Midterm 1 Exam - ECE 503 Fall 2016

- Date: Monday, September 26, 2016.

- Time: 11:00 am -11:50 am (in class)

- Maximum Credit: 100 points

1. [25 points] A continuous valued random variable, X has the following PDF:

$$f_X(x) = \begin{cases} k_1 x + k_2 x^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What condition must (k_1, k_2) satisfy so that $f_X(x)$ is a valid PDF ?

   (b) Suppose that you are given that $P(X \leq 1/2) = 1/2$. Determine k_1 and k_2.

   (c) Determine the CDF of X.

2. [25 points] A computer network connects a source (S) and a destination (D) through intermediate nodes X, Y, and Z as shown in the Figure below. For every pair of directly connected nodes, say i and j, the probability that the link from node i to node j is working is given by p_{ij}. These probabilities are shown in the figure. We assume that the link failures are independent of each other.

 (a) What is the probability that <u>all the paths</u> from S to D fail ?

 (b) What is the probability that there is <u>exactly one working path</u> connecting S to D ?

 (c) What is the probability that there is <u>at least one working path</u> from S to D ?

X

0.75   0.5

0.25   0.5

S   Y   D

Source   Destination

0.25   0.5

Z
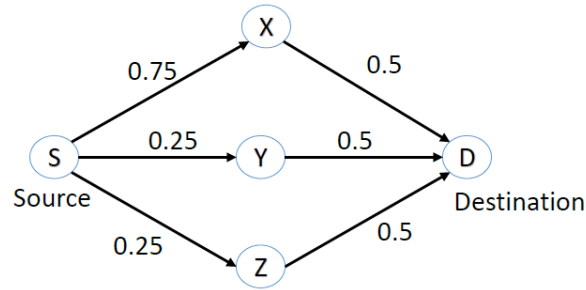
3. [25 points] Let X be a uniform random variable in $[0, 2]$. Compute the CDF and PDF of the random variable $Y = g(X)$, where the function $g(.)$ is shown in the figure below.
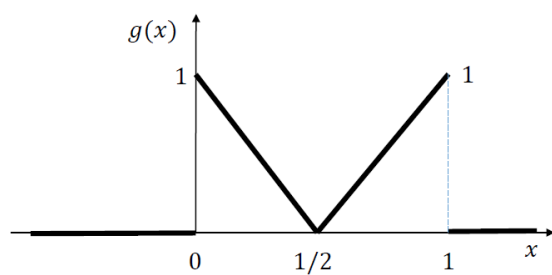
4. [25 points] The random variable X models the duration of the call made by a typical cell phone user. Assume that X is distributed as an exponential random variable, with parameter $\lambda = 1$, i.e., the PDF of the call duration is

$$f_X(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Verizon and AT&T have different mechanisms of charging a user based on the call duration. Verizon uses the following charging plan (i.e., if the call duration is X, then $Y_{\text{Verizon}}(X)$ denotes the amount of money charged as a function of X):

$$Y_{\text{Verizon}}(X) = \begin{cases} 3X, & 0 \leq X \leq 1 \\ 5, & X > 1 \end{cases}$$

On the other hand, AT&T uses the following charging plan:

$$Y_{\text{AT\&T}}(X) = \begin{cases} 4X, & 0 \leq X \leq 1 \\ 4, & X > 1 \end{cases}$$

(a) Find the expected amount you will pay if you pick the Verizon plan, i.e., $E[Y_{\text{Verizon}}(X)]$.

(b) Find $E[Y_{\text{AT\&T}}(X)]$. Which one would you prefer ?