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SECTION: B

ASSINGNMENT NO: SECOND

SUBMITTED TO:

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COMPUTER SYSTEM ENGINEERING

QNO 1: An FIR Filter is described by The difference Equation, $y[n] = \frac{1}{2} x[n] + \frac{1}{2} x[n-1]$ (a) compute and Sketch its Magnitude And phase Response (b) ....

part (a)

Solution:

$$y[n] = \frac{1}{2} [x[n]] + \frac{1}{2} x[n-1]$$

$$y[n] = \frac{1}{2} [x[n] + x[n-1]] \rightarrow (a)$$

Take Fourier Transform on (a)

$$Y(\omega) = \frac{1}{2}[X(\omega) + X(\omega) e^{-j\omega}]$$

$X(\omega) e^{-j\omega} \longrightarrow$ ~~Time~~ Shifting

$$Y(\omega) = \frac{1}{2} X(\omega) [1 + e^{-j\omega}]$$

$X(\omega) \longrightarrow$ common

$$\frac{Y(\omega)}{X(\omega)} = H(\omega) \longrightarrow \text{Transfer Function of a System}$$

$$H(\omega) = \frac{1}{2} [1 + e^{-j\omega}] \rightarrow (b)$$

we can write further Simplification on (b)

$$= \frac{1}{2} [1 + e^{-j\omega/2} \cdot e^{-j\omega/2}]$$

$$= \frac{1}{2} e^{-j\frac{\omega}{2}} [e^{j\omega/2} + e^{-j\omega/2}]$$

$$\Rightarrow e^{-j\frac{\omega}{2}} \left[\frac{e^{j\omega/2} + e^{-j\omega/2}}{2} \right]$$

Qno1 (2)

$$H(\omega) = e^{-j\omega/2}\left[\frac{e^{j\omega/2} + e^{-j\omega/2}}{2}\right]$$

As we know

$$\cos\phi = \frac{e^{j\phi} + e^{-j\phi}}{2}$$

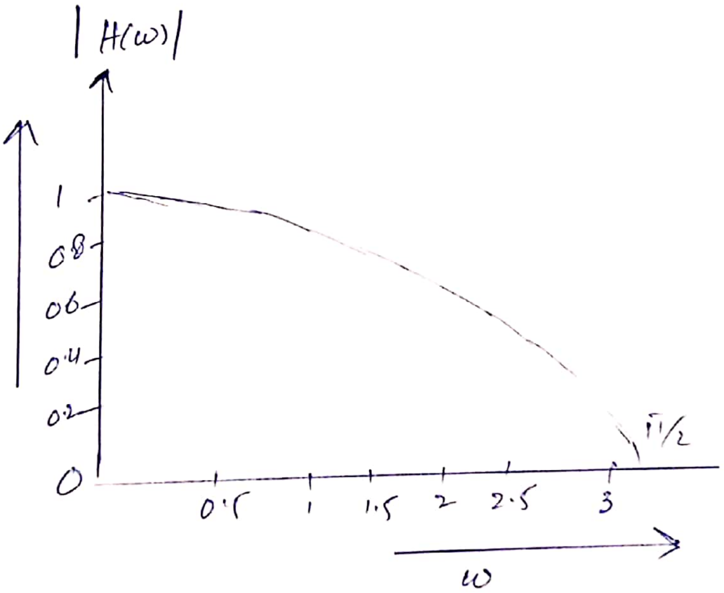
$$H(\omega) = e^{-j\omega/2}\cos(\omega/2)$$

