Homework 9 - ECE 503 Fall 2020

- Assigned on: Monday, November 30, 2020.
- Due Date: **Saturday, December 5, 2020 by 11:59 pm Tucson Time.**
- Maximum Credit: 150 points

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1. [10 points] Which of the following are valid auto-correlation functions of a WSS random process ?
   (a) $R_1(\tau) = \delta(\tau)$
   (b) $R_2(\tau) = \delta(\tau) + 10$
   (c) $R_3(\tau) = \delta(\tau) - 10$
   (d) $R_4(\tau) = \delta(\tau - 10)$

2. [20 points] $X(t)$ is a WSS random process with auto-correlation function $R_X(\tau) = 10\sin(2\pi 1000t)/(2\pi 1000t)$. The process $Y(t)$ is a delayed version of $X(t)$ by 50 microseconds, i.e., $Y(t) = X(t-t_0)$, where $t_0 = 5\times 10^{-5}$ seconds.
   (a) Find the autocorrelation function of $Y(t)$.
   (b) Find the cross-correlation function of $X(t)$ and $Y(t)$
   (c) Are $X(t)$ and $Y(t)$ jointly WSS ?

3. [20 points] Consider the random process
$$W(t) = X\cos(2\pi f_0 t) + Y\sin(2\pi f_0 t)$$
   where X and Y are uncorrelated random variables, each with expected value 0 and variance σ^2.
   (a) Find the auto-correlation function of the random process $W(t)$.
   (b) Is $W(t)$ wide sense stationary (WSS) ?

4. [20 points] $X(t)$ is a WSS random process with average power equal to 1. Let Θ denote a random variable with uniform distribution over $[0, 2\pi]$, and $X(t)$ and Θ are independent.
   (a) What is $E[X^2(t)]$?
   (b) What is $E[\cos(2\pi f_c t + \Theta)]$?
   (c) Let $Y(t) = X(t)\cos(2\pi f_c t + \Theta)$. What is $E[Y(t)]$?
   (d) What is the average power of $Y(t)$?

5. [10 points] A white Gaussian noise process $N(t)$ with auto-correlation $R_N(\tau) = \alpha\delta(\tau)$ is passed through an integrator yielding the output
$$Y(t) = \int_0^t N(u)du$$
   Find the mean and auto-correlation functions of $Y(t)$. Show that $Y(t)$ is a non-stationary process.

6. [10 points] A discrete-time random process X_n is WSS if $E[X_n]$ does not depend on n and if the correlation $E[X_n X_m]$ depends on n and m only through their difference. Show that if X_n is WSS, then so is $Y_n = X_n - X_{n-1}$.

7. [20 points] A popular music group produces a new hit song every 7 months on average. Assume that hit songs are produced according to a Poisson process.

   (a) Find the probability that the group produces more than two hit songs in 1 year.

   (b) How long do you expect it to take until the group produces its 10th hit ?

8. [20 points] Space shuttles are launched according to a Poisson Process. The average time between launches is 2 months.

   (a) Find the probability that there are no launches during a 4 month period.

   (b) Find the probability that during at least 1 month out of four consecutive months, there are at least two launches.

9. [20 points] Data packets depart from a router according to a Poisson process with rate λ per minute. Each packet arrives successfully at a receiver with probability p, independently of every other packet.

   (a) Find the distribution of the time until the first packet arrives.

   (b) Find the probability that no packets arrive successfully in any particular hour.

   (c) Find the expected number of packets that arrive successfully during a particular hour.