Student ID:		Lab Section:	
Name:		Lab Group:	

Experiment No. 7

Study of the Transient Behavior in RC Circuits

Objective

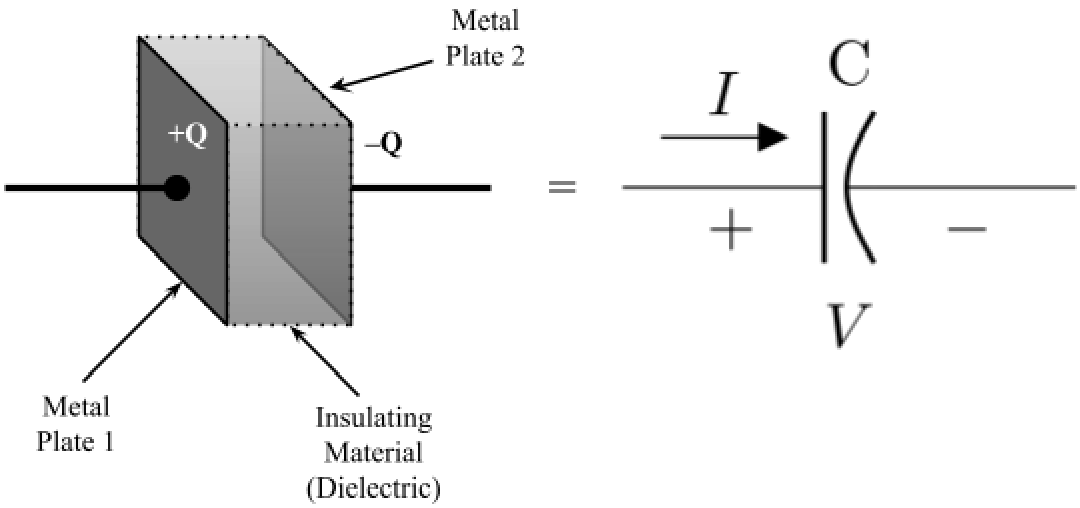
The aim of this experiment is to investigate the transient response of first-order circuits. In this experiment, students will find the time constant τ of an RC circuit.

Theory

The word 'transient' means something that only lasts for a short time *(short-lived)*. In circuit theory, transient response is the response of a system to a change from an equilibrium or a steady state. In the context of RC circuits *(a circuit only consisting of resistors and capacitors but no inductor)*, we will study how the voltage and current in an RC circuit change due to external excitation, such as switching or sudden change in input. In today's experiment, we will construct RC circuits and observe their response due to sudden changes in input voltage.

Capacitor

Capacitors are passive elements that can store energy within its own electric field. A capacitor can be as simple as an insulating material *(dielectric)* consisting of two parallel conductive plates. Charges can build up within these plates which creates an electric field across the plates and a voltage difference between them.



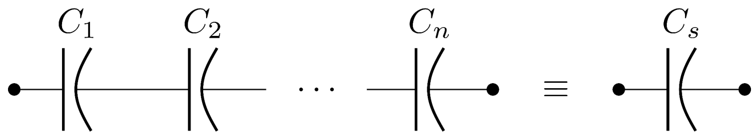
The amount of charge accumulated in each plate is directly proportional to the voltage difference applied across the two plates of a capacitor. If the voltage across the capacitor is v_C and the accumulated charge is Q, then we can write,

$$
\begin{aligned}
& Q \propto V \\
\Rightarrow\ & Q = CV \\
\Rightarrow\ & \frac{d}{dt}(Q) = \frac{d}{dt}(CV) = C\frac{d}{dt}(V) \\
& \boxed{\Rightarrow I = C\frac{dV}{dt}}
\end{aligned}
$$

Here, I is the current through the capacitor and C is the **capacitance** [*S.I. unit is* ***Farad (F)***]. This boxed equation dictates the behavior of a capacitor. As we can see, there is a current through the capacitor if and only if the voltage across the capacitor changes over time.

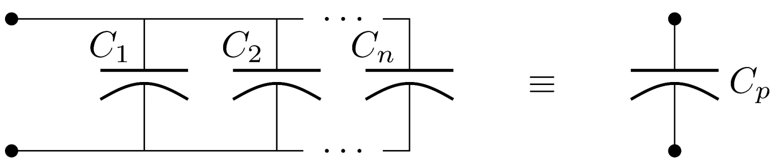
From this equation, we can find the equivalent series and parallel capacitance.

- ➢ Series combination:



$$\frac{1}{C_s} = \sum \frac{1}{C_n} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

- ➢ Parallel combination:

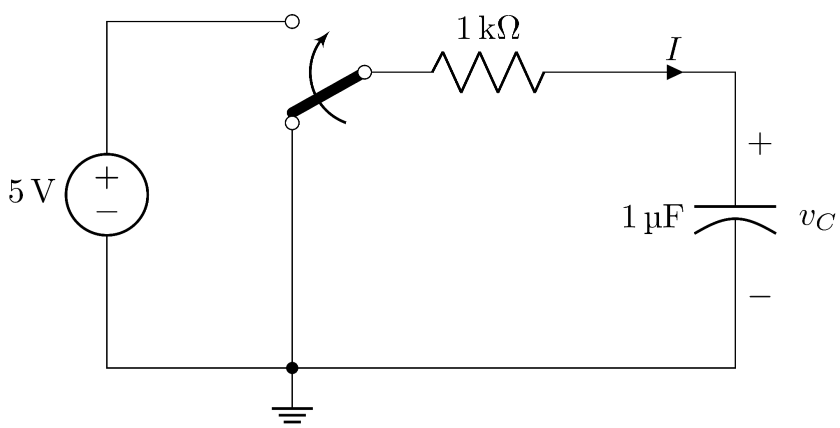


$$C_p = \sum C_n = C_1 + C_2 + \cdots + C_n$$

RC circuit

An RC circuit is an electric circuit composed of resistors and capacitors as the only passive components (may contain other active components). Such circuits exhibit transient behaviors if the input voltage is suddenly changed.

