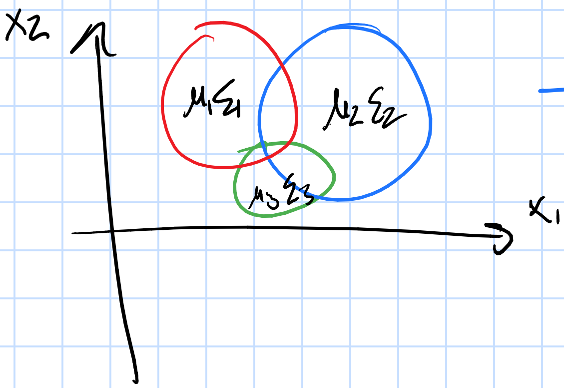
Lecture 01/29/2021



How do we get the likelihood?

$$\boxed{P(\omega_j \mid x) \propto p(x \mid \omega_j)\, p(\omega_j)}$$

$$\vec{x} = [x_1, x_2 \dots x_d]$$

$$p(x \mid \omega_j) = p(x_1 \mid \omega_j)\, p(x_2 \mid \omega_j) \cdots$$

$$\boxed{= \prod_{i=1}^{d} p(x_i \mid \omega_j)}$$

what could problematic here?

$$p(x \mid \omega_j) = \frac{1}{\sqrt{2\pi}\,\sigma_j} \exp\left(- \frac{(x-\mu_j)^2}{\sigma_j^2}\right)$$

$$\log p(x \mid \omega_j) = \log \prod_{i=1}^{d} p(x_i \mid \omega_j)$$

$$= \sum \log p(x_i \mid \omega_j)$$

Recap

We want to find a liner model

$$\boxed{g(x) = w^T x + b} \rightarrow y \{\pm 1\}$$

$$\ell(w) = \frac{1}{2} \sum_{i=1}^{n} (y_i - [w^T x_i + b])^2$$

Assume two classes $\{+1, -1\}$

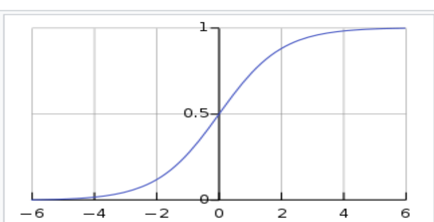
$g(x_1) = 0.125$

$g(x_2) = 22.95$

$g(x_3) = -0.01$

We want to interpret $g(x)$ as a probility, but that is not the current form of $g(x)$.

Logistic Regression



Standard logistic sigmoid function i.e. $L = 1, k = 1, x_0 = 0$

For the recurrence relation, see Logistic map.

A **logistic function** or **logistic curve** is a common S-shaped curve (sigmoid curve) with equation

$$f(x) = \frac{L}{1 + e^{-k(x - x_0)}},$$

where

x_0, the x value of the sigmoid's midpoint;

L, the curve's maximum value;

k, the logistic growth rate or steepness of the curve.[1]

Assume, that we have a binary prediction task

$y \in \{0,1\}$ y here is bernoulli

$$\mathbb{P}(Y=1 \mid x) = \frac{\mathbb{P}(x \mid Y=1)\,\mathbb{P}(Y=1)}{\mathbb{P}(x \mid Y=1)\,\mathbb{P}(Y=1) + \mathbb{P}(x \mid Y=0)\,\mathbb{P}(Y=0)} \cdot \frac{\frac{1}{\mathbb{P}(x \mid y=1)\mathbb{P}(y=1)}}{\frac{1}{\mathbb{P}(x \mid y=1)\,\mathbb{P}(y=1)}}$$

$$= \frac{1}{1 + \exp\left\{\log\left\{\frac{\mathbb{P}(Y=0)\,\mathbb{P}(x \mid y=0)}{\mathbb{P}(Y=1)\,\mathbb{P}(x \mid y=1)}\right\}\right\}}$$

$\log(a/b)$

$\log(a) - \log(b)$

$\log(ab)$

$\log(a) + \log(b)$

$$= \frac{1}{1 + \exp\left\{\log\left(\frac{\mathbb{P}(Y=0)}{\mathbb{P}(Y=1)}\right) + \underbrace{\log\left(\frac{\mathbb{P}(x \mid y=0)}{\mathbb{P}(x \mid y=1)}\right)}\right\}}$$

$$= \frac{1}{1 + \exp\left(-\log\left(\frac{P(Y=1)}{P(Y=0)}\right) + \sum_{i=1}^{d} \log\left(\frac{P(x_i \mid Y=0)}{P(x_i \mid Y=1)}\right)\right)}$$

$$\log\left(\frac{\mathbb{P}(Y=1)}{\mathbb{P}(Y=0)}\right) + \log\left(\frac{\mathbb{P}(x \mid Y=1)}{\mathbb{P}(x \mid Y=0)}\right) = w_0 + w^T x$$