1) Ordered Sampling with Replacement $\rightarrow n^K$

2) Ordered Sampling w/o Replacement $\rightarrow \frac{n!}{(n-k)!} = P^n_k$

Today
3) Unordered Sampling w/o Replacement
4) Unordered Sampling with Replacement

(3) Unordered Sampling w/o Replacement.

$A = \{1, 2, \dots, n\}$.

draw $\boxed{k}$ items w/o replacement.

\# of ways w/o rep when order mattered. $= \frac{n!}{(n-k)!}$

$(1,2) = (2,1)$

$(2,3) = (3,2)$

$k = 3$ items. $\rightarrow$ $(3,5,9)$, $(3,9,5)$ $(5,3,9)$, $(5,9,3)$, $(9,3,5)$, $(9,5,3)$

$$\underbrace{C^n_k = \binom{n}{k}}_{\text{\# of combinations of } k \text{ obj from a set of } n \text{ objects}} = \frac{n!/(n-k)!}{k!} = \frac{n!}{(n-k)!\,k!}$$

\# of combinations of k obj from a set of n objects.

$$\boxed{C^n_k = \frac{P^n_k}{k!}}$$

Binomial coefficient

$0 \le k \le n$.

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

Identities

1) $$\boxed{\sum_{k=0}^{n} \binom{n}{k}} \overset{?}{=} 2^n$$

(Plug $a=b=1$).

2). $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ $\boxed{\text{Pascal's rule}}$

$0 \leq k < n$

$LHS = \binom{n}{k} = \dfrac{n!}{(n-k)!\,k!}$

$RHS = \dfrac{(n-1)!}{(k-1)!\,(n-k)!} + \dfrac{(n-1)!}{k!\,(n-k-1)!}$

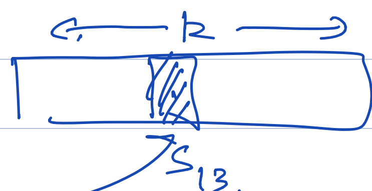
$= \binom{n}{k} = \underline{\underline{LHS}}$

$\Rightarrow \binom{n}{k} \longrightarrow$ # of ways of choosing $\underline{k}$ items out of $\underline{n}$.

$\{ S_1, S_2, S_3 \ldots \ldots, S_n \}$

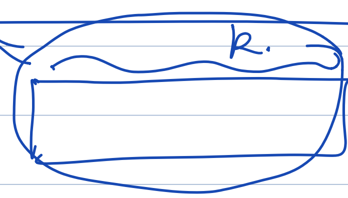
~~Kill~~ $\boxed{\text{Select } k \text{ Students}}$

$\Rightarrow$ S_{13} is always present.



$\Rightarrow \binom{n-1}{k-1}$.

(S_{13} is absent) (where S_{13} is never present)



$\Rightarrow \binom{n-1}{k}$

$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

(3) $\binom{m+n}{k} = \sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i}$

Eg: Deck of 52 cards..

Choose 3 cards from this deck $\boxed{w/o}$ rep.

$$A = \{\text{Containts at least one Ace}\}$$

$$P(A) \stackrel{?}{=} 1 - P(A^c)$$

$$A^c = \{\boxed{\text{No Ace}} \text{ in three cards}\}$$

n.
(A, Q, K)
(A, 9, 7)
⋮
(A, A, ...)
(A A A)

$$|A^c| \stackrel{?}{=} \boxed{\binom{48}{3}} \quad ??$$

$k = 3$ cards.

$n \Rightarrow 52.$

↓ remove all 4 Aces

$(n-4) \Rightarrow 48$

$$|S| = \binom{52}{3}.$$

$$P(A) = 1 - \frac{|A^c|}{|S|} = 1 - \frac{\binom{48}{3}}{\binom{52}{3}} = \ldots$$

$A \rightarrow$ Set of items.

$|A| \Rightarrow$ Size of the set / # of items in the set. / Cardinality of the set.

$$P(A^c) = \frac{\text{\# of way in which } A^c \text{ occur.}}{\text{Total \# of way in we select 3 cards}}$$

Eg How many distinct seq. we can write using 3 letter A's & 5 letter B's.

$$\{\underbrace{AAABBBBB}_{8},\ \underbrace{ABBABBAB}_{8},\ \ldots\ldots\}$$

all are length 8 sequences..

Can be filled using 3 A's & 5 B's.

we want to pick 3 slots for A. out of 8 slots

$$= \binom{8}{3} = \binom{8}{5}.$$

$$\binom{n}{k} = \binom{n}{n-k}.$$

Bernoulli Trials & Binomial Distrib.

→ random exp w/ two outcomes "success" / "failure".

Prob of "Success" = p

Prob of "failure" = $1-p = q$.

$p+q=1$

Binomial Experiment / Repeated Bernoulli Trials

$\hookrightarrow$ if we perform n independent Bernoulli trials.

H / T $\quad P(H) = p$

$P(T) = q = (1-p)$

Toss the coin 5 times.

Q1: $P(THHHH) \stackrel{?}{=} q \times p^4$.

$= P(T \cap H \cap H \cap H \cap H)$.

$= P(T)\,P(H) \cdots P(H)$

$q \times p^4$

Q2: $P(HTHHH) \stackrel{?}{=} q p^4$.

Q3: $B = \{$Event that we Observe exactly 4 H's & 1 Tail.$\}$

$$P(B) \stackrel{?}{=} \binom{5}{1} \times q\, p^4.$$

$B = \{$ THHHH, HTHHH, HHTHH, HHHTH, HHHHT $\}$

$$P(B) = \underbrace{P(\ ) + P(\ ) + \dots + P(\ )}$$

$$= \binom{5}{1} \times q \times p^4.$$

Q4: What is prob we observe. [flip 5 times]

$C = \{$3 heads & 2 tails$\}$?

$$P(C) \stackrel{?}{=} \binom{5}{2} \times q^2 p^3 = \binom{5}{3} q^2 p^3.$$

Q5: If we toss a coin n times.

& $E = \{ k$ $\boxed{\text{heads}}$ & $(n-k)$ $\boxed{\text{tails}} \}$

success / +ve covid — failure / −ve covid

$$P(E) = \binom{n}{k} \times p^k \times q^{(n-k)}.$$

Binomial Prob.

Multinomial

Eg: Food relief effor. 8 agencies that provide support.

- 4 agencies for food
- 3 " for shelter
- 1 " for security.

Q → How many ways can we divide agences into these groups.

8 | 4 food. | 3 shelt. | 1 | security slot

$$\binom{8}{4} \times \binom{4}{3} \times \binom{1}{1}$$

\# of rem ag = $\boxed{4}$ \# of " = 1.

Equal

$$\binom{5}{4} \times \binom{8}{3} \times \binom{1}{1}$$

$$\frac{8!}{4!\,\cancel{4!}} \times \frac{\cancel{4!}}{3!\,\cancel{1!}} \times \frac{\cancel{1!}}{1!\,0!}$$

$$= \frac{8!}{4!\,3!\,1!}$$