Control Systems - 7th Semester - Week 4

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Model - Recalling concepts

A model is representation or abstraction of reality/system

Who invent model? We, human beings, invent model based on our knowledge

This means the more knowledge a person has, the better he/she can write a model

What is mathematical model?

- A set of equations (linear or differential) that describes the relationship between input and output of a system

Types of Model

There are three types of mathematical models

- Black Box
- Grey Box
- White Box

Introduction to Transient Analysis

Sometimes we cannot write white box models for the systems

Either we do NOT know what is inside the system or either the system is too complex to verify the components

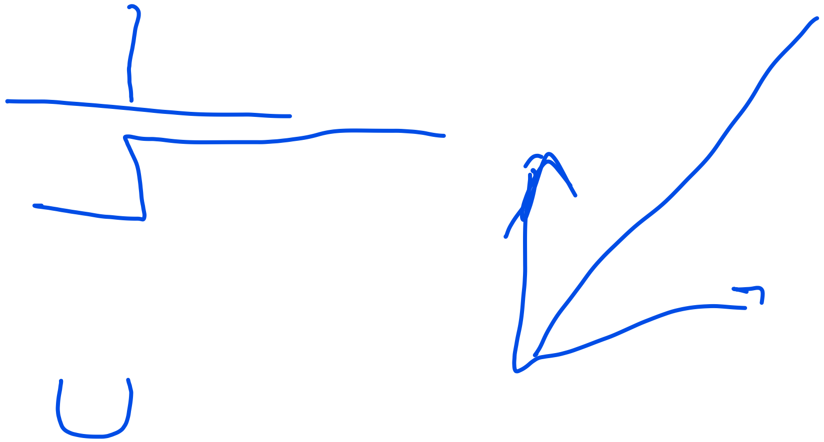
Perhaps sometimes the components are not easily identifiable and their configuration or layout is not readable

So another way to obtain model of a system is to apply a test input signal and obtain the output signal

Standard Input Signals

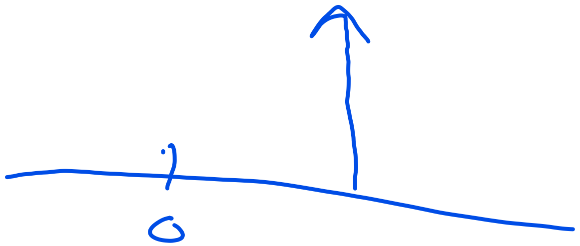
Though there are many possible combinations of input signals, the following are famous or popular input signals

- ❑ Impulse Signal
- ❑ Step Signal
- ❑ Ramp Signal
- ❑ Parabolic Signal



Impulse Signal

The impulse signal imitate the sudden shock characteristics of a signal



$$\delta(t) = \begin{cases} A, & if\ t = 0 \\ 0, & if\ t \neq 0 \end{cases}$$

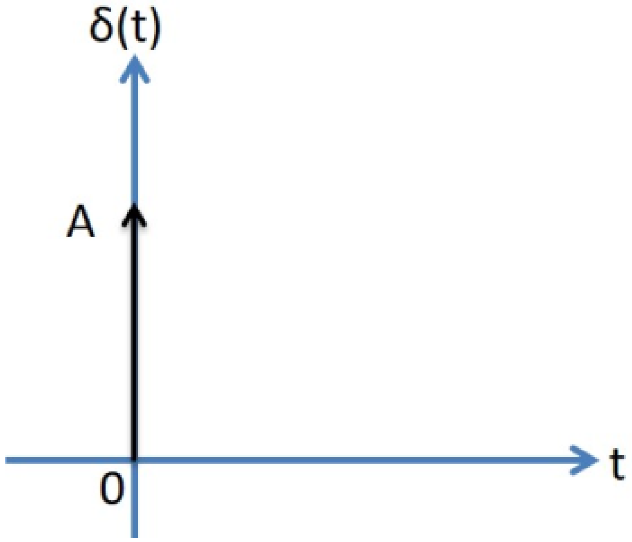


Figure: Impulse Signal

If A = 1, it is called unit impulse signal

Step Signal

The step signal is used to imitate the sudden change of a signal

$$\boldsymbol{u(t)} = \begin{cases} \boldsymbol{A}, & \boldsymbol{if\ t \geq 0} \\ \boldsymbol{0}, & \boldsymbol{if\ t < 0} \end{cases}$$

