$\boxed{\text{Today}}$

(1) White Noise

→ (2) White Gaussian Noise

3) Introduction to Markov Chains.

Recap → (SS) Stationary, WSS, WSS I/P → $\boxed{\text{LTI}}$ → o/p

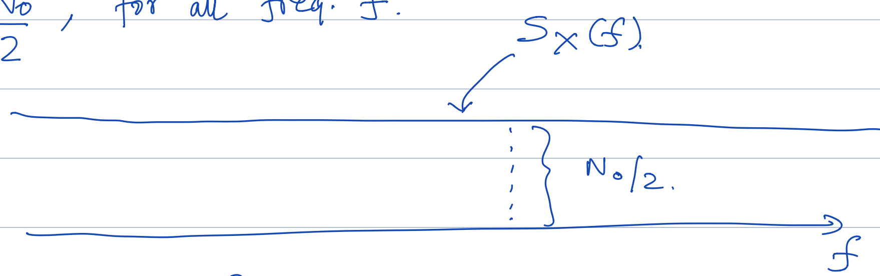
Jointly WSS

White Noise → to model thermal noise in electronic systems.

$X(t)$ is White Noise if.

PSD of $X(t)$, $S_X(f)$ $\boxed{\text{is constant for all frequencies.}}$

$S_X(f) = \frac{N_0}{2}$, for all freq. f.



Power of White Noise Random Process ?

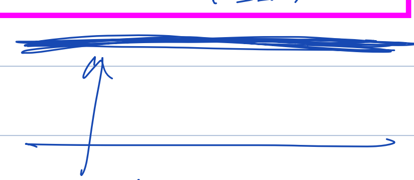
$$= \int_{-\infty}^{\infty} \frac{N_0}{2}\, df \overset{?}{=} \boxed{\infty}.$$

$$\boxed{\begin{aligned} P_X &= R_X(0) \\ &= E[X^2(t)] \\ &= \int_{-\infty}^{\infty} \underbrace{S_X(f)}_{\text{PSD}}\, df. \end{aligned}}$$

Power of I

"in reality"

$\approx$

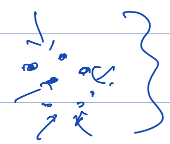


White Gaussian Noise (WGN)

$X(t) \rightarrow$ (i) Stationary Gaussian random process

(ii) Zero mean ($\mu_X = 0$)

(iii) Flat (constant) PSD,
$$S_X(f) = \frac{N_0}{2} \text{ for all freq. } f.$$



$$\rightarrow \frac{\sum_i \text{disturbances}}{\#\text{ of items}} \Rightarrow \frac{X_1+X_2+\dots+X_n}{n}$$

by CLT

$$\rightarrow \mathcal{N}(\mu, \sigma^2)$$

ACF of WGN : (F.T. pair)

