Database Management System

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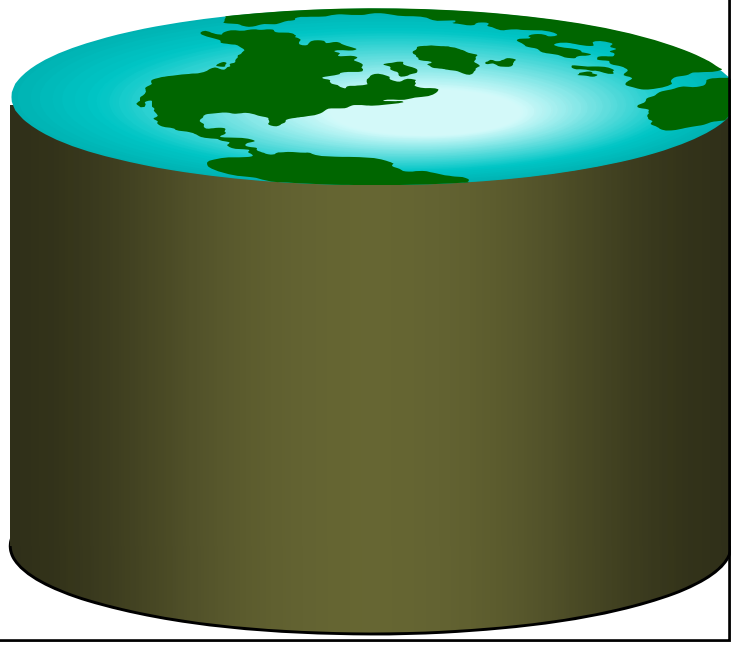
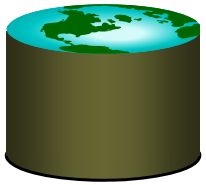




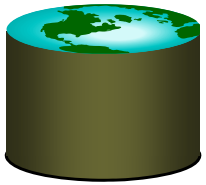
Objectives

- **Relational Algebra**
- **Relational Calculus**

Relational Algebra

By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and, in effect, *increases the mental power of the race.*

-- Alfred North Whitehead (1861 - 1947)



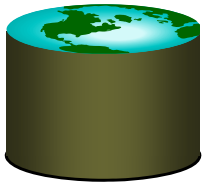
Formal Relational Query Languages

Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:

***Relational Algebra*: More operational, very useful for representing execution plans.**

***Relational Calculus*: Lets users describe what they want, rather than how to compute it. (even more *declarative*.)**

- *Understanding Algebra & Calculus is key to understanding SQL, query processing!*



Example: Joining Two Tables

Enrolled

cid	grade	sid
Carnatic101	C	53666
Reggae203	B	53666
Topology112	A	53650
History105	B	53666

Students

sid	name	login	age	gpa
53666	Jones	jones@cs	18	3.4
53688	Smith	smith@eecs	18	3.2
53650	Smith	smith@math	19	3.8

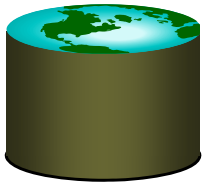
- **Pre-Relational:**
  write a program

- **Relational SQL**
  Select name, cid from students s, enrolled e where s.sid = e.sid

- **Relational Algebra**

$$\pi_{name,cid}(Students \bowtie_{sid} Enrolled)$$

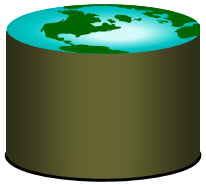
- **Relational Calculus**
  {S | S∈Students ∧ ∃E(E∈Enrolled ∧ E.sid = S.sid)}



Relational Algebra: 5 Basic Operations

- *Selection* (σ) Selects a subset of ***rows*** from relation (horizontal).
- *Projection* (π) Retains only wanted ***columns*** from relation (vertical).
- *Cross-product* (×) Allows us to combine two relations.
- *Set-difference* (—) Tuples in r1, but not in r2.
- *Union* (∪) Tuples in r1 and/or in r2.

