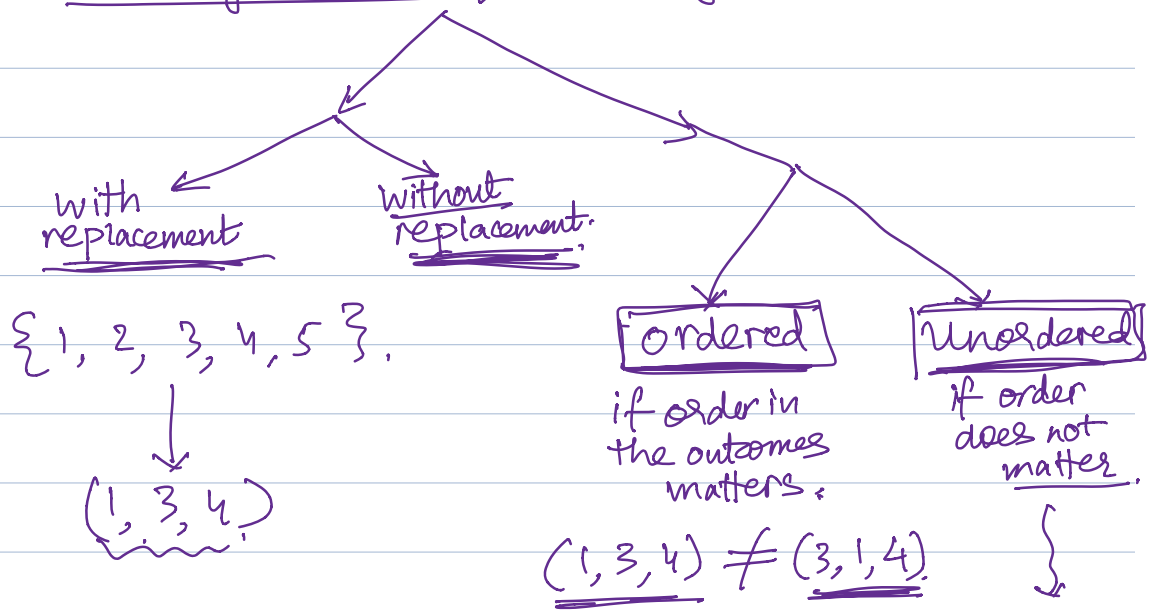
Counting Problems / Sampling



$\{1, 2, 3, 4, 5\}$.

$(1, 3, 4)$

$(1,3,4) \neq (3,1,4)$

$(1,3,4) \equiv (3,1,4)$

	Ordered	Unordered
with Replacemt	(I)	(III)
w/o Replacemt	(II)	(IV).

(I) Ordered Sampling with Replacemt

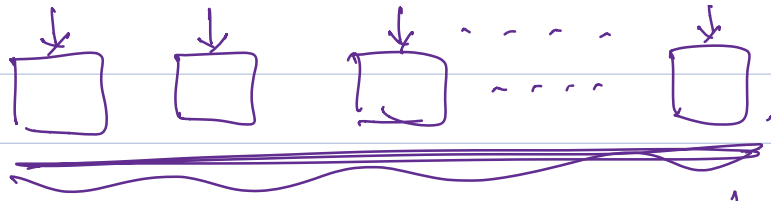
- $A = \{1, 2, 3, \ldots, n\}$.
- Draw k samples from A. — with replace — ordered.

$A = \{1, 2, 3\}$. $k = 2$ items.

$(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)$.

Q: Possible Answers: n^k

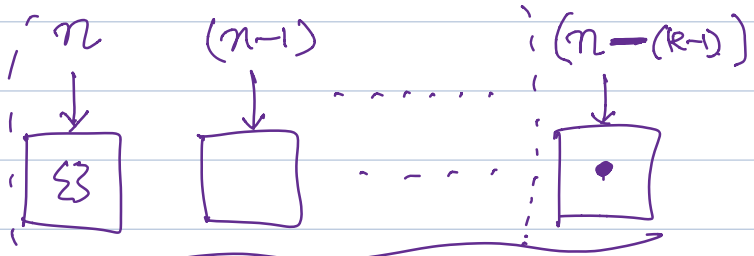
$n \times n \times \cdots \times n = n^k$.



k slots

(II) Ordered Sampling w/o replacement.

A → n items k sampled



k slots.

# of possib = $n \times (n-1) \times \cdots \times (n-(k-1))$

$= \dfrac{n(n-1)\cdots(n-(k-1))\,(n-k)!}{(n-k)!} = \dfrac{n!}{(n-k)!}$

$(n-k)\times(n-(k+1))\times\cdots\times 2\times 1$?

$n(n-1)\cdots(n-(k-1)) \times (n-k)\times(n-(k+1))\times\cdots\times 2\times 1$

$= n!$

$= P^{n}_{k}$

$A = \{1, 2, 3\}$ $k = 2$.

