Make-up Midterm 2 Exam - ECE 503 Fall 2017

- Due Date: Monday, November 13, 2017.
- Maximum Credit: **100 points**

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1. **[20 points]**

   (a) (4 points) $E\left[\frac{1}{X}+\frac{1}{Y}\right]=\frac{1}{E[X]}+\frac{1}{E[Y]}$
   True or False ?

   (b) (8 points) Let U be a uniformly distributed random variable in $[0,1]$. Let

   $$X_n=\begin{cases} n^5, & 0\le U\le 1/n \\ 0, & \text{otherwise}\end{cases}$$

   denote a sequence of r.v.s. Determine whether or not X_n converges to 0 in probability.

   (c) (8 points) Let A and B be independent random variables, and each is Bernoulli with parameter $1/2$. We define two random variables as $U=(A+B)$, and $V=(A-B)$.

   i. Are U and V independent ?

   ii. Are U and V uncorrelated ?

2. **[20 points]** Let $X_1, X_2, \ldots, X_n$ be a sequence of i.i.d. random variables, each uniformly distributed in the interval $[a,b]$ for some $0<a<b$. Let

   $$X_{(n)}=\min(X_1,X_2,\ldots,X_n)$$

   be the minimum of these n random variables.

   (a) Find the CDF of X_n.

   (b) Show that X_n converges in distribution to a.

3. **[20 points]** The random variables X and Y have the following joint probability density function

   $$f_{X,Y}(x,y)=\begin{cases} 6(y-x) & 0\le x\le y\le 1 \\ 0 & \text{otherwise}\end{cases}$$

   (a) Find the MMSE estimate of X given Y.

   (b) Find the LMMSE estimate of X given Y.

4. **[20 points]** The waiting time W for accessing one record from a database is a random variable, distributed uniformly between 0 and 10 milliseconds. The read time R (for moving the information from disk to main memory) is 3 milliseconds. The random variable X is the total access time (waiting + read time) to get one one block of information from the disk. Before performing a task, the computer must access 12 different blocks of information from the disk. The access times for different blocks are independent of each other. Let A denote the total access time for the task.

   (a) Find $E[X]$, the expected value of access time.

   (b) Find $\text{Var}[X]$, the variance of access time.

   (c) Find $E[A]$, the variance of the total access time for the task.

   (d) Find the standard deviation of the total access time for the task.

   (e) Approximate the probability that the total access time for the task exceeds 116 milliseconds.

5. **[20 points]** This problem has two parts.

   (a) For any two random variables A and B, show that

$$(E(AB))^2 \leq E(A^2)E(B^2)$$

   (b) Now, let X and Y be random variables, each with mean 0, variance 1 and covariance ρ. Show that

$$E[\max(X^2, Y^2)] \leq 1 + \sqrt{1-\rho^2}$$

[Hint for part (b): For any real u, v, we can write $\max(u, v) = \frac{(u+v)}{2} + \frac{|u-v|}{2}$]