Since each operation returns a relation, operations can be *composed!* (Algebra is "closed".)



Example Instances

R1

sid	bid	day
22	101	10/10/96
58	103	11/12/96

S1

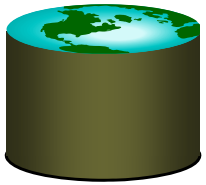
sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Boats

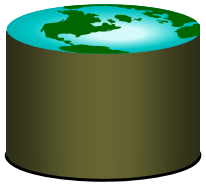
S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0



Projection

- Examples: $\pi_{age}(S2)$; $\pi_{sname,rating}(S2)$
- Retains only attributes that are in the "*projection list*".
- *Schema* of result:
  - exactly the fields in the projection list, with the same names that they had in the input relation.
- Projection operator has to *eliminate duplicates* (How do they arise? Why remove them?)
  - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it. (Why not?)



Projection

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

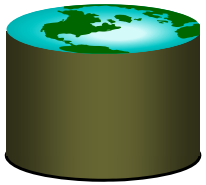
sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

$$\pi_{sname,rating}(S2)$$

age
35.0
55.5

$$\pi_{age}(S2)$$



Selection (σ)

- Selects rows that satisfy *selection condition*.
- Result is a relation.

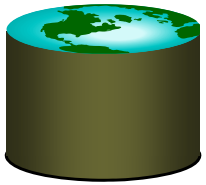
  Schema of result is same as that of the input relation.

- Do we need to do duplicate elimination?

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

sname	rating
yuppy	9
rusty	10

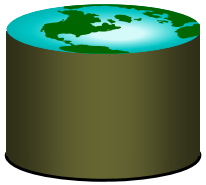
$$\sigma_{rating>8}(S2) \qquad \pi_{sname,rating}(\sigma_{rating>8}(S2))$$



Union and Set-Difference

- All of these operations take two input relations, which must be ***union-compatible***:
  - Same number of fields.
  - `Corresponding' fields have the same type.

- For which, if any, is duplicate elimination required?



Union

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

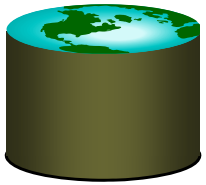
S1

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

$S1 \cup S2$



Set Difference

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

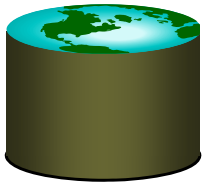
S2

sid	sname	rating	age
22	dustin	7	45.0

$S1 - S2$

sid	sname	rating	age
28	yuppy	9	35.0
44	guppy	5	35.0

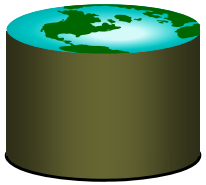
$S2 - S1$



Cross-Product

- S1 $\times$ R1: Each row of S1 paired with each row of R1.
- Q: How many rows in the result?
- *Result schema* has one field per field of S1 and R1, with field names `inherited' if possible.
  - *May have a naming conflict*: Both S1 and R1 have a field with the same name.
  - In this case, can use the *renaming operator*:

$$\rho\,(C(1 \to sid1, 5 \to sid2),\ S1 \times R1)$$



Cross Product Example

sid	bid	day
22	101	10/10/96
58	103	11/12/96

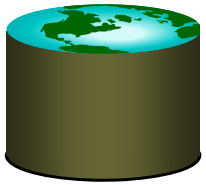
R1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

R1 X S1 =

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/96
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	22	101	10/10/96
31	lubber	8	55.5	58	103	11/12/96
58	rusty	10	35.0	22	101	10/10/96
58	rusty	10	35.0	58	103	11/12/96

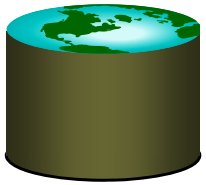


Compound Operator: Intersection

- In addition to the 5 basic operators, there are several additional "Compound Operators"
  - These add no computational power to the language, but are useful shorthands.
  - Can be expressed solely with the basic ops.