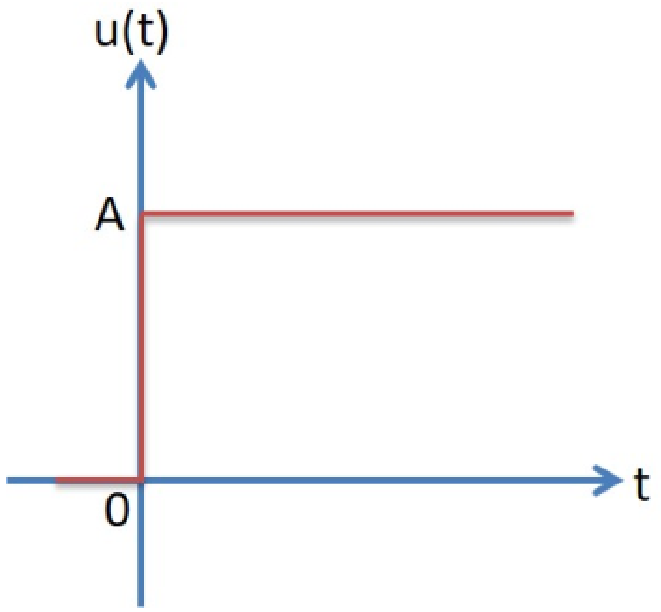


Figure: Step Signal

If A = 1, the step signal is called unit step signal

Ramp Signal

The ramp signal is used to imitate the constant velocity characteristic of a signal

$$\boldsymbol{r(t) = \begin{cases} At, & if\ t \geq 0 \\ 0, & if\ t < 0 \end{cases}}$$

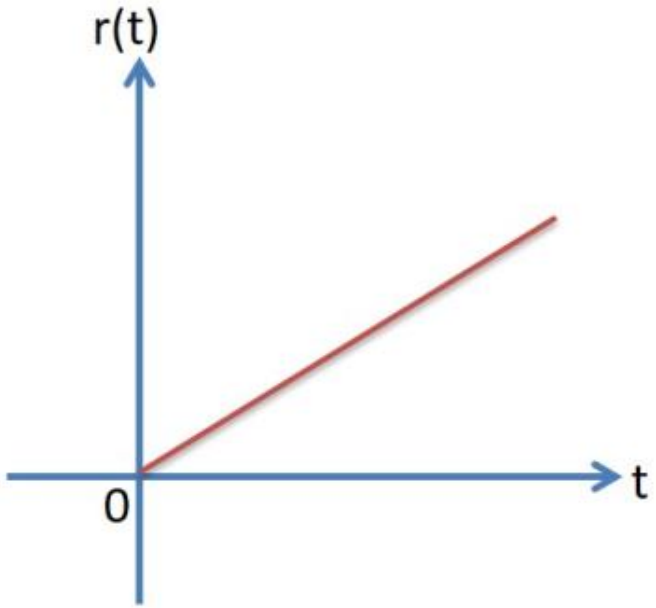


Figure: Ramp Signal

If A = 1, the ramp signal is called unit ramp signal

Parabolic Signal

The parabolic signal is used to imitate the constant acceleration characteristic of a signal

$$p(t) = \begin{cases} \dfrac{At^2}{2}, & if\ t \geq 0 \\ 0, & if\ t < 0 \end{cases}$$

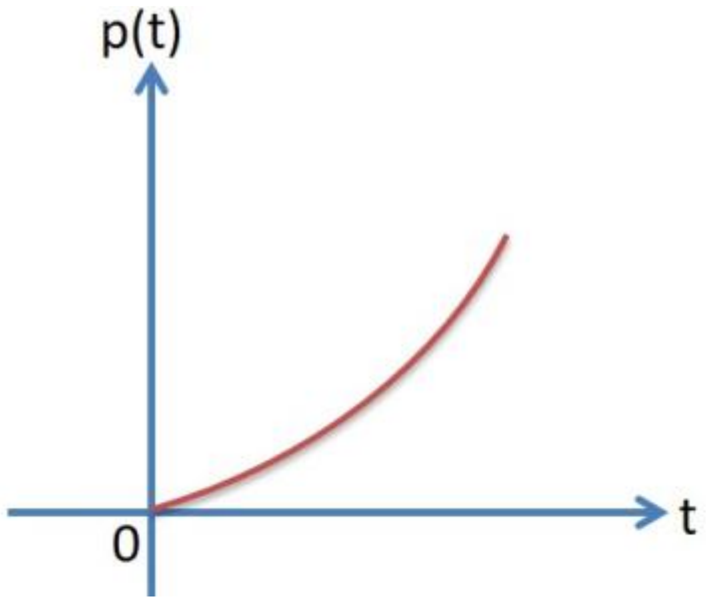


Figure: Parabolic Signal

If A = 1, the parabolic signal is called unit parabolic signal

Step Response

Impulse response is the best response because transfer function is defined as impulse response of a system

