

✓ 1) Ordered Sampling with Replacement  $\rightarrow C_n^k$

✓ 2) Ordered Sampling w/o Replacement  $\rightarrow \frac{n!}{(n-k)!} = P_k^n$

(3) Unordered Sampling w/o Replacement

Today  
4) Unordered Sampling with Replacement

(3) Unordered Sampling w/o Replacement.

$$A = \{1, 2, \dots, n\}.$$

draw  $k$  items w/o replacement.

# of ways w/o rep  
when order mattered.

$$\frac{n!}{(n-k)!}$$

$$(1, 2) = (2, 1)$$
$$(2, 3) = (3, 2).$$

$$k=3 \text{ items. } \rightarrow \begin{bmatrix} (3, 5, 9), (3, 9, 5), (5, 3, 9), \\ (5, 9, 3), (9, 3, 5), (9, 5, 3). \end{bmatrix}$$

$$C_{n,k} = \binom{n}{k} = \frac{n! / (n-k)!}{k!} = \frac{n!}{(n-k)! k!}$$

# of combinations  
of  $k$  obj from a set of  $n$  objects.

$$C_k^n = \frac{P_k^n}{k!}$$

Binomial Coefficient

$$0 \leq k \leq n.$$

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$$

Identities

$$1) \quad \sum_{k=0}^n \binom{n}{k} = ?$$

$$= 2^n$$

(Pls.  $a=b=1$ ).

....

$$2). \quad \binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$0 \leq k < n$

Pascal's rule

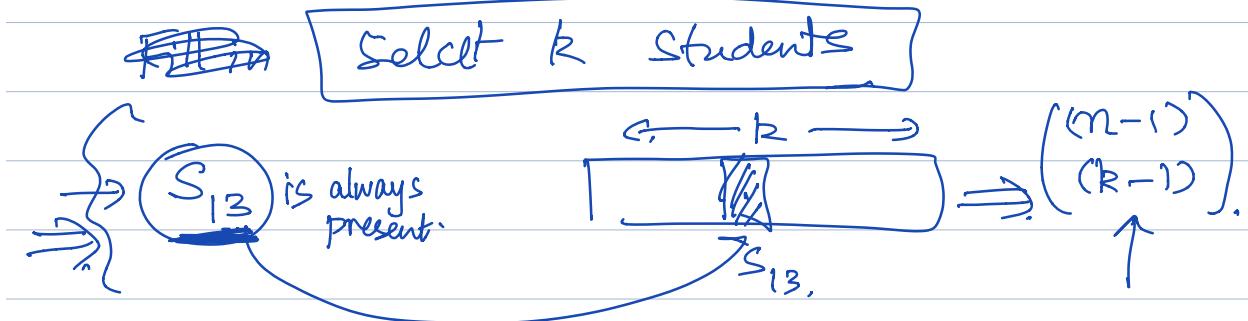
$$\text{LHS} = \binom{n}{k} = \frac{n!}{(n-k)! k!}$$

$$\text{RHS} = \frac{(n-1)!}{(k-1)! (n-k)!} + \frac{(n-1)!}{k! (n-k)!}$$

$$= \binom{n}{k} = \underline{\underline{\text{LHS}}}$$

$\Rightarrow \binom{n}{k} \rightarrow \# \text{ of ways of choosing } k \text{ items out of } n.$

$\{ S_1, S_2, S_3, \dots, S_n \}$



$\left\{ \begin{array}{l} (S_{13} \text{ is absent}) \\ (S_{13} \text{ is never present}) \end{array} \right\} \Rightarrow \boxed{k.} \Rightarrow \binom{n-1}{k}$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$\{3\} \quad \binom{m+n}{k} = \sum_{i=0}^{k.} \binom{m}{i} \binom{n}{k-i}$

Eg: Deck of 52 cards ..

Choose 3 cards from this deck w/o rep.

$A = \{ \text{contains. (at least) one Ace} \}$

$$P(A) = ? \quad | - P(A^c)$$

$A^c = \{ \text{No Ace} \text{ in these cards} \}$

$$|A^c| = ? \quad \boxed{\binom{48}{3}} \quad ??$$

$k = 3$  cards.

$n \Rightarrow 52$ .

