Online Series Examination – 2024 -'25
Mathematics
Cycle 2 – Descriptive – 22 December 2024
Grade 10 – Answer Key

Time: 2 Hours Maximum Marks: 60

General Instructions:

1. This Question Paper has 4 Sections B – E.
2. Section B has 5 questions carrying 02 marks each.
3. Section C has 6 questions carrying 03 marks each.
4. Section D has 4 questions carrying 05 marks each.
5. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
6. All Questions are compulsory. However, an internal choice has been provided in the 2 marks questions of Section E
7. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION B

Section B consists of 5 questions of 2 marks each.

1. Prove that $\sqrt{2}$ is an irrational number. [2]

Ans Let us assume, to the contrary, that $\sqrt{2}$ is rational.

So, we can find integers a and b such that $\sqrt{2} = \frac{a}{b}$ where a and b are coprime.

So, $b\sqrt{2} = a$.

Squaring both sides,

we get $2b^2 = a^2$.

Therefore, 2 divides a^2 and so 2 divides a.

So, we can write a = 2c for some integer c.

Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$.

This means that 2 divides b^2, and so 2 divides b

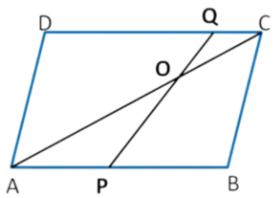
Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b have no common factors other than 1.

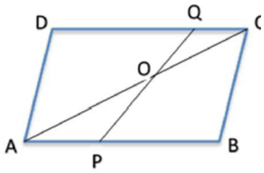
This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

So, we conclude that $\sqrt{2}$ is irrational.

2. ABCD is a parallelogram. Point P divides AB in the ratio 2:3 and point Q divides DC in the ratio 4:1. Prove that OC is half of OA. [2]



Ans: ABCD is a parallelogram.



AB = DC = a

Point P divides AB in the ratio 2:3

AP = $\frac{2}{5}$ a , BP = $\frac{3}{5}$ a

point Q divides DC in the ratio 4:1.

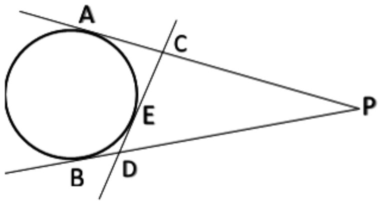
DQ = $\frac{4}{5}$ a , CQ = $\frac{1}{5}$ a

Δ APO ~ Δ CQO [AA similarity]

$$\frac{AP}{CQ} = \frac{PO}{QO} = \frac{AO}{CO}$$

$$\frac{AO}{CO} = \frac{\frac{2}{5}a}{\frac{1}{5}a} = \frac{2}{1} \Rightarrow OC = \tfrac{1}{2}\,OA$$

3. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At a point E on the circle, a tangent is drawn to intersect PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of ΔPCD. [2]



Ans: PA = PB; CA = CE; DE = DB [Tangents to a circle]

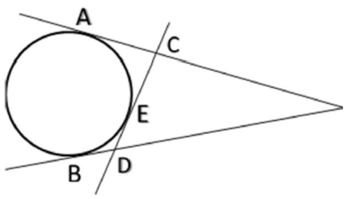
Perimeter of ΔPCD = PC + CD + PD

= PC + CE + ED + PD

= PC + CA + BD + PD

= PA + PB

Perimeter of ΔPCD = PA + PA = 2PA = 2(10) = 20 cm



4. If tan (A + B) = $\sqrt{3}$ and tan (A – B) = $\frac{1}{\sqrt{3}}$; 0° < A + B < 90°; A > B, find A & B. [2]

Ans: $\because \tan(A+B) = \sqrt{3} \quad \therefore A + B = 60^0$ ...(1)

$\because \tan(A-B) = \frac{1}{\sqrt{3}} \quad \therefore A - B = 30^0$ ...(2)

Adding (1) & (2), we get 2A=$90^0 \Longrightarrow A = 45^0$

Also (1) –(2), we get $2B = 30^0 \Longrightarrow B = 45^0$

5. With vertices A, B and C of ΔABC as centres, arcs are drawn with radii 14 cm and the three portions of the triangle so obtained are removed. Find the total area removed from the triangle. [2]

Ans: Total area removed = $\frac{\angle A}{360}\pi r^2 + \frac{\angle B}{360}\pi r^2 + \frac{\angle C}{360}\pi r^2$

$= \frac{\angle A + \angle B + \angle C}{360}\pi r^2$

$= \frac{180}{360}\pi r^2$

$= \frac{180}{360} \text{ X } \frac{22}{7} \text{ X } (14)^2$

= 308 cm^2

SECTION C

Section C consists of 6 questions of 3 marks each.

