1) Ordered with Replacement $\longrightarrow$ n^K

2) Ordered W/O Replacement $\longrightarrow$ $P_K^n = \dfrac{n!}{(n-K)!}$

3) Unordered W/O Replacement $\longrightarrow$ $C_K^n = \binom{n}{K} = \dfrac{n!}{K!\,(n-K)!}$

4) Unordered with Replacement $\longrightarrow$ TODAY

Bernoulli Trials : Exp $\to$ Succ. $\longrightarrow$ w.p. p

Failure $\longrightarrow$ w.p. $(1-p)$

Binomial Exp $\longrightarrow$ n repeated & indep. Bernoulli trials.

$$P(k \text{ success in } n \text{ trials}) = \underbrace{\binom{n}{k}}_{\substack{\#\text{ of}\\ \text{ways.}}} \times \underbrace{\boxed{p^K (1-p)^{n-K}}}_{\substack{\text{Prob of}\\ \text{each such}\\ \text{outcome}}}.$$

$$1 \overset{?}{=} \underbrace{\sum_{k=0}^{n} \binom{n}{k} p^K (1-p)^{n-K}}_{\text{Proof. Via Bino. th.}}.$$

S : $P(k=0) + P(k=1) + \dots + P(k=n)$

$P(S) = 1$

Exp $\to$ $S_1, S_2, \dots, S_r$: r possible outcomes.

$P(S_i) = P_i$

$i = 1, 2, \dots, r.$

$\left(\sum P_i = 1\right)$

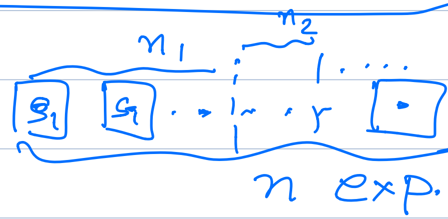
Suppose we do this Exp n times independently.

$$\boxed{P(S_1 \text{ occurs } n_1 \text{ times},\ S_2 \text{ occurs } n_2 \text{ times},\ \dots\dots,\ S_r \text{ occurs } n_r \text{ times}) = ?}$$

$$\boxed{\begin{array}{l} n_1 + n_2 + \cdots + n_r = n. \\ 0 \le n_i \le n. \end{array}}$$

$$\boxed{\begin{array}{l} \#\text{ of ways in which } S_1 / n_1,\ S_2 / n_2, \ldots, S_r / n_r \text{ times} \\ = \dfrac{n!}{n_1!\, n_2! \cdots n_r!} \end{array}}$$

only 3 possible outcomes per experiment.
(S_1, S_2, S_3)



$$\boxed{n = 8 \text{ times}}$$

\# of ways $(S_1 \to 4,\ S_2 \to 3,\ S_3 \to 1)$

$$\binom{8}{4} \times \binom{4}{3} \times \binom{1}{1}$$

$$= \frac{8!}{\cancel{4!}\, 4!} \times \frac{\cancel{4!}}{3!\ 1!} \times \frac{1!}{1!\ 0!}$$

$$= \frac{8!}{4!\ 3!\ 1!}$$

$$\underbrace{\binom{n}{n_1}}_{S_1} \times \underbrace{\binom{n-n_1}{n_2}}_{S_2} \times \binom{n-n_1-n_2}{n_3} \times \cdots$$

$$= \frac{n!}{n_1!\ \cancel{(n-n_1)!}} \times \frac{\cancel{(n-n_1)!}}{n_2!\ \cancel{(n-n_1-n_2)!}} \times \cdots$$

$$= n! \,/\, (n_1!\ n_2! \cdots n_r!)$$

$P(\ldots\ldots)$ (from above) $= \left(\dfrac{n!}{n_1!\,n_2!\ldots n_r!}\right) \times \boxed{P_1^{n_1} \times P_2^{n_2} \ldots \times P_r^{n_r}}$

\# of ways/possib.

Prob of any one of them

$S_1 \to n_1$, $S_2 \to n_2$, ..., $S_r \to n_r$ times

8

$S_1\ S_1\ S_1\ S_2\ S_3\ S_1\ S_2\ S_2$.

$P(\quad) = P_1 \times P_1 \times P_1 \times P_2 \times P_3 \times P_1 \times P_2 \times P_2$

$= P_1^{n_1} \times P_2^{n_2} \times P_3^{n_3}$.

$S_1\ S_2\ S_2\ S_2\ S_3\ S_1\ S_1\ S_1$

(4) Unordered Sampling with Replacement

$A = \{1, 2, \ldots, n\}$

$A = \{1, 2, 3\}$ draw $k = 2$ items.

