Digital Image Processing

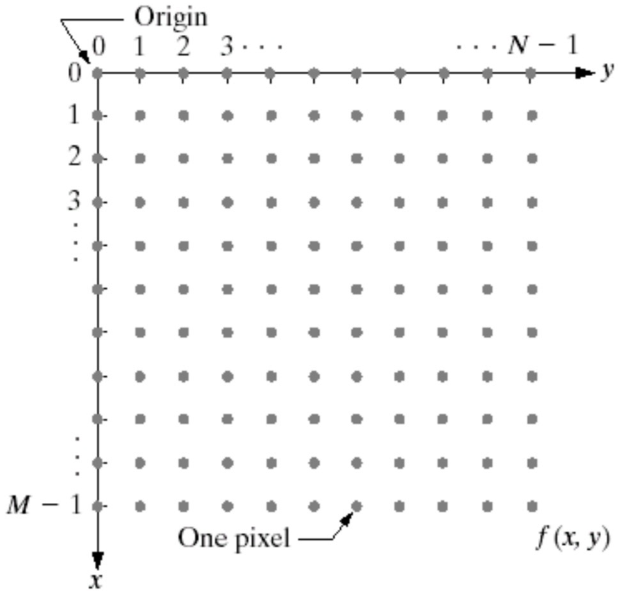
Dr. Muhammad Abeer Irfan
Department of Computer Systems Engineering
UET Peshawar

Lecture No: 04

Readings: Chapter 1, Chapter 2 (sections 2.1 to 2.4)

Digital Image Fundamentals

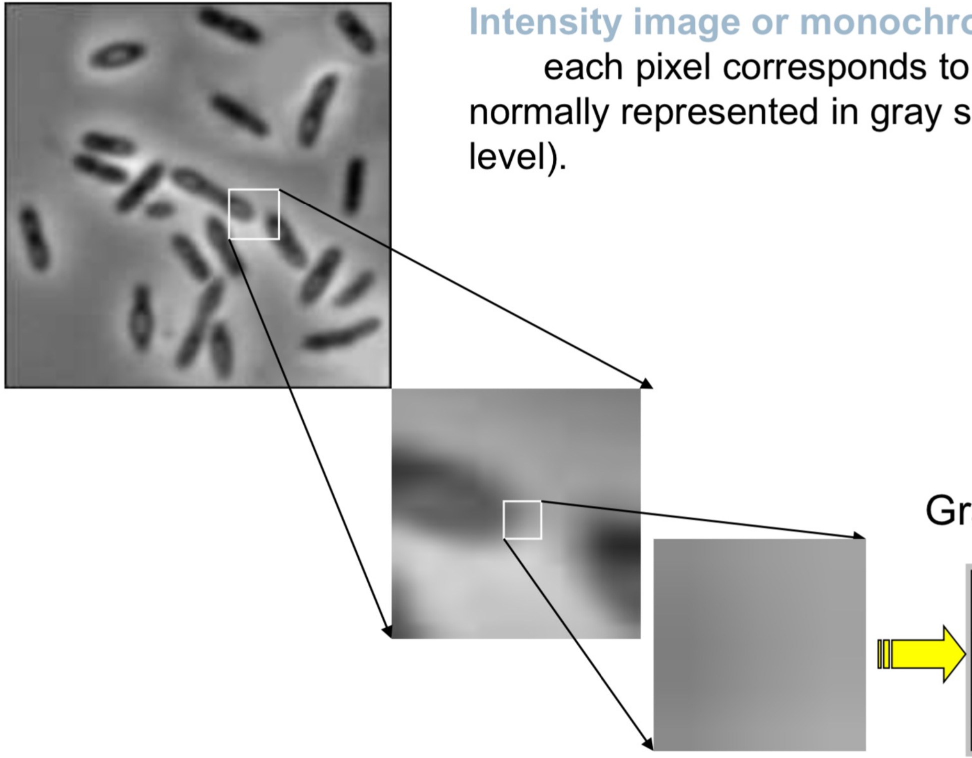
Conventional Coordinate for Image Representation



Digital Image Types : Intensity Image

Intensity image or monochrome image

each pixel corresponds to light intensity normally represented in gray scale (gray level).



Gray scale values

$$\begin{bmatrix} 10 & 10 & 16 & 28 \\ 9 & 6 & 26 & 37 \\ 15 & 25 & 13 & 22 \\ 32 & 15 & 87 & 39 \end{bmatrix}$$

Digital Image Types : RGB Image

Color image or RGB image: each pixel contains a vector representing red, green and blue components.

RGB components

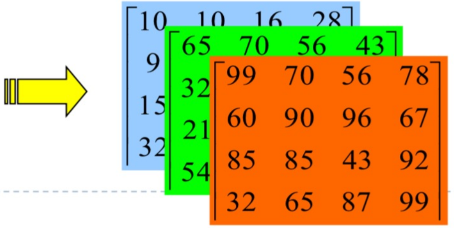


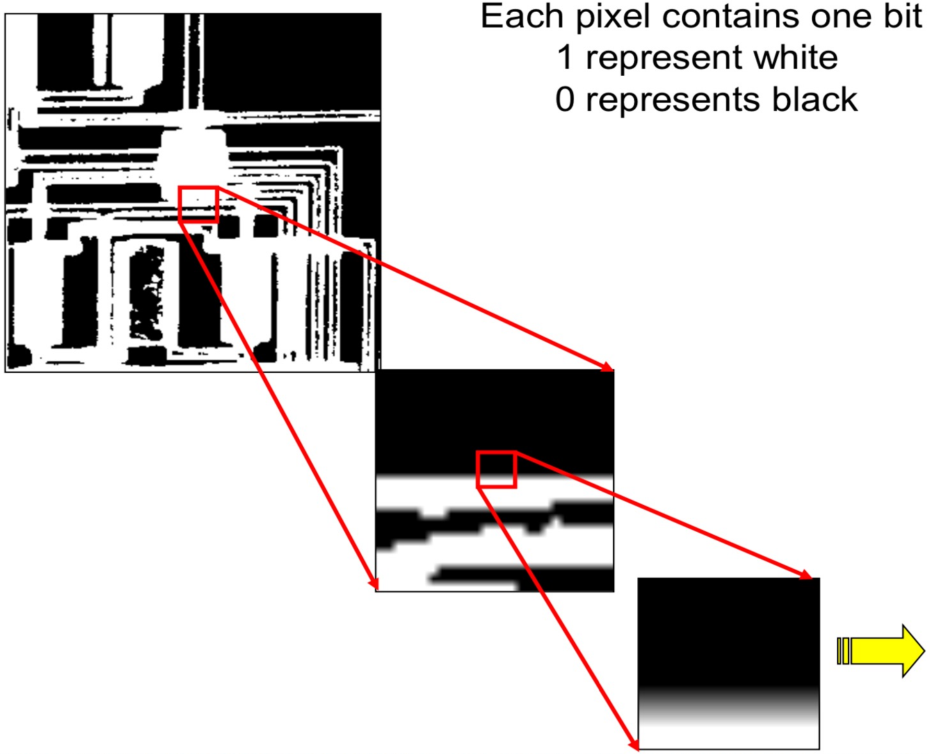
Image Types : Binary Image

Binary image or black and white image

Each pixel contains one bit :

1 represent white

0 represents black



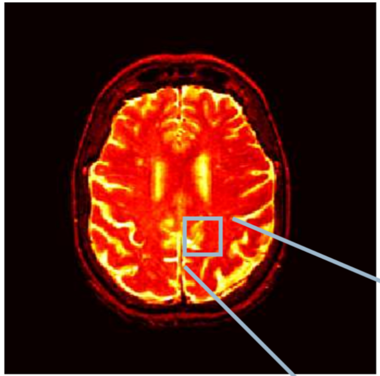
Binary data

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Image Types : Index Image

Index image

Each pixel contains index number pointing to a color in a color table



$$\begin{bmatrix} 1 & 4 & 9 \\ 6 & 4 & 7 \\ 6 & 5 & 2 \end{bmatrix}$$

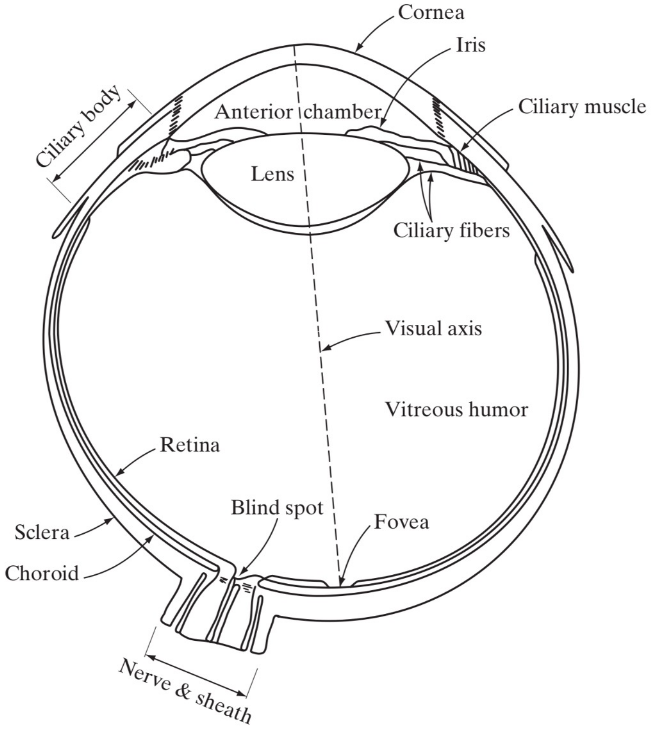
Index value

Color Table

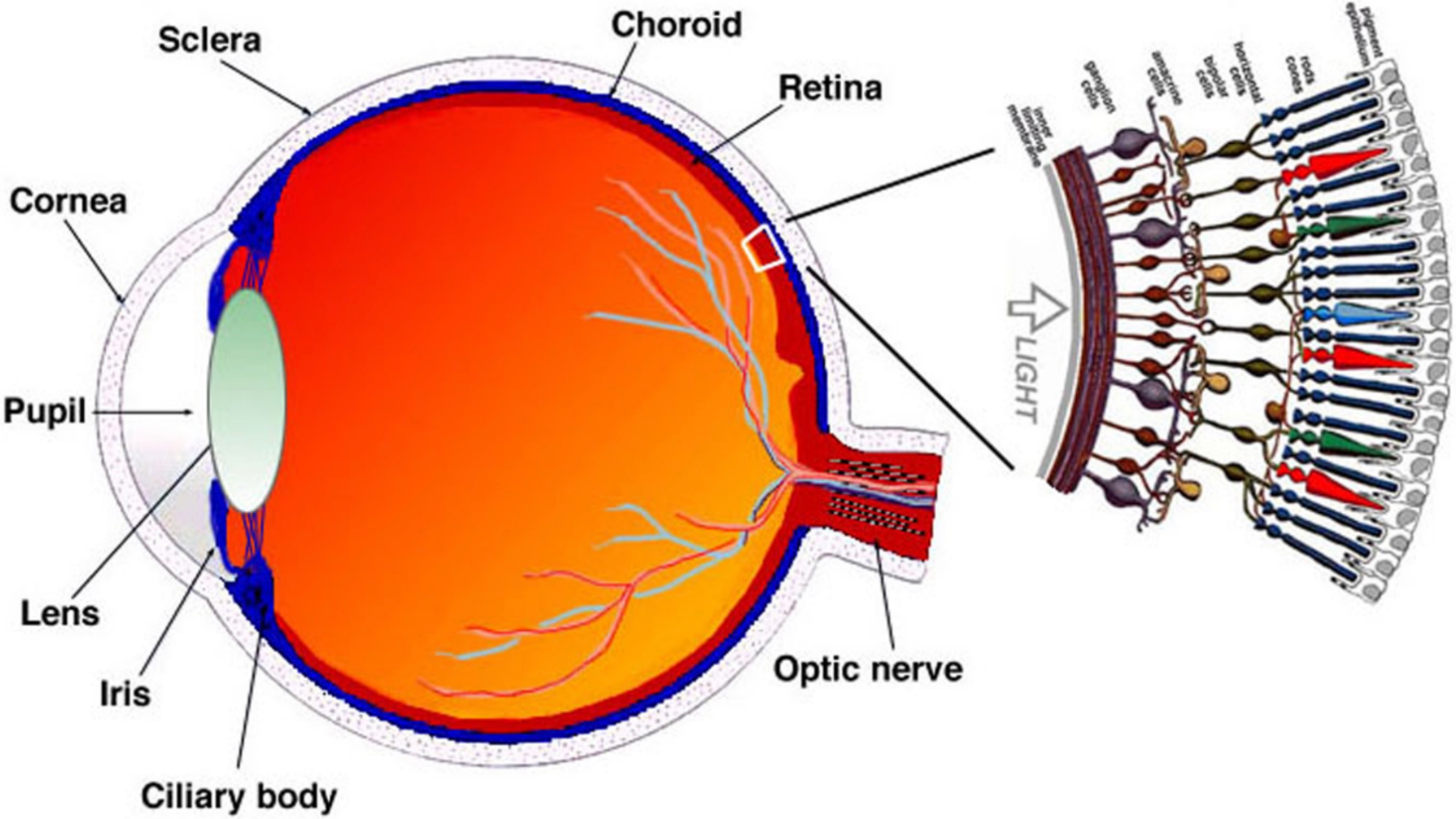
Index No.	Red component	Green component	Blue component
1	0.1	0.5	0.3
2	1.0	0.0	0.0
3	0.0	1.0	0.0
4	0.5	0.5	0.5
5	0.2	0.8	0.9
…	…	…	…

Structure of Human Eye (Cross section)

- The eye is nearly a sphere, with an average diameter of approx; 22mm.
- Three membranes enclose the eye:
  - Cornea and sclera – outer cover.
  - Choroid; and
  - Retina.
- Cornea: It is a tough, transparent tissue that covers the anterior surface of the eye.
- Sclera is an opaque membrane that encloses the remainder of the optic globe.



