Probability Methods in Engineering

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Lecture 19

Properties of cdf

$$0 \le F_X(x) \le 1$$

$$\lim_{x \to \infty} F_X(x) = 1$$

$$\lim_{x \to -\infty} F_X(x) = 0$$

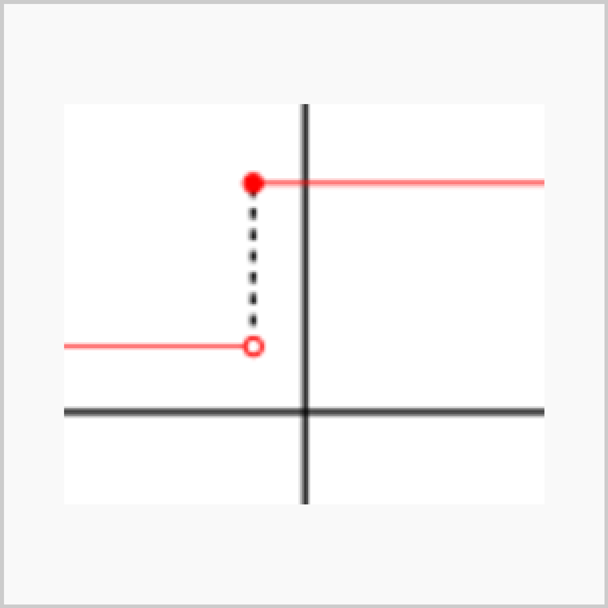
- Non-decreasing function, if $a < b$, then

$$F_X(a) \le F_X(b)$$

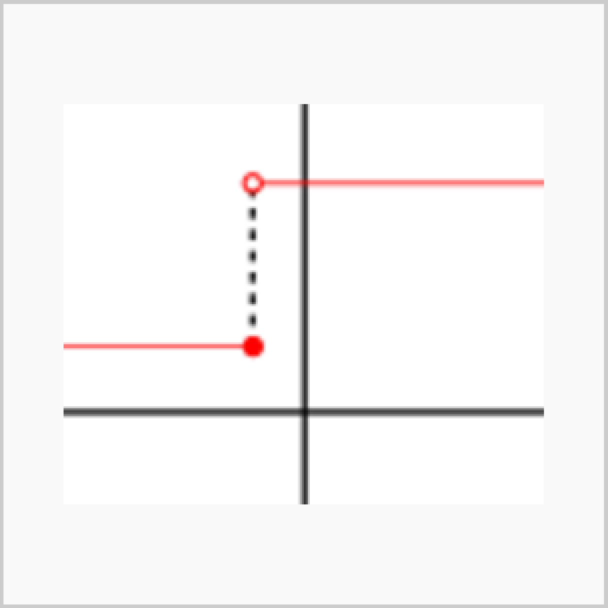
- Continuous from the right, for $h > 0$,

$$F_X(b) = \lim_{h \to 0} F_X(b + h) = F_X(b^+)$$

Properties of cdf (cont.)



A right-continuous function



A left-continuous function

Source: *https://en.wikipedia.org/wiki/Continuous_function*

Properties of cdf (cont.)

- Properties for calculating the probability of events involving intervals and single values of X

$$P[a < X \leq b] = F_X(b) - F_X(a)$$

$$P[X = b] = F_X(b) - F_X(b^-)$$

$$P[X > x] = 1 - F_X(x)$$

Examples

- Let X be the number of heads in three tosses of a fair coin. Use the cdf to find the probability of the events

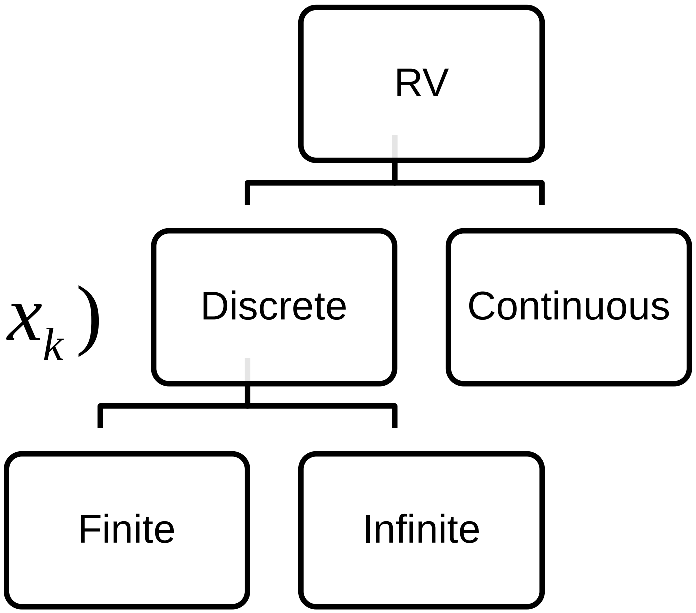
$$P[1 < X \le 2]$$

$$P[0.5 \le X \le 2.5]$$

$$P[1 \le X < 2]$$

Continuous RV

- cdf of discrete RV
  - right-continuous
  - staircase function
  - jumps at countable set of points



$$F_X(x) = \sum_{x_k \le x} p_X(x_k) = \sum_k p_X(x_k) u(x - x_k)$$

- cdf of continuous RV
  - Continuous everywhere
  - Integration instead of summation

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt$$

  - where $P[X = x] = 0$ at all values

Probability Density Function

- The pdf is the derivative of $F_X(x)$ if it exists

$$f_X(x) = \frac{dF_X(x)}{dx}$$

Properties of pdf

$$f_X(x) \geq 0$$

$$P[a \leq X \leq b] = \int_a^b f_X(x)dx$$

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt$$

$$1 = \int_{-\infty}^{\infty} f_X(t)dt$$

Examples

- The probability density function of a R. V is given as:

$$f(x)=3x^2, \qquad 0 < X < 1$$

What is the cumulative distribution function F(x)?

Examples

- Let's X has the following probability density function:

$$f(x) = \frac{x^3}{4} \text{for } 0 < X < 2$$

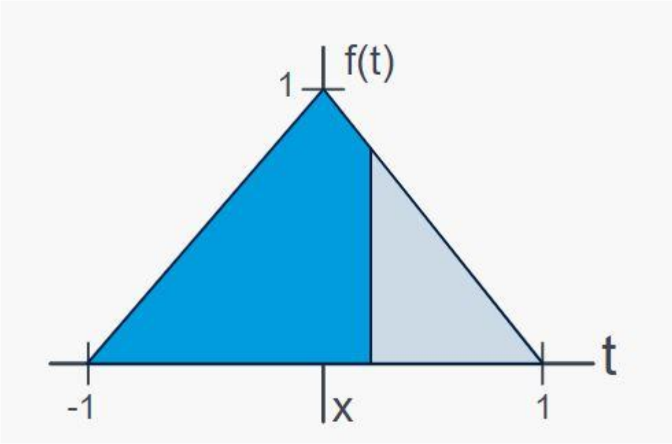
What is the cumulative distribution function of X?

Examples

- The pdf of uniform RV is given by

$$f(x) = \begin{cases} x+1, & -1 < x < 0 \\ 1-x, & 0 \le x < 1 \end{cases}$$

Find its cdf.



Examples

- The pdf of uniform RV is given by

$$f_X(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & x < a \text{ and } x > b \end{cases}$$

Find its cdf.

Examples (cont.)

- The transmission time X of messages in a communication system has an exponential distribution with cdf

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$$

Find its pdf.