

Homework 7 - ECE 503 Fall 2017

- Assigned on: Wednesday, November 15, 2017.
 - Due Date: **Wednesday, November 22, 2017 by 11:00 am Tucson Time.**
 - Maximum Credit: **150 points**
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1. **[10 points]** Which of the following are valid auto-correlation functions of a WSS random process ?

- (a) $R_1(\tau) = \delta(\tau)$
- (b) $R_2(\tau) = \delta(\tau) + 10$
- (c) $R_3(\tau) = \delta(\tau) - 10$
- (d) $R_4(\tau) = \delta(\tau - 10)$

2. **[20 points]** $X(t)$ is a WSS random process with auto-correlation function $R_X(\tau) = 10 \sin(2\pi 1000t) / (2\pi 1000t)$. The process $Y(t)$ is a delayed version of $X(t)$ by 50 microseconds, i.e., $Y(t) = X(t - t_0)$, where $t_0 = 5 \times 10^{-5}$ seconds.

- (a) Find the autocorrelation function of $Y(t)$.
- (b) Find the cross-correlation function of $X(t)$ and $Y(t)$
- (c) Are $X(t)$ and $Y(t)$ jointly WSS ?

3. **[20 points]** Consider the random process

$$W(t) = X \cos(2\pi f_0 t) + Y \sin(2\pi f_0 t)$$

where X and Y are uncorrelated random variables, each with expected value 0 and variance σ^2 .

- (a) Find the auto-correlation function of the random process $W(t)$.
- (b) Is $W(t)$ wide sense stationary (WSS) ?

4. **[20 points]** $X(t)$ is a WSS random process with average power equal to 1. Let Θ denote a random variable with uniform distribution over $[0, 2\pi]$, and $X(t)$ and Θ are independent.

- (a) What is $E[X^2(t)]$?
- (b) What is $E[\cos(2\pi f_c t + \Theta)]$?
- (c) Let $Y(t) = X(t) \cos(2\pi f_c t + \Theta)$. What is $E[Y(t)]$?
- (d) What is the average power of $Y(t)$?

5. **[10 points]** A white Gaussian noise process $N(t)$ with auto-correlation $R_N(\tau) = \alpha \delta(\tau)$ is passed through an integrator yielding the output

$$Y(t) = \int_0^t N(u) du$$

Find the mean and auto-correlation functions of $Y(t)$. Show that $Y(t)$ is a non-stationary process.

6. [10 points] A discrete-time random process X_n is WSS if $E[X_n]$ does not depend on n and if the correlation $E[X_n X_m]$ depends on n and m only through their difference. Show that if X_n is WSS, then so is $Y_n = X_n - X_{n-1}$.
7. [20 points] A popular music group produces a new hit song every 7 months on average. Assume that hit songs are produced according to a Poisson process.
- (a) Find the probability that the group produces more than two hit songs in 1 year.
 - (b) How long do you expect it to take until the group produces its 10th hit ?
8. [20 points] Space shuttles are launched according to a Poisson Process. The average time between launches is 2 months.
- (a) Find the probability that there are no launches during a 4 month period.
 - (b) Find the probability that during at least 1 month out of four consecutive months, there are at least two launches.
9. [20 points] Data packets depart from a router according to a Poisson process with rate λ per minute. Each packet arrives successfully at a receiver with probability p , independently of every other packet.
- (a) Find the distribution of the time until the first packet arrives.
 - (b) Find the probability that no packets arrive successfully in any particular hour.
 - (c) Find the expected number of packets that arrive successfully during a particular hour.