Anatomy of Human Eye



Human Visual System

Human vision:

- **Cornea** acts as a protective lens that roughly focuses incoming light
- **Iris** controls the amount of light that enters the eye
- The **lens** sharply focuses incoming light onto the retina
  - Absorbs both infrared and ultra-violet light which can damage the lens
- The **retina** is covered by photoreceptors (light sensors) which measure light

Photoreceptors

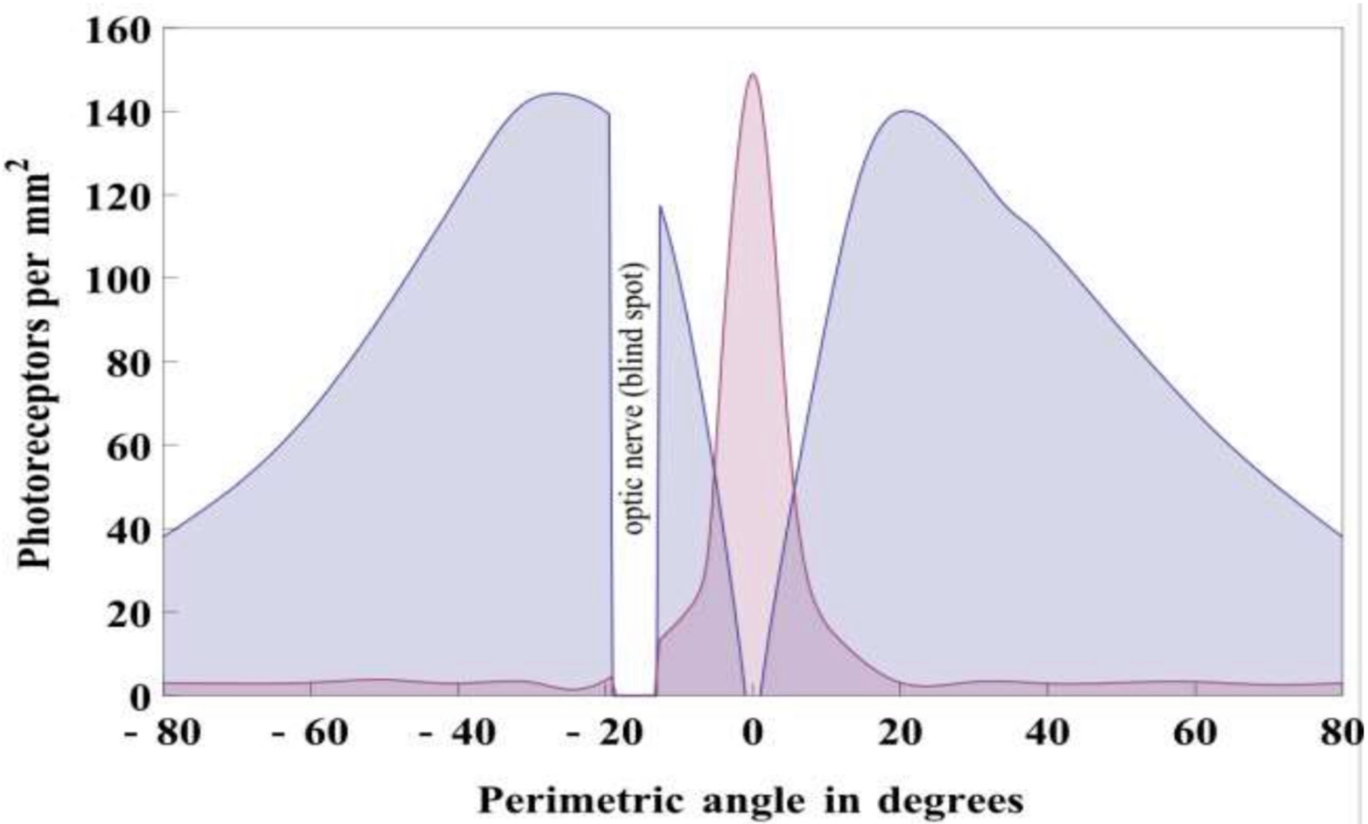
Rods:

- Approximately 100-150 million rods
- Non-uniform distribution across the retina
- Sensitive to low-light levels (scotopic vision)
- Lower resolution

Cones:

- Approximately 6-7 million cones
- Sensitive to higher-light levels (photopic vision)
- High resolution
- Detect color by the use of 3 different kinds of cones each of which is sensitive to red, green, or blue frequencies
    - Red (L cone) : 564-580 nm wavelengths (65% of all cones)
    - Green (M cone) : 534-545 nm wavelengths (30% of all cones)
    - Blue (S cone) : 420-440 nm wavelengths (5% of all cones)

Photoreceptor density across retina



Comparison between rods and cones

Rods	Cones
Used for night vision	Used for day vision
Loss causes night blindness	Loss causes legal blindness
Low spatial resolution with higher noise	High spatial resolution with lower noise
Not present in fovea	Concentrated in fovea
Slower time response to light	Quicker time response to light
One type of photosensitive pigment	Three types of photosensitive pigment
Emphasis on motion detection	Emphasis on detecting fine detail

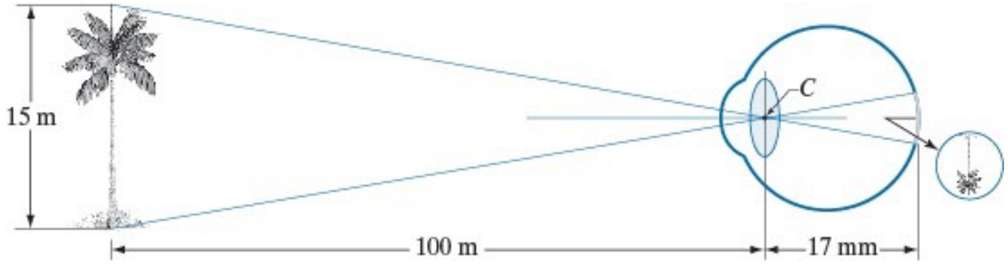
Color and Human Perception

- **Chromatic light**
  - has a color component

- **Achromaticlight**
  - has no color component
  - has only one property – intensity

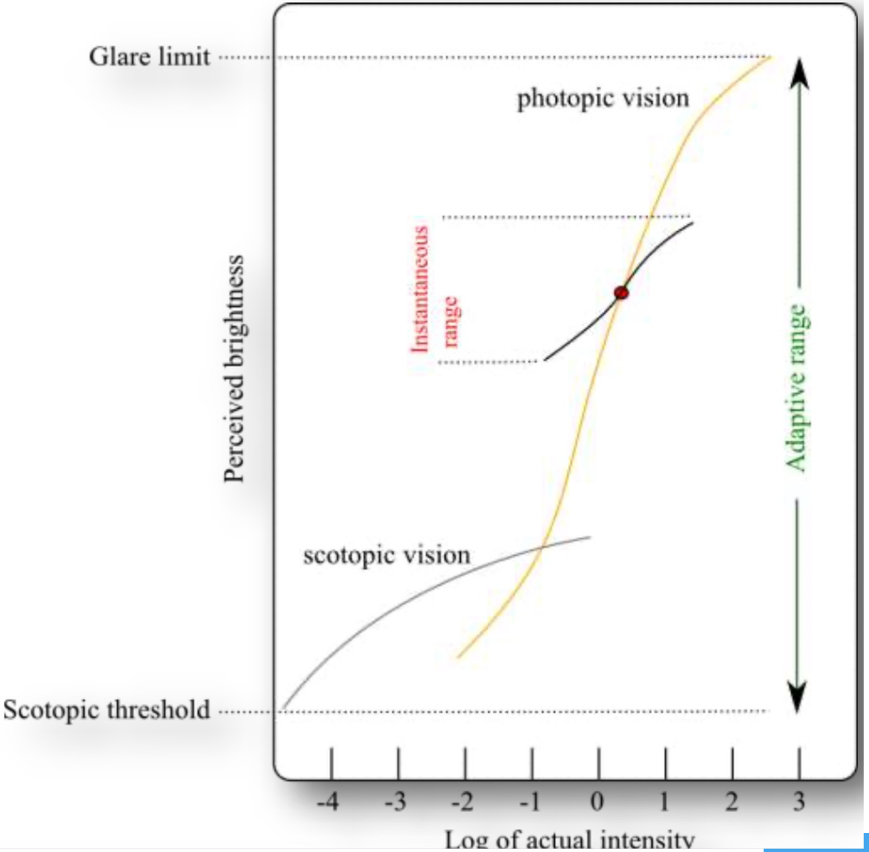
Human Eye

- In regular cameras, the lens has a fixed focal length, and focusing achieved by varying the distance between the lens and the imaging plane.
- In human eye, however, the distance between the center of the lens and the retina is fixed (approximately 17 mm), and the focal length is changed by varying the shape of the lens (14-17 mm)!



Brightness Adaptation

- Actual light intensity is (basically) log-compressed for perception.
- Human vision can see light between the glare limit and scotopic threshold but not all levels at the same time.
- The eye adjusts to an average value (the red dot) and can simultaneously see all light in a smaller range surrounding the adaptation level.
- Light appears black at the bottom of the instantaneous range and white at the top of that range.



Weber Ratio ΔI/I

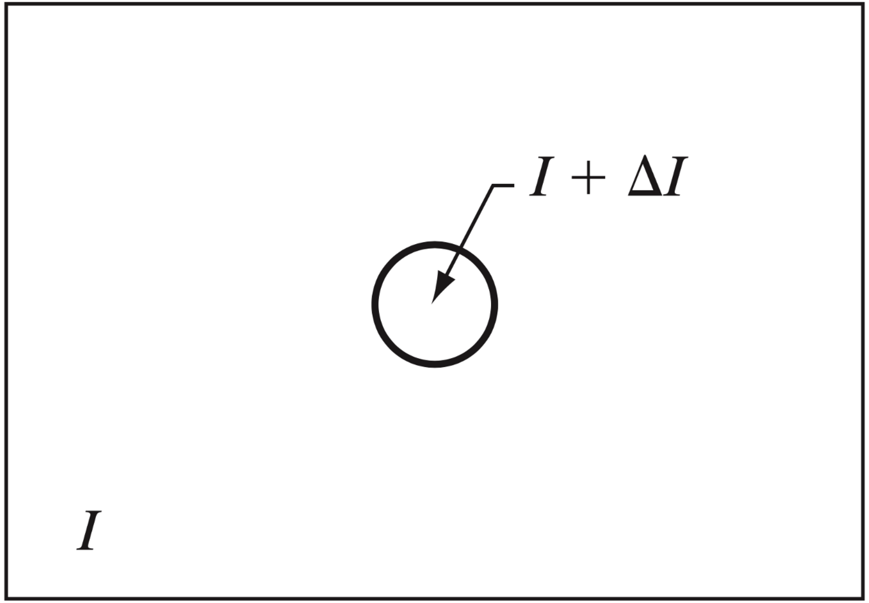


FIGURE 2.5 Basic experimental setup used to characterize brightness discrimination.

