Midterm 2 Exam - ECE 503 Fall 2016

- Date: Wednesday, November 2, 2016.
- Time: 11:00 am -11:50 am (in class)
- Maximum Credit: 100 points

1. [25 points] Let the random variables (X, Y) have the following joint PDF:

$$f_{X,Y}(x,y) = \begin{cases} 6(y-x) & 0 \leq x \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the marginal PDF of X.

   (b) Find the conditional distribution $f_{Y|X}(y|x)$.

   (c) What is the optimal MMSE estimator of Y given X ?

   (d) Is the above estimator linear ?

2. [25 points] A 3-dimensional random vector $X = [X_1 \;\; X_2 \;\; X_3]$ has zero mean, i.e., $E[X] = [0 \;\; 0 \;\; 0]$, and a auto-correlation matrix as follows:

$$R_X = \begin{bmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{bmatrix}$$

We observe a 2-dimensional random vector $Y = [Y_1 \;\; Y_2]$, where

$$Y_1 = X_1 + X_2$$
$$Y_2 = X_2 + X_3$$

Find the Linear Minimum Mean Squared Error (LMMSE) Estimator for X_1 from Y.

(Hint: Use the orthogonality principle, i.e., the error must be orthogonal to the observations.)

3. [30 points] Let $X_1, X_2, \ldots, X_n$ be a sequence of i.i.d. (independent and identically distributed) random variables. Each one of these random variables is uniformly distributed over $[-1, 1]$.

- Show that $Y_n = \frac{X_n}{n}$ converges to 0 in probability.
- Show that $Y_n = X_1 \cdot X_2 \cdots X_n$ converges to 0 in the mean square sense.
- Show that $Y_n = \max\{X_1, X_2, \ldots, X_n\}$ converges to 1 in distribution.

4. [20 points] Let p be the fraction of Arizona voters who will support a particular candidate in the 2016 presidential election. We survey n randomly selected voters, and record M_n, the fraction of the voters who support this candidate. We can view M_n as our estimate of p and would like to investigate its properties. In particular, we can interpret the responses of voters as i.i.d. Bernoulli random variables, with probability of voting for the candidate as p. How many voters should we include in the survey, so that our estimate M_n is within 0.02 confidence interval of p, and with high confidence (probability at least 95%) ?