Results so far

- Homework #3 due 03/08/2021
- Project Proposal Due 03/10/2021

① $Z_t = 2\sqrt{\varepsilon_t(1-\varepsilon_t)}$

$$\varepsilon_t = \sum_{i=1}^{n} D_t(i) [\![h_t(x_i) \neq y_i]\!]$$

② $\displaystyle\sum_{h_t(x_i)\neq y_i} D_{t+1}(i) = \frac{1}{2}$

③ Bounds on the training error

ensemble decision

$$\widehat{err}(H) = \boxed{\frac{1}{n}\sum_{i=1}^{n} [\![H(x_i) \neq y_i]\!]} \leq ?$$

$$H(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$$D_{t+1}(i) = \frac{\boxed{D_t(i)}}{Z_t} \exp\left(-\alpha_t h_t(x_i) y_i\right)$$

$$= \frac{D_{t-1}(i)}{Z_t Z_{t-1}} \exp\left(-\alpha_t h_t(x_i) y_i\right) \exp\left(-\alpha_t h_t(x_i) y_i\right)$$

$\dots$

From the previous lecture

$$\boxed{\star\ D_1(i) = \frac{1}{n}}$$

$$D_{t+1}(i) = \frac{D_1(i)}{\prod_{r=1}^{t} Z_r} \exp\Big(-y_i \underbrace{\sum_{r=1}^{t} \alpha_r h_r(x_i)}_{H(x)}\Big)$$

Let us examine what happens when we make a mistake on x_i,

$$\text{sign}\left(\sum \alpha_t h_t(x_i)\right) \neq y_i \Rightarrow \text{sign}\left(y_i \sum_{t=1}^{T} \alpha_t h_t(x_i)\right) = -1$$

$H(x_i) \neq y_i \qquad \exp\left(-y_i \sum \alpha_t h_t(x_i)\right) \geqslant 1$ *

$H(x_i) = y_i \qquad \exp\left(-y_i \sum \alpha_t h_t(x_i)\right) \geqslant 0$

$$\hat{err}(H) \doteq \frac{1}{n} \sum_{i=1}^{n} [\![H(x_i) \neq y_i]\!]$$

$$\leqslant \frac{1}{n} \sum_{i=1}^{n} \exp\left(-y_i \sum_{t=1}^{T} \alpha_t h_t(x_i)\right)$$

$$= \frac{1}{\cancel{n}} \sum_{i=1}^{n} \cancel{n}\, D_{t+1}(i) \prod_{t=1}^{T} Z_t$$

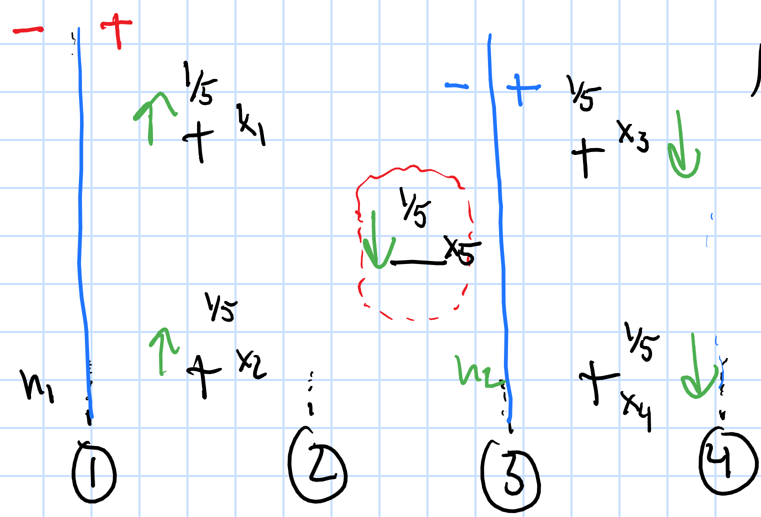
$$= \left(\sum_{i=1}^{n} D_{t+1}(i) \right) \prod_{t=1}^{T} Z_t$$

$$= \prod_{t=1}^{T} 2\sqrt{\varepsilon_t(1-\varepsilon_t)} = 2^T \prod_{t=1}^{T} \sqrt{\varepsilon_t(1-\varepsilon_t)}$$

→ This error drops off exponentially fast

$$\left(2\sqrt{\varepsilon_1(1-\varepsilon_1)}\right)\left(2\sqrt{\varepsilon_2(1-\varepsilon_2)}\right) = 2^2 \prod_{t=1}^{2} \sqrt{\varepsilon_t(1-\varepsilon_t)}$$

$$\begin{cases} i = \text{randsample}(D, N) \\ X_{tr}, y_{tr} = X[i], y[i] \\ h_t = \text{CART}(X_{tr}, y_{tr}, \text{params}) \end{cases}$$



$N = 5$

$\varepsilon_1 = 1/5$

$\alpha_t =$

$$\sum_{i=1}^{4} D_2(i) = 1/2$$

$$D_2(5) = 1/2$$