Weber Ratio (Cont'd…)

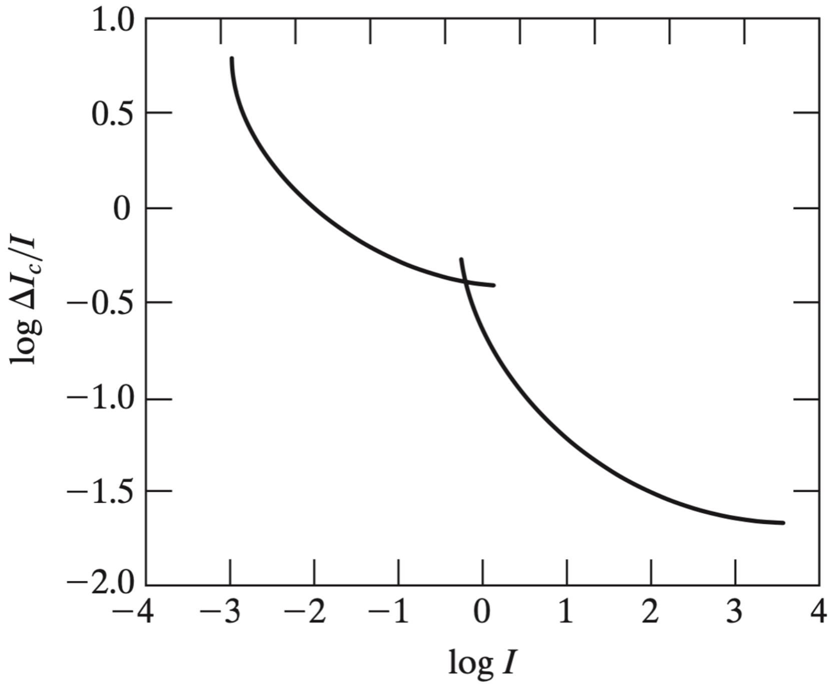


FIGURE 2.6 Typical Weber ratio as a function of intensity.

Human Visual Perception

- **Light intensity:**
  - The lowest (darkest) perceptible intensity is the scotopic threshold
  - The highest (brightest) perceptible intensity is the glare limit
  - The difference between these two levels is on the order of 10^{10}
  - We can't discriminate all these intensities at the same time! We adjust to an average value of light intensities and then discriminate around the average.

Human Visual Perception (Cont'd…)

- **Log compression:**
  - Experimental results show that the relationship between the perceived amount of light and the actual amount of light in a scene is generally related logarithmically.
    - The human visual system perceives brightness as the logarithm of the actual light intensity and interprets the image accordingly.
    - Consider, for example, a bright light source that is approximately 6 times brighter than another. The eye will perceive the brighter light as approximately twice the brightness of the darker.

Brightness Adaptation and Mach Banding

- **When viewing any scene:**
  - The eye rapidly scans across the field of view while coming to momentary rest at each point of particular interest.
  - At each of these points the eye adapts to the average brightness of the local region surrounding the point of interest.
  - This phenomena is known as local brightness adaptation.
  - Mach banding is a visual effect that results, in part, from local brightness adaptation.
  - The eye over-shoots/under-shoots at edges where the brightness changes rapidly. This causes ‘false perception’ of the intensities

Brightness Adaptation and Mach Banding (Con’td…)

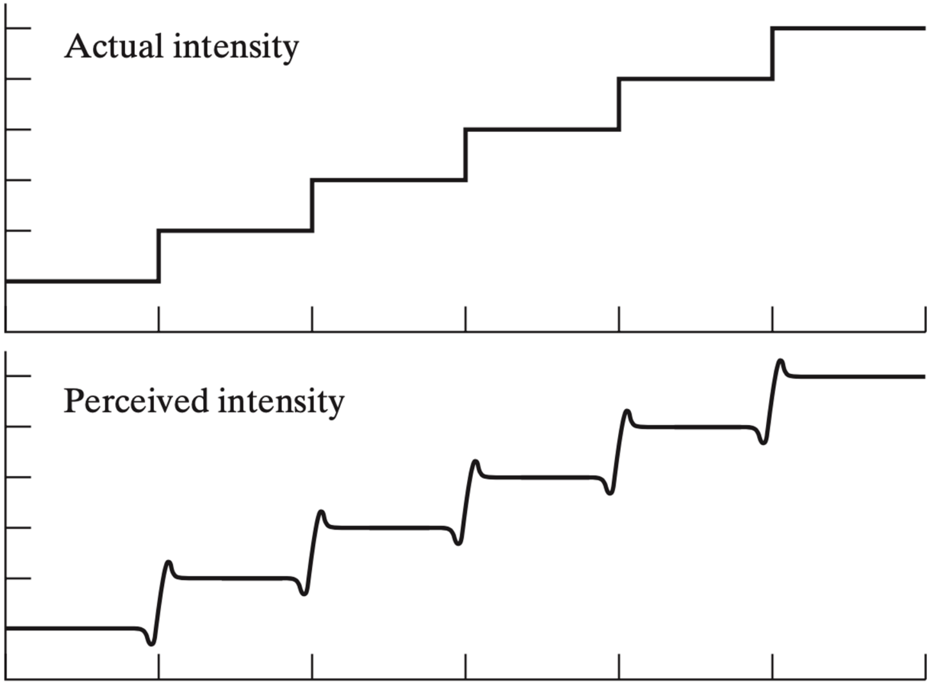
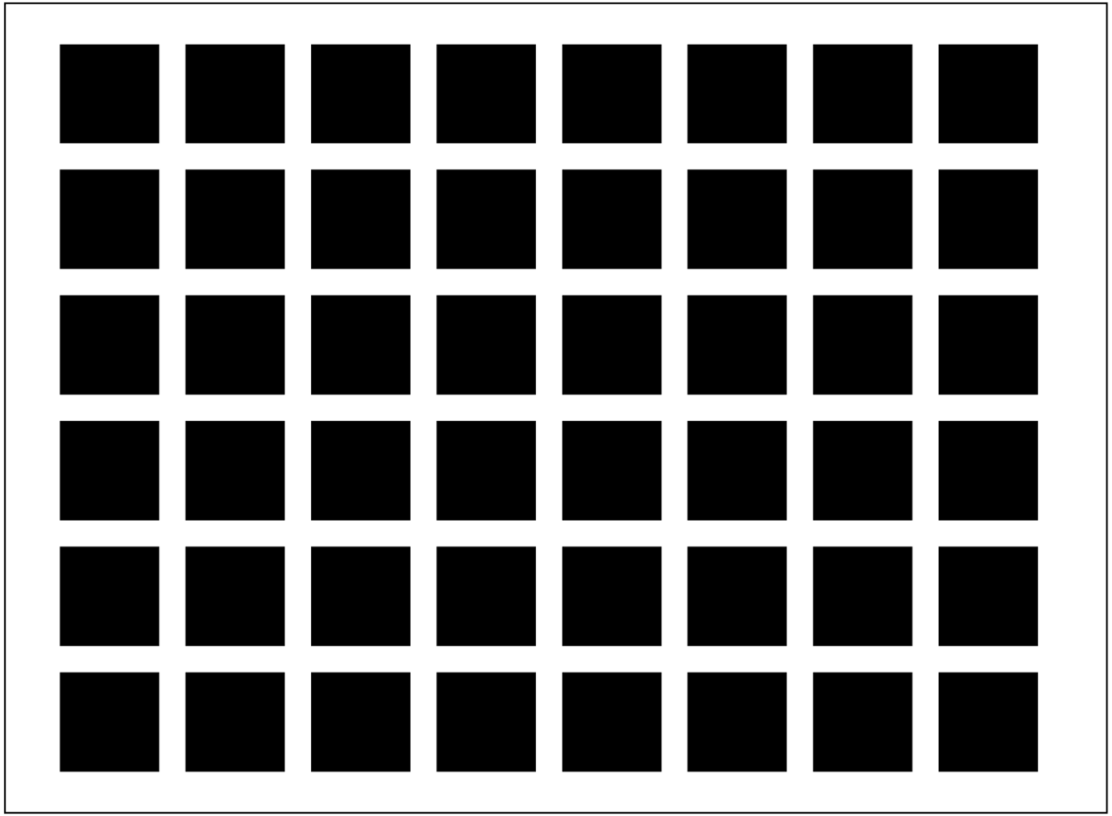
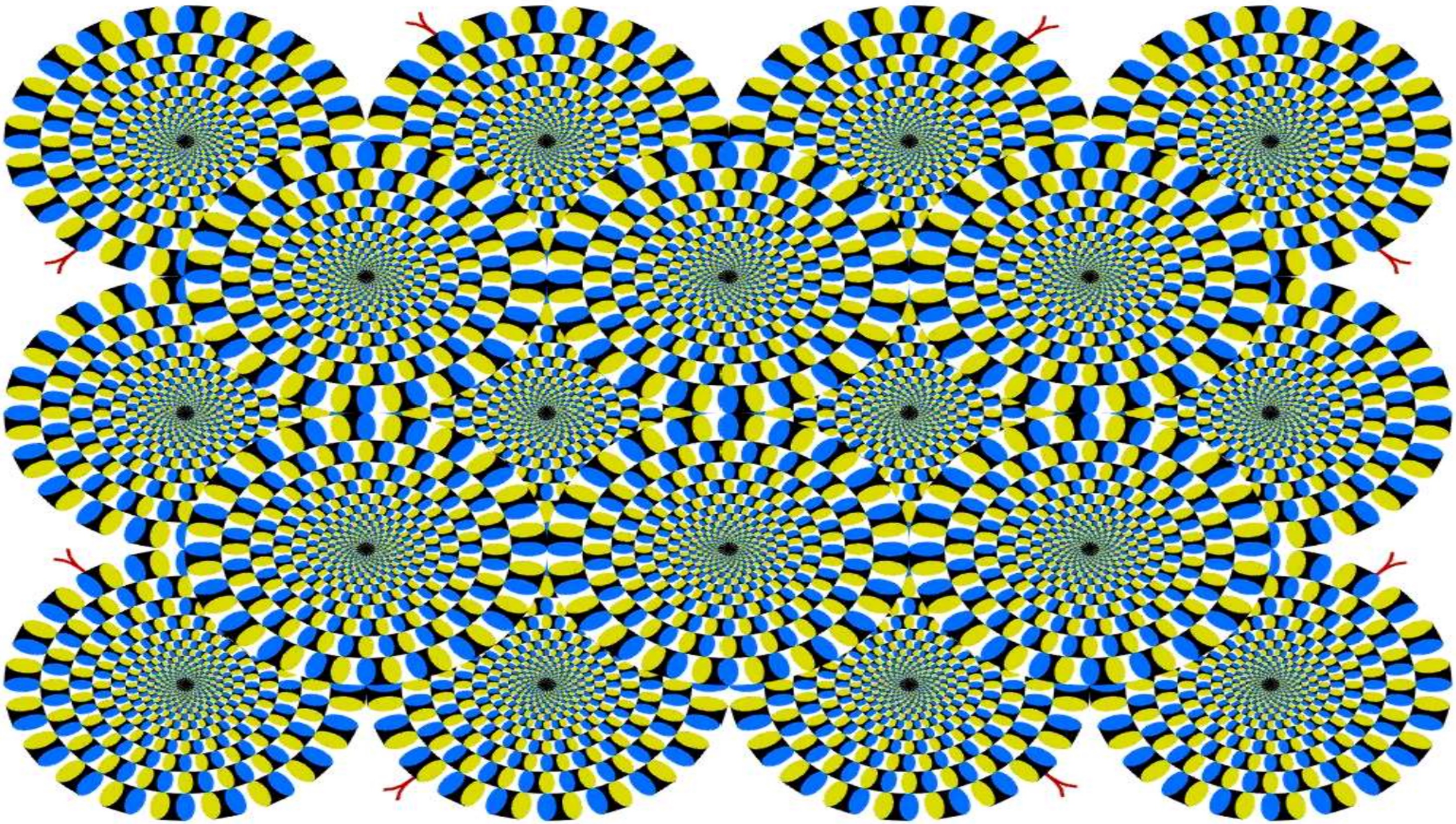


FIGURE 2.7 Illustration of the Mach band effect. Perceived intensity is not a simple function of actual intensity.