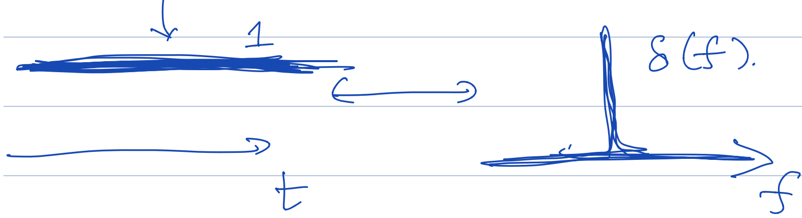
ACF $\Longleftrightarrow$ PSD

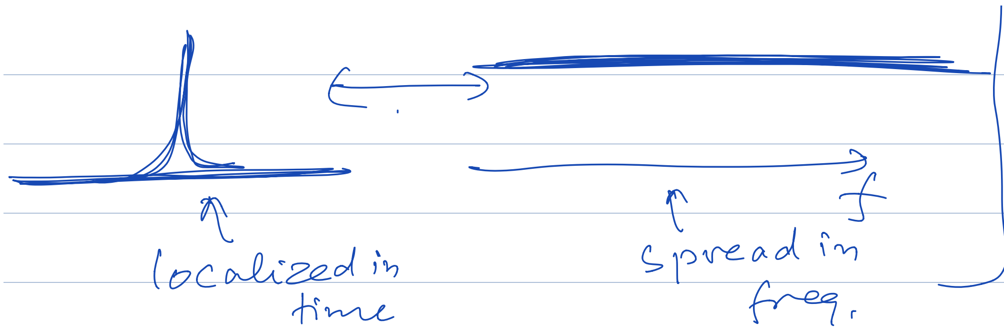
$R_X(\tau)$ $\qquad$ $S_X(f)$.

$$R_X(\tau) = \mathcal{F}^{-1}(\text{PSD})$$
$$= \mathcal{F}^{-1}\left(\frac{N_0}{2}\right) = \frac{N_0}{2} \cdot \delta(\tau)$$

Dirac delta function

$$x(t) \longleftrightarrow X(f)$$





WGN. Are $X(t_1)$, $X(t_2)$ independent of each other?

is $\rho = 0$? or $\underbrace{\mathrm{cov}(X(t_1), X(t_2))}$ $\overset{?}{=} 0$

$$E\left[(X(t_1) - 0)(X(t_2) - 0)\right]$$

$R_X(\tau) = \frac{N_0}{2}\delta(\tau)$

$$= E\left[X(t_1)X(t_2)\right]$$

$$= \underbrace{R_X(t_1 - t_2)} = \frac{N_0}{2}\underbrace{\delta(t_1 - t_2)}_{\substack{\| \\ 0 \\ \text{for all } t_1 \neq t_2}}$$

Eg: $X(t)$ is a WGN

$\mu_Y = \mu_X H(0)$

$$\underset{(WGN)}{X(t)} \longrightarrow \boxed{LTI} \longrightarrow Y(t)$$

PSD $\sim \frac{N_0}{2}$

$$H(f) = \begin{cases} 2 & \text{if } 1 < |f| < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find $P\left(\underbrace{Y(5)} < \sqrt{N_0} \right)$

PSD of $Y(t)$?

$$S_Y(f) = |H(f)|^2 \times \underbrace{S_X(f)}_{\text{input PSD}}$$

$$= 4 \times \frac{N_0}{2} = 2N_0.$$

$$\boxed{S_Y(f) = \begin{cases} 2N_0, & 1 < |f| < 2 \\ 0 & \text{otherwise.} \end{cases}}$$

$\boxed{\text{PSD of } Y(t).}$

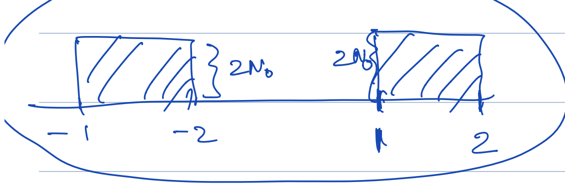
Distribution of $Y(t)$?