Determine its Magnitude Response

$$|H(\omega)| = \left|\cos(\omega/2)\, e^{-j\omega/2}\right|$$

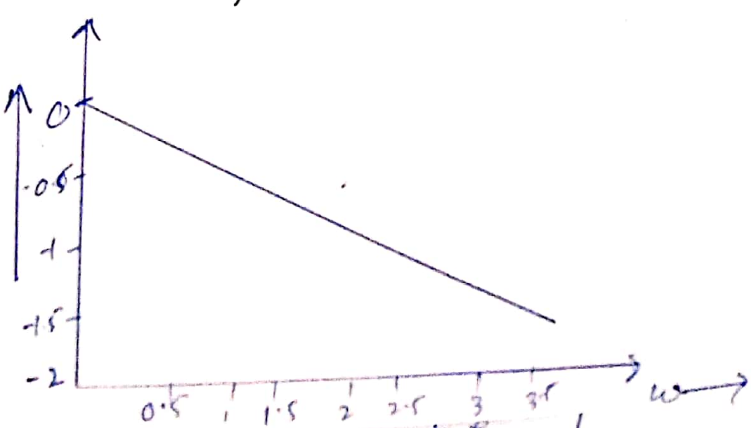
$$= \cos(\omega/2)\left|e^{-j\omega/2}\right|$$

$$= \cos(\omega/2)$$



Determine The Phase Response of The System

$$\angle H(\omega) = -\omega/2$$



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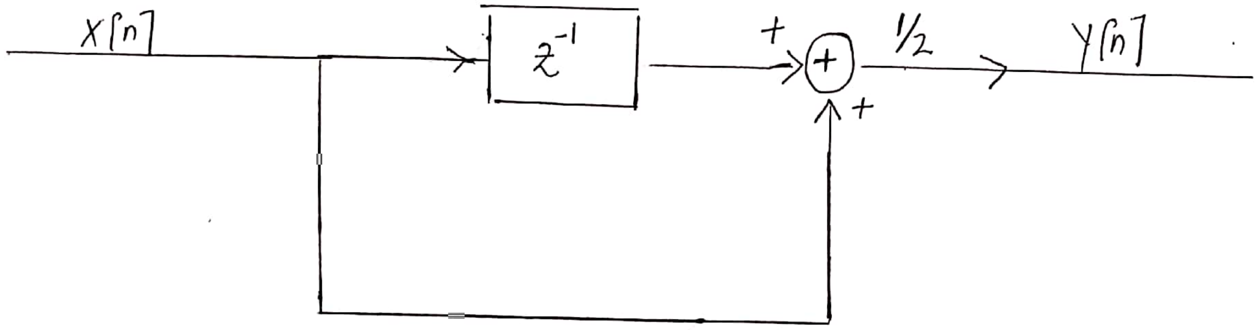
part (b)

QUESTION: 02 Determine And sketch The Magnitude And phase Response OF the System shown in The Following



SOLUTION: From The above figure we can write the following difference Equation.

$x[n] \longrightarrow$ input

$y[n] \longrightarrow$ output

$$y(n) = \left(x(x-1) + x(n)\right)\frac{1}{2}$$

$$y(n) = \frac{1}{2}\left[x(n) + x(n-1)\right] \rightarrow (a)$$

Apply Fourier Transform on both side.

$$Y(\omega) = \frac{1}{2}\left[X(\omega) + e^{-j\omega}X(\omega)\right]$$

$e^{-j\omega}X(\omega) \longrightarrow$ using shifting properties

$$Y(\omega) = \frac{1}{2}X(\omega)\left[1 + e^{-j\omega}\right]$$

$X(\omega) \longrightarrow$ take common.

$$\frac{Y(\omega)}{X(\omega)} = \frac{1}{2}\left[1 + e^{-j\omega}\right]$$

As we know that

$$\frac{Y(w)}{X(w)} = H(w)$$

Then

$$H(w) = \frac{1}{2}\left(1+e^{-jw}\right)$$

Simplify the system Function further.