Brightness Adaptation(Hermann Grid)





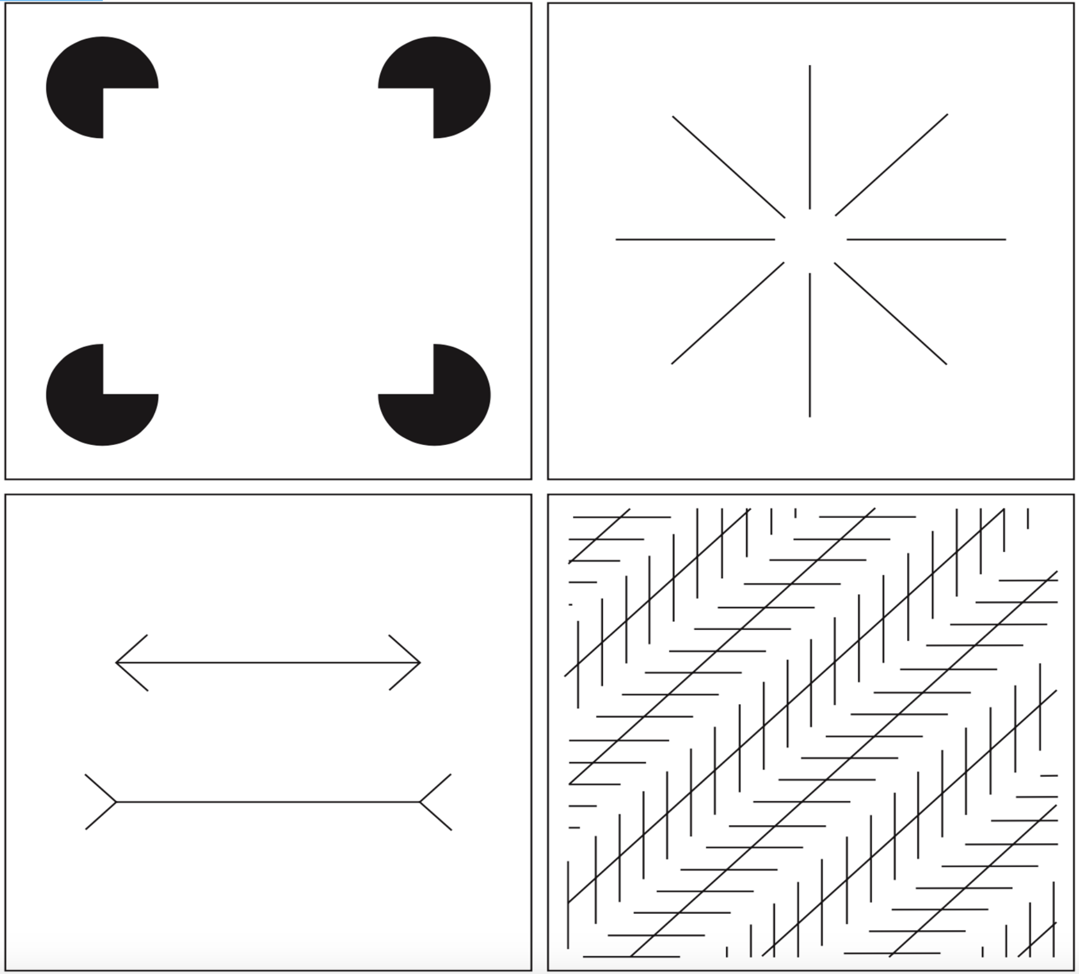


FIGURE 2.9 Some well-known optical illusions.

Simultaneous Contrast

- Simultaneous contrast refers to the way in which two adjacent intensities (or colors) affect each other
  - Example: Note that a blank sheet of paper may appear white when placed on a desktop but may appear black when used to shield the eyes against the sun.
  - **Figure 2.8** is a common way of illustrating that the perceived intensity of a region is dependent upon the contrast of the region with its local background.
  - The four inner squares are of identical intensity but are contextualized by the four surrounding squares
  - The perceived intensity of the inner squares varies from bright on the left to dark on the right.

Simultaneous Contrast (Cont'd…)

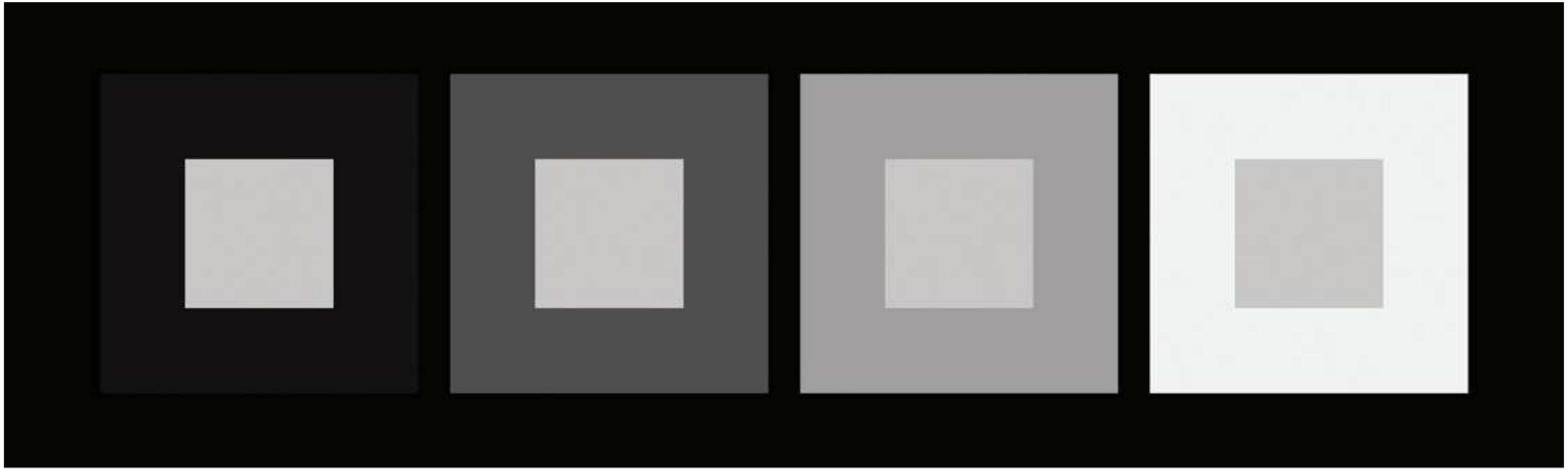


FIGURE 2.8 Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.

Light and the Electromagnetic Spectrum

- The colors perceived in an object are determined by the nature of the light reflected by it.
- **Monochromatic** (*achromatic*) light, is a light that is void of color, represented only by its intensity (gray level), ranging from black to white.
- **Chromatic** light spans the electromagnetic energy spectrum from 0.43 to 0.79 micro-meter.
- **Radiance**: total amount of energy that flows from the light source, measured in watts (W).
- **Luminance**: amount of energy an observer perceives from a light source, measured in lumens (lm).
- **Brightness**: a subjective descriptor of light perception, impossible to measure, representing the achromatic notion of intensity.

Image Sensing and Acquisition

- Images are generated by the combination of an “illumination” source and the reflection or absorption of energy from the source by the elements of the “scene”.
- Illumination can be from a source of electromagnetic energy, or from less traditional sources such as ultrasound, acoustics, or even computer-generated.
- Scene elements can be familiar objects, or molecules, rock formations, human brain etc.

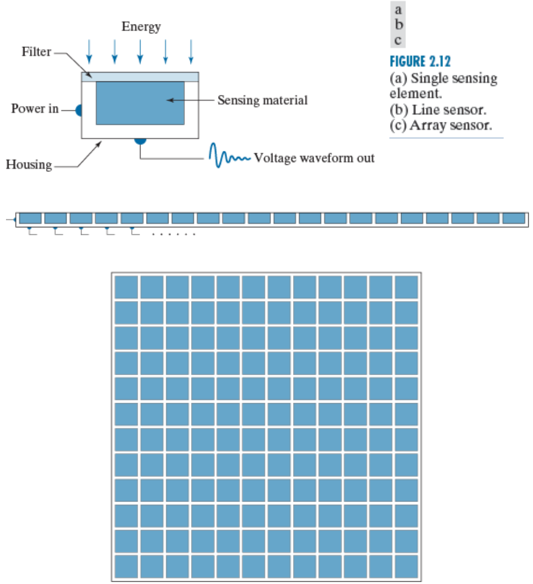
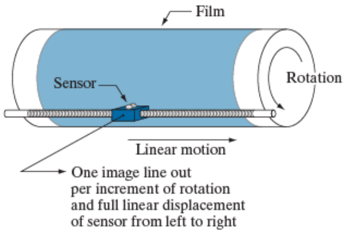


FIGURE 2.12
(a) Single sensing element.
(b) Line sensor.
(c) Array sensor.

Image Sensing and Acquisition

FIGURE 2.13 Combining a single sensing element with mechanical motion to generate a 2-D image.



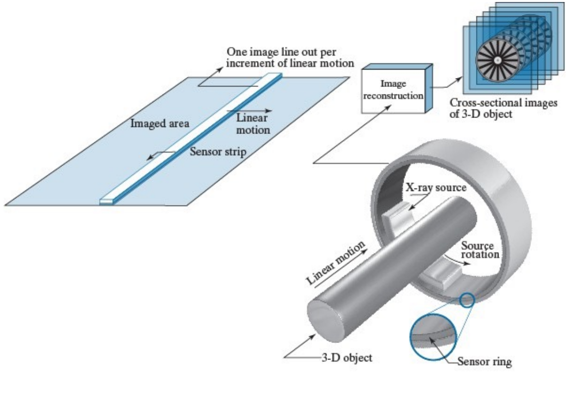
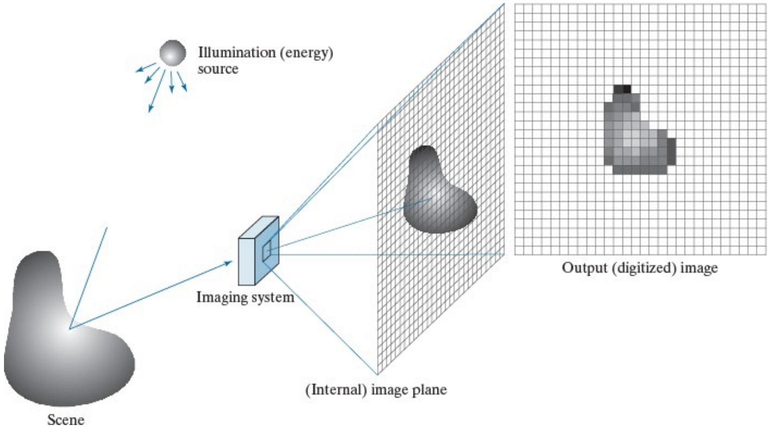


FIGURE 2.14 (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

Image Sensing and Acquisition



a
b c d e

FIGURE 2.15 An example of digital image acquisition. (a) Illumination (energy) source. (b) A scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Image Formation Model

- An image is denoted by a function ***f(x, y)***, which the value of ***f*** at spatial coordinates ***(x, y)*** is a scalar quantity proportional to energy radiated by a physical source.
- The values of ***f*** are non-negative, and finite: ***0 ≤ f(x,y) < inf***.
- Function ***f(x, y)*** is characterized by two components:
  - **Illumination**: the amount of source illumination incident on the scene being viewed, represented by ***i(x, y)***.
  - **Reflectance**: the amount of illumination reflected by the objects in the scene, ***r(x,y)***.

$$f(x, y) = i(x, y) * r(x, y)$$
$$0 \leq i(x, y) < inf$$
$$0 \leq r(x, y) \leq 1$$

- In some cases, for example X-ray imaging, we have transmissivity instead of reflectance.