$$\underbrace{Y(t)} \sim \mathcal{N}(0, 4N_0).$$

$$\underbrace{\mathrm{Var}[Y(t)]} = E[Y^2(t)]$$

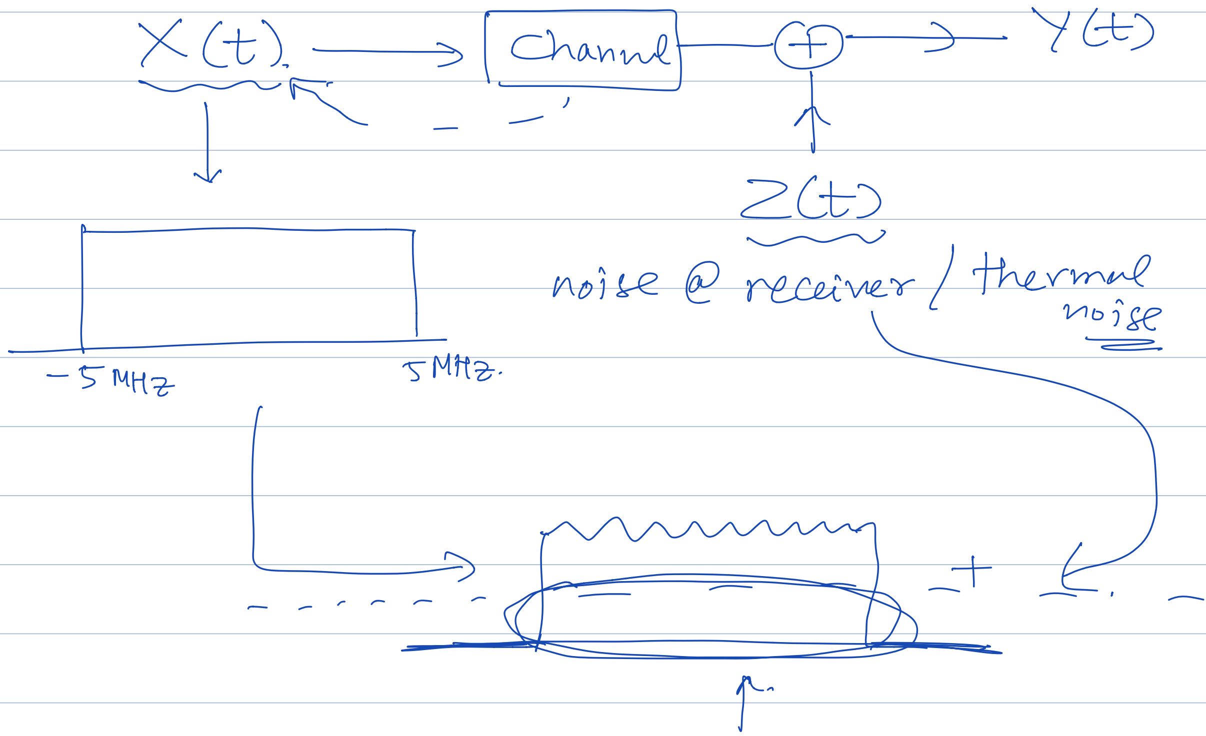
$$= R_Y(0). = \text{Power of } Y(t).$$

$$= \int_{-\infty}^{\infty} S_Y(f)\,df = 4N_0.$$



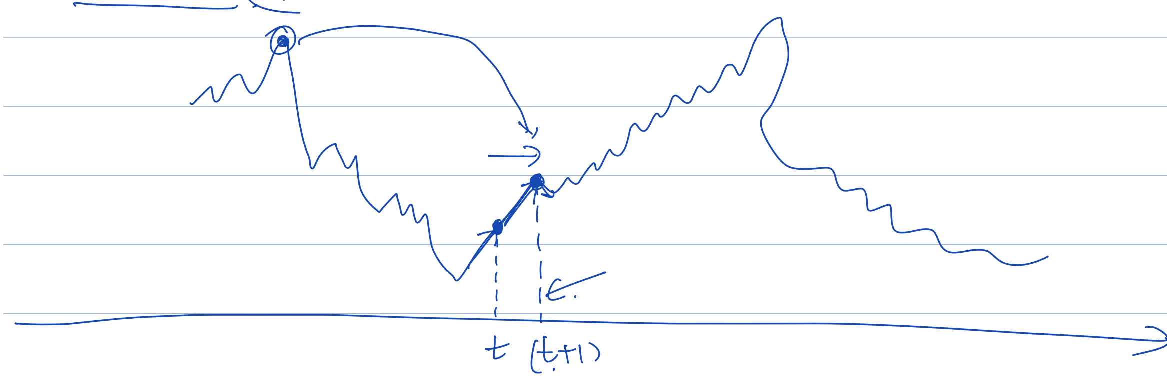
$$P\left(\underbrace{Y(5)}_{\mathcal{N}(0,4N_0).} < \sqrt{N_0} \right) = \Phi\left(\frac{\sqrt{N_0} - 0}{\sqrt{4N_0}} \right)$$

$$= \Phi(1/2). \approx 0.69.$$



Markov Random Processes

Stocks (prices)



Weather. (temp/ Sunny /Rainy)

Close dependence of R.P. over time

* Want to incorporate depencene on recent past.

