Homework 2: ECE 503 Fall 2017

- Assigned on: Saturday, September 2, 2017.

- Due Date: **Friday, September 8, 2017 by 11:00 am Tucson Time.**

- Mode of submission: Drop in my mailbox (ECE Building, 2nd Floor)

- Maximum Credit: 100 points

1. [10 points] Box 1 contains 40 white balls and 960 red balls. Box 2 contains 200 red and 800 white balls. A ball is picked from a randomly selected box.

    - If the ball is red, then what is the probability it came from Box 1 ?
    - If the ball is white, then what is the probability it came from Box 2 ?

2. [10 points] Consider the following three events:

    - At least 1 six is obtained when six dice are rolled.
    - At least 2 sixes are obtained when 12 dice are rolled.
    - At least 3 sixes are obtained when 18 dice are rolled.

    Which of these events is more likely ?

3. [20 points] Let p represent the probability of an event A. What is the probability that

    (a) Event A occurs at least twice in n independent trials.
    (b) Event A occurs at least thrice in n independent trials.

4. [20 points] You choose r of the first n positive integers, and a lottery chooses a random subset L of the same size r. What is the probability that:

    (a) The set L includes no consecutive integers ?
    (b) The set L includes exactly one pair of consecutive integers ?
    (c) Your choice of numbers is the same as L ?
    (d) The numbers in L are drawn in an increasing order ?

5. [10 points] A box contains m white and n black balls. Suppose that k balls are drawn (without replacement). What is the probability that we draw at least one white ball ?

6. [10 points] A class consisting of 4 graduate students and 12 undergraduate students is randomly divided into four groups of 4 students each. What is the probability that each group includes a graduate student ?

7. [20 points] Your DNA is composed of a series of four nucleotides: adenine, guanine, thymine and cytosine (A, G, T, and C, respectively). Suppose that each of these nucleotides appears independently in the DNA sequence of length $N = 1000$ with equal probability.

    (a) How many DNA sequences of length N are possible ?
    (b) What is the probability that the nucleotide B appears at least once in a sequence of length $N = 1000$?
    (c) What is the probability that there will be exactly 5 A's appearing in a sequence of length $N = 1000$?
    (d) Repeat parts (a), (b) and (c) if the probabilities of nucleotides were $P(A) = 1/2, P(G) = 1/4, P(C) = 1/8, P(T) = 1/8$.