Midterm 1 Exam - ECE 503 Fall 2020

- Due Date and Time: Monday, Oct. 5, 2020, by Noon.
- Submit your answers on D2L.
- Maximum Credit: 100 points

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1. [25 points]

   (a) (5 points) Mutually exclusive events are always independent. (True or False ?)

   (b) (10 points) Six cards are drawn at random (with replacement) from a deck of 52 cards. What is the probability that there are at least two Aces?

   (c) (10 points) Let X be a discrete random variable with the following PMF:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{(n-k)}, \quad k = 0, 1, \ldots, n$$

   Compute the mean of X.

2. [25 points] Let X be a random variable with the following CDF:

$$F_X(x) = \begin{cases} 0, & x < -3 \\ \frac{x}{12} + \frac{1}{2}, & -3 \leq x < 0 \\ \frac{x}{12} + \frac{3}{4}, & 0 \leq x < 3 \\ 1, & 3 \leq x. \end{cases}$$

(a) Find $P(X = -2), P(X = 0)$ and $P(0 < X \leq 2)$

(b) Find $P(X \leq 2 | X > -1)$.

(c) If $Y = X^2$, find the CDF of the random variable Y.

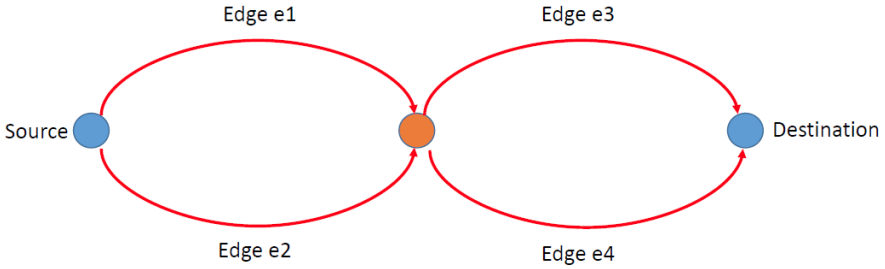
3. [25 points] To sign up for a new COVID-19 contact tracing app, users are asked to pick a password of length 8, with the following guidelines. The password must have

   - exactly 4 upper-case letters (can be chosen with replacement) from $\{A, B, \cdots, Z\}$
   - exactly 2 lower-case letters (can be chosen with replacement) from $\{a, b, \cdots, z\}$
   - exactly 2 special characters (chosen without replacement) from the following list $\{\#, \$, \%, \&, !, @\}$

   (a) How many distinct passwords are possible?

   (b) Suppose there are N users that sign up for the app. Each user independently picks a valid password at random. What is the probability that none of the users share the same password?

4. [25 points] Consider a source (S) and a destination (D) connected through the network shown in the figure below. A path from S to D is defined as a sequence of edges that connect S to D. For instance, the path $P_{1,3} = e_1 \rightarrow e_3$ is a valid path that can allow data transfer from S to D. Each edge in the network is functional independently with probability p (and does not work with probability $1-p$). In order to send data from S to D, one needs a working path, i.e., a path with all functional edges. For instance, the path $P_{1,3}$ is a working path only if both the edges e_1 and e_3 are functional.



(a) Enumerate all the valid paths for this network.

(b) What is the probability that there is at least one working path from S to D?

(c) What is the expected number of working paths?