- Intersection takes two input relations, which must be ***union-compatible***.
- Q: How to express it using basic operators?

$$R \cap S = R - (R - S)$$



Intersection

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

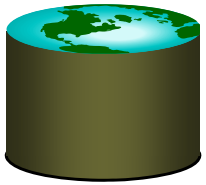
S1

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

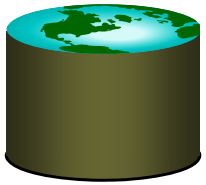
sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

$$S1 \cap S2$$



Compound Operator: Join

- Joins are compound operators involving cross product, selection, and (sometimes) projection.
- Most common type of join is a "*natural join*" (often just called "join"). R⋈ S conceptually is:
  - Compute R × S
  - Select rows where attributes that appear in both relations have equal values
  - Project all unique atttributes and one copy of each of the common ones.
- Note: Usually done much more efficiently than this.
- Useful for putting "normalized" relations back together.



Natural Join Example

sid	bid	day
22	101	10/10/96
58	103	11/12/96

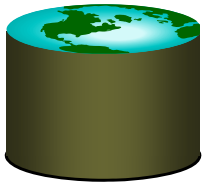
R1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

R1 ⋈ S1 =

sid	sname	rating	age	bid	day
22	dustin	7	45.0	101	10/10/96
58	rusty	10	35.0	103	11/12/96



Other Types of Joins

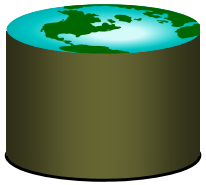
- ***Condition Join (or "theta-join")*:**

$$R \bowtie_C S = \sigma_C (R \times S)$$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	58	103	11/12/96

$$S1 \bowtie_{S1.sid < R1.sid} R1$$

- *Result schema* same as that of cross-product.
- May have fewer tuples than cross-product.
- ***Equi-Join*: Special case: condition *c* contains only conjunction of *equalities*.**

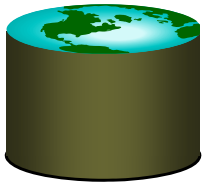


Compound Operator: Division

- Useful for expressing "for all" queries like:
  Find sids of sailors who have reserved all boats.
- For A/B attributes of B are subset of attrs of A.
  - May need to "project" to make this happen.
- E.g., let *A* have 2 fields, *x* and *y*; *B* have only field *y*:

$$A/B = \{\langle x\rangle \mid \forall \langle y\rangle \in B(\exists \langle x, y\rangle \in A)\}$$

***A/B* contains all tuples (x) such that for *every y* tuple in *B*, there is an *xy* tuple in *A*.**



Examples of Division A/B

sno	pno
s1	p1
s1	p2
s1	p3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

A

pno
p2

B1

pno
p2
p4

B2

pno
p1
p2
p4

B3

sno
s1
s2
s3
s4

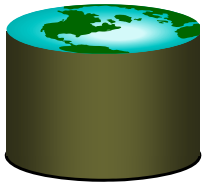
A/B1

sno
s1
s4

A/B2

sno
s1

A/B3

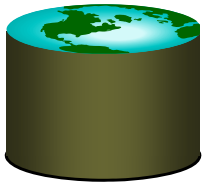


Expressing A/B Using Basic Operators

- **Division is not essential op; just a useful shorthand.**
  - (Also true of joins, but joins are so common that systems implement joins specially.)
- ***Idea*: For *A/B*, compute all *x* values that are not `disqualified' by some *y* value in *B*.**
  - *x* value is *disqualified* if by attaching *y* value from *B*, we obtain an *xy* tuple that is not in *A*.