Image Sampling and Quantization

- To create a digital image, we need to convert the continuous sensed data into a digital format.
- Two processes are required:
  - Sampling: digitization in the spatial domain
  - Quantization: digitization in the function domain

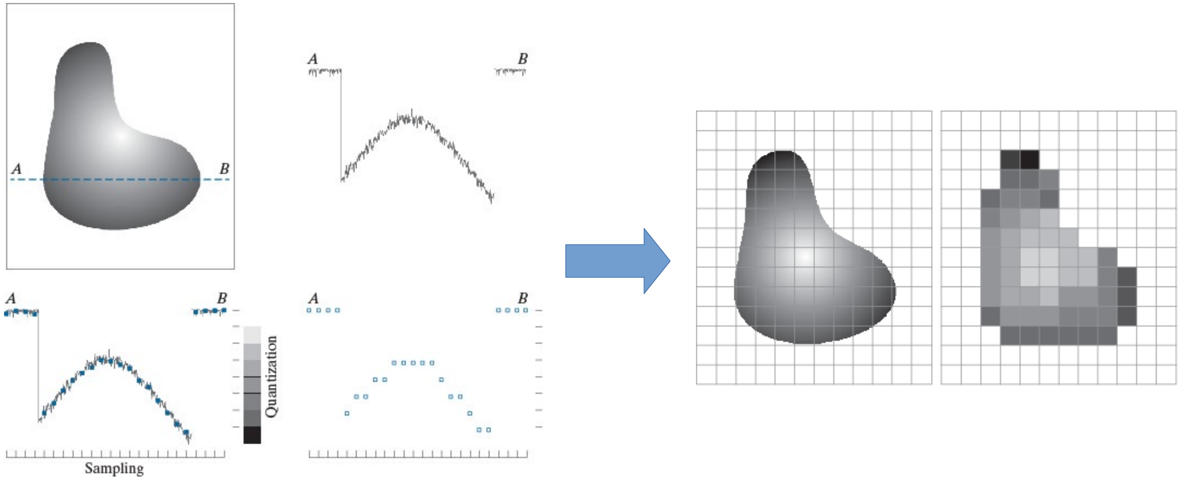


Image Sampling and Quantization

- Assuming ***f(s, t)*** as a continuous image function, using sampling and digitization, we create the image ***f(x, y)***, containing ***M*** rows and ***N*** columns.

- The spatial coordinate values are shown by integers as: ***x=0, 1, 2, …, M-1*** and ***y=0, 1, 2, …, N-1***.

- For image ***f(x, y)***, we have ***L*** number of intensity levels, represented as a power of 2. For example, in an 8-bit image, we have 256 intensity levels:

$$L = 2^k$$

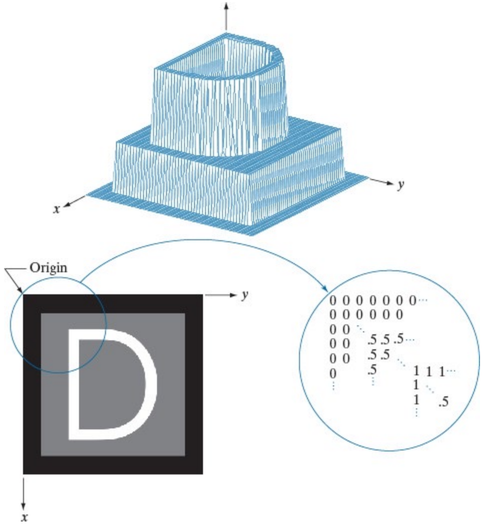
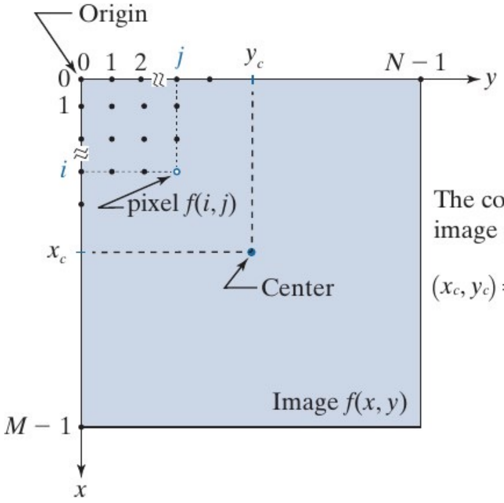


Image Sampling and Quantization

$$f(x,y)=\begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$



The coordinates of the image center are

$$(x_c, y_c) = \left(\text{floor}\left(\frac{M}{2}\right), \text{floor}\left(\frac{N}{2}\right)\right)$$

$$\mathbf{A}=\begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

Saturation and Noise

- Saturation and noise

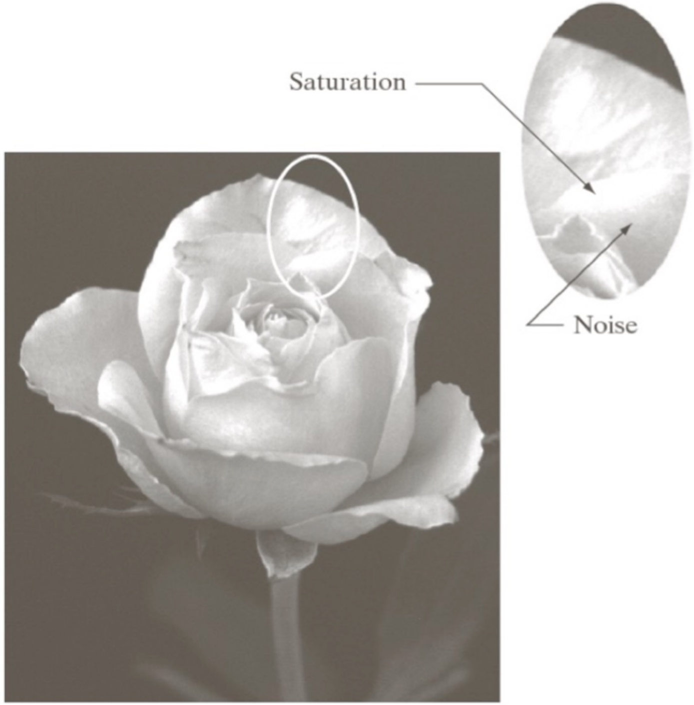


FIGURE 2.19 An image exhibiting saturation and noise. Saturation is the highest value beyond which all intensity levels are clipped (note how the entire saturated area has a high, *constant* intensity level). Noise in this case appears as a grainy texture pattern. Noise, especially in the darker regions of an image (e.g., the stem of the rose) masks the lowest detectable true intensity level.

- Number of storage bits

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Spatial Resolution

- It is a measure of the smallest discernible detail in an image
- Can be stated in line pairs per unit distance, and dots(pixels) per unit distance
- Dots per unit distance commonly used in the printing and publishing industry (dots per inch)
- Newspapers are printed with a resolution of 75 dpi, magazines at 133 dpi, and glossy brochures at175 dpi
- examples

- Spatial and gray-level resolution

FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

- Subsampled and resampled

a b c
d e f

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

- Reducing spatial resolution

a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

Intensity Resolution

- It refers to the smallest discernible change in intensity level
- The number of intensity levels usually is an integer power of two
- Also refers to the number of bits used to quantize intensity as the intensity resolution
- Which intensity resolution is good for human perception 8 bit, 16-bit, or 32 bit

- Varying the number of gray levels

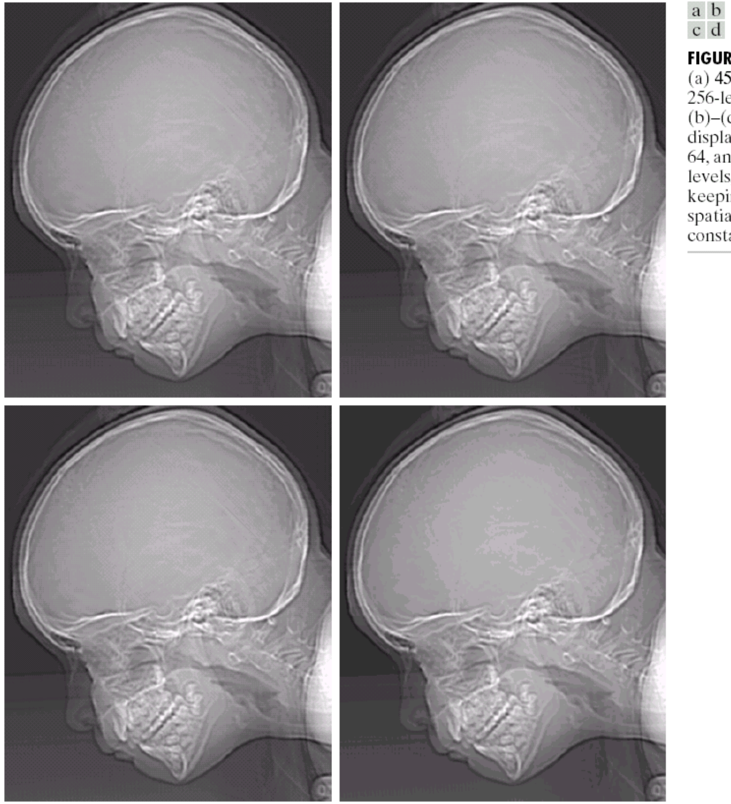
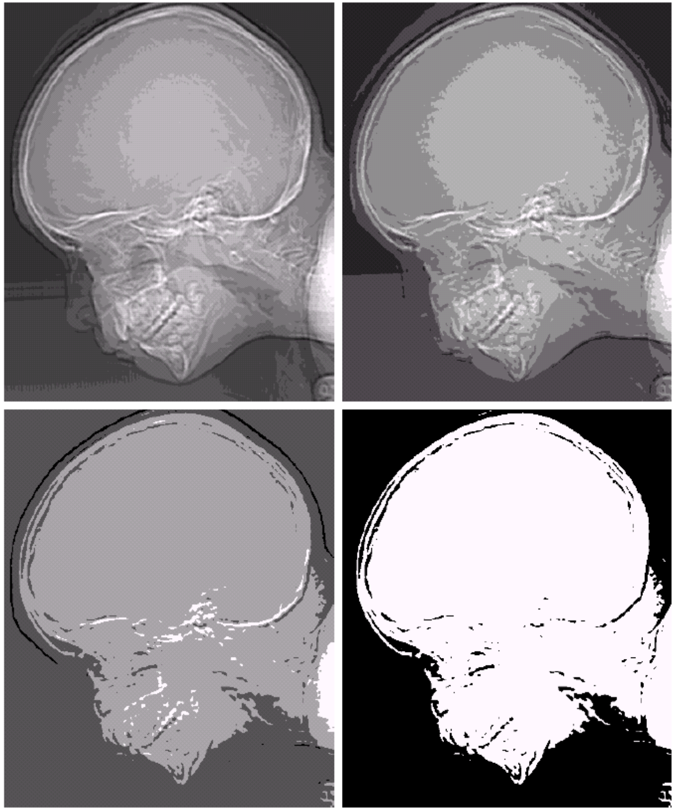


FIGURE 2.21 (a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

- Varying the number of gray levels



e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

Image Interpolation

- Used in image resizing (zooming and shrinking), rotating, and geometric corrections
- Interpolation is the process of using known data to estimate values at unknown locations
- **Nearest Neighbor interpolation**
  - It assigns to each new location the intensity of its nearest neighbor in the original image
  - Produce undesirable artifacts, such as severe distortion of straight edge
- **Bilinear Interpolation**
  - We use the four nearest neighbors to estimate the intensity
  - $V(x, y) = ax + by + cxy + d$

Image Interpolation (Cont'd…)

- Need to solve four equations
- Better results than nearest neighbor interpolation, with a modest increase in computational burden
- **Bicubic Interpolation**
    - Involves sixteen neighbors to estimate intensity
    - $V(x, y) = \sum\sum a_{ij}\, x^i y^j$ (i, j = 0 to 3)
    - Need to solve sixteen equations
    - Gives better results than other methods
    - More complex

- Interpolations

a	b	c
d	e	f

FIGURE 2.24 (a) Image reduced to 72 dpi and zoomed back to its original size (3692 × 2812 pixels) using nearest neighbor interpolation. This figure is the same as Fig. 2.20(d). (b) Image shrunk and zoomed using bilinear interpolation. (c) Same as (b) but using bicubic interpolation. (d)–(f) Same sequence, but shrinking down to 150 dpi instead of 72 dpi [Fig. 2.24(d) is the same as Fig. 2.20(c)]. Compare Figs. 2.24(e) and (f), especially the latter, with the original image in Fig. 2.20(a).

