

Homework 4: ECE 503 Fall 2020

- Assigned on: Friday, September 25, 2020.
 - Due Date: **Friday, October 2, 2020 by 11:59 pm Tucson Time.**
 - Mode of submission: D2L
 - Maximum Credit: **100 points**
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1. **[10 points]** Suppose that X is a uniform random variable in $(-5, 5)$. Given the event $B = \{|X| \leq 3\}$,
 - Find the conditional PDF $f_{X|B}(x)$.
 - Find the conditional expected value $E[X|B]$.
 - Find the conditional variance $\text{Var}[X|B]$.
2. **[10 points]** The voltage across a resistor is modeled as a uniform random variable V with mean 0 and variance 3. The instantaneous power is $P = V^2$. Find the CDF and PDF of P .
3. **[10 points]** Let U be uniform random variable in $(0, 1)$ and $X = -\ln(1 - U)$ (\ln denotes the natural log).
 - Find the CDF and PDF of X .
 - Compute the expected value of X .
4. **[10 points]** Let V be a Poisson random variable with mean λ .
 - Compute the variance of V .
 - Compute the expected value of $Y = V^2 - 2V + 3$.
5. **[10 points]** Let X be a Gaussian random variable with mean μ and variance σ^2 .
 - Compute the PDF of $Y = aX + b$. Is Y also a Gaussian random variable?
 - Compute the PDF of $Z = X^2$. Is Z also a Gaussian random variable?
6. **[10 points]** Let Y be a random variable with the PDF

$$f_Y(y) = \begin{cases} \frac{c}{(1+y^2)}, & y \in (-2, 2) \\ 0, & \text{otherwise} \end{cases}$$

Find c , μ_Y and σ_Y^2 .

7. **[20 points]** A coin with probability of heads p is tossed repeatedly till a heads appears. Each coin toss is independent. Let X be the random variable denoting the number of tosses till a heads appears.
 - What values does X take?
 - Write down the PMF of the random variable X .
 - Compute the expected number of tosses till you see a heads (i.e., compute $E[X]$).
8. **[20 points]** An ad-company makes 2\$ profit for every ad-link that is clicked, whereas it loses 1\$ in operational costs if it is not clicked. Assume that the probability of an ad being clicked by a user is p , and each ad is either clicked/not-clicked independently. The number of ads shown to a user follows a Poisson distribution with rate $\lambda = 4$ ads/hour. What is the expected profit that the company makes over a 24 hour period? Compute your answer as a function of p .