However, in practical life generating impulse signal is not easy

We use unit step signal as test input signal and then analyze the output of the system

First Order System

A first order system has a single pole (irrespective of number of zeros)

Many systems are of first order. Examples include

- velocity of a car on road
- control of angular velocity of rotating systems
- an RLC circuit with only one capacitor and no inductor
- an RLC circuit with only one inductor and no capacitor
- fluid flow in a pipeline
- level control in a tank
- pressure control in a gas cylinder

First Order System - Examples

Examples of first order system from computing system domain:

Many systems are of first order. Examples include

- speed control of dc motor in hard disk system
- time taken by queries in database management system, e.g.

  SELECT * from STUDENT

  where ATTENDANCE PERCENTAGE > 75;
- time taken to read a temperature sensor interfaced with an embedded system
- energy consumed by an IoT device
- control of collision detection in networks e.g. CSMA/CA
- time taken by PS4 or Xbox one device to boot

Step Response of First Order System

A general first order system without zeros can be written as follows:

$$\boldsymbol{G(s) = \frac{b}{s+a}}$$

$\Rightarrow b/s+a$

Let *C(s)* be the output of a system having transfer function *G(s)* (expressed above). If the input to *G(s)* is a unit step, then the output can be expressed as follows:

$$Output\ Signal = Input\ Signal\ \times Transfer\ function$$

$1/s$

We can further write the following:

$$\boldsymbol{C(s) = Unit\ step\ signal\ \times G(s)}$$

$$\boldsymbol{C(s) = \frac{1}{s} \times \frac{b}{s+a}}$$

Step Response of First Order System

$$C(s) = \frac{1}{s} \times \frac{b}{s+a}$$

$$C(s) = \frac{b}{(s)(s+a)}$$

$T = \frac{1}{a}$

Sometimes in numerator, we have a gain term. If we can identify the gain and time-constant from step response of a system, then we can obtain its transfer function also

$K = b/a$

Step Response of First Order System

The term $\boldsymbol{a}$ is important time. The inverse of $\boldsymbol{a}$ is called time constant i.e.

$$\boldsymbol{\tau} = \frac{\mathbf{1}}{\boldsymbol{a}}$$

where $\boldsymbol{\tau}$ is called time-constant of first order systems. For example compute $\boldsymbol{\tau}$ of the following system:

$$\boldsymbol{G(s)} = \frac{\mathbf{3}}{\boldsymbol{s} + \mathbf{2}}$$

1/2 3/2

Here $\tau = \frac{1}{2} = 0.5$ and gain K is computed as $\frac{3}{2} = 1.5$

b/a

The value of gain ***K*** indicates the final steady-state value of the step response

Step Response of First Order System

In order to compute transfer function from a plot, we need to define a few more terminologies

Rise Time: T_r, time taken to reach **90%** or **0.9** of final value from **10%** or **0.1**

90%.

Rise Time

Mathematically:

$$T_r = \frac{2.2}{a}$$

Settling Time: T_s, time taken to stay within **2%** of its final value (or reach **98%** of final value).

Mathematically:

98.

$$T_s = \frac{4}{a}$$

Step Response of First Order System

Can you compute the transfer functions?