6. National Art convention got registrations from students from all parts of the country, of which 60 are interested in music, 84 are interested in dance and 108 students are interested in handicrafts. For optimum cultural exchange, organisers wish to keep them in minimum number of groups such that each group consists of students interested in the same artform and the number of students in each group is the same. Find the number of students in each group. Find the number of groups in each art form. How many rooms are required if each group will be allotted a room? [3]

Ans: Number of students in each group subject to the given condition = HCF (60,84,108

HCF (60,84,108) = 12

Number of groups in Music = $\frac{60}{12} = 5$

Number of groups in Dance = $\frac{84}{12} = 7$

Number of groups in Handicrafts = $\frac{108}{12} = 9$

Total number of rooms required = 21

7. If α, β are zeroes of quadratic polynomial $5x^2 + 5x + 1$, find the value of: [3]

(a) $\alpha^2 + \beta^2$

(b) $\alpha^{-1} + \beta^{-1}$

Ans: P(x) = $5x^2 + 5x + 1$

$\alpha+\beta = \frac{-b}{a} = \frac{-5}{5} = -1$

$\alpha\beta = \frac{c}{a} = \frac{1}{5}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-1)^2 - 2\left(\frac{1}{5}\right)$

$= 1 - \frac{2}{5} = \frac{3}{5}$

$\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$

$= \frac{(\alpha+\beta)}{\alpha\beta} = \frac{(-1)}{\frac{1}{5}} = -5$

8. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there? [3]

Ans: Let the ten's and the unit's digits in the first number be x and y, respectively.

So, the original number = 10x + y

When the digits are reversed, x becomes the unit' s digit and y becomes the ten's Digit.

So, the obtain by reversing the digits = 10y + x

According to the given condition: (10x + y) + (10y + x) = 66

i.e., 11(x + y) = 66

i.e., x + y = 6 ---- (1)

We are also given that the digits differ by 2,

therefore, either **x – y = 2 ---- (2)**

or **y – x = 2 ---- (3)**

If x – y = 2, then solving (1) and (2) by elimination, we get x = 4 and y = 2.

In this case, we get the number 42.

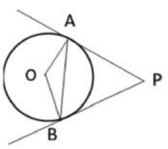
If y – x = 2, then solving (1) and (3) by elimination, we get x = 2 and y = 4.

In this case, we get the number 24.

Thus, there are two such numbers 42 and 24.

9. PA and PB are tangents drawn to a circle of centre O from an external point P. Chord AB makes an angle of 30° with the radius at the point of contact. If length of the chord is 6 cm, find the length of the tangent PA and the length of the radius OA. [3]

Ans:



$\angle OAB = 30°$

$\angle OAP = 90°$ [Angle between the tangent and the radius at the point of contact]

$\angle PAB = 90° - 30° = 60°$

AP = BP [Tangents to a circle from an external point]

$\angle PAB = \angle PBA$ [Angles opposite to equal sides of a triangle]

In ΔABP, $\angle PAB + \angle PBA + \angle APB = 180°$ [Angle Sum Property]

$60° + 60° + \angle APB = 180°$

$\angle APB = 60°$

$\therefore$ ΔABP is an equilateral triangle, where AP = BP = AB.

PA = 6 cm

In Right ΔOAP, $\angle OPA = 30°$

$$\tan 30° = \frac{OA}{PA}$$

$$\frac{1}{\sqrt{3}} = \frac{OA}{6}$$

$$OA = \frac{6}{\sqrt{3}} = 2\sqrt{3}cm$$

10. If $1 + sin^2\theta = 3sin\theta.cos\theta$, then prove that $tan\theta = 1$ or $\frac{1}{2}$ [3]

Ans: Given, $1 + \sin^2\theta = 3 \sin\theta \cos\theta$

Dividing both sides by $\cos^2\theta$,

$$\frac{1}{\cos^2\theta} + \tan^2\theta = 3\tan\theta$$

$$\sec^2\theta + \tan^2\theta = 3\tan\theta$$

$$1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$$

$$1 + 2\tan^2\theta = 3\tan\theta$$

$$2\tan^2\theta - 3\tan\theta + 1 = 0$$

If $\tan\theta = x$, then the equation becomes $2x^2 - 3x + 1 = 0$

$\Rightarrow (x-1)(2x-1) = 0$ $x = 1$ or $\frac{1}{2}$

$\tan\theta = 1$ or $\frac{1}{2}$

11. The length of 40 leaves of a plant are measured correct to nearest millimetre, and the data obtained is represented in the following table. [3]

Length in (mm)	118 – 126	127 – 135	136 – 144	145 – 153	154 – 162	163 – 171	172 – 180
No.of Leaves	3	5	9	12	5	4	2

Find the mean length of the leaves.