↓ remove all

4 Aces

$$(n-4) \Rightarrow \boxed{48}$$

$$P(A) = 1 - \frac{|A^c|}{|S|} = 1 - \frac{\binom{48}{3}}{\binom{52}{3}} = \dots$$

$A \rightarrow$  Set of items.

$|A| \Rightarrow$  size of the set / # of items in the set.

/ Cardinality of

$$P(A^c) = \frac{\# \text{ of way in whch } A^c \text{ can be}}{\text{Total # of way in we select 3 cards}}$$

Eg How many distinct seq. we can write using 3 letter A's & 5 letter B's.

$\left\{ \underbrace{\text{AAA BBBBB}}_8, \underbrace{\text{ABBABBBAB}, \dots}_8 \right\}$

all are length 8 sequences..



Can be filled using  
3 A's &  
5 B's.

We want to pick  
3 slots for A.  
out of 8 slots.

$$= \binom{8}{3} = \binom{8}{5}$$

$$\binom{n}{k} = \binom{n}{n-k}$$

Bernoulli Trials

Binomial Distroib.

↳ random exp w/ two outcomes  
"succes" / "fails".

Prob of "success" =  $P$

Prob of "failure" =  $1 - P = q.$

$$P + q = 1$$

{ Binomial Experiment } / Repeated Bernoulli Trials  
↳ if we perform  $n$  independent Bernoulli trials.

H / T

$$P(H) = P$$

$$P(T) = q = (1 - P)$$

Toss the coin 5 times.

Q1:  $P(\underline{T H H H H}) = ?$   
 $= P(T \cap H \cap H \cap H \cap H).$   
 $= \underbrace{P(T)}_{q} \underbrace{P(H) \dots P(H)}_{P^4}$

Q2:  $P(H T H H H) = ?$

Q3:  $B = \{ \text{Event that we observe exactly 4 H's \& 1 Tail.} \}$

$$\underset{\sim}{\downarrow} P(B) \stackrel{?}{=} \binom{5}{1} \times q \times p^4.$$

$B = \left\{ \text{THHHH}, \text{HTHHH}, \text{HHTHH}, \text{HHHTH}, \text{HHHHT} \right\}$

$$P(B) = \underbrace{P(\text{ })}_{\sim} + \underbrace{P(\text{ })}_{\sim} + \dots + \underbrace{P(\text{ })}_{\sim}$$

$$= \binom{5}{1} \times q \times p^4.$$

Q4: What is prob we observe. [flip 5 times]

$C = \left\{ \text{3 heads \& 2 tails} \right\}$ ?

$$P(C) \stackrel{?}{=} \binom{5}{2} \times q^2 p^3 = \binom{5}{3} q^2 p^3.$$

Q5: If we toss a coin  $n$  times. failure.

&  $\Sigma = \left\{ k \begin{matrix} \text{heads} \\ \text{succes} \\ \text{in covid.} \end{matrix} \text{ \& } (n-k) \begin{matrix} \text{tails} \\ \text{failure} \\ -n \text{ covid} \end{matrix} \right\}$

$$\underline{P(\Sigma)} = \binom{n}{k} \times p^k \times q^{(n-k)}$$

Binomial Prob.

## Multinomial

Eg : Food relief eff. 8 agencies that provide support.

- 4 agencies for food
- 3 " for shelter
- 1 " for security.

Q → How many ways can we divide, agencies into these groups.

The diagram shows a large bracket grouping three boxes: "4 food.", "3 shel.", and "1 security". To the left of this group is a large bracket containing the number 8. A curved arrow points from the number 8 to the bracket under the "4 food." box. Another curved arrow points from the bracket under "4 food." to the bracket grouping all three boxes. Below this, a double-headed vertical arrow connects the bracket under "4 food." to the text "equal". To the right of the "security" box, there is a small bracket labeled "slot". Below the main grouping bracket, the expression  $\binom{8}{4} \times \binom{4}{3} \times \binom{1}{1}$  is written. Below this, the text "# of rem ag = 4" is written next to a bracket under the "4 food." box, and "# of n = 1" is written next to a bracket under the "1 security" box. Below these, another expression  $\binom{5}{4} \times \binom{8}{3} \times \binom{1}{1}$  is shown. At the bottom, the full multinomial coefficient is calculated as  $\frac{8!}{4! 3! 1!} = \frac{8!}{4! 3! 1!}$ .

$$\frac{8!}{4! 3! 1!} = \frac{8!}{4! 3! 1!}$$

