ME2100 – Applied Thermal Engineering

Module 1: Second law analysis

- → Lecture 1: Review of entropy (II^{nd} law of TD)
- → Lecture 2: Idea of exergy (available work)
- → Lecture 3: II^{nd} law analysis
- → Lecture 4: Summary and discussion of tutorials

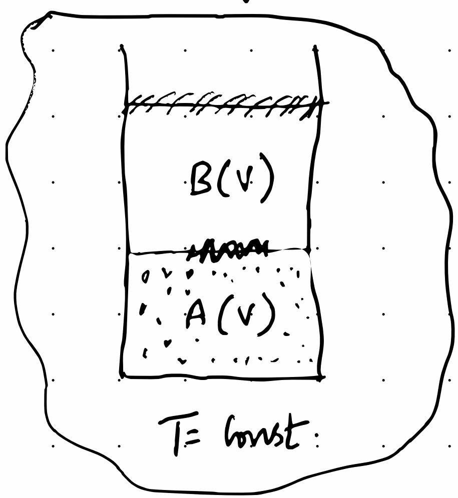
# of tutorials for this Module: 2

Continued...

Mod 1 Lecture 1 → Review of entropy (II^{nd} law of TD)

Switching to slides...

Switching back to note book...



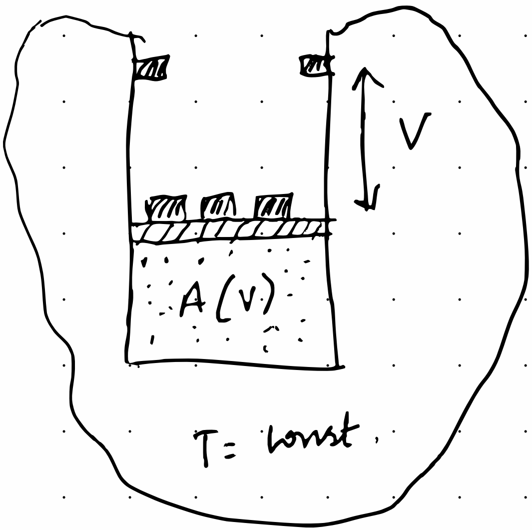
$$\boxed{\delta Q - \delta W = dU} \rightarrow I^{st} \text{ law of TD}$$

heat work Change in internal energy

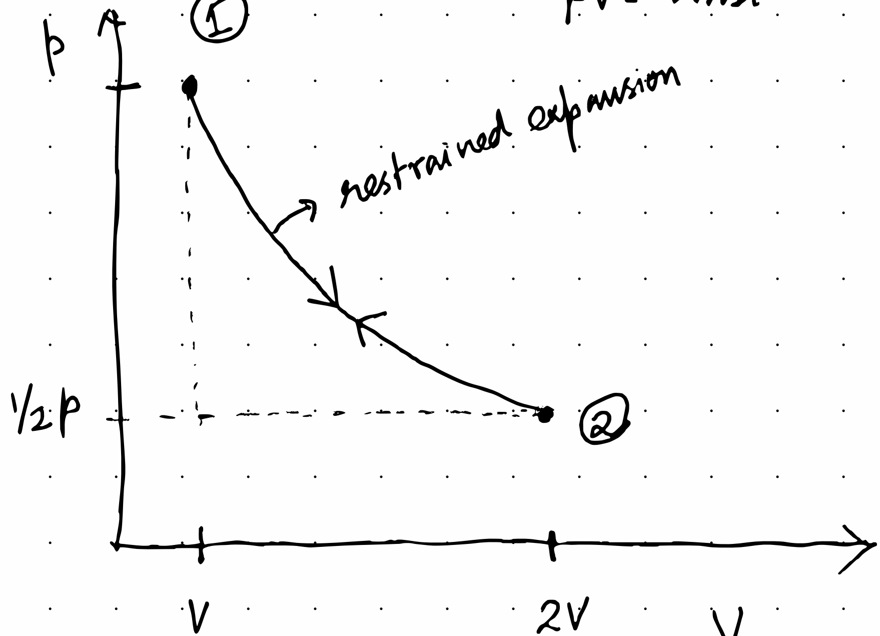
$\delta Q = 0$; $\delta W = 0$; $\Rightarrow$ $dU = 0$

Sudden expansion $dU = 0$

moving on to the restrained expansion expt...