- Zooming and shrinking

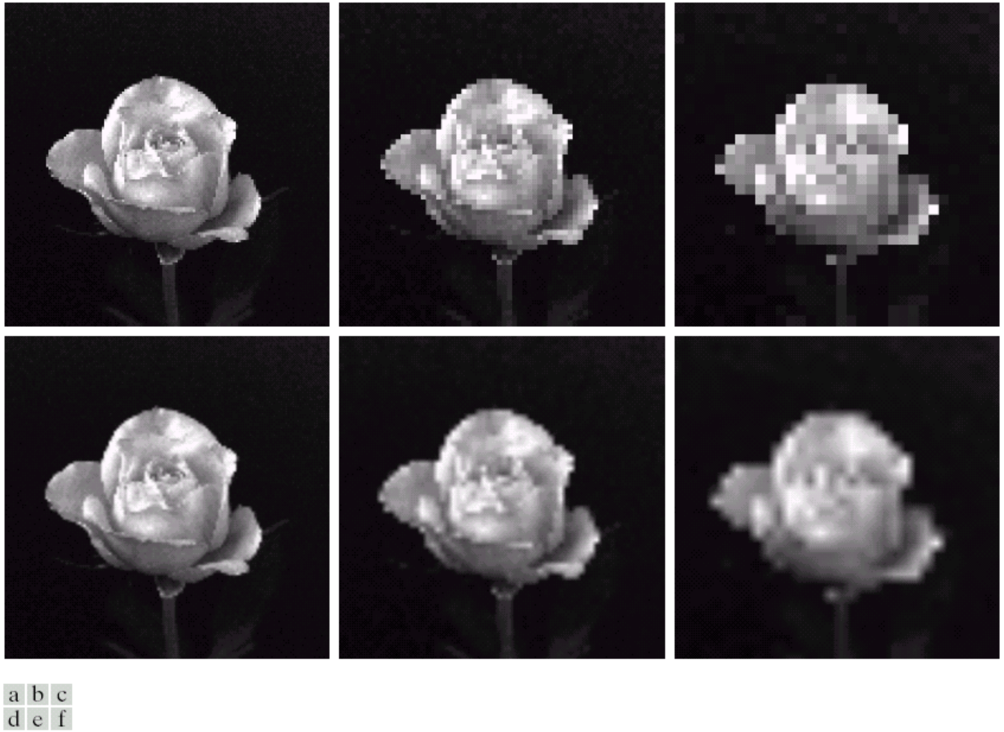
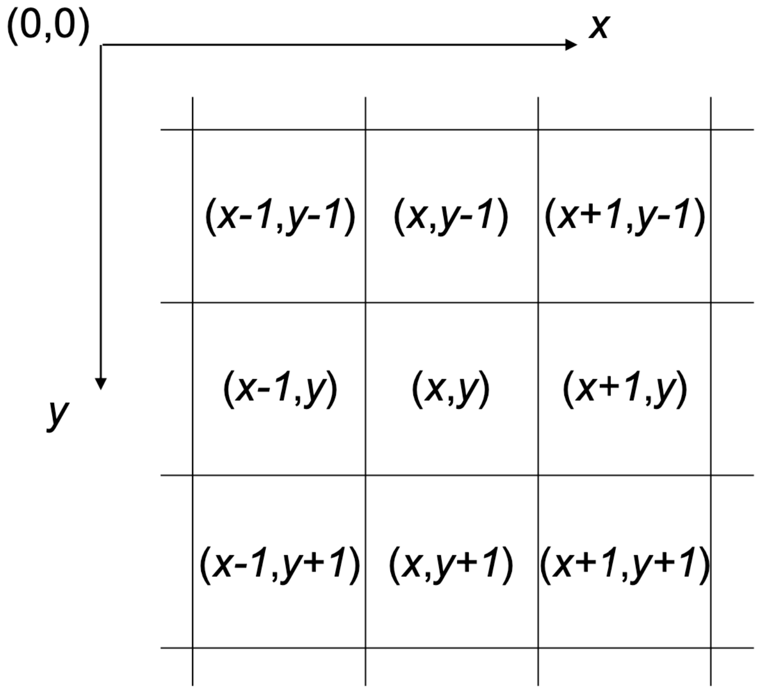


FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

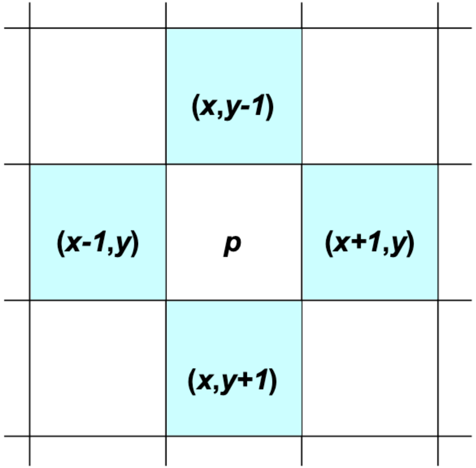
Some Basic Relationships Between Pixels



Conventional indexing method

Neighbors of a Pixel

Neighborhood relation is used to tell adjacent pixels. It is useful for analyzing regions.



4-neighbors of *p*:

$$N_4(p) = \left\{ \begin{array}{c} (x-1,y) \\ (x+1,y) \\ (x,y-1) \\ (x,y+1) \end{array} \right\}$$

4-neighborhood relation considers only vertical and horizontal neighbors.

Note: $q \in N_4(p)$ implies $p \in N_4(q)$

Neighbors of a Pixel (cont'd…)

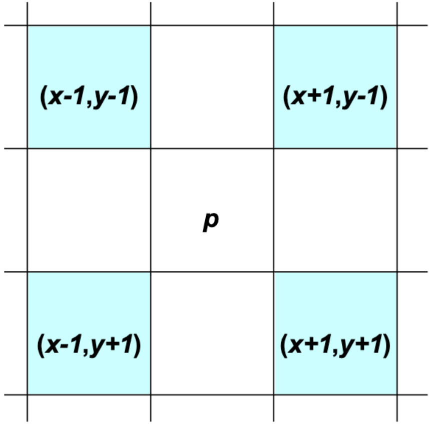
(x-1,y-1)	*(x,y-1)*	*(x+1,y-1)*
(x-1,y)	*p*	*(x+1,y)*
(x-1,y+1)	*(x,y+1)*	*(x+1,y+1)*

8-neighbors of *p*:

$$N_8(p) = \left\{ \begin{array}{c} (x-1,y-1) \\ (x,y-1) \\ (x+1,y-1) \\ (x-1,y) \\ (x+1,y) \\ (x-1,y+1) \\ (x,y+1) \\ (x+1,y+1) \end{array} \right\}$$

8-neighborhood relation considers all neighbor pixels.

Neighbors of a Pixel (cont'd...)



Diagonal neighbors of *p*:

$$N_D(p) = \left\{ \begin{array}{c} (x-1,y-1) \\ (x+1,y-1) \\ (x-1,y+1) \\ (x+1,y+1) \end{array} \right\}$$

Diagonal -neighborhood relation considers only diagonal neighbor pixels.

Connectivity

Connectivity is adapted from neighborhood relation. Two pixels are connected if they are in the same class (i.e. the same color or the same range of intensity) and they are neighbors of one another.

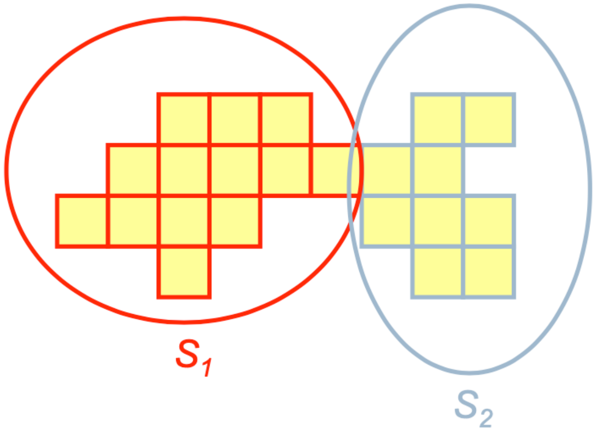
For p and q from the same class

- 4-connectivity: p and q are 4-connected if $q \in N_4(p)$
- 8-connectivity: p and q are 8-connected if $q \in N_8(p)$
- mixed-connectivity (m-connectivity):

  p and q are m-connected if $q \in N_4(p)$ or
  $q \in N_D(p)$ and $N_4(p) \cap N_4(q) = \varnothing$

Adjancency

A pixel *p* is *adjacent* to pixel *q* is they are connected. Two image subsets S_1 and S_2 are adjacent if some pixel in S_1 is adjacent to some pixel in S_2



We can define type of adjacency: 4-adjacency, 8-adjacency or m-adjacency depending on type of connectivity.

Path

A ***path*** from pixel p at (x,y) to pixel q at (s,t) is a sequence of distinct pixels:

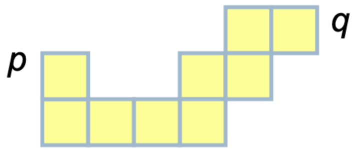
$$(x_0,y_0), (x_1,y_1), (x_2,y_2),\ldots, (x_n,y_n)$$

such that

$$(x_0,y_0) = (x,y) \text{ and } (x_n,y_n) = (s,t)$$

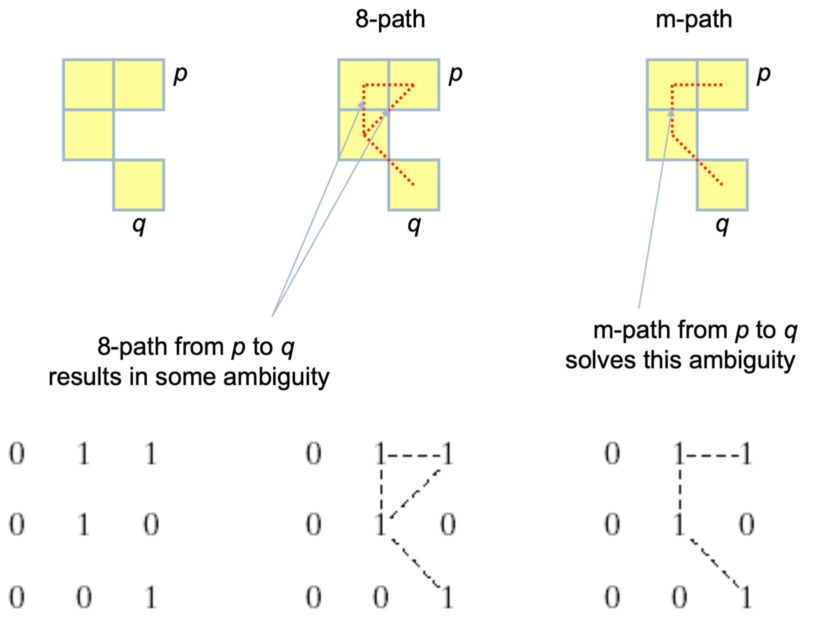
and

$$(x_i,y_i) \text{ is adjacent to } (x_{i-1},y_{i-1}), \qquad i = 1,\ldots,n$$



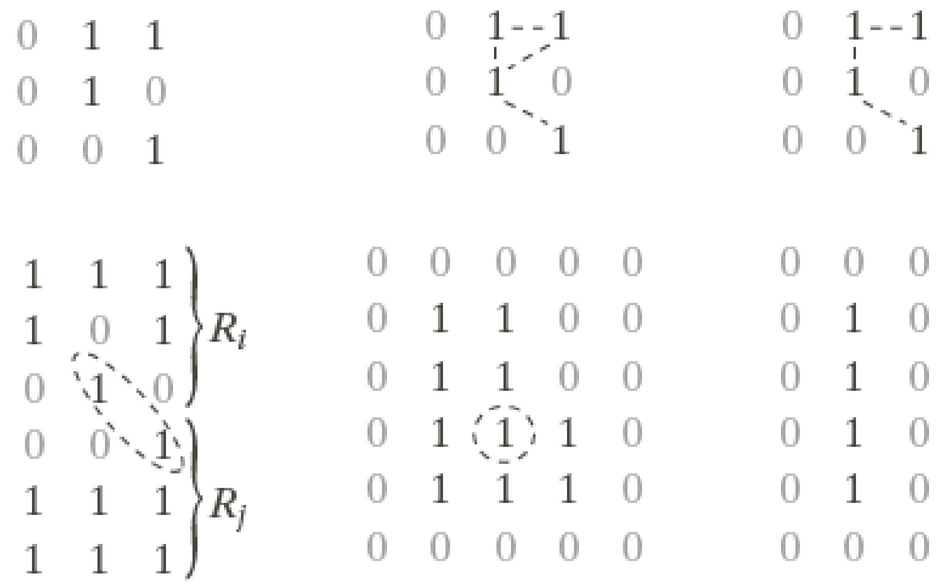
We can define type of path: 4-path, 8-path or m-path depending on type of adjacency.

