Final Exam - ECE 503 Fall 2016

- Date: Wednesday, December 14, 2016.
- Time: 10:30 am -12:30 pm
- Maximum Credit: 100 points

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1. [10 points] In the Arizona State Lottery, a player picks 6 different numbers from a sequence of 1 through 15. At a lottery drawing, 6 balls are drawn at random from a box containing 15 balls numbered 1 through 15.

   (a) What is the probability that the player has 0 matches and wins nothing ?

   (b) What is the probability that the player has exactly 6 matches and wins the jackpot ?

2. [20 points] Suppose X and Y are independent random variables, and each one of them is uniformly distributed in $[0, 1]$. Let $Z = \min(X, Y)$, and $W = \max(X, Y)$.

   (a) Find the joint density (PDF) of (Z, W).

   (b) Are Z and W independent random variables ?

3. [20 points] A wireless communication channel suffers from a clustered error pattern. Whenever a packet has an error, the next packet will have an error with probability 0.9. Whenever a packet is error-free, the next packet is error free with probability 0.99. We want to use a Markov chain to analyze the above process, assuming two states, E (error), and NE (no error).

   (a) Draw a state-transition diagram showing the transition probabilities, and write the one-step transition matrix P.

   (b) Find the stationary probability distribution (and comment on its uniqueness).

   (c) Suppose that the initial probability distribution was $p(0) = [p_E(0) \;\; p_{NE}(0)] = [0 \;\; 1]$. Find the probability distribution at time $n = 5$.

4. [15 points]

(a) Let $X(t)$ be a wide sense stationary random process, and we define $Y(t) = X(t+5)$. Are $X(t)$ and $Y(t)$ jointly wide sense stationary ?

(b) Let X_n have the following PDF,

$$f_n(x) = g_n(x)(1 - 1/n^3) + h_n(x)/n^3$$

where $g_n(x) \sim N(0, 1/n^2)$, and $h_n \sim N(n, 1)$. Does X_n converge to 0 in the mean square sense ?

(c) A W.S.S. process $X(t)$, of mean $\mu_X = 10$ is the input to a LTI filter, with impulse response

$$h(t) = \begin{cases} e^{t/0.2} & 0 \leq t \leq 0.2 \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected value of the output process $Y(t)$?

5. [20 points] Weather analysis in Florida revealed that hurricanes occur according to a Poisson process of intensity $\lambda = 2$ per week.

   (a) What is the average time between two hurricanes ?

   (b) What is the probability that no hurricanes occur during a given 2 week period ?

   (c) If the peak of hurricane season lasts 12 weeks, what is the expected number of hurricanes ?

   (d) Find the probability that during at least one of the 12 weeks of peak season, there are at least five hurricanes.

6. [15 points] For a zero-mean WSS random process $X(t)$, with auto-correlation function $R_X(\tau) = 1/(1+\tau^2)$, we perform the following sequence of operations. The continuous time process is first sampled at the rate of $f_s = 1$ sample per second. This yields a discrete time random process X_n. We then pass X_n through a discrete-time filter, whose output is denoted by $Y_n = X_n - X_{n-1}$.

   (a) Find the mean and auto-correlation function of X_n.

   (b) Is the discrete time process X_n also WSS?

   (c) Find the mean and auto-correlation function of Y_n.

   (d) Are X_n and Y_n jointly WSS ?