Probability Methods in Engineering

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Lecture 6

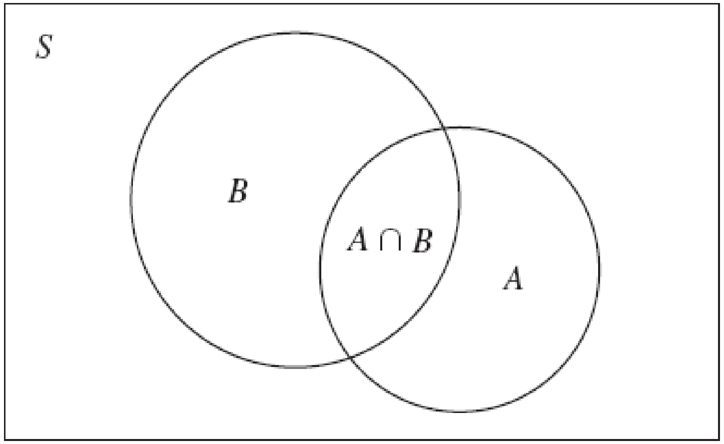
Conditional Probability

- Are two events A and B related?
- Is the occurrence of one event effecting the likelihood of other?
- **Conditional probability** of event A given that B has occurred

$$\boxed{P[A|B] = \frac{P[A \cap B]}{P[B]}}$$

- Knowledge of occurrence of event B reduces sample space
  - Occurrence of event A now depends on $A \cap B$
  - If $A \cap B = \emptyset$, occurrence of A is ruled out, given B occurred

Conditional Probability (cont.)



- If $A = B$, then due to the reduced sample space

$$\boxed{P[B \mid B] = 1}$$

- Similarly

$$\boxed{P[A \cap B] = P[A \mid B]P[B]}$$

$$\boxed{P[A \cap B] = P[B \mid A]P[A]}$$

Examples

- A ball is selected from an urn containing two black balls, numbered 1 and 2, and two white balls, numbered 3 and 4. The number and color of the ball is noted, so the sample space is {(1, b), (2, b), (3, w), (4, w)}. Assuming that the four outcomes are equally likely, find $P[A|B]$ and $P[A|C]$ where A, B, and C are the following events:

  A = {(1, b), (2, b)}, "black ball selected,"

  B = {(2, b), (4, w)}, "even-numbered ball selected," and

  C = {(3, w), (4, w)}, "number of ball is greater than 2."

Examples (cont.)

- An urn contains two black balls and three white balls. Two balls are selected at random from the urn without replacement and the sequence of colors is noted. Find the probability that both balls are black.

Example (cont.)

- Many communication systems can be modeled in the following way. First, the user inputs a 0 or a 1 into the system, and a corresponding signal is transmitted. Second, the receiver makes a decision about what was the input to the system, based on the signal it received. Suppose that the user sends 0s with probability $1-p$ and 1s with probability p, and suppose that the receiver makes random decision errors with probability ε. For i = 0, 1, let A_i be the event "input was i," and let B_i be the event "receiver decision was i." Find the probabilities $P[A_i \cap B_i]$ for i = 0, 1.

Examples (cont.)

- A teacher gave his class two quizzes. 25% of the class passed both quizzes and 42% of the class passed the first quiz. What percent of those who passed the first quiz also passed the second quiz?

Source: *http://www.mathgoodies.com/lessons/vol6/conditional.html*