Path (Cont’d…)



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.



a	b	c
d	e	f

FIGURE 2.25 (a) An arrangement of pixels. (b) Pixels that are 8-adjacent (adjacency is shown by dashed lines; note the ambiguity). (c) m-adjacency. (d) Two regions that are adjacent if 8-adjecency is used. (e) The circled point is part of the boundary of the 1-valued pixels only if 8-adjacency between the region and background is used. (f) The inner boundary of the 1-valued region does not form a closed path, but its outer boundary does.

Distance

For pixel p, q, and z with coordinates (x,y), (s,t) and (u,v), D is a ***distance function*** or ***metric*** if

- $D(p,q) \geq 0$ $\quad$ ($D(p,q) = 0$ if and only if $p = q$)
- $D(p,q) = D(q,p)$
- $D(p,z) \leq D(p,q) + D(q,z)$

Example: Euclidean distance

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

Distance (Cont'd…)

D_4-distance (*city-block distance*) is defined as

$$D_4(p,q) = |x-s| + |y-t|$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Pixels with $D_4(p) = 1$ is 4-neighbors of *p*.

Distance (Cont'd…)

D_8-distance (*chessboard distance*) is defined as

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Pixels with $D_8(p) = 1$ is 8-neighbors of p.

Boundary

- Boundary (Border or Contour)
  of a region, R is the set of points that are adjacent to points in the complement of R.
- A region is the set of pixels in the region that has at least one background neighbor.
  - **Inner Border**
  - **Outer Border**

- Arithmetic operations
    - Addition

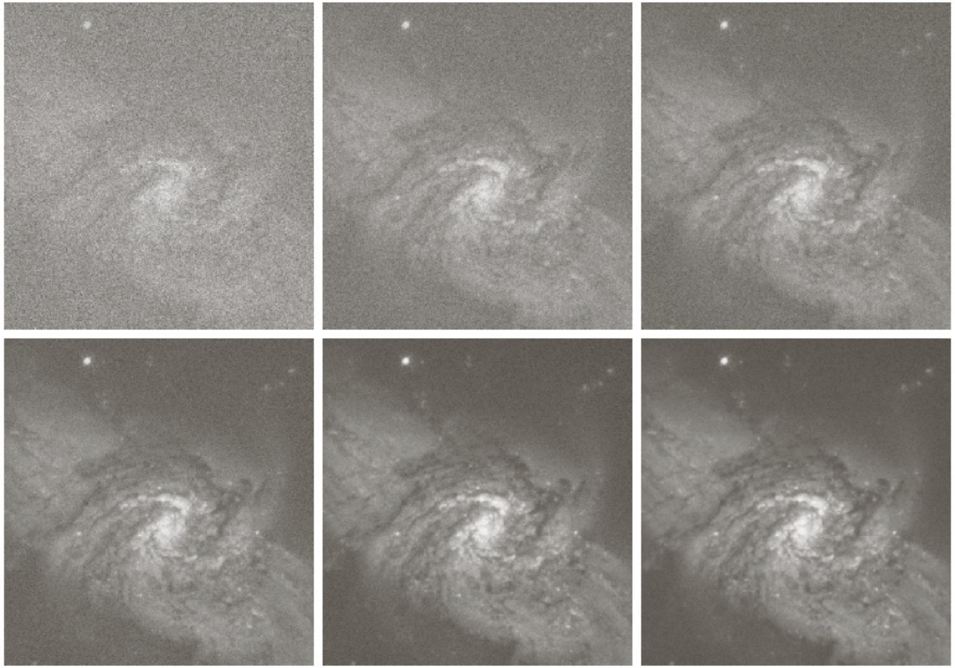
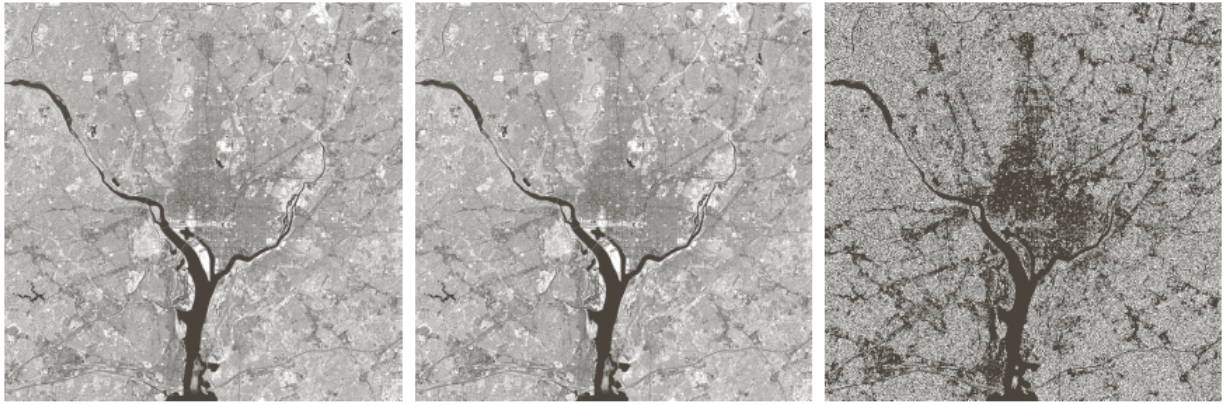


FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

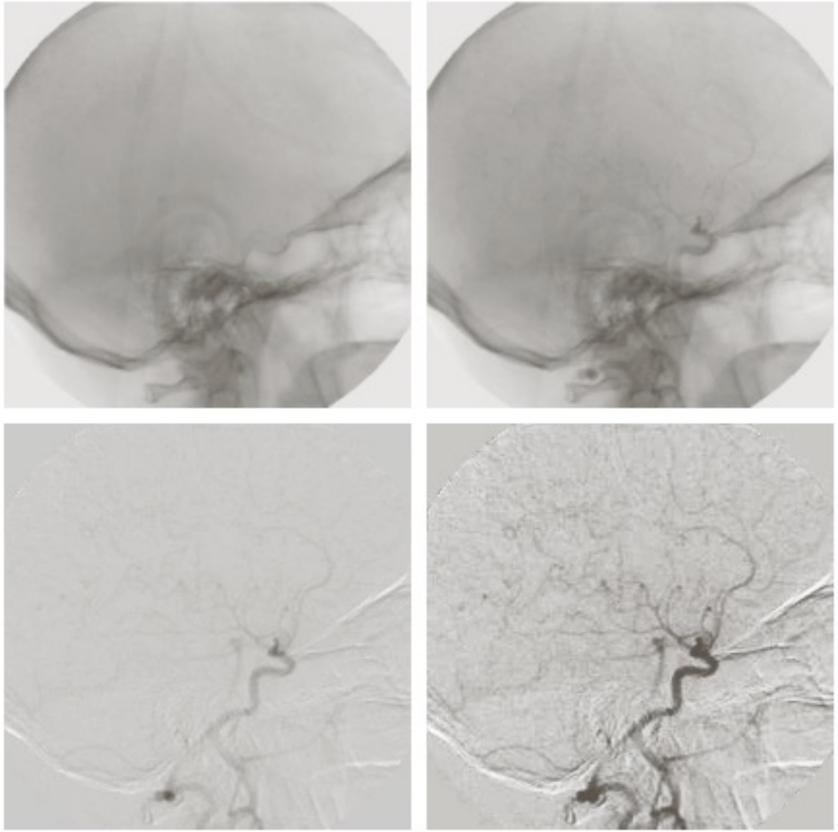
- Arithmetic operations
    - Subtraction



a b c

FIGURE 2.27 (a) Infrared image of the Washington, D.C. area. (b) Image obtained by setting to zero the least significant bit of every pixel in (a). (c) Difference of the two images, scaled to the range [0, 255] for clarity.

- Digital subtraction angiography



a b
c d

FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

- Shading correction

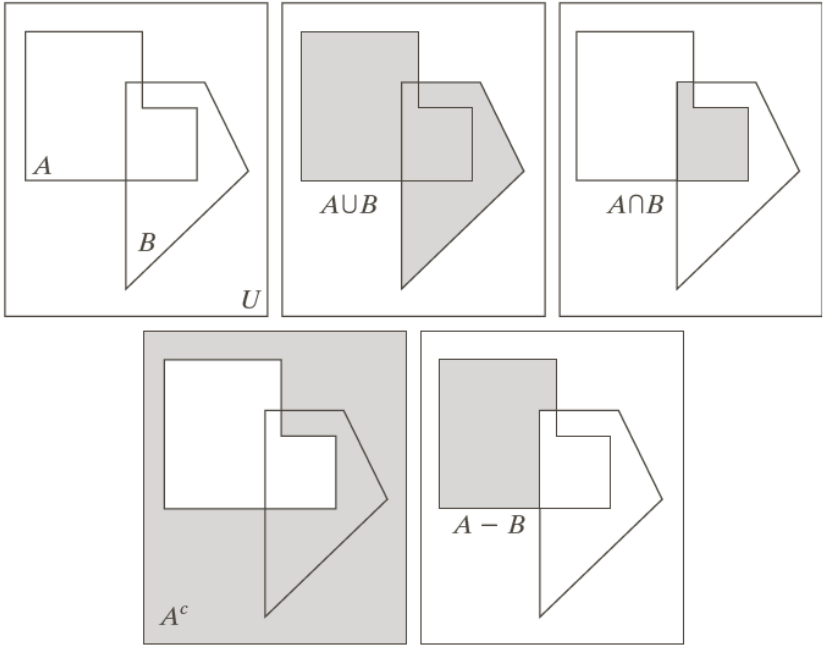
FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