$pV = \text{Const}$



$$\delta Q - \delta W = dU = 0 \Rightarrow \delta Q = \delta W = p\,dV$$

$$\delta Q = \delta W = \text{Const}\,\frac{dV}{V} \Rightarrow \int \delta Q = \int \delta W = \text{Const} \cdot \int_V^{2V} \frac{dV}{V}$$

$$\boxed{\int \delta Q = \int \delta W = \text{Const} \cdot \ln 2}$$

Sudden expansion

$$\boxed{\delta Q = \delta W = dU = 0} \checkmark$$

$$\boxed{U_1 = U_2}$$

Restrained expansion

$$\delta Q = \delta W = \text{const} \cdot \frac{dV}{V} ;$$

$$\boxed{dU = \delta Q - \delta W = 0} \checkmark$$

$$\int \delta Q = \int \delta W = nRT \ln 2$$

$$\boxed{U_1 = U_2}$$

Entropy

entropy change for $1 \to 2 \equiv \frac{\Delta S_{1\to 2}}{n} = RT \ln 2$

entropy change for $2 \to 1 \equiv \frac{\Delta S_{2\to 1}}{n} = - RT \ln 2$

$$\rightarrow \boxed{dS = \frac{\delta Q_{rev}}{T}} \rightarrow \text{from Clausius inequality}$$

$$\int_{1\to 2} dS = S_2 - S_1 = \frac{nRT \ln 2}{T} = nR \ln 2$$

$$\frac{\Delta S_{1\to 2}}{n} = R \ln 2 \,; \quad \frac{\Delta S_{2\to 1}}{n} = -R \ln 2 \,;$$

<u>Direction of a process</u> (Clausius inequality)

$$dS \geq \frac{\delta Q}{T} \quad \checkmark$$

<u>Sudden expansion</u> $\delta Q = 0$

$$\int \frac{dS}{n} = R \ln 2 > 0$$

Reverse of Sudden expansion $(2 \to 1)$

$$\int \frac{ds}{n} = -R\ln 2 < 0 \to \text{violation of CI}$$

$\therefore$ impossible process

Restrained expansion

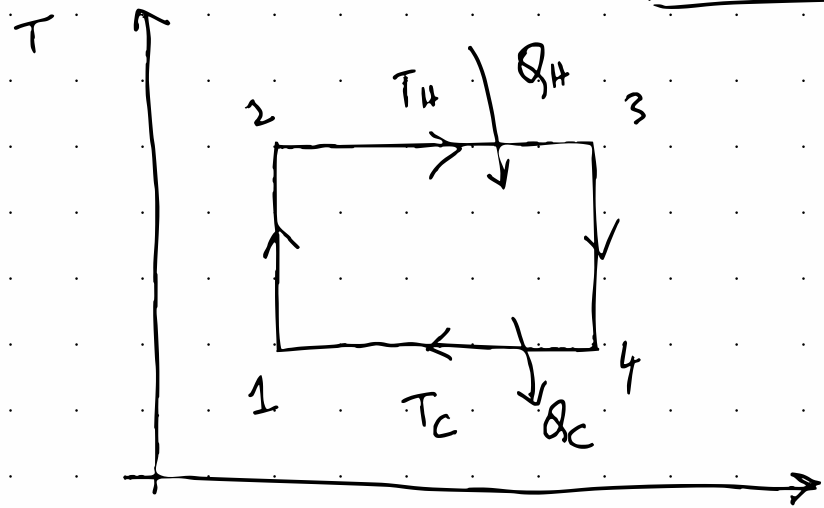
$$\int_{1\to 2} \frac{ds}{n} = R\ln 2 = \int \frac{\delta Q}{T} = R\ln 2 \to \text{forward}$$

$$\int_{2\to 1} \frac{ds}{n} = -R\ln 2 = \int \frac{\delta Q}{T} = -R\ln 2 \to \text{reverse}$$

No violation of CI

$\therefore$ both forward and reverse are allowed //

Carnot Cycle



$1 \to 2$: isentropic comp.

$2 \to 3$: isothermal Heat add.