K = 0.7

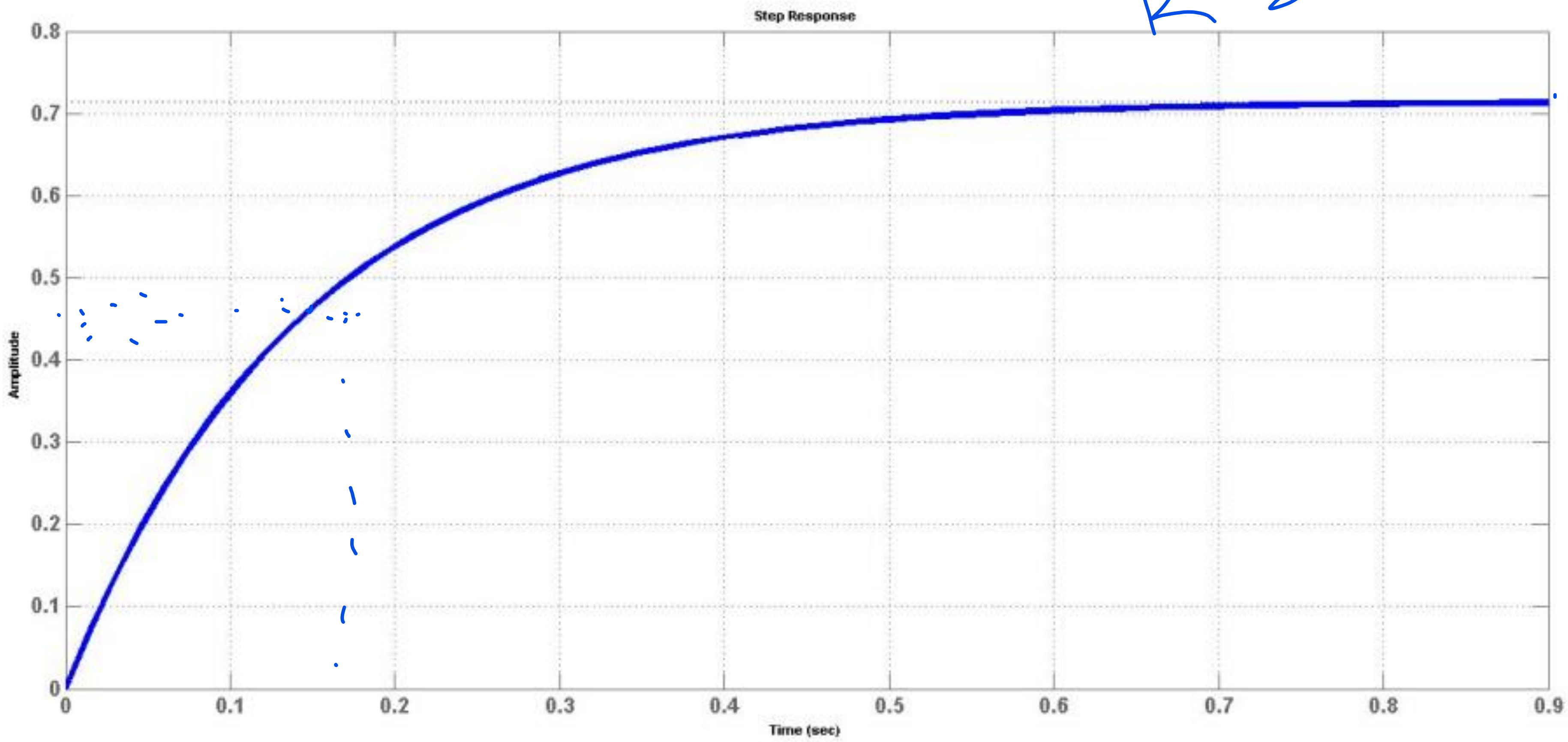


Figure: Step Response of a transfer function

Step Response of First Order System

Time constant: Time to reach **63%** of final value. Compute the transfer function from the previous plot

$0.63 \times 0.72 = 0.45$

Final value = steady-state value = gain ***K*** = **0.72**

63% of final value is **0.63 × 0.72** = **0.4464**

$a = 0.45$

Time taken to reach **0.45** value is **0.15** seconds

The final transfer function is

$b = \frac{K}{a}$

$$G(s) = \frac{\frac{0.72}{0.15}}{s + \frac{1}{0.15}}$$

$\frac{k}{s+c}$

$\frac{1}{s}$

Step Response of First Order System

The final transfer function is

$G(s) = \frac{b}{s+a}$

$K = \frac{b}{a}$

$\Rightarrow \frac{b}{s + \frac{1}{0.15}}$

$b = Ka$

$$G(s) = \frac{\frac{0.72}{0.15}}{s + \frac{1}{0.15}}$$

$0.72 \times \frac{1}{0.15}$

$\frac{1}{a}$

Pole is inverse of time constant which comes out to be $\frac{1}{0.15}$ **= 6.67**

Another way of writing the transfer function is

$$G(s) = \frac{4.802}{s + 6.67}$$

Step Response of First Order System

The previous step-response was obtained for the following actual transfer function:

$$G(s) = \frac{5}{s+7}$$

MATLAB code for obtaining step response

```
num = [5] ;

den = [1 7] ;

step(num, den)
```