(1,2) (1,3) (2,1) (2,3)

(3,1) (3,2)

{ # of k-perm. of n elements.

Eg: Group k students

$\Rightarrow$ What is prob that at least two of them have the same birthday?

($n = 365$ days)

A = Event that (at least) two pers share birthday..

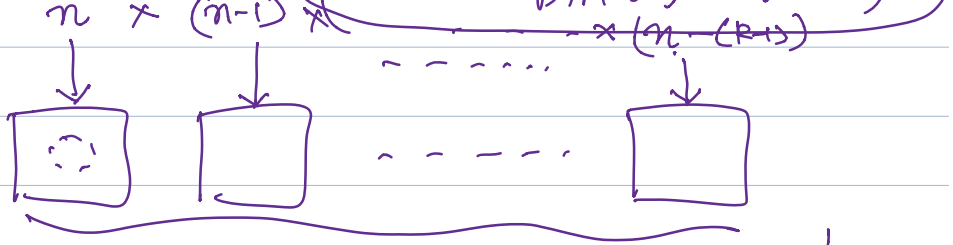
Case 1: if $\underbrace{k}_{367} > \underbrace{n}_{365}$ $\quad P(A) \overset{?}{=} 1$.

Case 2: if $k \leq n$.

A^c = Event that no one shares a birthday

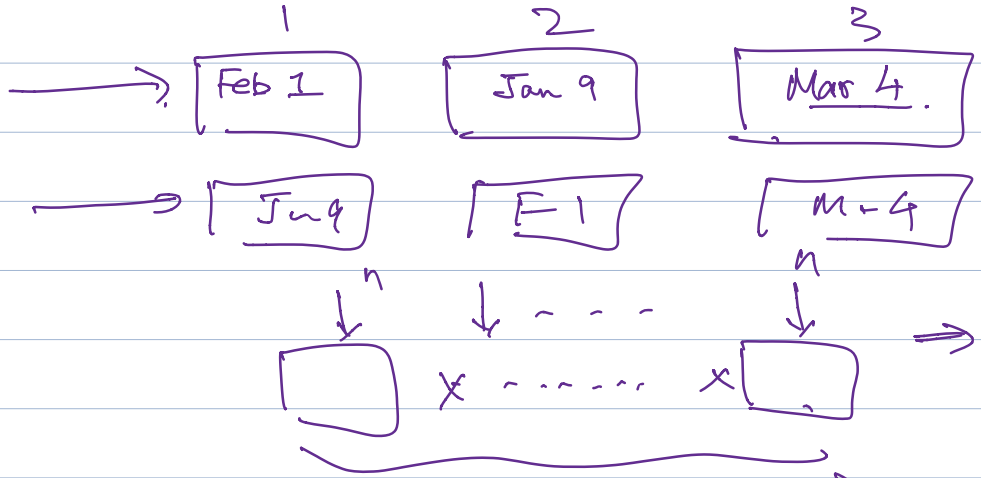
$P(A) = 1 - P(A^c)$.

$P(A^c) = \dfrac{(\#\text{ of elements in } A^c)}{(\text{total \# of possible birthday sequences})}$



k slots/students.

# of ways no one sh. a birthday $= \dfrac{n!}{(n-k)!} = P^n_k$.



$$P(A^c) = \frac{(n!/(n-k)!)}{n^k}.$$

$$P(A) = 1 - \frac{n!}{(n-k)!\, n^k} \approx 0.5073$$

$k = 23$.

Eg2: Group of k people.
What is the prob that (at least) one person has the same birthday as yours?

Sep 2

A = at least one person same b/d as Sep 2.

A^c = No one has Sep 2 as their Birthday.

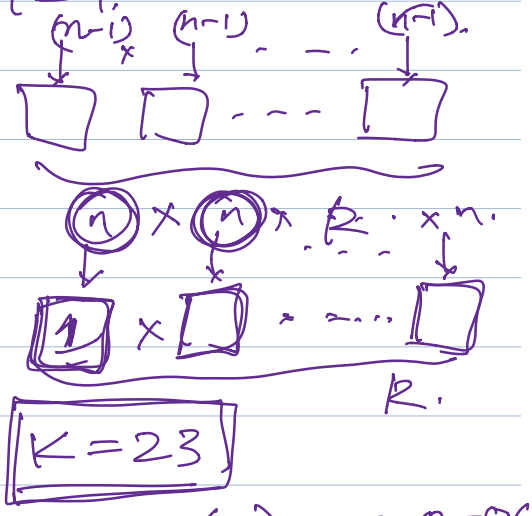
$$P(A) = 1 - P(A^c).$$

$= 1 - \frac{|A^c|}{|S|} = 1 - \frac{(n-1)^k}{n^k}$

$|A^c| \overset{?}{=} (n-1)^k$

k people.

$|S| \overset{?}{=} n^k$



$K = 23$

$P(A) \approx 0.0586$