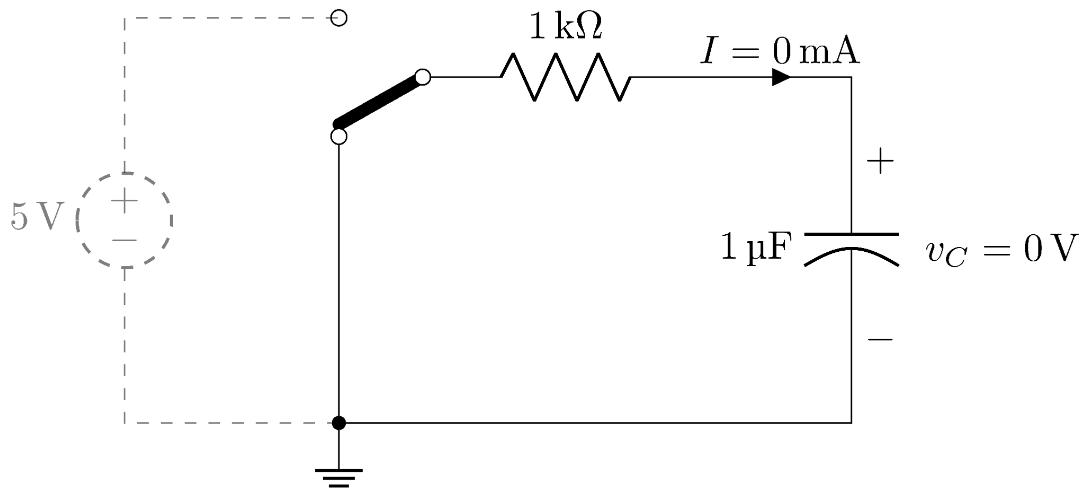
Consider this RC circuit with a switch (arrow indicates the direction of switching):



We can break this circuit into two separate circuits:

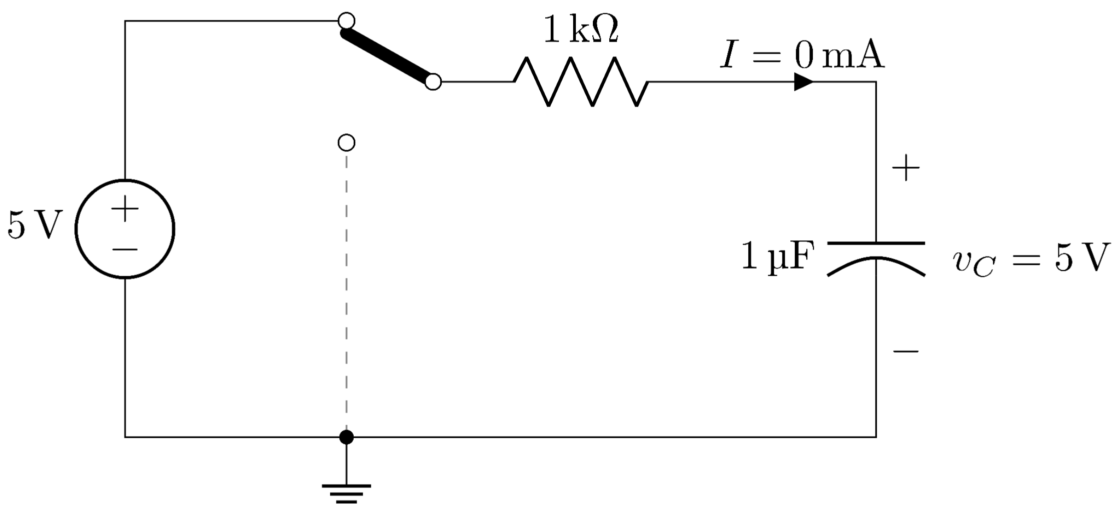
- ➢ Initial circuit
- ➢ Final circuit

The initial position of the switch indicates the voltage source was open and the resistor was grounded. Since there is no source in the circuit, the elements will have no current. Furthermore, **at steady-state conditions, a capacitor acts like an open circuit.** As a result, the voltage across the capacitor v_C will be 0V.



Initial Circuit

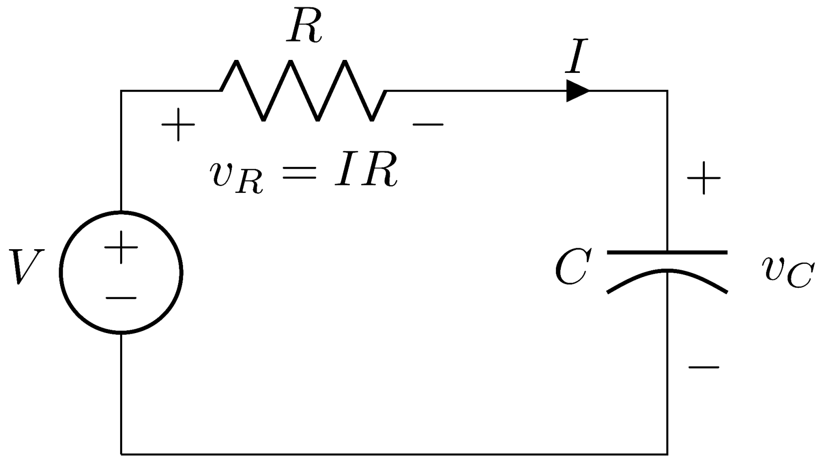
On the other hand, the final position of the switch indicates that the voltage source will now supply voltage. However, **after reaching a steady-state condition, the capacitor will again act like an open circuit.** As a result, the voltage across the capacitor v_C will be 5V.



Final Circuit (after reaching steady-state)

Transient Behavior

In the previous circuit, the voltage across the capacitor v_C rises from 0V to 5V. Unlike resistors, it takes a significant amount of time for the voltage across a capacitor to change. This behavior is called transient behavior. We can figure out how the voltage will change over time using KVL and KCL.



Applying KVL on the circuit we get,

$$v_R + v_C - V = 0.$$

$$\Rightarrow IR + v_C - V = 0$$

$$\Rightarrow \left(C\frac{d}{dt}v_C\right)\cdot R + v_C - V = 0$$

$$\Rightarrow v_C + RC\frac{d}{dt}v_C - V = 0$$

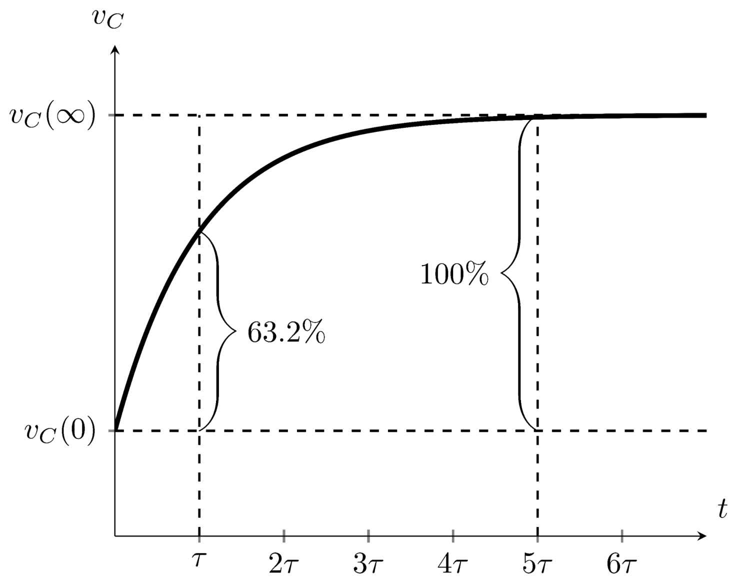
$$\Rightarrow v_C + \tau\frac{d}{dt}v_C - V = 0$$