Step Response of First Order System

Effects of decreasing time constant

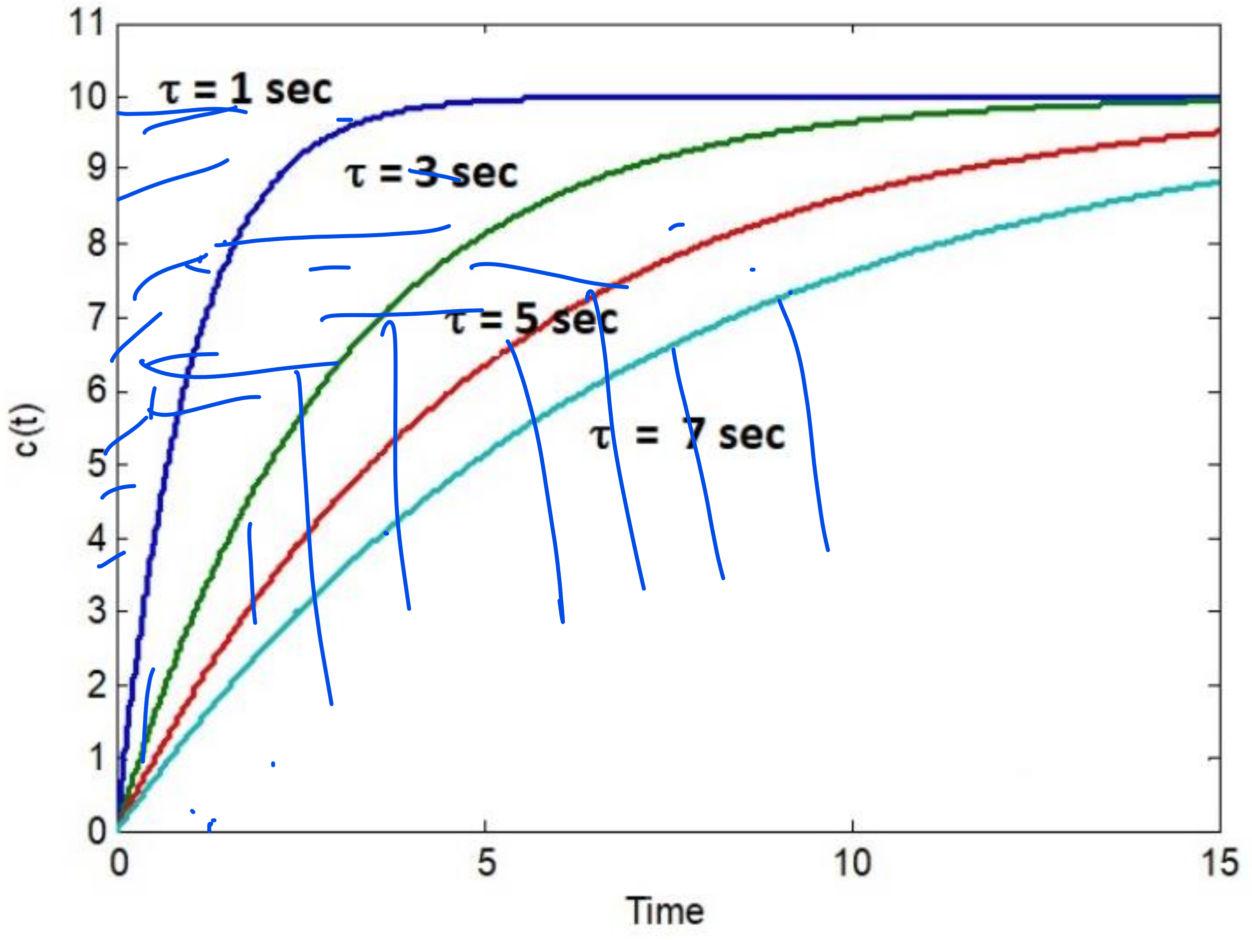


Figure: Effects of decreasing time constants of first order transfer function

Step Response of First Order System

Effects of increasing gains (remember its ***K*** not the term ***b***)

