Homework 1 - ECE 503 Fall 2017

- Assigned on: Friday, August 25, 2017.
- Due Date: **Friday, September 1, 2017 by 11:00 am Tucson Time.**
- Mode of submission: Drop in my mailbox (ECE Building, 2nd Floor)
- Maximum Credit: 100 points

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1. [10 points] Recall that in class, we showed that for any two events A_1 and A_2, the following holds: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$. In this problem, we are interested in generalizing this fact to more that two events. For any three events A_1, A_2 and A_3, prove that
$$P(A_1 \cup A_2 \cup A_3) = \sum_{i=1}^{3} P(A_i) - \sum_{i \neq j} P(A_i \cap A_j) + P(A_1 \cap A_2 \cap A_3)$$
How would you generalize this result for any n events $(A_1, A_2, \ldots, A_n)$?

2. [10 points] Let $P(A) = 0.7$ and $P(B) = 0.6$, then show that $P(A \cap B) \geq 0.3$.

3. [10 points] Show that if the events A, B and C are independent, then the events A and $B \cup C$ are also independent.

4. [10 points] In an experiment, a fair coin is tossed repeatedly till a head appears.
   (a) Write the sample space of this experiment
   (b) Find the probability that the first head appears on the kth toss
   (c) Find the probability that the first head appears on an odd-numbered toss. Is it different from the probability of observing the first head on an even-numbered toss ?

5. [10 points] Consider the experiment of throwing two fair dice. Let A be the event that the first die is odd, B be the event that the second die is odd, and C be the event that the sum is odd. Show that the events A, B and C are pairwise independent, but A, B and C are not independent.

6. [20 points] For a completely booked United Airlines flight from Phoenix to Honolulu, n passengers have been told their seat numbers. They get on the plane one by one. The first person sits on a wrong seat. Subsequent passengers sit in their assigned seat if they find it available, or select another empty seat at random. What is the probability that the last passenger finds his (originally assigned) seat free ?

7. [20 points] Because of the stress of the first week of school, you decide to take a little vacation and go to Las Vegas on Labor Day weekend. There, you want to spend only $m = \$100$ on gambling. Being an inexperienced gambler, you choose to play the following simple game. A dealer tosses a fair coin. If it comes up heads you win a dollar, but if it comes up tails you pay the dealer one dollar. You decide to play this game repeatedly until either you win $N = \$1000$, or you lose all your money. We are interested in finding the probability of the good event, W of winning $N = \$1000$ dollars.
   (a) Let us denote $P_m(W)$ as the probability of winning given that you have m dollars to spend. Compute $P_m(W)$ as a function of m and N.
   (b) What are the consequences of being greedy and increasing N ?

8. [10 points] Alice and Bob participate in a *Game of Hats*. Each one of them is donned with either a black hat or a white hat (equally likely). They cannot see the color of their own hat, but they can see the other persons' hat. Alice and Bob must play as a team and take a guess for the color of their own hat. They win if both Alice and Bob guess correctly. They can strategize before the start of the game, but are not allowed to communicate once the game starts.
   - Suppose that Alice and Bob decide to take independent random guesses about the color of their own hat. Find the winning probability of this strategy.
   - Can you devise a different strategy which is better than random guessing ?