Probability Methods in Engineering

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Lecture 10

Discrete Distribution

- A discrete distribution is a probability distribution that depicts the occurrence of discrete (individually countable) outcomes, such as 1, 2, 3...
- A discrete probability distribution counts occurrences that have countable or finite outcomes.
- Common examples of discrete distribution include
  - Bernoulli distributions.
  - Binomial distributions.
  - Poisson distributions.

Bernoulli Trial

- **Bernoulli trial** is a random experiment with exactly two possible outcomes, "success" and "failure".
- A **Bernoulli trial** is one of the simplest experiments you can conduct. It's an experiment where you can have one of two possible outcomes. For example, "Yes" and "No" or "Heads" and "Tails."
- **Coin tosses**: record how many coins land heads up and how many land tails up.
- **Rolling Dice**: the probability of a roll of two die resulting in a double six.

- Bernoulli trial is that each action must be independent. That means the probabilities must remain the same throughout the trials; each event must be completely separate and have nothing to do with the previous event.

Source: *http://www.zweigmedia.com/RealWorld/Summary6.html*

Bernoulli Trial

- Experiment with only two outcomes
- Either success or failure
  - Flip a coin
  - Take a penalty shot on goal
  - Test a randomly selected circuit to see whether it is defective
  - Roll a die and determine whether it is a 6 or not
  - Determine whether there was flooding this year at Warsak

Source: *http://www.zweigmedia.com/RealWorld/Summary6.html*

Bernoulli Trial

- **Example:** Eight balls are drawn from a bag containing 10 white and 10 black balls. Predict whether the trials are Bernoulli trials if the ball drawn is replaced and not replaced.

Source: *http://www.zweigmedia.com/RealWorld/Summary6.html*

Binomial Process

- A binomial distribution can be thought of as simply the probability of a SUCCESS or FAILURE outcome in an experiment or survey that is repeated multiple times.
- The binomial is a type of distribution that has two possible outcomes (the prefix "bi" means two, or twice).
- Binomial Random Variable

- $S_X = \{0, 1, 2, \ldots, n\}$
- $P_k = C^n_k \, p^k \, q^{n-k}$

- **b(x; n, P)**
- Where:
  b = binomial probability
  k = total number of "successes" (pass or fail, heads or tails etc.)
  P = probability of a success on an individual trial
  n = number of trials

Source: *http://www.zweigmedia.com/RealWorld/Summary6.html*

The Binomial Probability Law

- Sequence of independent Bernoulli trials
  - k number of successes
  - n number of independent Bernoulli trials
- k successes in n trials
- Probabilities of k given by binomial probability law

$$p_n(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

- Graphical representation online tool

http://www.zweigmedia.com/RealWorld/stats/bernoulli.html

The Binomial Probability Law (cont.)

- Properties:
  - 2^n possible outcomes of experiments with n Bernoulli trials
  - Binomial probabilities sum to 1
  - If $p_n(k)$ is given, $p_n(k+1)$ can be determined as

$$p_n(k+1) = \frac{(n-k)p}{(k+1)(1-p)} p_n(k)$$

Examples

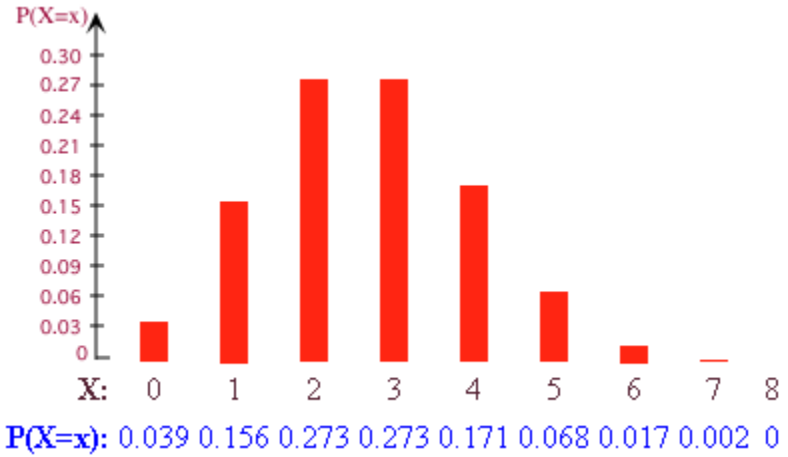
- Suppose that a coin is tossed three times. If we assume that the tosses are independent and the probability of getting a heads is 0.4. Find the probabilities of 0, 1, 2 and 3 heads.

Examples (cont.)