Let, $\tau = RC$. This quantity is called the **<u>time constant</u>** and the S.I unit is **seconds (s)**. In this example, $\tau = 1k\Omega \times 1\mu F = 1ms$. Time constant has physical significance. It determines how fast the transient response dies out.

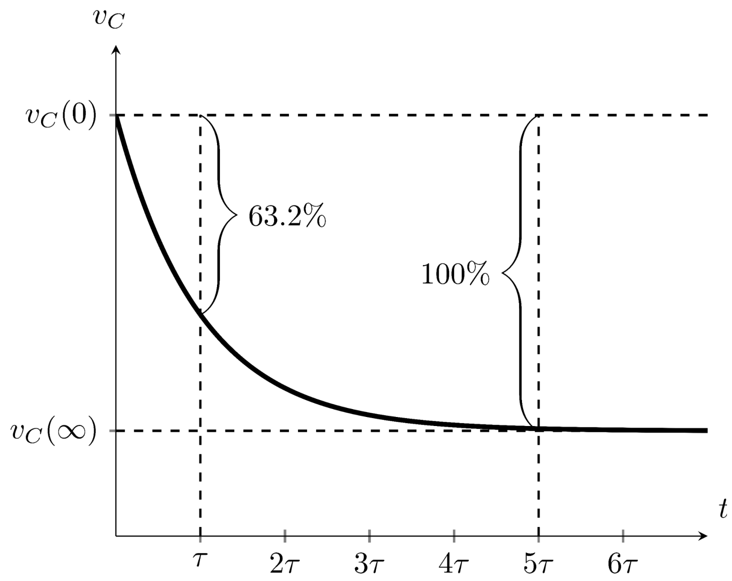
Solving the above differential equation, we get,

$$\boxed{v_c(t) = v_C(\infty) + \left[v_C(0) - v_C(\infty)\right]e^{-t/\tau}}$$

Here, $v_C(t)$ is the voltage across the capacitor at time t. Therefore, $v_C(0)$ **refers to the capacitor voltage of the initial circuit and** $v_C(\infty)$ **refers to the capacitor voltage of the final circuit after it has reached steady-state.** If $v_C(0) < v_C(\infty)$, then the RC circuit is said to be in the **charging phase**. And the RC circuit is in the **discharging phase** if $v_C(0) > v_C(\infty)$.



Charging Phase



Discharging Phase

Time Constant

For a given circuit with a resistance of R and a capacitance of C, the time constant is $\tau = RC$. However, it is also possible to find the time constant from the plot of transient response. Higher the value of time constant, the longer it takes for the voltage to reach steady-state. At time $t = \tau$,

$$v_C(\tau) = v_C(\infty) + \left[v_C(0) - v_C(\infty)\right]e^{-\tau/\tau} = v_C(\infty) + \left[v_C(0) - v_C(\infty)\right]e^{-1}$$

$$\therefore \frac{v_C(\infty) - v_C(\tau)}{v_C(\infty) - v_C(0)} = 1 - e^{-1} \approx 0.632 = 63.2\%$$

For example, if $\tau = 1ms$, then $1ms$ after switching, the voltage has already reached 63.2% of its way to the final steady-state voltage.

A similar analysis shows that, after $t = 5\tau$, the voltage almost reaches the final steady-state voltage. So we can conclude **it takes approximately** 5τ **time for a transient circuit to reach steady-state.**

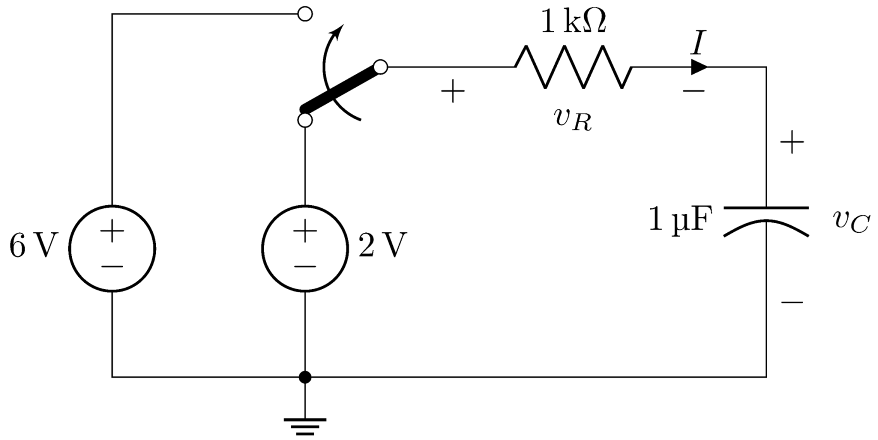
Apparatus

- Multimeter
- Resistors
- Capacitors
- Breadboard
- Jumper wires
- DC power supply
- Function Generator
- Oscilloscope

Procedures

- ➢ Measure the resistances and capacitances of the provided resistors and capacitors and fill up the Data Table 1.

