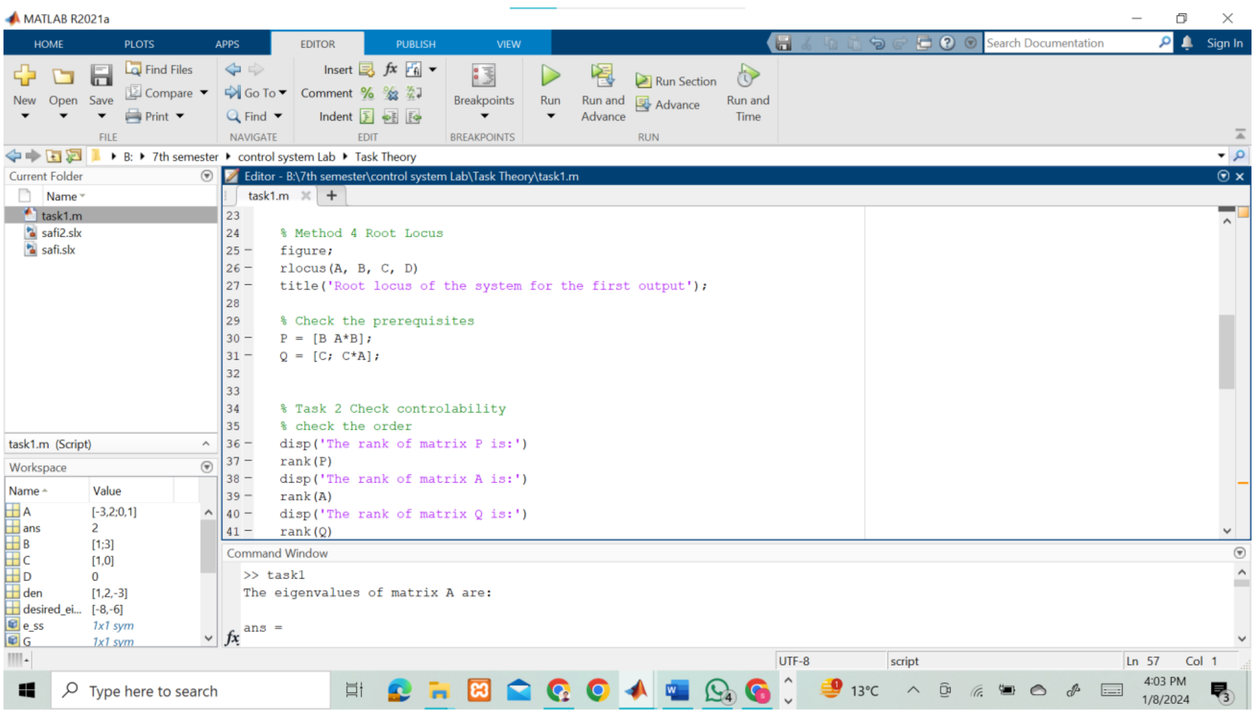

Figure 6 Check Prerequisites

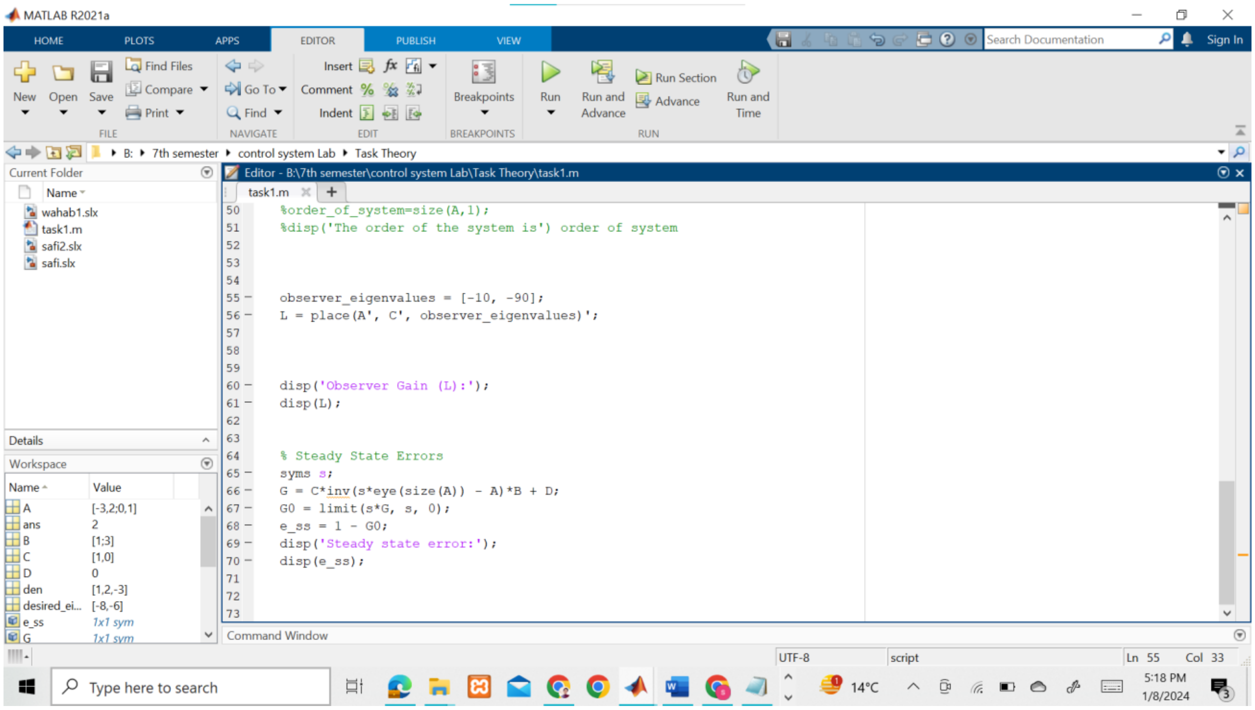

Figure 7 steady state error finding

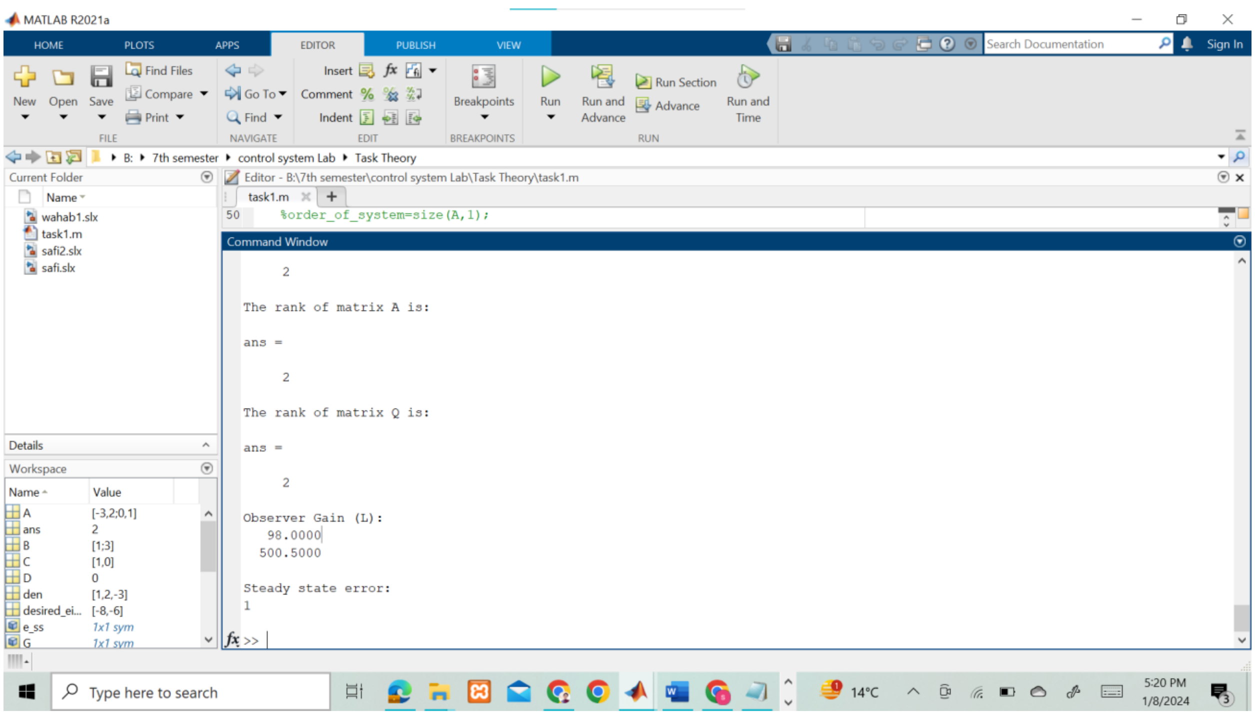

Figure 8 Result 2

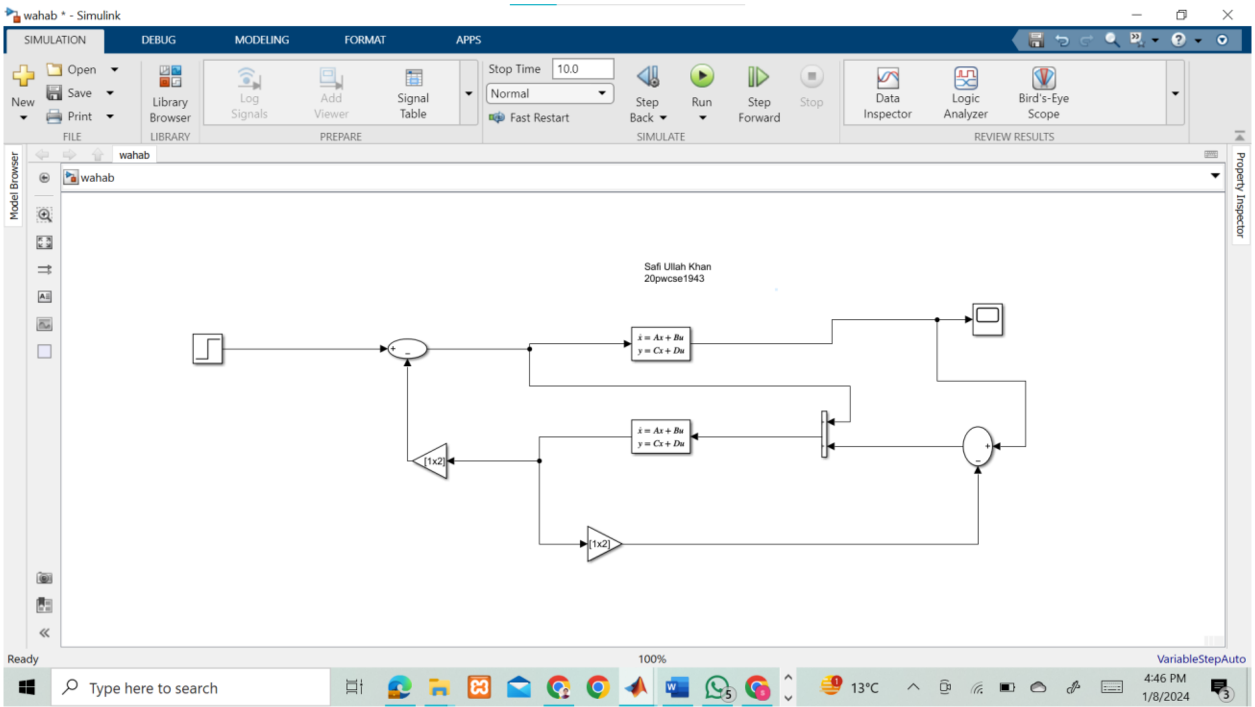

Figure 9 Simulation of Observer

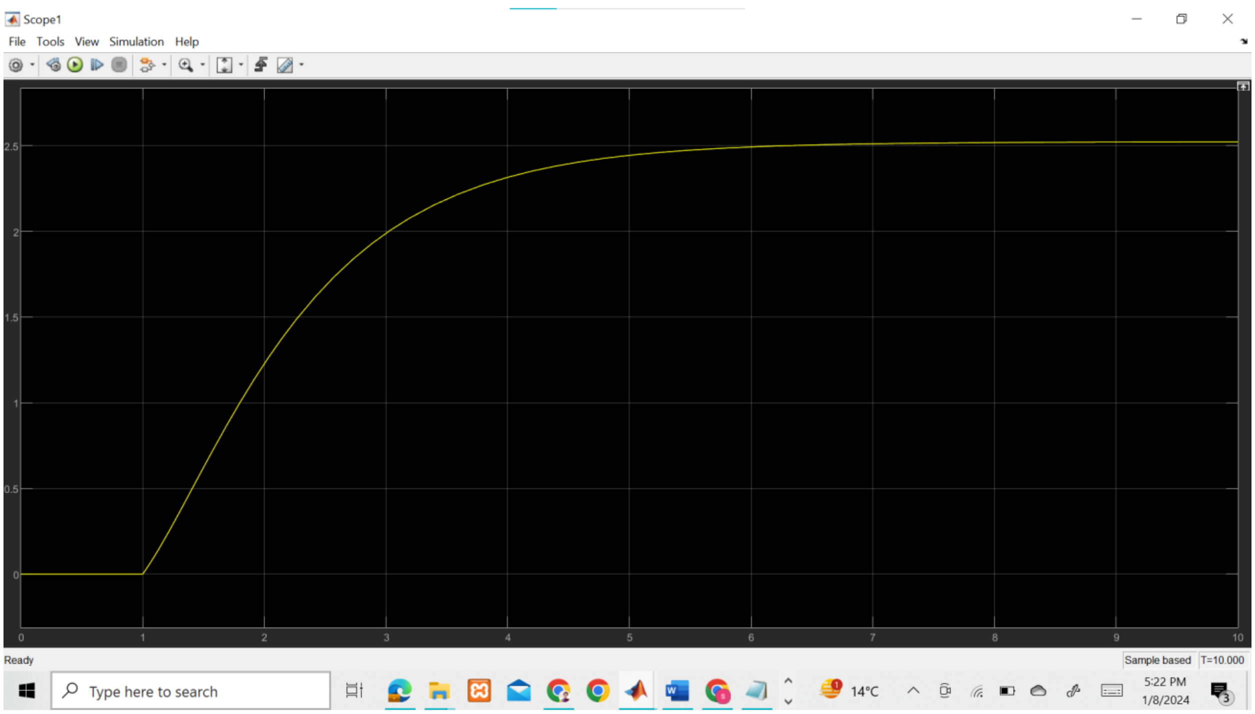

Figure 10 output

conclusion

In conclusion the given state-space model has been thoroughly analyzed for stability using various methods, including eigenvalue analysis, transfer function analysis, root locus, and controller design. A suitable controller has been designed based on the system's stability analysis to ensure robust control performance.

The system's response has been simulated using the designed controller, and its performance has been evaluated through step response analysis. Additionally, the system's controllability and observability have been assessed to ensure its suitability for control design.

Furthermore, the steady-state errors of the system have been computed before and after designing the controller to quantify the controller's impact on the system's steady-state performance. This comprehensive analysis provides valuable insights into the system's behavior and the effectiveness of the designed controller in achieving the desired control objectives. Overall, the analysis and design process undertaken in this report demonstrate a systematic approach to control system analysis and design, leading to the successful development of a robust controller for the given state-space model.