Ans:

Length [in mm]	Number of leaves (f)	CI	Mid x	d	fd
118 – 126	3	117.5- 126.5	122	-27	-81
127 – 135	5	126.5– 135.5	131	-18	-90
136 – 144	9	135.5– 144.5	140	-9	-81
145 – 153	12	144.5 – 153.5	a = 149	0	0
154 – 162	5	153.5 – 162.5	158	9	45
163 – 171	4	162.5 – 171.5	167	18	72
172 – 180	2	171.5 – 180.5	176	27	54

$$\text{Mean} = a + \frac{\sum fd}{\sum f} = 149 + \frac{-8}{40}$$

$$= 149 - 2.025 = 146.975$$

Average length of the leaves = 146.975

SECTION D

12. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream. [5]

Ans: Let the speed of the stream be x km/h.

The speed of the boat upstream = (18 – x) km/h and

the speed of the boat downstream = (18 + x) km/h.

The time taken to go upstream $= \frac{distance}{speed} = \frac{24}{18-x}$ hours

the time taken to go downstream $= \frac{distance}{spe} = \frac{24}{18+x}$ hours

According to the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$24(18 + x) - 24(18 - x) = (18 - x)(18 + x)$$

$$x^2 + 48x - 324 = 0$$

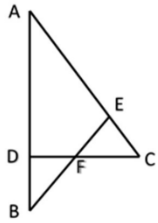
$$x = 6 \text{ or } -54$$

Since x is the speed of the stream, it cannot be negative.
Therefore, x = 6 gives the speed of the stream = 6 km/h.

13. (a) State and prove Basic Proportionality theorem. [5]

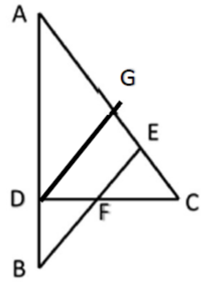
(b) In the given figure ∠CEF = ∠CFE. F is the midpoint of DC.

Prove that $\frac{AB}{BD} = \frac{AE}{FD}$.



Ans: **Proof of BPT**

(b)



Draw DG || BE

In Δ ABE, $\frac{AB}{BD} = \frac{AE}{GE}$ [BPT]

CF = FD [F is the midpoint of DC] ---(i)

In Δ CDG, $\frac{DF}{CF} = \frac{GE}{CE} = 1$ [Mid point theorem]

GE = CE ---(ii)

∠CEF = ∠CFE [Given]

CF = CE [Sides opposite to equal angles] ---(iii)

From (ii) & (iii) CF = GE ---(iv)

From (i) & (iv) GE = FD

$\therefore \frac{AB}{BD} = \frac{AE}{GE} \Rightarrow \frac{AB}{BD} = \frac{AE}{FD}$

14. A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 3 m and 14 m respectively, and the total height of the tent is 13.5 m, find the area of the canvas required for making the tent, keeping a provision of 26 m^2 of canvas for stitching and wastage. Also, find the cost of the canvas to be purchased at the rate of ₹ 500 per m^2. [5]