- ➢ Construct the following circuit on a breadboard. Try to use minimum number of jumper wires:

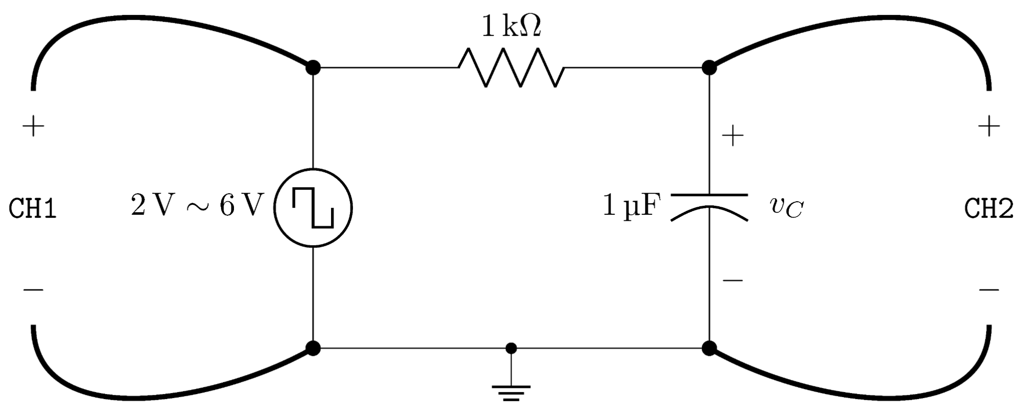


Circuit 1

- ➢ For **Circuit 1**, apply the specified supply voltages using the DC power supply.

- ➢ Keep the switch to the initial position (connect to 2V and keep 6V open). Measure the initial voltages, $v_C(0)$ across the capacitor, $v_R(0)$ across the 1 kΩ resistor using the multimeter and use Ohm's law to calculate the current $I(0)$ through the resistor and capacitor.

- ➢ Then change the switch to the final position (connect to 6V and keep 2V open). Measure the initial voltages, $v_C(\infty)$ across the capacitor, $v_R(\infty)$ across the 1 kΩ resistor using the multimeter. Then use Ohm's law to calculate the current $I(\infty)$ through the resistor and capacitor, and fill up the data tables.

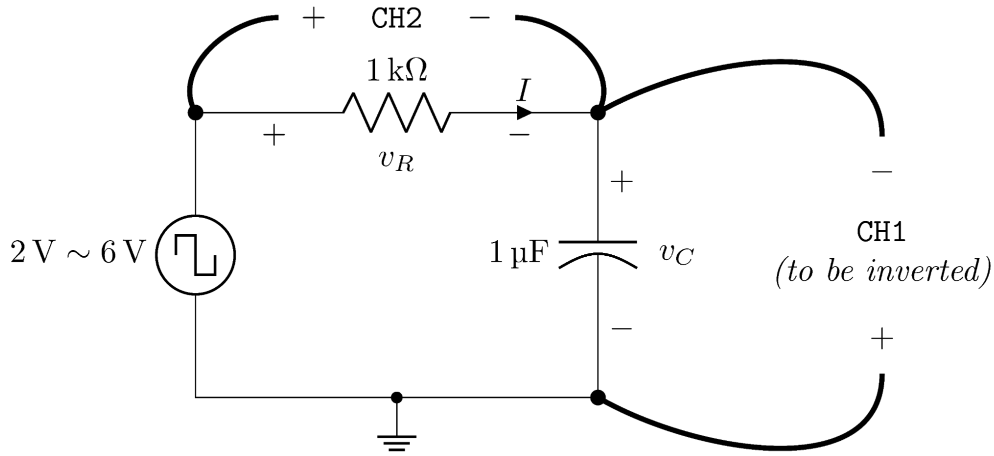


Circuit 2

➢ Construct **Circuit 2** on a breadboard. Use the function generator to generate **4V peak-peak signal with square waveform and 50 Hz frequency** along with a **DC offset of 4V**.

➢ Connect the two oscilloscope channels as shown in the schematic. Make sure the x axis of both graphs are at the same position *(push both position knobs)*.

➢ Take a picture of the graphs. Identify the charging and discharging phase from the graph.



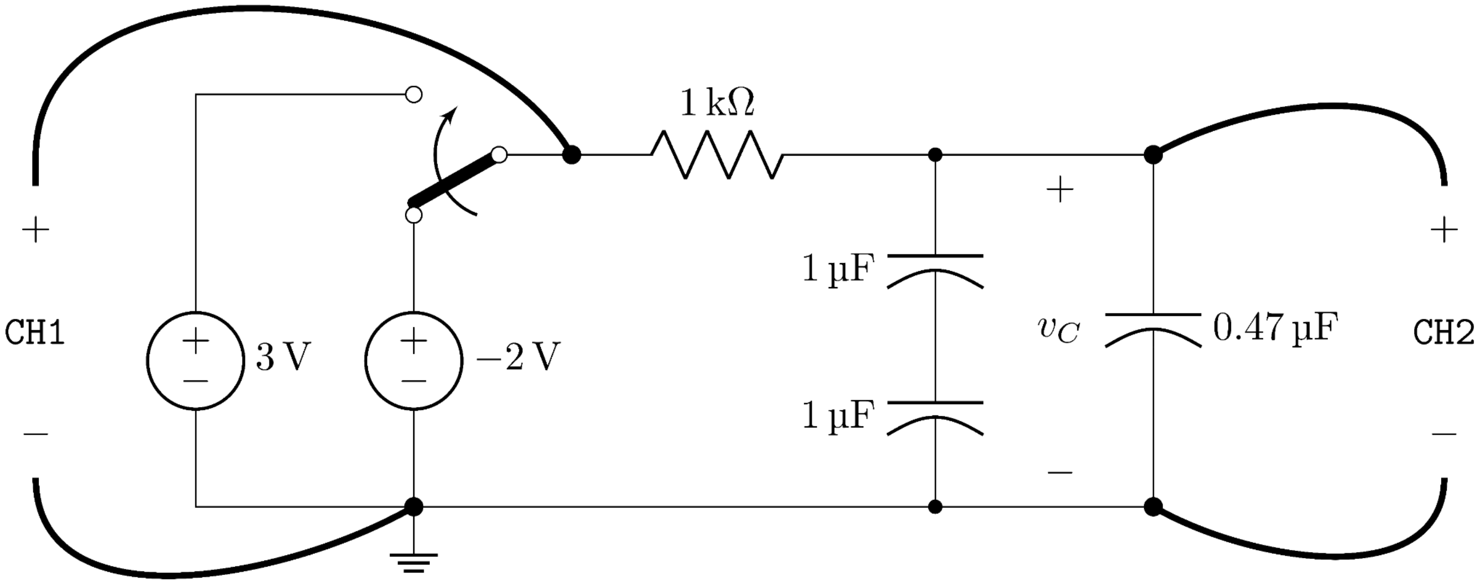
Circuit 3

➢ Construct **Circuit 3** on a breadboard. Use the function generator to generate **4V peak-peak signal with square waveform and 50 Hz frequency** along with a **DC offset of 4V**.

➢ Connect the two oscilloscope channels as shown in the schematic. Invert the signal in channel 1. Make sure the x axis of both graphs are at the same position *(push both position knobs)*.