- Verify the previous example using the binomial probability law

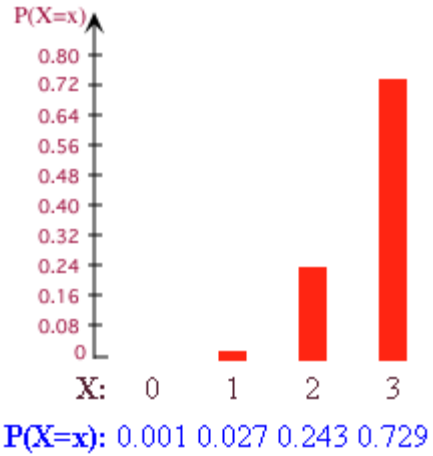
Examples (cont.)

- Let k be the number of active speakers in a group of 8 speakers. Suppose that a speaker is active with probability 1/3. Find the probability that the number of active speakers is greater than 6.



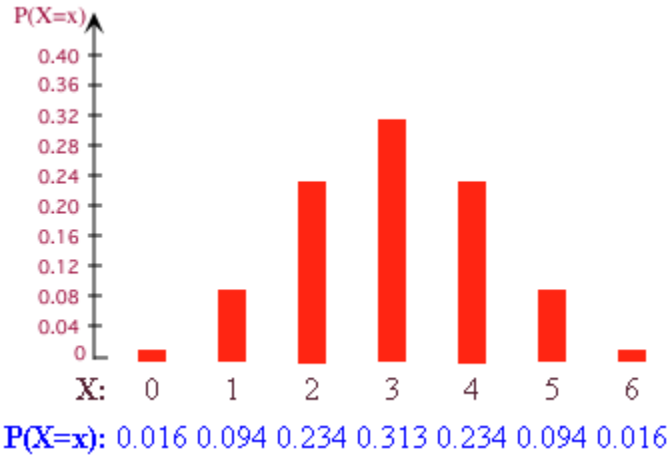
Examples (cont.)

- A communication system transmits binary information over a channel that introduces random bit errors with probability $\varepsilon = 0.1$. The transmitter transmits each information bit three times, and a decoder takes a majority vote of the received bits to decide on what the transmitted bit was. Find the probability that the receiver will make an incorrect decision.



Examples (cont.)

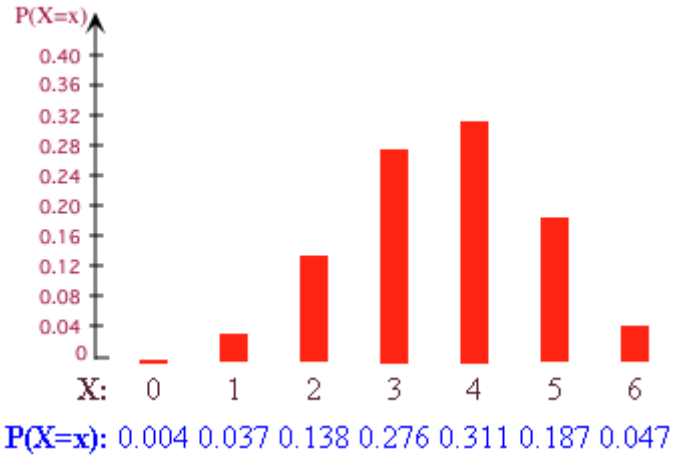
- What is the probability of getting heads exactly twice if you flip a fair coin 6 times?



Source: *http://www.zweigmedia.com/ThirdEdSite/tutstats/frames8_2.html*

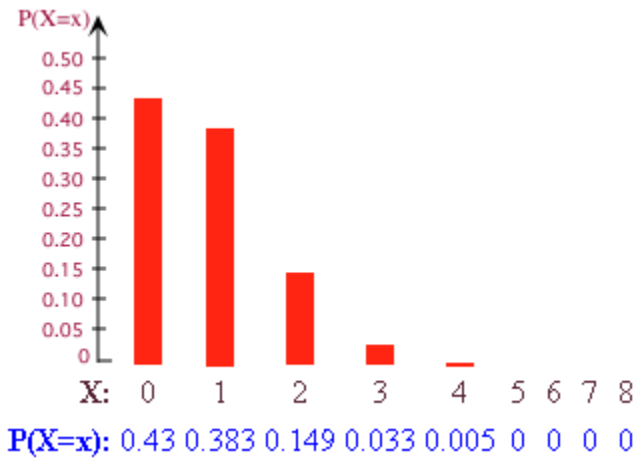
Examples (cont.)

- What is the probability of getting heads more than 4 times if you flip a coin 6 times having 0.6 as the probability of heads?



Examples (cont.)

- A salesman has a 10% chance of persuading a randomly selected person to buy his product. He calls 8 persons. What is the probability that exactly one person is persuaded?



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