$$H(w) = \frac{1}{2}\left(1+e^{-j\frac{w}{2}} \cdot e^{j\frac{w}{2}}\right)$$

because $e^{-jw/2} \cdot e^{-jw/2} = e^{-jw}$

$$= \frac{1}{2} e^{-jw/2}\left(e^{jw/2} + e^{-jw/2}\right)$$

$e^{-jw/2} \longrightarrow$ common

$$\Rightarrow \quad e^{-jw/2}\left(\frac{e^{jw/2} + e^{-jw/2}}{2}\right)$$

As we know

$$\cos\phi = \frac{e^{j\phi} + e^{-j\phi}}{2}$$

$$\Rightarrow \quad \cos(w/2)\, e^{-jw/2}$$

Determine The Magnitude Response of the System

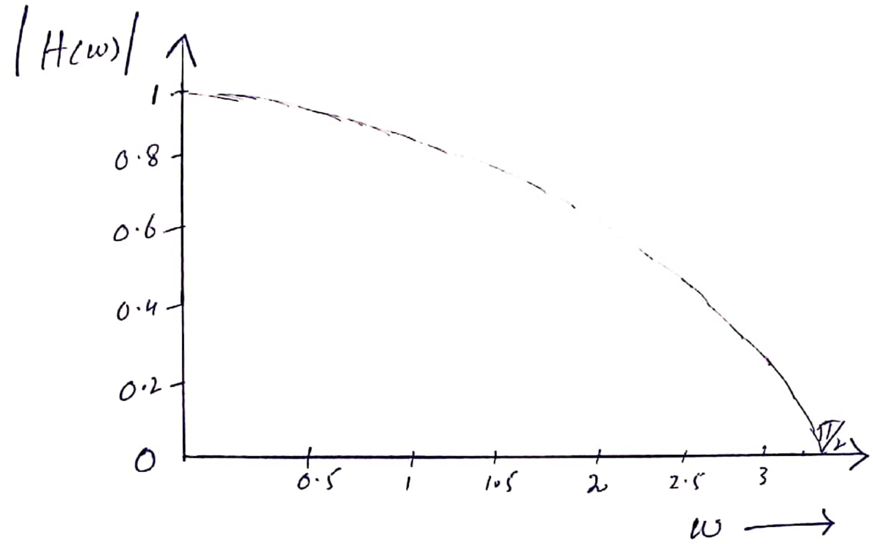
$$|H(w)| = \left|\cos(w/2)\, e^{-jw/2}\right|$$

$$\Rightarrow \quad \cos(w/2)\left|e^{-jw/2}\right|$$

$$\Rightarrow \quad \cos(w/2)$$

$Q_2(3)$

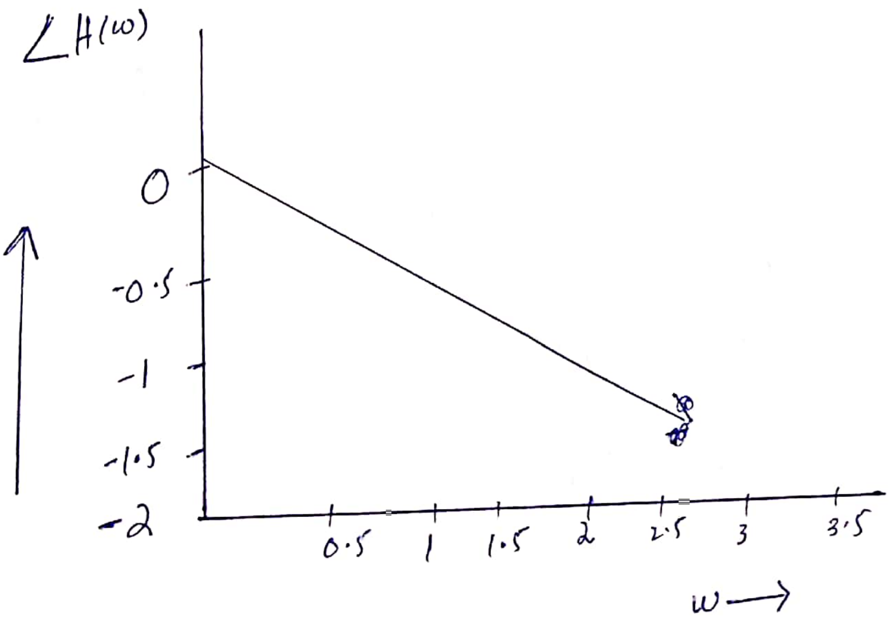
Draw The Magnitude Response of The system.