* Ignore dependence on much later Past.

(X_m ⇝ m is time index)

$$X_0,\ X_1,\ X_2,\ X_3 \ldots\ldots, X_t,\ X_{t+1} \ldots$$

1st order M.C.

$$P(X_t = j \mid X_{t-1} = i_1, X_{t-2} = \ldots, X_1 = \ldots, X_0 = \ldots)$$

$$= \underbrace{P(X_t = j \mid X_{t-1} = i_1)}_{\text{transition probabilities}}$$

(*) Discrete time

(*) Time-invariant transition probabilities.

(*). State Space of a M.C. (Markov Chain).

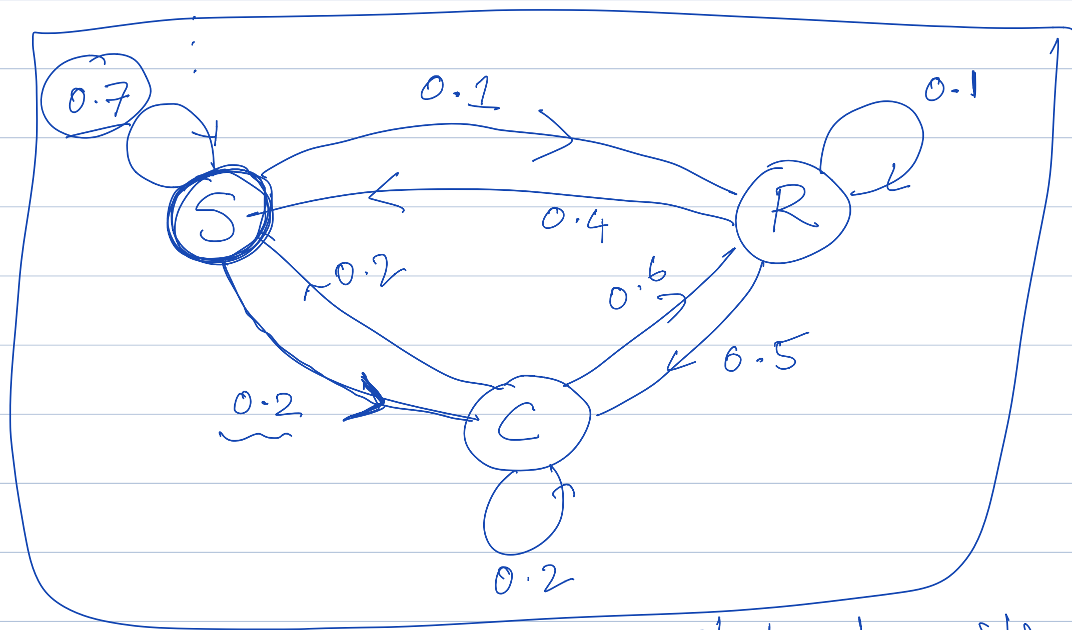
Weather ⟶ $\{S, R, C\}$ = State space of MC.

X_t takes values either → S, R, C.

$$P(X_t = S \mid X_{t-1} = S) \stackrel{?}{=}$$

$$P(X_t = S \mid X_{t-1} = R) \stackrel{?}{=}$$

$$P(X_t = S \mid X_{t-1} = C) \doteq ?$$



State-transition diagram

$$\begin{array}{c c} & \begin{array}{ccc} S & R & C \end{array} \\ \begin{array}{c} S \rightarrow \\ R \rightarrow \\ C \rightarrow \end{array} & \begin{bmatrix} 0.7 & 0.1 & 0.2 \\ 0.4 & 0.1 & 0.5 \\ 0.2 & 0.6 & 0.2 \end{bmatrix} \end{array} \Longrightarrow \text{state transition matrix} \leftarrow P$$