Ans: Radius of the cylindrical tent (r) = 14 m

Total height of the tent = 13.5 m

Height of the cylinder = 3 m

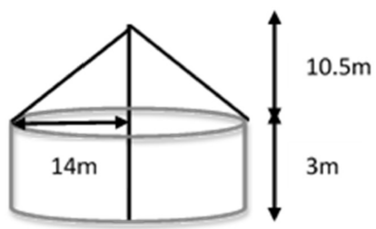
Height of the Conical part = 10.5 m

Slant height of the cone (l) = $\sqrt{h^2 + r^2}$

$= \sqrt{(10.5)^2 + (14)^2}$

$= \sqrt{110.25 + 196}$

$= \sqrt{306.25} = 17.5$ m



Curved surface area of cylindrical portion

$= 2\pi rh$

$= 2x\frac{22}{7}\times 14\times 3$

$= 264\ m^2$

Curved surface area of conical portion

$=\pi rl$

$= \frac{22}{7}\times 14\times 17.5$

$=770\ m^2$

Total curved surface area = $264\ m^2 + 770\ m^2 = 1034\ m^2$

Provision for stitching and wastage = $26\ m^2$

Area of canvas to be purchased = $1060\ m^2$

Cost of canvas = Rate × Surface area

= 500 x 1060 = ₹ 5,30,000/-

15. The median of the following data is 50. Find the values of 'p' and 'q', if the sum of all frequencies is 90. Also find the mode of the data. [5]

Marks Obtained	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of Students	p	15	25	20	q	8	10

Ans:

Marks obtained	Number of students	Cumulative frequency
20 – 30	p	p
30 – 40	15	p + 15
40 – 50	25	p + 40
50 – 60	20	p + 60
60 – 70	q	p + q + 60
70 – 80	8	p + q + 68
80 - 90	10	p + q + 78
	90	

$p + q + 78 = 90$

$p + q = 12$

Median $=(l) + \frac{\frac{n}{2}-c}{f}\ .\ h$

$50 = 50 + \frac{45-(p+40)}{20}\ .\ 10$

$\frac{45-(p+40)}{20}\ .\ 10 = 0$

$45 - (p + 40) = 0$

$P = 5$

$5 + q = 12$

$q = 7$

Mode $= l + \frac{f1-f0}{2f1-f0-f2}\ .\ h$

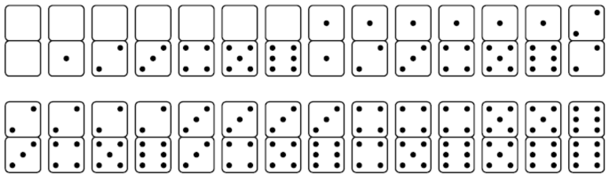
$= 40 + \frac{25-15}{2(25)-15-20}\ .\ 10$

$= 40 + \frac{100}{15} = 40 + 6.67 = 46.67$

SECTION E

Section E consists of 3 questions of 4 marks each.

16. Double-six Dominos: It is a game played with the 28 numbered tiles shown in the diagram. [4]
The 28 dominos are placed in a bag, shuffled, and then one domino is randomly drawn. Give the following answer.



(i) What is the probability the total number of dots on the domino is three or less?
(ii) What is the probability the total number of dots on the domino is greater than three?
(iii) What is the probability the total number of dots on the domino does not have a blank half?

OR

What is the probability the total number of dots on the domino is not a "double" (both sides the same)?

Ans: Total possible outcomes in all case is 28 because there are total 28 dominos.

Therefore $\quad n(S) = 28$

(i) Total number of dots on the domino is three or less,

Let E_1 be the event that the total number of dots on the domino is three or less.

Favourable outcome, $\quad n(E_1) = 6$

$$\text{Probability,} \quad P(E_1) = \frac{n(E_1)}{n(S)}$$

$$= \frac{6}{28} = \frac{3}{14}$$

(ii) Total number of dots on the domino is greater than three,

Let E_2 be the event that the total number of dots on the domino is greater than three.

Favourable outcome, $n(E_2) = 22$

Probability, $P(E_2) = \frac{n(E_2)}{n(S)}$

$= \frac{22}{28} = \frac{11}{14}$

Alternative :

$$P(E_2) = 1 - P(E_1) = 1 - \frac{3}{14} = \frac{11}{14}$$

(iii) Total number of dots on the domino does not have a blank half,

Let E_3 be the event that the total number of dots on the domino does not have a blank half . Let $\overline{E_3}$ be the event that the total number of dots on the domino have a blank half .

Favourable outcome, $n(\overline{E_3}) = 7$

Probability, $P(\overline{E_3}) = \frac{n(\overline{E_3})}{n(S)}$

$= \frac{7}{28} = \frac{1}{4}$

Probability, $P(E_3) = 1 - P(\overline{E_3})$

$= 1 - \frac{1}{3} = \frac{3}{4}$

OR

Favourable outcome, $n(\overline{E_4}) = 7$

Probability, $P(E_4) = \frac{n(\overline{E_4})}{n(S)}$

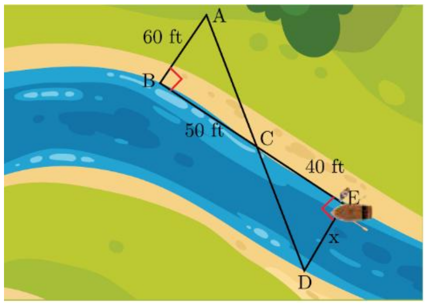
$= \frac{7}{28} = \frac{1}{4}$

Probability, $P(E_4) = 1 - P(\overline{E_4})$

$= 1 - \frac{1}{3} = \frac{3}{4}$

17. Tania is very intelligent in maths. She always tries to relate the concept of maths in daily life. One day she plans to cross a river and want to know how far it is to the other side. [4]

