

Homework 2: ECE 503 Fall 2020

- Assigned on: Friday, September 4, 2020.
 - Due Date: **Friday, September 11, 2020 by 11:59 pm Tucson Time.**
 - Mode of submission: D2L
 - Maximum Credit: **100 points**
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1. **[10 points]** Box 1 contains 40 white balls and 960 red balls. Box 2 contains 200 red and 800 white balls. A ball is picked from a randomly selected box.
 - If the ball is red, then what is the probability it came from Box 1 ?
 - If the ball is white, then what is the probability it came from Box 2 ?
2. **[10 points]** Consider the following three events:
 - At least 1 six is obtained when six dice are rolled.
 - At least 2 sixes are obtained when 12 dice are rolled.
 - At least 3 sixes are obtained when 18 dice are rolled.

Which of these events is more likely ?
3. **[20 points]** Let p represent the probability of an event A . What is the probability that
 - (a) Event A occurs at least twice in n independent trials.
 - (b) Event A occurs at least thrice in n independent trials.
4. **[20 points]** You choose r of the first n positive integers, and a lottery chooses a random subset L of the same size r . What is the probability that:
 - (a) The set L includes no consecutive integers ?
 - (b) The set L includes exactly one pair of consecutive integers ?
 - (c) Your choice of numbers is the same as L ?
 - (d) The numbers in L are drawn in an increasing order ?
5. **[10 points]** A box contains m white and n black balls. Suppose that k balls are drawn (without replacement). What is the probability that we draw at least one white ball ?
6. **[10 points]** A class consisting of 4 graduate students and 12 undergraduate students is randomly divided into four groups of 4 students each. What is the probability that each group includes a graduate student ?
7. **[20 points]** Your DNA is composed of a series of four nucleotides: adenine, guanine, thymine and cytosine (A, G, T, and C, respectively). Suppose that each of these nucleotides appears independently in the DNA sequence of length $N = 1000$ with equal probability.
 - (a) How many DNA sequences of length N are possible ?
 - (b) What is the probability that the nucleotide A appears at least once in a sequence of length $N = 1000$?
 - (c) What is the probability that there will be exactly 5 A's appearing in a sequence of length $N = 1000$?
 - (d) Repeat parts (a), (b) and (c) if the probabilities of nucleotides were $P(A) = 1/2, P(G) = 1/4, P(C) = 1/8, P(T) = 1/8$.