Disqualified x values: $\pi_x((\pi_x(A) \times B) - A)$

A/B: $\pi_x(A)$ − Disqualified x values



Examples

Reserves

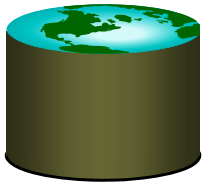
sid	bid	day
22	101	10/10/96
58	103	11/12/96

Sailors

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

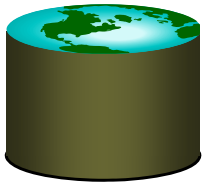
Boats

bid	bname	color
101	Interlake	Blue
102	Interlake	Red
103	Clipper	Green
104	Marine	Red



E1: Find names of sailors who've reserved boat #103

- **Solution 1:** $\pi_{sname}((\sigma_{bid=103} Reserves) \bowtie Sailors)$
- **Solution 2:** $\pi_{sname}(\sigma_{bid=103}(Reserves \bowtie Sailors))$



E2: Find names of sailors who've reserved a red boat

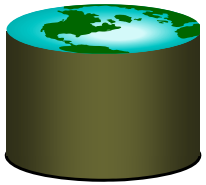
- **Information about boat color only available in Boats; so need an extra join:**

$$\pi_{sname}((\sigma_{color='red'} Boats) \bowtie Reserves \bowtie Sailors)$$

- A more efficient solution:

$$\pi_{sname}(\pi_{sid}((\pi_{bid}\sigma_{color='red'} Boats) \bowtie Res) \bowtie Sailors)$$

- *A query optimizer can find this given the first solution!*

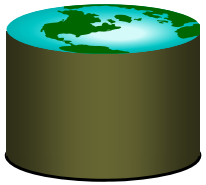


E3: Find sailors who've reserved a red or a green boat

- **Can identify all red or green boats, then find sailors who've reserved one of these boats:**

$$\rho\ (Tempboats, (\sigma_{color='red' \vee color='green'}\ Boats))$$

$$\pi_{sname}(Tempboats \bowtie \mathrm{Re}serves \bowtie Sailors)$$



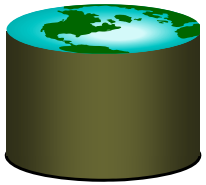
E4: Find sailors who've reserved a red and a green boat

- **Previous approach won't work! Must identify sailors who've reserved red boats, sailors who've reserved green boats, then find the intersection (note that *sid* is a key for Sailors):**

$$\rho\,(Tempred, \pi_{sid}((\sigma_{color='red'} Boats) \bowtie Reserves))$$

$$\rho\,(Tempgreen, \pi_{sid}((\sigma_{color='green'} Boats) \bowtie Reserves))$$

$$\pi_{sname}((Tempred \cap Tempgreen) \bowtie Sailors)$$



E5: Find the names of sailors who've reserved all boats

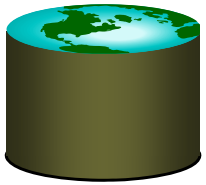
- **Uses division; schemas of the input relations to / must be carefully chosen:**

$$\rho\ (Tempsids, (\pi_{sid,bid} \text{Re}serves) / (\pi_{bid} Boats))$$

$$\pi_{sname} (Tempsids \bowtie Sailors)$$

- To find sailors who've reserved all 'Interlake' boats:

$$\ldots\ / \pi_{bid}(\sigma_{bname='Interlake'} Boats)$$



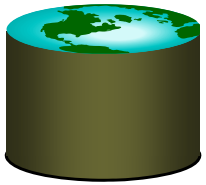
Summary

- **Relational Algebra: a small set of operators mapping relations to relations**
  - Operational, in the sense that you specify the explicit order of operations
  - A *closed* set of operators! Can mix and match.
- **Basic ops include:** σ, π, $\times$, $\cup$, $-$
- **Important compound ops:** $\cap$, $\bowtie$, /

Relational Calculus

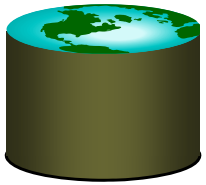
We will occasionally use this arrow notation unless there is danger of no confusion.