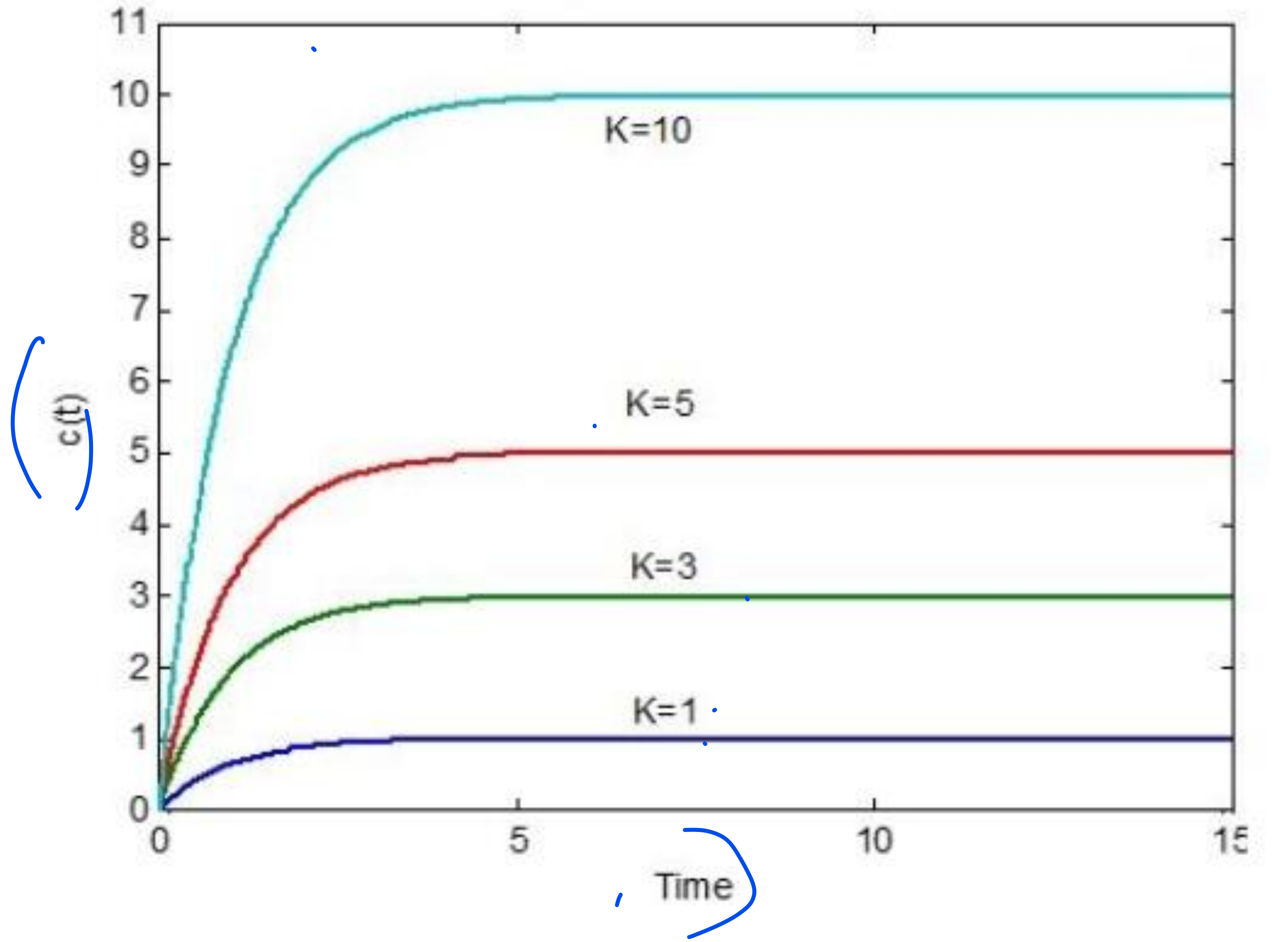


Figure: Effects of increasing gains of first order transfer function

Question: NADRA manages the registration database of Pakistani citizens. Previously, till 2001 people had 11 digit NIC numbers. Each citizen of Pakistan is issued a 13 digit CNIC number. The first 5 digits in a CNIC are based on a citizen locality, the next 7 numbers are random, and the final last digit is gender based (even for females, and odd for males). The current database entries in NADRA are estimated as 90 million. If a person details are required and his/her CNIC is entered in the NADRA database, its take 1 min to search 98% of the records in a NADRA database. Assuming the query search process to be a first-order system, find the time constant.

$a = ?$

0.98

60 sec

settling

$T_s = \frac{4}{a}$

$60 = \frac{4}{a}$

$a = \frac{4}{60} = \frac{1}{15}$