



Probability Methods in Engineering

Prepared By
Dr. Safdar Nawaz Khan Marwat
DCSE, UET Peshawar

Lecture 18



Important Discrete RVs

➤ Poisson Random Variable

$$S_X = \{0, 1, 2, \dots\}$$

$$P_k = (\alpha^k/k!)e^{-\alpha}$$

$$E[X] = \alpha, \text{VAR}[X] = \alpha$$

- Counting number of occurrences of an event in a time period
- Arises in situations where events occur at random
 - ❑ E.g. counts of emissions from radioactive substances
- Here, α is number of arrivals in time interval of length t
 - ❑ α is unitless and given as $\alpha = \lambda t$
 - ❑ λ is arrival rate with unit of jobs/time (e.g. packets/sec)



Poisson Random Variable

- A Poisson random variable will relatively describe a phenomenon if there are few successes over many trials.
- The Poisson distribution is used as a limiting case of the binomial distribution when the trials are large indefinitely.

➤ Applications

- To count the number of defects of a finished product
- To count the number of deaths in a country by any disease or natural calamity
- To count the number of infected plants in the field
- To count the number of bacteria in the organisms or the radioactive decay in atoms
- To calculate the waiting time between the events.



Examples

- The number N of queries arriving in t seconds at a call centre is a Poisson random variable with $\alpha = \lambda t$ where λ is the average arrival rate in queries/second. Assume that the arrival rate is 4 queries per minute. Find the probability of the following events:
- ☐ 4 queries in 10 seconds
 - ☐ More than 4 queries in 10 seconds
 - ☐ Less than or equal to 5 queries in 2 minutes



Examples (cont.)

- The number N of packet arrivals in t seconds at a multiplexer is a Poisson random variable with $\alpha = \lambda t$ where λ is the average arrival rate in packets/second. Find the probability that there are no packet arrivals in t seconds.



The Cumulative Distribution Function

- The cdf is the probability of event $\{X \leq x\}$

$$F_X(x) = P[X \leq x] = P[\{\zeta : X(\zeta) \leq x\}]$$

- Probability that RV X takes on a value in set $(-\infty, x]$



Examples

- Find the cdf of RV X , such that X is the outcome of rolling a fair dice.



Examples (cont.)

- Find the cdf of RV X , where X is the number of heads in three tosses of a fair coin.