-- Ronald Graham (Elements of Ramsey Theory)



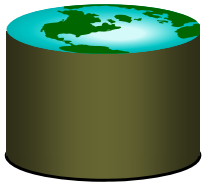
Relational Calculus

- **High-level, first-order logic description**
- **English:**
  Find all sailors with a rating above 7
- **More logical English:**
  From the universe of all things, find me the set of things that happen to be tuples in the Sailors relation and whose rating field is a number greater than 7.
- **Relational Calculus (TRC)**
  $\{S \mid S \in Sailors \wedge S.rating > 7\}$



So what is Relational Calculus?

- **A formal, logical description, of what you want from the database.**



Relational Calculus

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

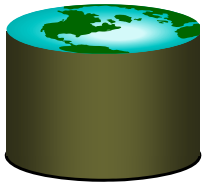
- **Comes in two flavors:**
  - *Tuple relational calculus* (TRC)
  - *Domain relational calculus* (DRC)

- **English Example: Find all sailors with a rating above 7**
  - Tuple R.C.: From the universe of all things, find me the set of things that happen to be tuples in the Sailors relation and whose rating field is a number greater than 7.

    $\{S \mid S \in Sailors \wedge S.rating > 7\}$

  - Domain R.C.: From the universe of all things, find me S, N, R, and A, where S is an integer, N is a string, R is an integer greater than 7, and A is a floating point number, and <S, N, R, A> is a tuple in the Sailors relation.

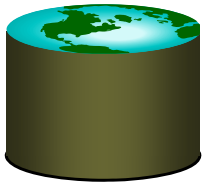
    $\{\langle S,N,R,A\rangle \mid \langle S,N,R,A\rangle \in Sailors \wedge R > 7\}$



Relational Calculus (cont.)

- **Calculus has**
  - *variables*
    - *TRC*: Variables range over (i.e., get bound to) *tuples*. Like SQL.
    - *DRC*: Variables range over *domain elements* (= field values). Like Query-By-Example (QBE)
  - *constants,* e.g.7, "Foo", 3.14159, etc.
  - *comparison ops,* e.g. =, <>, <, >, etc.
  - *logical connectives*
    - $\neg$ - not
    - $\wedge$ – and
    - $\vee$ - or
    - $\Rightarrow$- implies
    - $\in$ - is a member of
  - *quantifiers*
    - $\forall X(p(X))$ – p(X) must be true for every X
    - $\exists X(p(X))$ – p(X) is true for some X

- **Both TRC and DRC are simple subsets of first-order logic. We'll focus on TRC here**

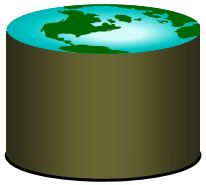


Tuple Relational Calculus

- ***Query*** **has the form: {*T* | *p*(*T*)}**
  - ***p(T)*** denotes a formula in which tuple variable ***T*** appears.

- ***Answer*** **is the set of all tuples *T* for which the *formula p*(*T*) evaluates to *true*.**

- ***Formula*** **is recursively defined:**
  - start with simple *atomic formulas* (get tuples from relations or make comparisons of values)
  - build bigger and better formulas using the *logical connectives*.



TRC Formulas

- ***An Atomic formula is one of the following:***

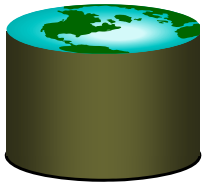
  $R \in Rel$

  $R.a\ op\ S.b$

  $R.a\ op\ constant$

  op is one of $<, >, =, \leq, \geq, \neq$

- **A *formula* can be*:***
  - an atomic formula
  - $\neg p, p \wedge q, p \vee q$ where p and q are formulas
  - $\exists R(p(R))$ where variable R is a tuple variable
  - $\forall R(p(R))$ where variable R is a tuple variable

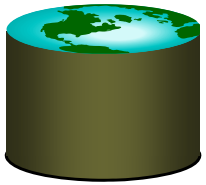


Free and Bound Variables

- **The use of quantifiers $\forall$X and $\exists$X in a formula is said to *bind* X in the formula.**
  - A variable that is not bound is free.