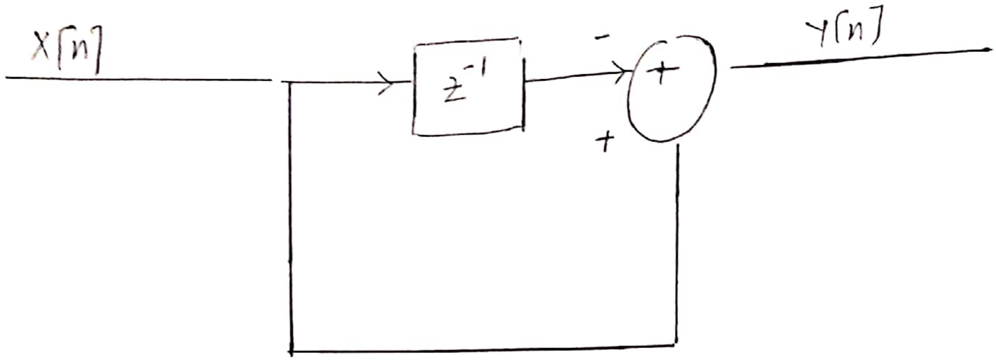
Determine The phase Response of The system

$$\angle H(w) = -w/2$$

Draw The phase Response of the system.



Q2 (4)



Solution: write The difference Equation for the above fig.

$$Y[n] = \left(-x(n-1) + x(n)\right)\frac{1}{2}$$

$$= \frac{1}{2}\left[x[n] - x(n-1)\right]$$

Apply Fourier Transform ON both side,

$$Y(\omega) = \frac{1}{2}\left[X(\omega) - e^{-j\omega}X(\omega)\right]$$

$e^{-j\omega}X(\omega) \longrightarrow$ using shifting properties.

$$Y(\omega) = \frac{1}{2}X(\omega)\left[1 - e^{-j\omega}\right]$$

$$\frac{Y(\omega)}{X(\omega)} = H(\omega) \longrightarrow \text{System Transfer Function}$$

$$H(\omega) = \frac{1}{2}\left[1 - e^{-j\omega}\right]$$

$$H(\omega) = \frac{1}{2}\left[1 - e^{-j\omega/2} \cdot e^{-j\omega/2}\right]$$

because

$$e^{-j\omega/2} \cdot e^{-j\omega/2} = e^{-j\omega}$$

$$= \frac{1}{2}\, e^{-j\omega/2}\left(e^{j\omega/2} - e^{-j\omega/2}\right]$$

$$H(\omega) = e^{-j\omega/2}\left[\frac{e^{j\omega/2} - e^{-j\omega/2}}{2}\right]$$

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$y(n)$

multiply And divide by j Q_2 (5)

$$H(\omega) = je^{-j\omega/2}\left[\frac{e^{j\omega/2} - e^{-j\omega/2}}{2j}\right]$$