- Image multiplication

a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

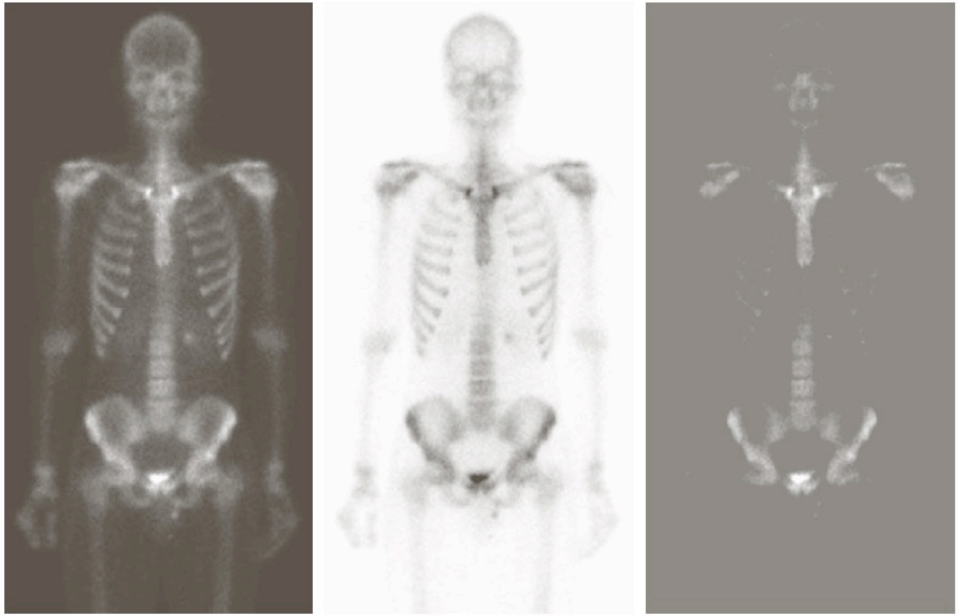
- Set operations



a b c
d e

FIGURE 2.31
(a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

- Complements



a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

- Logical operations

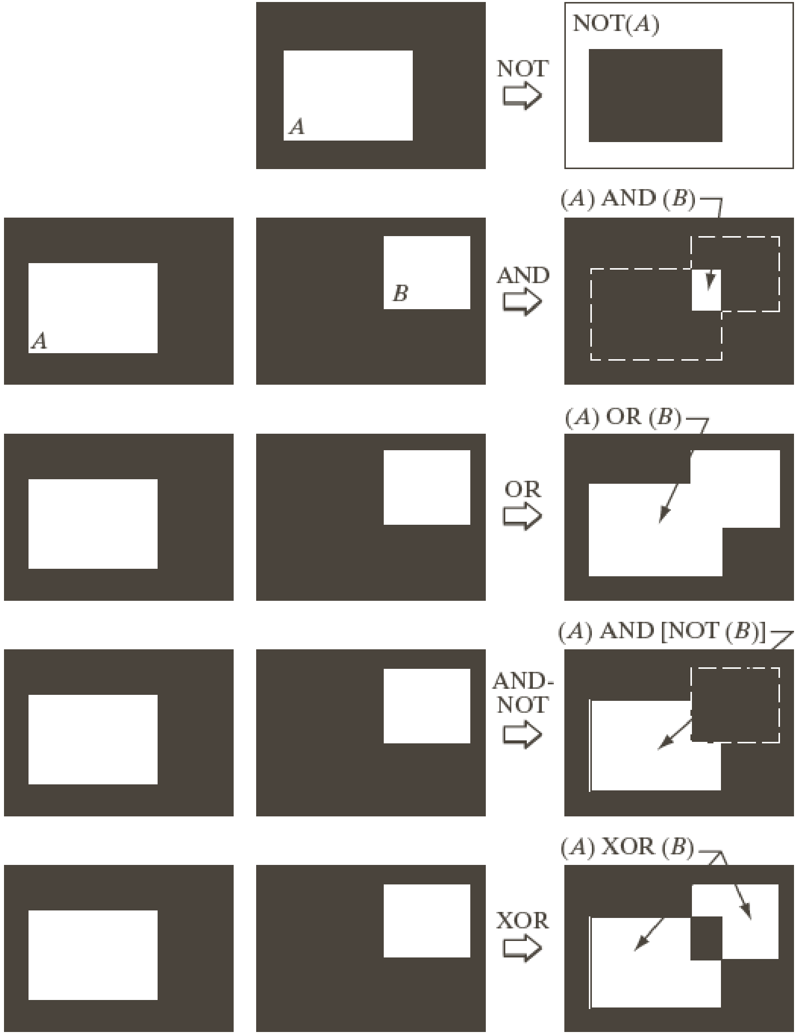


FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

- Single-pixel operations

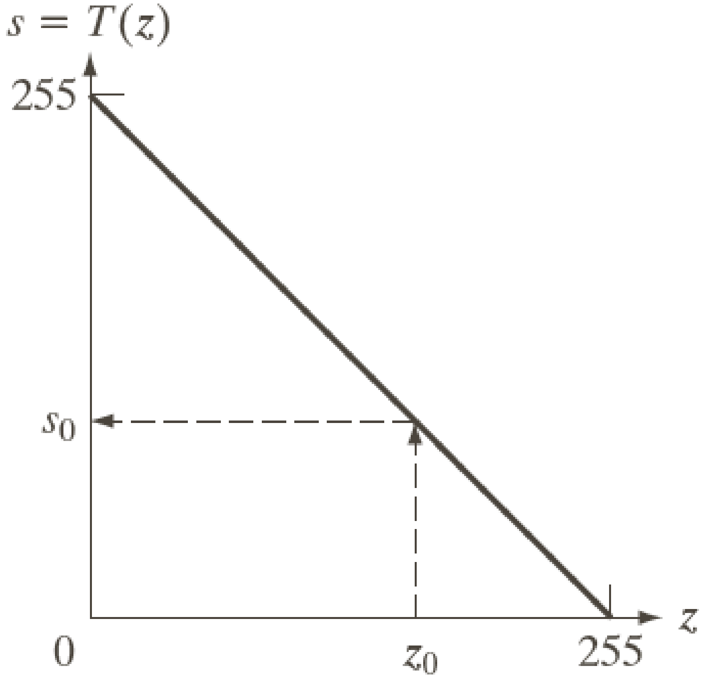


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0.

- Neighborhood operations

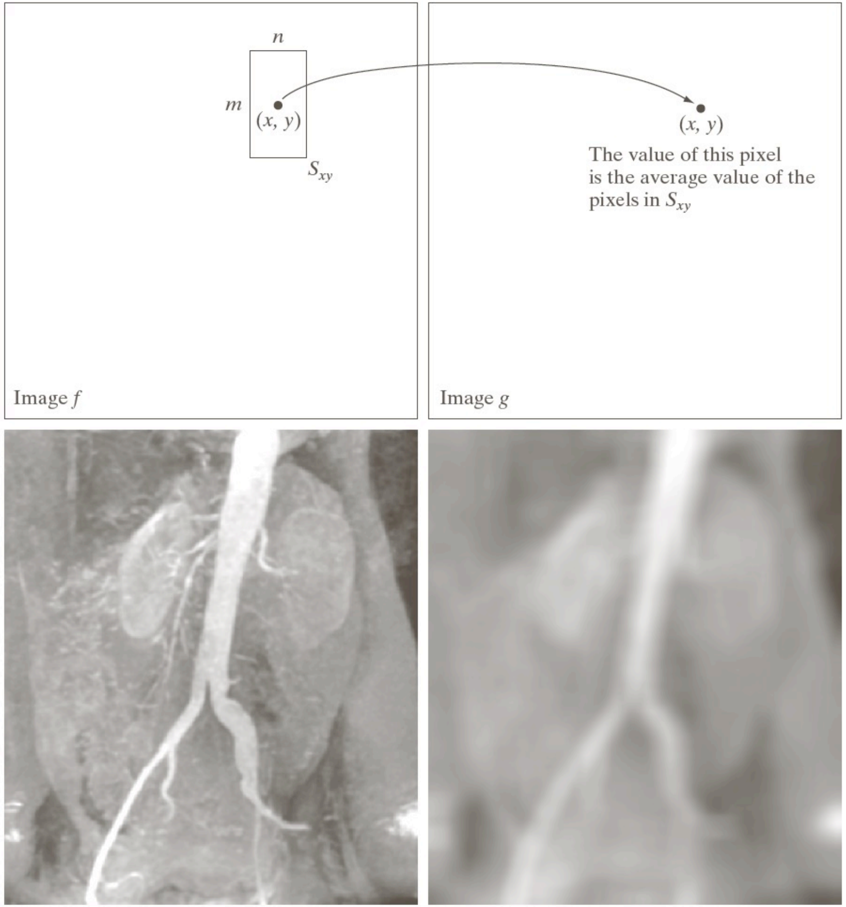
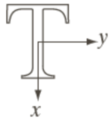
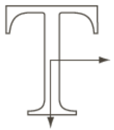
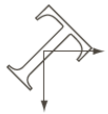
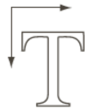
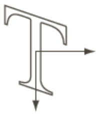


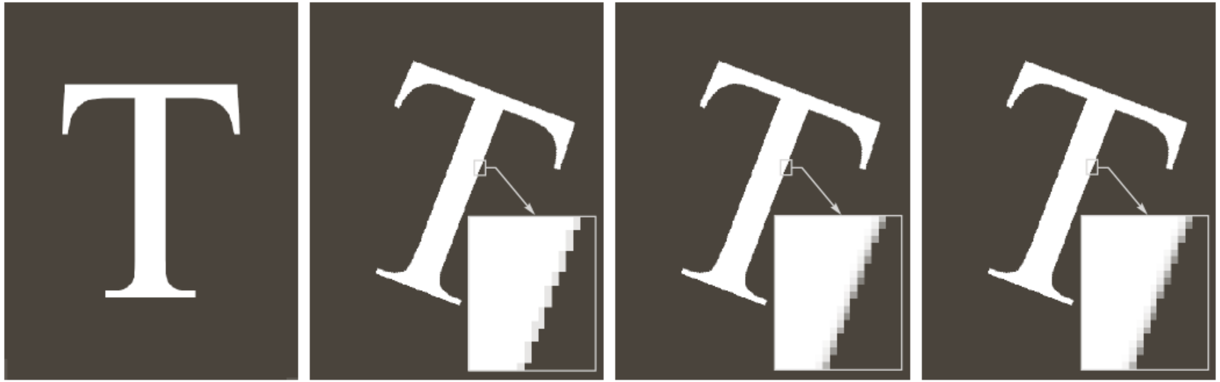
FIGURE 2.35 Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with $m = n = 41$. The images are of size 790×686 pixels.

Affine transformations

TABLE 2.2
Affine transformations based on Eq. (2.6.–23).

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

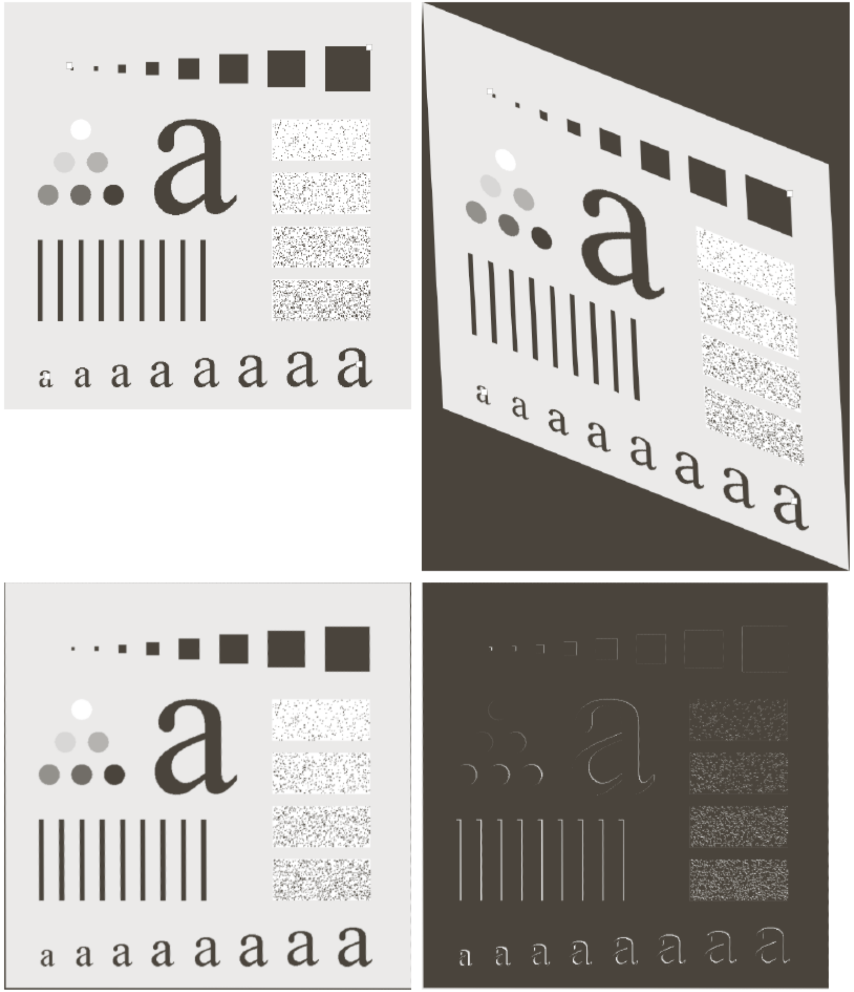
- Inverse mapping



a b c d

FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

- Registration



a b
c d

FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the borders). (d) Difference between (a) and (c), showing more registration errors.

- Vector operations

FIGURE 2.38
Formation of a vector from corresponding pixel values in three RGB component images.