- **Let us revisit the definition of a query:**
  - $\{T \mid p(T)\}$

- **There is an important restriction**
  - the variable T that appears to the left of `|' must be the *only* free variable in the formula $p(T)$.
  - in other words, all other tuple variables must be bound using a quantifier.



Use of ∀

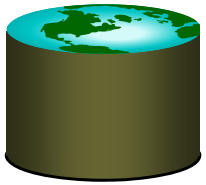
- ∀x (P(x)) - is only true if P(x) is true for *every* x in the universe
- Usually:

  ∀x ((x ∈ Boats) ⇒ (x.color = "Red"))

- ⇒ logical implication,

  a ⇒ b means that if a is true, b must be true

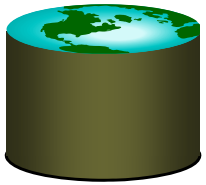
  a ⇒ b is the same as ¬a ∨ b



$a \Rightarrow b$ is the same as $\neg a \vee b$

a \ b	T	F
T	T	F
F	T	T

- **If a is true, b must be true!**
  - If a is true and b is false, the implication evaluates to false.
- **If a is not true, we don't care about b**
  - The expression is always true.



Quantifier Shortcuts

$$\forall x\ ((x \in \textbf{Boats}) \Rightarrow (x.\textbf{color} = \text{“Red”}))$$

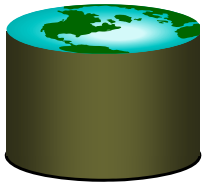
can also be written as:

$$\forall x \in \textbf{Boats}(x.\textbf{color} = \text{“Red”})$$

$$\exists x\ ((x \in \textbf{Boats}) \wedge (x.\textbf{color} = \text{“Red”}))$$

can also be written as:

$$\exists x \in \textbf{Boats}(x.\textbf{color} = \text{“Red”})$$



Selection and Projection

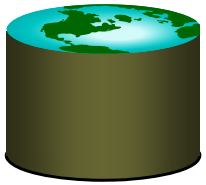
- **Find all sailors with rating above 8**

$$\{S \mid S \in Sailors \wedge S.rating > 8\}$$

- **Find names and ages of sailors with rating above 8.**

$$\{S \mid \exists S1 \in Sailors(S1.rating > 8 \wedge S.sname = S1.sname \wedge S.age = S1.age)\}$$

  - Note: *S* is a tuple variable of 2 fields (i.e. {S} is a projection of S*ailors)*
    - only 2 fields are ever mentioned and *S* is never used to range over any relations in the query.

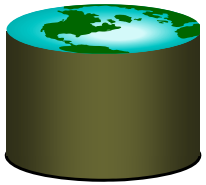


Joins

Find sailors rated > 8 who've reserved boat #103

$$\{S \mid S \in Sailors \wedge S.rating > 8 \wedge \exists R(R \in Reserves \wedge R.sid = S.sid \wedge R.bid = 103)\}$$

Note the use of ∃ to find a tuple in Reserves that `joins with' the Sailors tuple under consideration.

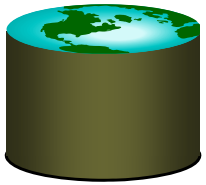


Joins (continued)

$$\{S \mid S \in Sailors \wedge S.rating > 8 \wedge$$
$$\exists R(R \in Reserves \wedge R.sid = S.sid$$
$$\wedge \exists B(B \in Boats \wedge B.bid = R.bid$$
$$\wedge B.color = \text{‘red’}))\}$$

Find sailors rated > 8 who’ve reserved a red boat

- **Observe how the parentheses control the scope of each quantifier’s binding.**
- **This may look cumbersome, but it’s not so different from SQL!**



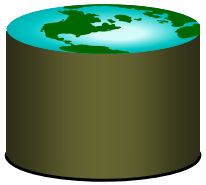
Division (makes more sense here???)