➢ Take a picture of the graphs. Identify the charging and discharging phase from the graph.

➢ Find how much time it takes for the circuit to stabilize (5τ) and from there, find the time constant.

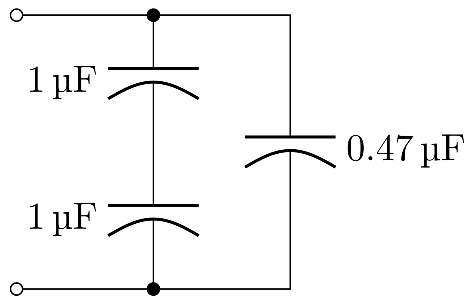


Circuit 4

➢ Construct **Circuit 4** using the function generator and observe the plots on the oscilloscope.

➢ Find how much time it takes for the circuit to stabilize (5τ) and from there, find the time constant.

➢ From the value of time constant, find the equivalent capacitance. $\left(\tau = RC_{eq}\right)$



Circuit 5

➢ Measure the equivalent capacitance using a multimeter and find the percentage of error. Make sure all the capacitors are completely discharged before measuring their capacitance. This can be done by shorting them.

Data Tables

Signature of Lab Faculty: **Date:**

** ***<u>For all the data tables, take data up to three decimal places, round to two, then enter into the table.</u>***

Table 1: Resistance and Capacitance Data

For all your future calculations, please use the observed values only **(even for theoretical calculations)**.

Notation	**Expected Resistance**	**Observed Resistance (kΩ)**	**Notation**	**Expected Capacitance**	**Observed Capacitance (μF)**
R	1 kΩ		C_1	1 μF	
			C_2	1 μF	
			C_3	0.47 μF	

Table 2: Data from Circuit 1 *(Initial)*

Keep the switch to the **initial position** (connect to 2V and keep 6V open).

Initial Circuit	Initial DC Supply Voltage $V_S(0)$ (V)			$v_C(0)$ (V)	$v_R(0)$ (V)	$I(0) = \frac{v_R(0)}{R}$ (mA)
	Expected Voltage	From DC power supply	Using multimeter			
Experi-mental	2.0					
Theo-retical						

Table 3: Data from Circuit 1 *(Final)*

Change the switch to the **final position** (connect to 6V and keep 2V open).

<table>
<tr><th rowspan="2">Final Circuit</th><th colspan="3">Final DC Supply Voltage
$V_S(\infty)$
(V)</th><th rowspan="2">$v_C(\infty)$
(V)</th><th rowspan="2">$v_R(\infty)$
(V)</th><th rowspan="2">$I(\infty) = \frac{v_R(\infty)}{R}$
(mA)</th></tr>
<tr><th>Expected Voltage</th><th>From DC power supply</th><th>Using multimeter</th></tr>
<tr><td>Experi-mental</td><td rowspan="2">6.0</td><td rowspan="2"></td><td rowspan="2"></td><td></td><td></td><td></td></tr>
<tr><td>Theo-retical</td><td></td><td></td><td></td></tr>
</table>

Table 4: Data from Circuit 2

Use the function generator for the supply voltage and observe all values from the oscilloscope.

Time constant, $\tau = RC =$ | | ms

Theoretical charging / discharging time $= 5\tau = 5RC =$ | | ms

<table>
<tr><th rowspan="2">Circuit 2</th><th colspan="2">Supply Voltage
V_S</th><th colspan="2">Capacitor Voltage
v_C</th><th rowspan="2">Charging / Discharging time
t_{full}
(ms)</th></tr>
<tr><th>Minimum Value
$V_{S_{min}}$
(V)</th><th>Maximum Value
$V_{S_{max}}$
(V)</th><th>Minimum Value
$v_{C_{min}}$
(V)</th><th>Maximum Value
$v_{C_{max}}$
(V)</th></tr>
<tr><td>Experimental (from oscilloscope)</td><td rowspan="2"></td><td rowspan="2"></td><td></td><td></td><td></td></tr>
<tr><td>Theoretical</td><td></td><td></td><td></td></tr>
</table>

Table 5: Data from Circuit 3

Use the function generator for the supply voltage and observe all values from the oscilloscope.

Circuit 3	Charging Phase				Discharging Phase			
	Resistor Voltage		Circuit Current $I = \frac{v_R}{R}$		Resistor Voltage		Circuit Current $I = \frac{v_R}{R}$	
	$v_{R_{min}}$ (V)	$v_{R_{max}}$ (V)	I_{min} (mA)	I_{max} (mA)	$v_{R_{min}}$ (V)	$v_{R_{max}}$ (V)	I_{min} (mA)	I_{max} (mA)
Experi-mental								
Theo-retical								

Table 6: Data from Circuit 4 & Circuit 5

Use the function generator for the supply voltage and observe all values from the oscilloscope.

Peak to peak voltage to set on the function generator = ______ V

DC offset to set on the function generator = ______ V

<table>
<tr><th rowspan="2">Circuit 4</th><th colspan="2">Capacitor Voltage v_C</th><th rowspan="2">Charging / Discharging time t_{full} (ms)</th><th rowspan="2">Time constant $\tau = \frac{t_{full}}{5}$ (ms)</th><th colspan="3">Equivalent Capacitance</th></tr>
<tr><th>Minimum Value $v_{C_{min}}$ (V)</th><th>Maximum Value $v_{C_{max}}$ (V)</th><th>(from osc.) $C = \frac{\tau}{R}$ (μF)</th><th>From Circuit 5 using multimeter C_{eq} (μF)</th><th>Error $\frac{\lvert C - C_{eq}\rvert \times 100\%}{C_{eq}}$ (%)</th></tr>
<tr><td>Experi-mental</td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Theo-retical</td><td></td><td></td><td></td><td></td></tr>
</table>

Questions

1. A capacitor stores energy-

☐ Magnetically ☐ Electrically ☐ Chemically ☐ Electro-chemically

2. If the capacitance (C) of a capacitor is related with the voltage (V) and the current (I) of the capacitor as $C = \frac{q}{V}$, which one of the following statements is correct? Capacitance of a capacitor can be increased by–

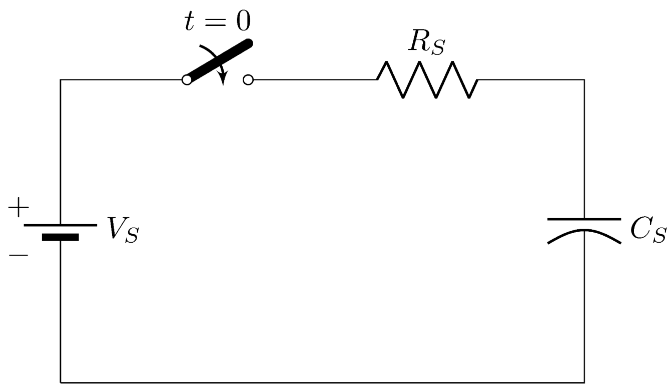
☐ decreasing the applied voltage across the capacitor.