(1, 1)
(1, 2)
(1, 3)
(2, 2)
(2, 3)
(3, 3)

6 ways

General Answer.

$$\binom{(n-1)+k}{k}$$

$n = 3$ $\quad k = 2$

$(n-1) = 2$ $\quad k = 2$

$$\binom{2+2}{2} = \binom{4}{2} = \frac{4!}{2!\,2!} = 6.$$

((1, 1)

$$\left\{\begin{array}{l}(1, 2)\\(1, 3)\\(2, 2)\\(2, 3)\\(3, 3)\end{array}\right.$$

drawing k items out of n

(x_1, x_2, x_3)

x_1: # of times item 1 is drawn

x_2: # of time 2

$$\left\{\begin{array}{lcl}(1,1) & \longleftrightarrow & (2, 0, 0)\\(1, 2) & \longleftrightarrow & (1, 1, 0)\\(1, 3) & \longleftrightarrow & (1, 0, 1)\\(2, 2) & \longleftrightarrow & (0, 2, 0)\\(2, 3) & \longleftrightarrow & (0, 1, 1)\\(3, 3) & \longleftrightarrow & (0, 0, 2)\end{array}\right.$$

x_i's are integers

$\boxed{0 \le x_i \le 2}$

$\boxed{0 \le x_i \le k}$

$$x_1 + x_2 + x_3 \overset{?}{=} 2$$

$$x_1 + x_2 + \cdots + x_n = k.$$

$$\boxed{\begin{array}{c}x_1 + x_2 + x_3 = 2\\0 \le x_i \le 2\end{array}} \Longrightarrow$$

How many distinct solutions to this equation

Find the # of distinct solutions to

$$x_1 + x_2 \cdots + x_n = k$$

where $x_i \in \{0, 1, 2, \ldots, k\}$

$n = 4$, $k = 6$.

four items. $(x_1\ x_2\ x_3\ x_4) = (3, 0, 2, 1)$.

$x_1 + x_2 + x_3 + x_4 = 3 + 0 + 2 + 1 = 6$

$\boxed{|||\ +\ \ +\ ||\ +\ |} = 6.$

using k sticks

& $(n-1)$ + signs.

$x_1 + x_2$

How many distinct seq. can we form using k sticks & $(n-1)$ + signs?

Ans :
$$\binom{(n-1)+k}{k} = \binom{(n-1)+k}{n-1}.$$

k sticks. & $(n-1)$ + ?

$$\binom{8}{5}$$

$8 = 5 + 3.$

(HW2 #4)

n integers

$\{1, 2, \ldots, n\}$.

lottery picks $\longrightarrow$ a subset of size r $\boxed{\text{w/o}}$ replacement (unordered)

$\{1, 2, 3, 4, \ldots, 10\}$.

$\boxed{r=3}$

$(2, 3, 7)$

$(2, 9, 10)$

$(1, 8, 3)$

Total # of ways $\Longrightarrow$ $\binom{n}{r}$

Total # of ways s.t. r items have **NO** consecutive integers.

$\cancel{(1, 2, 8)}$?

$\cancel{(2, 3, 7)}$

$\boxed{(1, 3, 5)}$

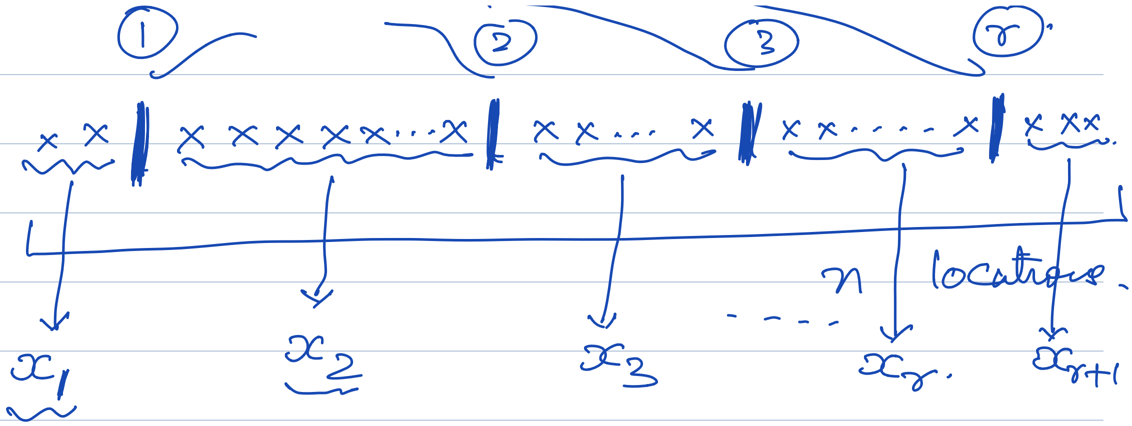
$1 \quad 2 \quad \boxed{3} \quad 4 \quad \ldots \quad \boxed{\,} \quad \boxed{n}$

Picking r numbers.

$\Downarrow$

Placing r sticks.

r sticks.



1) $x_1 \geq 0$

$x_2 \geq 1$

$x_3 \geq 1$

$\vdots$

$x_r \geq 1$

$x_{r+1} \geq 0$.

$$\underbrace{(x_1 + x_2 + \dots + x_{r+1})}_{\text{slots occp. by } x\text{'s.}} + \underbrace{r}_{\text{occupied by sticks.}} = n$$

$$\boxed{x_1 + x_2 \dots + x_{r+1} = (n - r)}$$