Find sailors who've reserved all boats

(hint, use ∀)

$$\{S \mid S \in Sailors \wedge \forall B \in Boats\ (\exists R \in Reserves\ (S.sid = R.sid \wedge B.bid = R.bid))\}$$

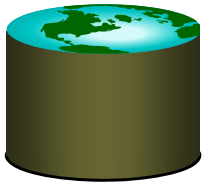
- **Find all sailors *S* such that for all tuples *B* in Boats there is a tuple in Reserves showing that sailor *S* has reserved *B*.**



Exercises

- Find the names of sailors who've reserved boat #103

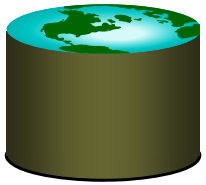
$$\{N \mid \exists S \in \mathit{Sailors}\ (S.name = N.name \wedge \exists R \in Reserves(S.sid = R.sid \wedge R.bid = 103))\}$$



Exercises

- Find the names of sailors who've reserved any red boat

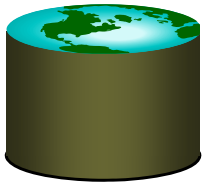
$$\{N \mid \exists S \in Sailors$$
$$(S.name = N.name \wedge$$
$$\exists R \in Reserves$$
$$(S.sid = R.sid \wedge$$
$$\exists B \in Boats(B.color = \text{“Red”} \wedge$$
$$B.bid = R.bid)))\}$$



Exercises

- Find sailors who've reserved a red boat or a green boat

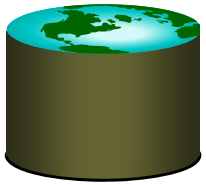
$$\{S \mid S \in Sailors \wedge (\exists R \in Reserves (S.sid = R.sid \wedge \exists B \in Boats(B.bid = R.bid \wedge (B.color = \text{“Red”} \vee B.color = \text{“Green”}))))\}$$



Exercises

- Find sailors who've reserved a red boat and a green boat

$$\{S \mid S \in Sailors \wedge$$
$$((\exists R \in Reserves$$
$$(S.sid = R.sid \wedge$$
$$\exists B \in Boats(B.color = \text{“Red”} \wedge B.bid = R.bid)))$$
$$\wedge$$
$$(\exists R \in Reserves$$
$$(S.sid = R.sid \wedge$$
$$\exists B \in Boats(B.color = \text{“Green”} \wedge B.bid = R.bid))))\}$$

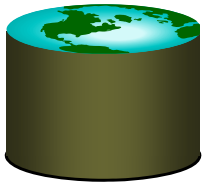


Exercises

- Find sailors who've reserved all red boats

$$\{S \mid S \in Sailors \wedge \forall B \in Boats(B.color = \text{'red'} \Rightarrow \exists R(R \in Reserves \wedge S.sid = R.sid \wedge B.bid = R.bid))\}$$

$$\{S \mid S \in Sailors \wedge \forall B \in Boats (B.color \neq \text{'red'} \vee \exists R(R \in Reserves \wedge S.sid = R.sid \wedge B.bid = R.bid))\}$$

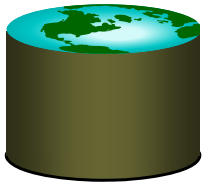


Unsafe Queries, Expressive Power

- ∃ **syntactically correct calculus queries that have an infinite number of answers!** ***Unsafe*** **queries.**
  - e.g., $\{S \mid \neg(S \in Sailors)\}$
  - Solution???? Don't do that!

- **Expressive Power (Theorem due to Codd):**
  - every query that can be expressed in relational algebra can be expressed as a *safe* query in DRC / TRC; the converse is also true.

- ***Relational Completeness***: Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus. (actually, SQL is more powerful, as we will see…)



Summary

- **The relational model has rigorously defined query languages — simple and powerful.**
- **Relational algebra is more operational**
  - useful as internal representation for query evaluation plans.
- **Relational calculus is non-operational**
  - users define queries in terms of what they want, not in terms of how to compute it. (*Declarative*)
- **Several ways of expressing a given query**
  - a *query optimizer* should choose the most efficient version.
- **Algebra and safe calculus have same *expressive power***
  - leads to the notion of *relational completeness*.