☐ increasing the charge accumulation on the plates of the capacitor.

☐ increasing the surface area of the plates.

☐ decreasing the size of the capacitor.

3. When the switch in the following circuit is closed at $t = 0$, the following energy conversions happen–



*[use the keywords **electrical/mechanical/chemical/electro-chemical/thermal/heat** to answer (a) (b) and (c)]*

(a) The battery converts ________________ energy to _________________ energy.

(b) The capacitor receives __________________ energy from the battery and stores in the form of _______________ energy.

(c) The resistor dissipates energy into __________________ energy.

(d) Upon being fully charged by the battery (not to be dead so quickly), the capacitor–

☐ spontaneously releases the stored energy after some time to the resistor connected.

☐ gives the stored energy back to the battery after some time.

☐ holds the energy until some other circuit elements are connected to receive it.

☐ the capacitor can better tell what it wants to do.

4. Why was it necessary to short the two terminals of a capacitor before measuring the capacitance in the laboratory?

Because

5. We know the time constant (τ) depends on the equivalent resistance and the capacitance as $\tau = R_{eq}C$. Let's say, for a particular circuit, under a certain dc bias, **the time it requires** for increasing the voltage of a capacitor from $0\ V$ to $5\ V$ is $5\ ms$. If there were an initial voltage in the capacitor of $2\ V$, will the time now to increase the voltage to $5\ V$ be the same?

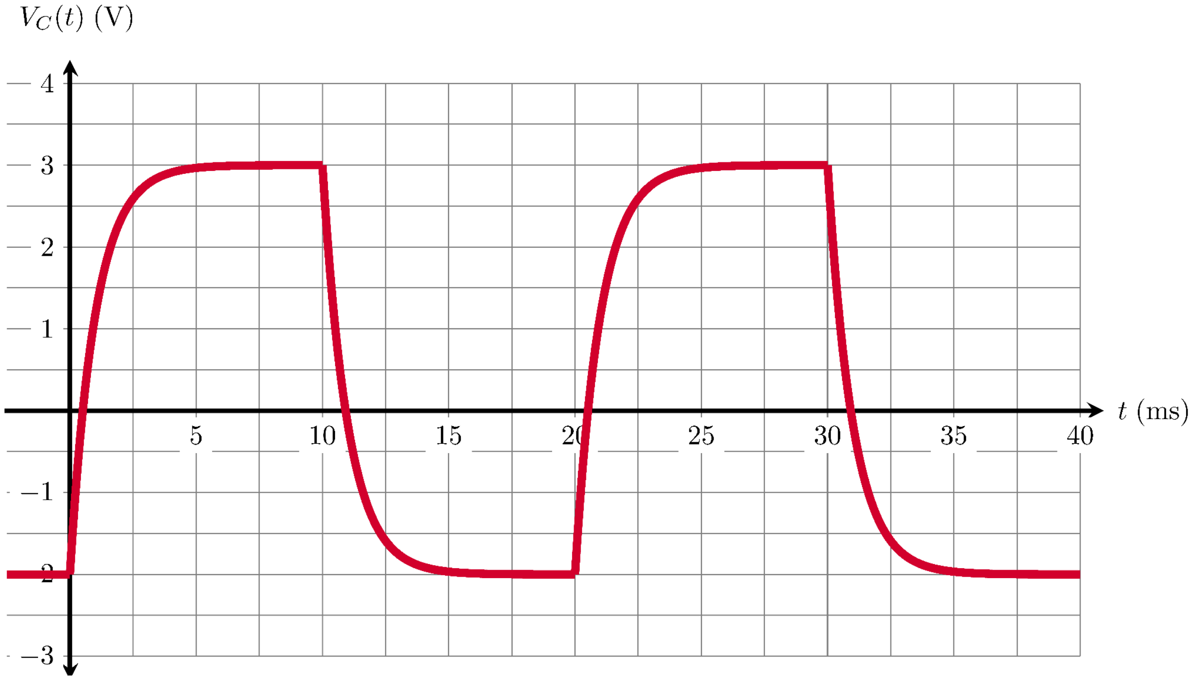
☐ Yes ☐ No

Why?

6. Based on your choice and understanding in question **5**, write briefly the significance of the time constant (τ) related to charging and discharging in a RC circuit?

The significance of τ is that

7. The capacitor voltage waveform you observed in the laboratory for **Circuit 3** is shown below where the excitation to the capacitance alternates between $-2\ V$ to $5\ V$ at a frequency of $50\ Hz$. The capacitor gets charged and discharged periodically.



(a) Mark the following things in the diagram:

(i) Charging and discharging phases (or timestamps)

(ii) The time constant (τ) for both charging and discharging phases

(iii) Initial and final voltages for both charging and discharging phases and

(iv) The times where the capacitor gets fully charged and discharged.

(b) Explain how can you change the time period of the voltage waveform?

The time period of the waveform can be changed by

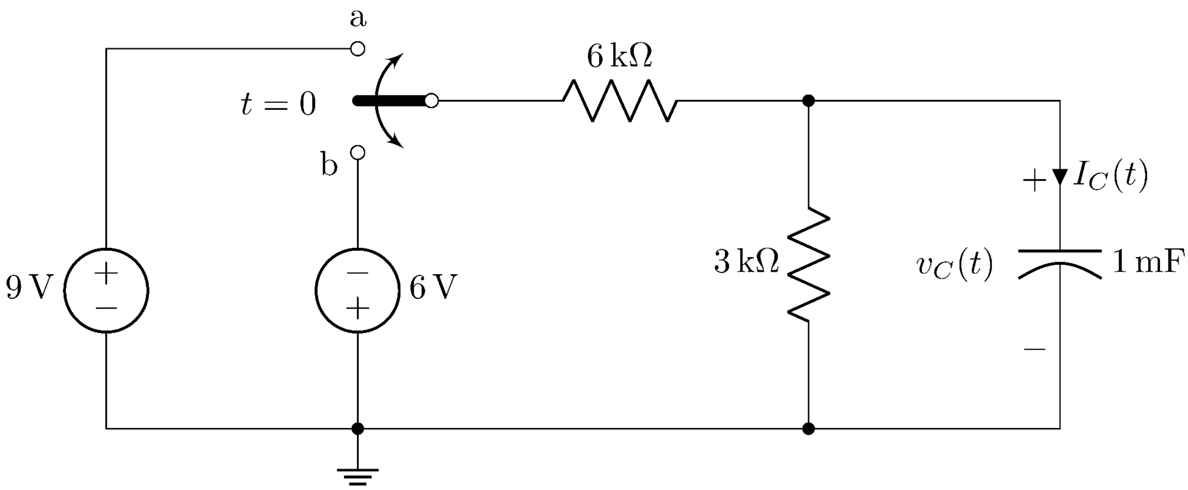
(c) Can the duty cycle of the voltage waveform be changed? Why?