we also Know That

$$\sin\phi = \frac{e^{j\phi} - e^{-j\phi}}{2j}$$

$$= je^{-j\omega/2}\sin(\omega/2)$$

Recall Euler's definition,

$$e^{j\phi} = \cos\phi + j\sin\phi$$

$$e^{j\pi/2} = \cos(\pi/2) + j\sin(\pi/2)$$

$$e^{j\pi/2} = 1$$

$$H(\omega) = \sin(\omega/2)\, e^{-j(\omega/2)}$$

Determine The Magnitude Response of The system

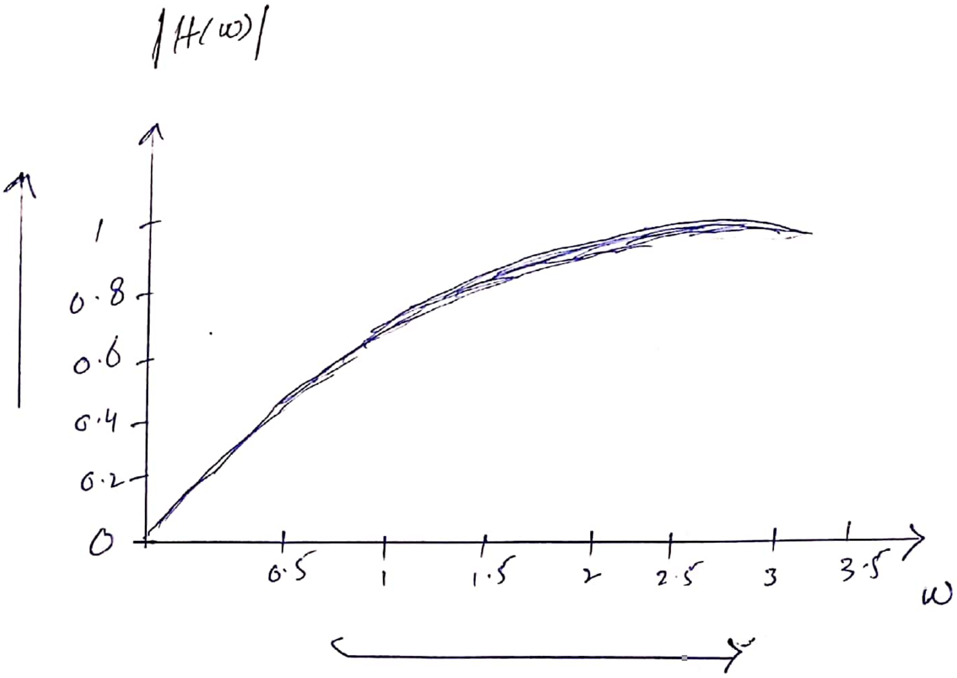
$$|H(\omega)| = \left|\sin(\omega/2)\, e^{-j(\omega/2)}\right|$$

$$\sin(\omega/2)\left|e^{-j(\omega/2)}\right|$$

$$|H(\omega)| = \sin(\omega/2)$$

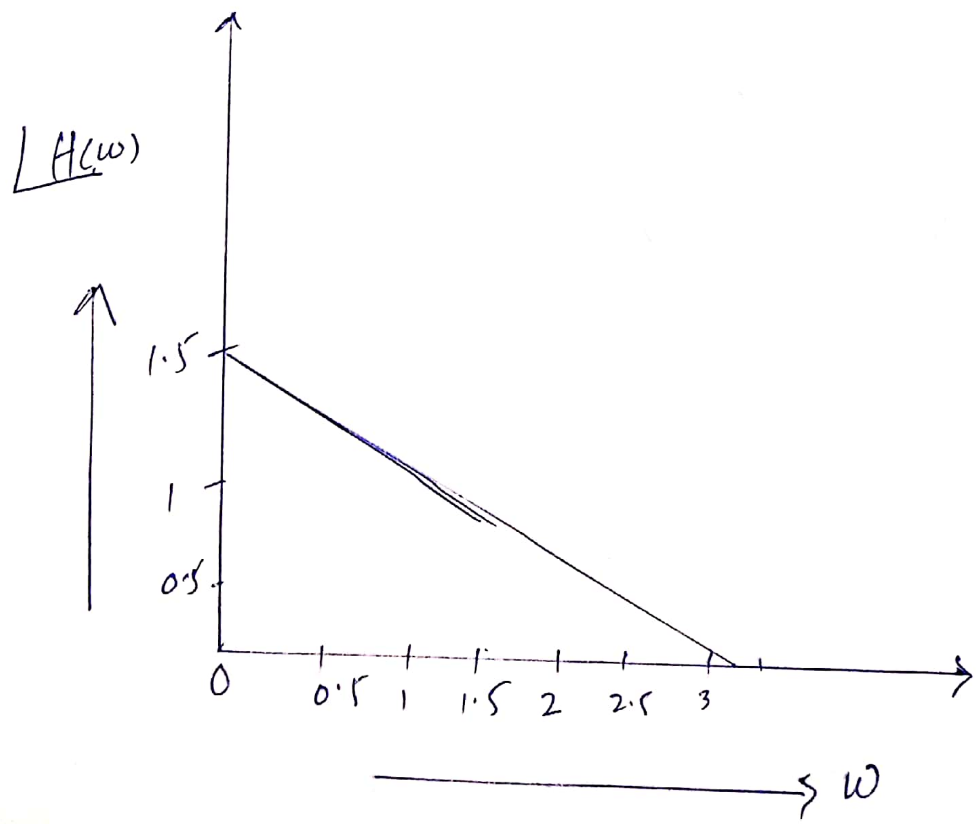
Q2(6)

Draw The Magnitude Response of The system.