She takes measurements on her side of the river and make the drawing as shown below.



(i) Which similarity criterion is used in solving the above problem?

(ii) Consider the following statement:

S_1: $\angle ACB = \angle DCE$

S_2: $\angle BAC = \angle CDE$

Which of the above statement is/are correct.

(a) S_1 and S_2 both (b) S_1 (c) S_2 (d) None

(iii) Consider the following statement

S_3: $\frac{AB}{DE} = \frac{CA}{CD}$ **S_4**: $\frac{BC}{CE} = \frac{AB}{DE}$ **S_5**: $\frac{CA}{CD} = \frac{DE}{AB}$

Which of the above statements are correct?

(a) S3 and S5 (b) S4 and S5

(c) S3 and S4 (d) All three

(iv) What is the distance x across the river?

OR

What is the approximate length of AD shown in the figure?

(i) We have used AA similarity criterion.

(ii) Here, $\angle ABC = \angle DEC$ (90° each)

Since vertical opposite angle are equal,

$$\angle ACB = \angle DCE$$

Thus due to AA similarity criterion,

$$\Delta ABC \sim \Delta DEC$$

and $\angle BAC = \angle CDE$

Therefore both are correct.

Thus (a) is correct option.

(iii) Since ΔABC and ΔDEC are similar triangle,

$$\frac{AB}{DE} = \frac{BC}{CE} = \frac{CA}{CD}$$

Here S_5 is not correct because $\frac{AB}{DE} = \frac{CA}{CD}$

Thus (c) is correct option.

(iv) We have $\dfrac{AB}{DE} = \dfrac{BC}{CE}$

$$\frac{60}{x} = \frac{50}{40} \Rightarrow x = 48 \text{ ft}$$

or

(v) $$AC = \sqrt{60^2 + 50^2}$$

$$= \sqrt{6100} = 71.8$$

$$CD = \sqrt{40^2 + 48^2}$$

$$= \sqrt{3904} = 62.5$$

$$AD = AC + CD$$

$$= 71.8 + 62.5$$

$$= 140.6 \text{ (Approx)} \approx 140$$

18. **100 Metres Race:** The 100 metres is a sprint race in track and field competitions. The shortest common outdoor running distance, it is one of the most popular and prestigious events in the sport of athletics. [4]

It has been contested at the summer Olympics since 1896 for men and since 1928 for women. The World Championships 100 metres has been contested since 1983. The reigning 100 m Olympic or world champion is often named “the fastest man or woman in the world”. A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (in sec)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
No. of Students	8	10	13	6	3

Based on the above information, answer the following questions.

(i) Estimate the mean time taken by a student to finish the race.

(ii) What will be the upper limit of the modal class?

(iii) What is the sum of lower limits of median class and modal class?

OR

How many students finished the race within 1 minute?

Ans: (i) We prepare the following commutative frequency distribution table.

Time (in sec)	**Number of students (f_i)**	**Mid-value x_i**	$f_i x_i$	**Cumulative frequency cf**
0-20	8	10	80	8
20-40	10	30	300	18
40-60	13	50	650	31
60-80	6	70	420	37
80-100	3	90	270	40
Total	$\sum f_i = 40$		$\sum f_i x_i = 1720$	

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1720}{40} = 43$$

Mean time taken by a student to finish the race is 43 sec.

(ii) Since 40-60 has highest frequency i.e. 13 upper limit of modal class is 60.

(iii) Cumulative frequency just greater than $\frac{N}{2} = \frac{40}{2} = 20$ is 31 and the corresponding class is 40-60. Thus median class is 40-60 and lower limit is 40.

Sum of lower limits of median class and modal class $= 40 + 40 = 80$.

or

The number of students finished the race within 1 min (i.e. 60 sec)

$= $ cumulative frequency of class 40-60

$= 31$