☐ Yes ☐ No, the duty cycle cannot be changed. Because

(d) Will increasing the frequency of switching have any effect on the charging or discharging time of the capacitor?

Increasing the frequency of switching will have ☐ **no** ☐ **an** effect on the charging and discharging time of the capacitor. Because

8. Consider the RC circuit shown below. At $t = 0$, the switch starts to alternate between positions a and b at a frequency of **500 Hz**.



(a) Which one of the following instruments do you need in the laboratory for setting up the **switching operation** as fast as 500 Hz shown in the circuit diagram above?

☐ Two separate DC Power supplies.

☐ A Function Generator with the functionality of providing a dc offset.

☐ An Oscilloscope.

☐ A DC Power Supply with two channels.

(b) Based on your selection in (a) and the values of the voltages in the circuit diagram, list down the parametric (amplitude of voltage/frequency/offset/scale etc.) values that you will set on the selected instrument.

As I will use *(instrument name:)*__________________________________ to model the switching mechanism as described in the question statement, I will need to set only the (*list:)*______________________________________ on the instrument as follows *(leave those blank which are not required to set)*:

1. Amplitude of the voltage:	V
2. Frequency:	Hz
3. DC Offset:	V

4. Others *(specify with values)*:

(c) Draw the active portion of the circuit when the switch is in position a and determine the voltage across the capacitor, $v_C(t,\ switch\ \rightarrow a)$. *See the **Theory** section of this sheet if necessary.*

(d) Draw the active portion of the circuit when the switch is in position b and determine the voltage across the capacitor, $v_C(t,\ switch\ \rightarrow b)$. *See the **Theory** section of this sheet if necessary.*

(e) So, the capacitor voltage $v_C(t)$ alternates between the values __________(V) and _________(V).

(f) Now, determine the equivalent resistance as seen from the capacitor terminals (for $t > 0$). Recall that, while determining equivalent resistance, we must **kill** the independent sources.

$R_{eq} =$ $(k\Omega)$

(g) The time constant τ is thus−

$\tau = R_{eq}C =$ (ms)

(h) If the time constant (τ) is ___________(ms), it will take ______________(ms) for the capacitor to be fully charged or discharged.

(i) In general, the voltage across a capacitor under a sudden change in the applied dc bias is,

$$v_C(t) = v_C(final) + [v_C(initial) - v_C(final)]e^{-\frac{t}{\tau}}$$ or

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-\frac{t}{\tau}}$$

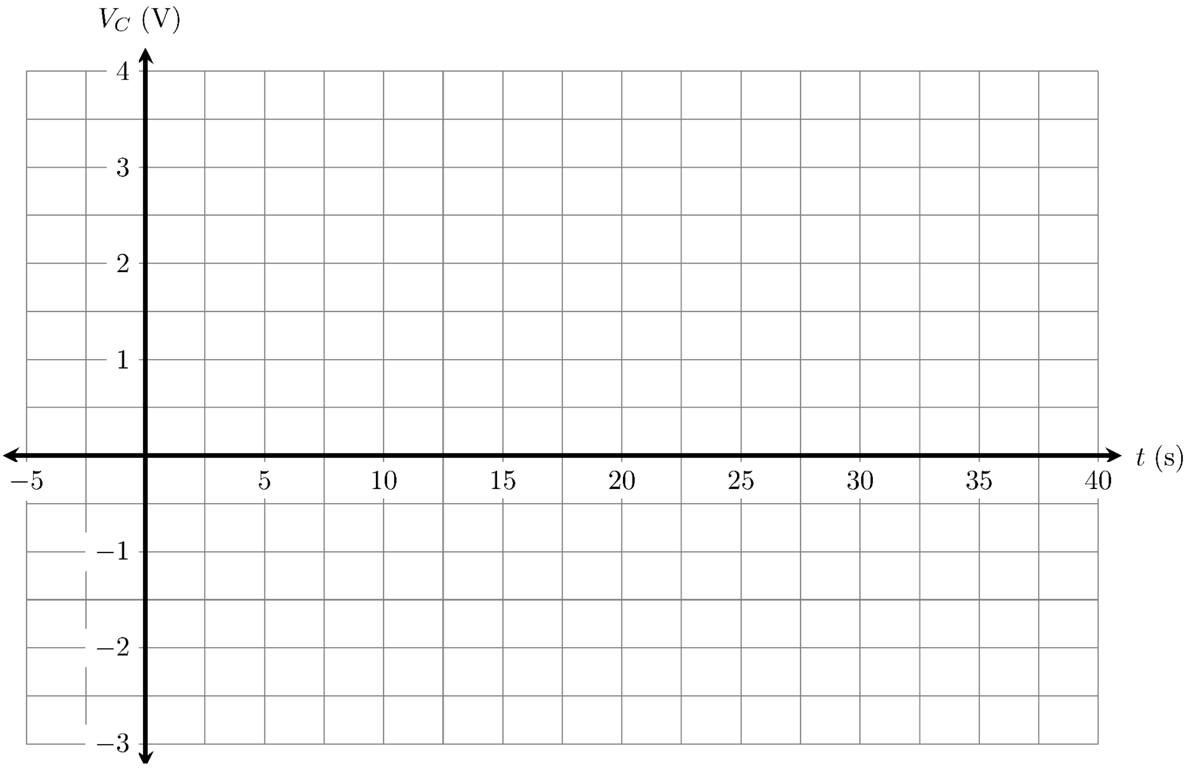
So, when $v_C(final) > v_C(initial)$, the capacitor gets charged and when $v_C(final) < v_C(initial)$, the capacitor gets discharged.