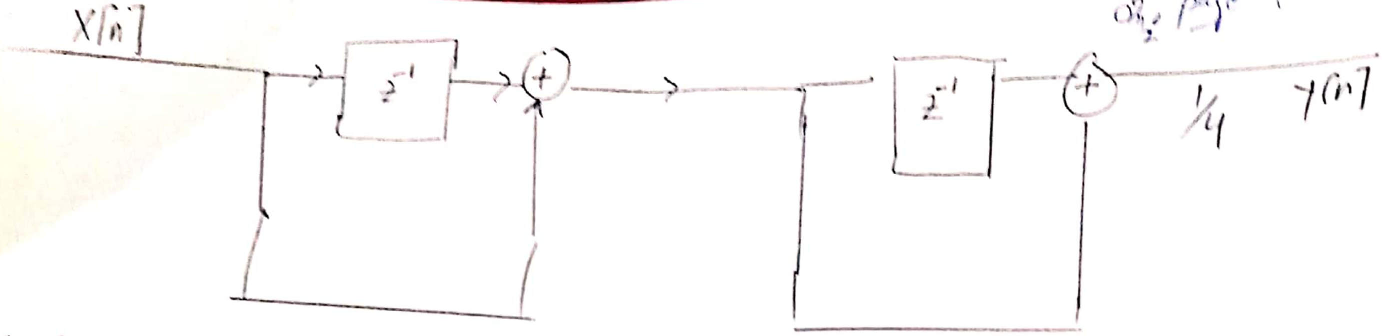


Determine The phase Response of the System.

$\angle H(w) = -w/2$



Q2, page (b)



SOLUTION: First of all write the difference Equation for the above fig.

Y[n] → out put

X[n] → input

$$Y[n] = \frac{1}{4}\left[\left[x[n] + x[n-1]\right] + \left[x[n] + x[n-2]\right]\right]$$

$$= \frac{1}{4}\left[x[n] + x[n-1] + x[n] + x[n-2]\right]$$

$$Y(n) = \frac{1}{4}\left[2x[n] + x[n-1] + x[n-2]\right]$$

Apply Fourier Transform on both sides.

$$Y(\omega) = \frac{1}{4}\left[2X(\omega) + e^{-j\omega}X(\omega) + e^{-2j\omega}X(\omega)\right]$$

X(ω) → common.

$$Y(\omega) = \frac{2}{4}X(\omega)\left[1 + e^{-j\omega} + e^{-2j\omega}\right]$$

$$\frac{Y(\omega)}{X(\omega)} = H(\omega) \rightarrow \text{system Transfer function.}$$

$$H(\omega) = \frac{1}{2}\left[1 + e^{-j\omega} + e^{-2j\omega}\right]$$

$$= \frac{1}{2}\left[1 + e^{-j\omega} + e^{-j\omega}\cdot e^{-j\omega}\right]$$

$e^{-j\omega}$ → common.

$$= \frac{1}{2}\left[1 + e^{-j\omega}\left(e^{j\omega} + e^{-j\omega}\right)\right]$$

$$\Rightarrow \left[1 + e^{-j\omega}\left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right)\right]$$

