Homework 8 - ECE 503 Fall 2020

- Assigned on: Thursday, November 19, 2020.
- Due Date: **Wednesday, November 25, 2020 by 11:59 pm Tucson Time.**
- Maximum Credit: 110 points

1. [20 points] The duration of a phone call is modeled as an exponential random variable with expected value 150 seconds. A subscriber has a calling plan that includes 300 minutes per month at a cost of \$30 plus \$0.40 for each minute that the total calling time exceeds 300 minutes. In a certain month, the subscriber makes 120 cellular calls.
   (a) Use the central limit theorem to estimate the probability that the subscriber's bill is greater than \$36. (Assume that the durations of all phone calls are mutually independent and that the telephone company measures call duration exactly and charges accordingly, without rounding up fractional minutes.)
   (b) Suppose the telephone company does charge a full minute for each fractional minute used. Recalculate your estimate of the probability that the bill is greater than \$36.

2. [20 points] Integrated circuits from a certain factory pass the quality test with probability 0.8. The outcomes of all tests are mutually independent.
   (a) What is the expected number of tests necessary to find 500 acceptable circuits?
   (b) Use the central limit theorem to estimate the probability of finding 500 acceptable circuits in a batch of 600 circuits.
   (c) Use Matlab to calculate the actual probability of finding 500 acceptable circuits in a batch of 600 circuits. (Note: Matlab has a built in command to evaluate CDF of commonly encountered random variables).
   (d) Use the central limit theorem to calculate the minimum batch size for finding 500 acceptable circuits with probability 0.9 or greater.

3. [10 points] Let W_1, W_2,... be i.i.d. random variables with unknown mean μ and known variance $\sigma^2 = 4$. We observe 100 samples and find that the sample mean is $\bar{W}_{100} = 14.846$. Find the 95% confidence interval estimate for the mean.

4. [10 points] Let X_1, X_2,... be i.i.d. random variables with unknown, finite mean μ and unknown variance σ^2. We observe 100 samples and find that the sample mean is $M_{100} = 10.083$ and the sample variance is $S_{100} = 0.568$. Find the 95% confidence interval estimate for the mean.

5. [10 points] Suppose that 100 engineering freshmen are selected at random and X_1,...,X_{100} are their times (in years) to graduation. If the sample mean is 4.422 years and the sample variance is 0.957, find the 93% confidence interval for their expected time to graduate.

6. [20 points] Suppose we observe outcomes from i.i.d. trials of an experiment, where each one is a Bernoulli random variable with $P(Success) = \theta$, and $P(Failure) = 1 - \theta$. After n trials, we count N_F = Number of failures and N_S = Number of successes. Find the Maximum Likelihood (ML) estimator of θ. If you want to estimate θ within an absolute error of 0.1 (i.e., $|\hat{\theta}_{ML} - \theta| < 0.1$) with probability at least 0.99, what is the minimum number of trials you should conduct?

7. [20 points] Suppose that the signal $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$ is transmitted over a communication channel. Assume that the received signal is given by $Y = X + W$, where $W \sim \mathcal{N}(\mu_2, \sigma_2^2)$ is independent of X.
   (a) Find the MMSE estimate of X, given $Y = y$ is observed.
   (b) Find the ML estimate of X, given $Y = y$ is observed.
   (c) Find the MAP estimate of X, given $Y = y$ is observed.