Based on this criteria, plug-in the values you got in (e) and (g) appropriately in the equation for $v_C(t)$ and write down the expression of $v_C(t)$ for−

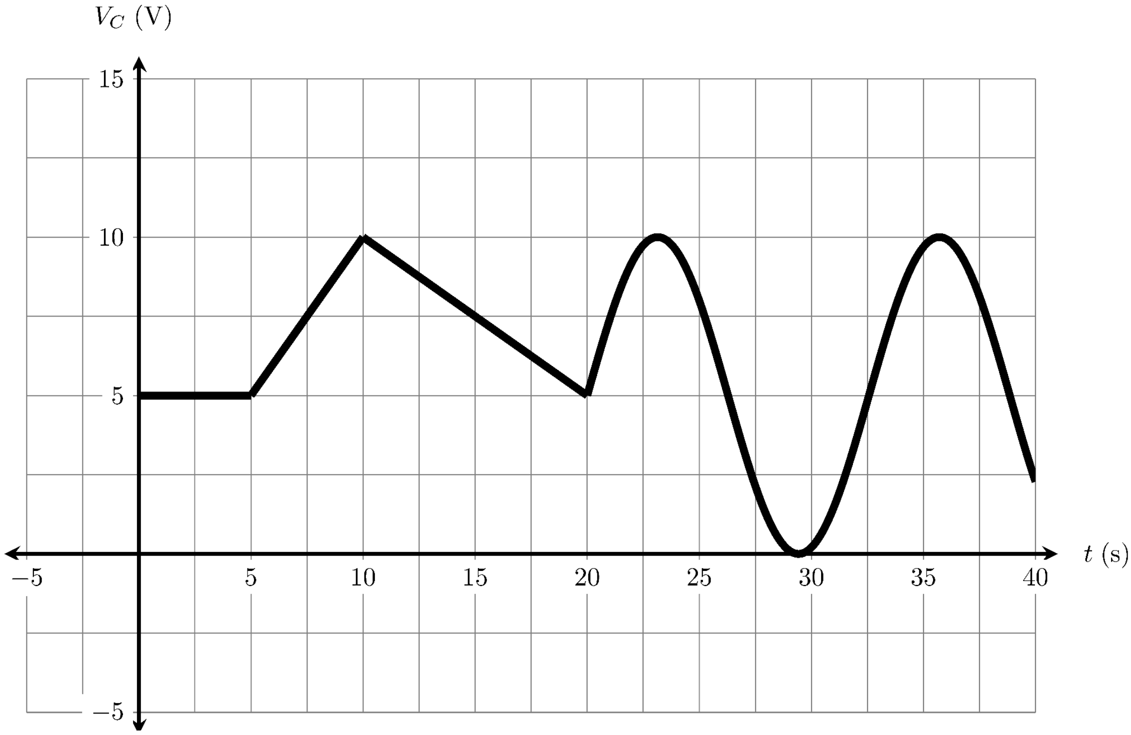
Charging: $v_C(t) =$

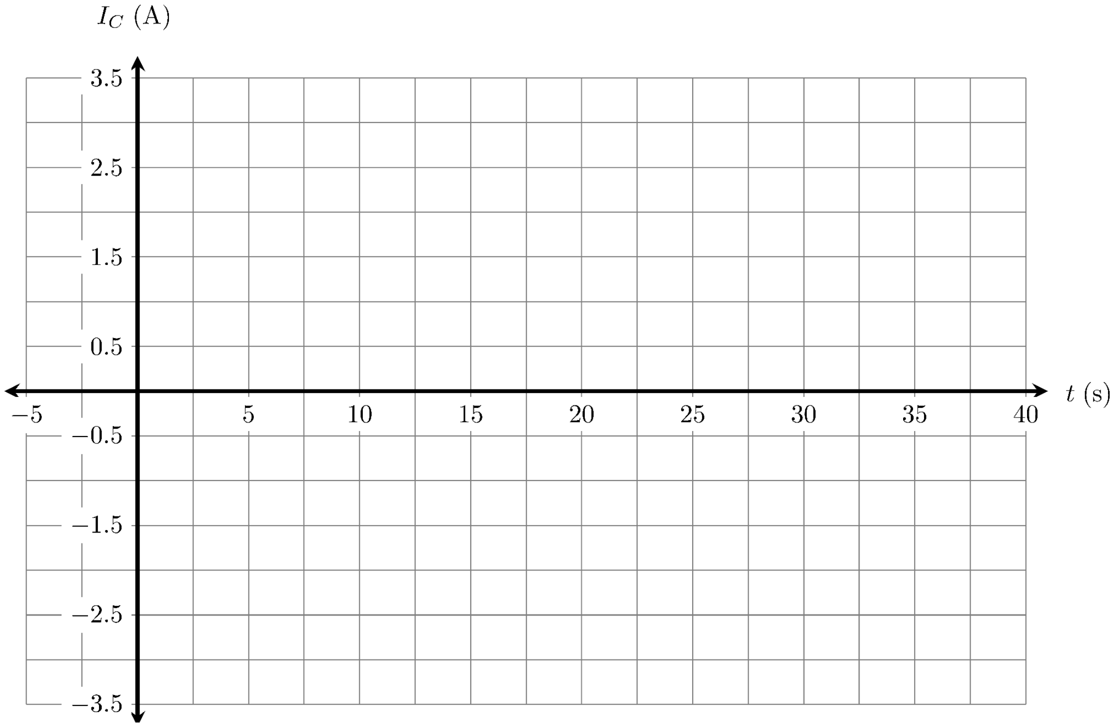
Discharging: $v_C(t) =$

(j) Based on the values in (e), (h), and the frequency (500 Hz), draw the waveform of the voltage across the capacitor v_C that we could observe in an Oscilloscope as a function of time for $t > 0$ as it gets charged and discharged continuously.



9. Draw (in the blank grid provided below) the current through a 1 F capacitor whose voltage is as shown in the following figure. Recall that, the voltage (V_C) across a capacitor and the current (I_C) through the capacitor (entering to the ' + ' polarity of the voltage) are related by the differential equation, $I_C = C\frac{dV_C}{dt}$.





Report

1. Fill up the theoretical parts of all the data tables.
2. Answer to the questions.
3. Attach the captured images of the plots observed in oscilloscope for **Circuits 2**, **3,** and **4**.
4. Discussion *[comment on the obtained results and discrepancies]*. Write in the next page.