$$H(\omega) = 1 + e^{-j\omega}(\cos\omega)$$

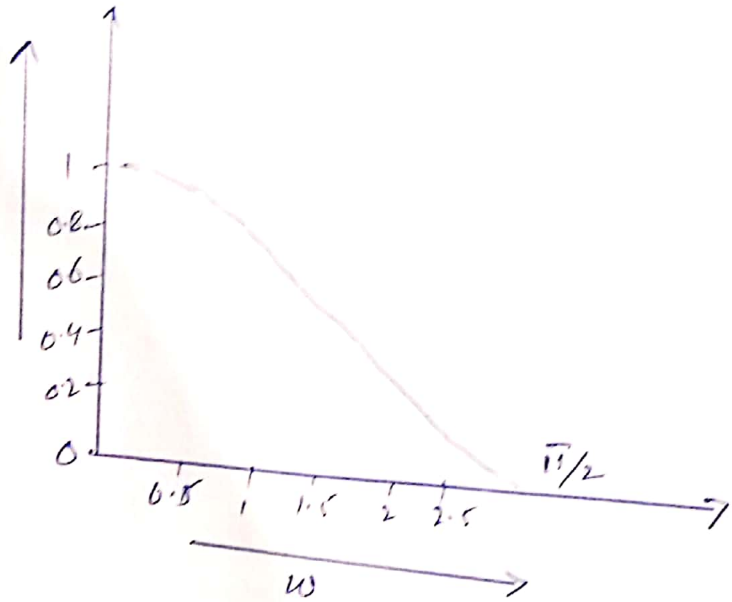
$$|H(\omega)| = |1 + e^{-j\omega}(\cos\omega)|$$

$$= \cos(\omega)\,|e^{-j\omega}|$$

$$= \cos(\omega)$$

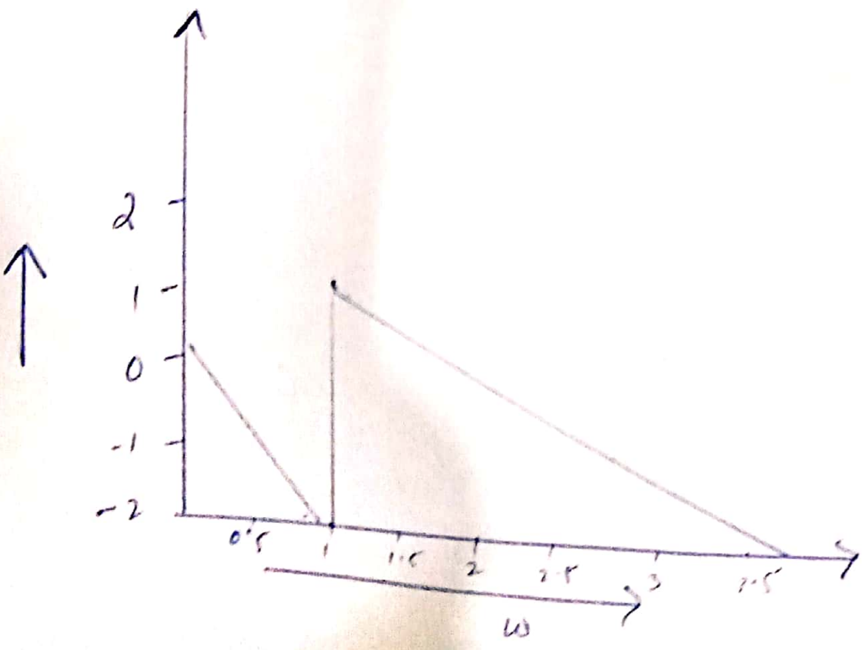
Draw The magnitude Response of The System

$|H(\omega)|$



Determine The phase Response of The system.

$\angle H(\omega) = \omega$



Q NO: 3: The Fourier Transform of a signal is given by $X(\omega) = 1/1-\frac{1}{3}e^{-j\omega}$, Find the Fourier Transform of the following signals using properties of the Fourier Transform,

SOLUTION:

$$X(\omega) = 1/1-\frac{1}{3}e^{-j\omega}$$

The signal has Fourier Transform as

$$X[n] \xrightarrow{F.T} X(\omega)$$

$$X(\omega) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

(a) $X(n-2)$:

As we know

$$X[n] \xrightarrow{F.T} X(\omega)$$

$$X[n-2] \xrightarrow{F.T} e^{-2j\omega}X(\omega)$$

$$e^{-2j\omega}X(\omega) = \frac{1}{1-\frac{1}{3}e^{-j\omega}} \times e^{-2j\omega}$$ SHIFTING properties

We also discrete Fourier transform of a signal is given by.

$$X(\omega) = \sum_{n=-\infty}^{\infty} X[n]\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} X(n-2)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} X(\omega)\, e^{-2j\omega n} \cdot e^{-j\omega n} \Rightarrow \boxed{X(\omega)\, e^{-2j\omega}}$$

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(b) $e^{\pi/3 n} x[-n+1]$:

As we know

$$x[n] \xrightarrow{F.T} X(\omega)$$

$$X(\omega) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

from given

$$X[n+1] \xrightarrow{F.T} X(\omega)e^{j\omega} \longrightarrow$$ Shifting properties

$$X[-n+1] \xrightarrow{F.T} X(-\omega)e^{-j\omega} \longrightarrow$$ Reversal Properties

$$e^{\pi/3 n} x[-n+1] \xrightarrow{F.T} X(-\omega-\pi/3)\, e^{-j(\omega-\pi/3)} \longrightarrow$$ Scaling properties.

$$X[n+1] = \frac{(e^{j\omega})}{1-\frac{1}{3}e^{j\omega}}$$

$$X(-n+1) = X(-\omega)\, e^{-j\omega}$$

$$x(-\omega-\pi/3)\, e^{-j(\omega-\pi/3)} = \left(\frac{1}{1-\frac{1}{3}e^{-j(\omega-\pi/3)}}\right) e^{-j(\omega-\pi/3)}$$

We also know That a discrete Fourier Transform of a Signal is Given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} X[n]\, e^{-j\omega_o n}$$

$$= \sum_{n=-\infty}^{\infty} e^{\pi/3 n} x[-n+1] \rightarrow (a)$$

Apply properties of shifting and Reversal we get

$$X(\omega) = \sum_{n=-\infty}^{\infty} e^{\frac{\pi}{3}n} x[-n+1]\, e^{-j\omega_0 n}$$

$$X(\omega) = \left(\frac{1}{1-\frac{1}{3}e^{j\omega}}\right) e^{j\omega}, \quad X(-\omega) = \left(\frac{1}{1-\frac{1}{3}e^{j\omega}}\right) \times e^{-j\omega}, \quad X(\omega) = \frac{1}{\left(1-\frac{1}{3}e^{-j(\omega-\pi/3)}\right)} \times e^{-j(\omega-\pi/3)}$$

Shifting — Reversal — Scaling.

(c) $n[x[n] * x[n-1]]$:-

Solution:-

properties used

(i) Convolution $\longrightarrow x[n] * x[n-1]$

(ii) Differentiation $\longrightarrow n[x[n] * x[n-1]]$

(iii) Shifting $\longrightarrow n[x[n] * x[n-1]]$.

Convolution:-

$x[n] * x[n-1]$

$$X(\omega) = \sum_{n} x[n] * x(n-1)\, e^{-j\omega n}$$

From the properties of Fourier Transform

$$x_1(n) * x_2(n) \overset{F}{\longleftrightarrow} X_1(\omega) \times X_2(\omega)$$

$$X(\omega) = X_1(\omega) \times X_2(\omega)$$

$$X(\omega)\left[X(\omega)\, e^{-j\omega}\right]$$

$$= X^2(\omega)\, e^{-j\omega} \Rightarrow \frac{1}{(1-ae^{-j\omega})^2} \times e^{-j\omega}$$

~~$X(\omega) = e^{-j\omega}$~~

Shifting:

$$x(n-1) \xrightarrow{F.T} X(\omega)\, e^{-j\omega}$$

As we know that

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x[n-1]\, e^{-j\omega}$$

Apply shifting properties.

$$= \sum_{n=\infty}^{\infty} X(\omega)\, e^{-j\omega}\, e^{-j\omega}$$

$$= X(\omega)\, e^{-j\omega}$$

Differentiation:

$$x[n] \xleftrightarrow{F.T} X(\omega)$$

$$n\,x[n] \longleftrightarrow j\frac{d\,X(\omega)}{d\omega}$$

but in our case

$$x[x[n] * x[n-1]] \xleftrightarrow{F} j\frac{d}{d\omega}\left[X(\omega).\ X(\omega)\, e^{-j\omega}\right]$$

$$\Rightarrow -j\frac{d}{d\omega}\left(\frac{1}{1-\frac{1}{3}e^{j\omega}}\right)\left(\frac{e^{-j\omega}}{(1-\frac{1}{3}e^{j\omega})^2}\right)$$