$3 \to 4$: isentropic expansion

$4 \to 1$: isothermal heat reject

$$\oint \frac{\delta Q}{T} = \cancelto{0}{\int_{1\to2} \frac{\delta Q}{T}} + \int_{2\to3}^{S} \frac{\delta Q}{T} + \cancelto{0}{\int_{3\to4} \frac{\delta Q}{T}} + \int_{4\to1} \frac{\delta Q}{T}$$

$$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_C}{T_C} = 0 = \oint ds = 0;$$

$$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_C}{T_C} < \oint dS = 0 \rightarrow \text{Actual (irr.) engine}$$

$$\oint \frac{\delta Q}{T} \leq dS = 0$$

for a simple state change $\boxed{dS \geq \frac{\delta Q}{T}}$

Mod-1 - Lecture 2 $\boxed{\text{Idea of exergy}}$

$$dS = \frac{\delta Q}{T} + dS_{gen} \, ; \quad dS_{gen} \geq 0 \, ;$$

for an isolated system, $\boxed{dS = dS_{gen} \geq 0}$

$dS_{gen} \geq 0$

Sources of entropy generation

(1) Friction (useful work to heat)

(2) Heat transfer across boundaries due to finite ΔT.

The idea of exergy

An Example to motivate the idea of exergy (e.g. 8-4 from C & B)

$T_0 = 200^\circ C$

$T_{ambient} = 27^\circ C$

determine "reversible work" and the "irreversibility" for this process.

Reversible work → using an heat engine

↓

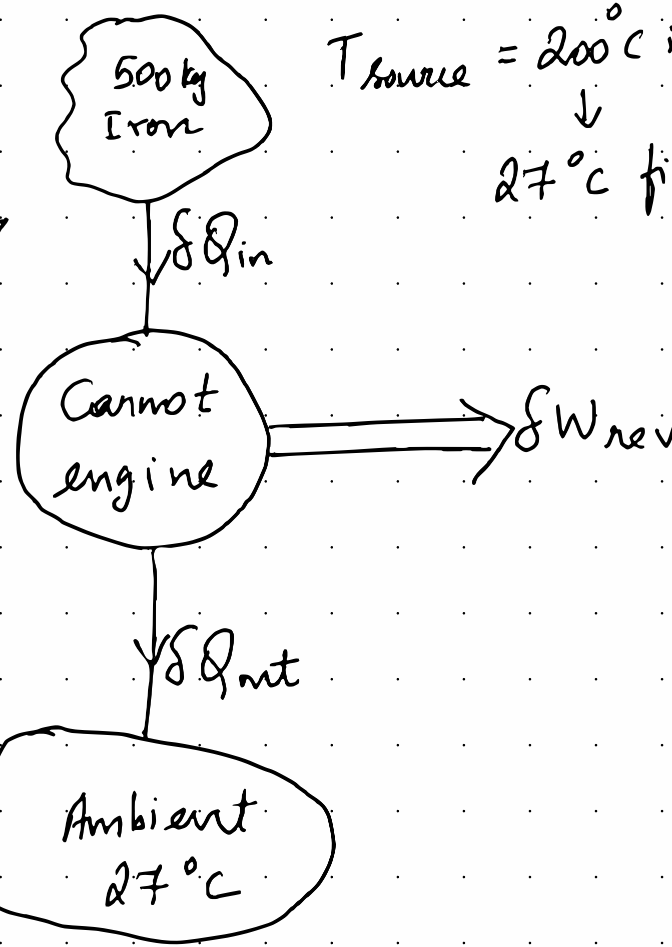
No entropy generation

$dS_{gen} = 0$

$T_{source} = 200^\circ C$ initially

↓

$27^\circ C$ finally



$$\eta_{th} = 1 - \frac{T_{ambient}}{T_{source}} \checkmark$$

$$\checkmark \; \delta Q_{in} = -mc\, dT_{source} \checkmark$$

$$\delta W_{rev} = \eta_{th} \delta Q_{in} \checkmark$$

$$\delta W_{rev} = -\left(1 - \frac{300}{T_{source}}\right) mc\, dT_{source} \checkmark$$

$$W_{rev} = \int \delta W_{rev} = -\int_{473\,K}^{300\,K} \left(1 - \frac{300}{T_{source}}\right) mc\, dT_{source}$$

$$W_{rev} = -mc(300 - 473) + mc300 \ln\left(\frac{300}{473}\right)$$

$$= mc\left[173 - 300 \ln\left(\frac{473}{300}\right)\right]$$

$$= 500 \times 450 \frac{J}{kg\text{-}K}\left[173 - 300 \ln\left(\frac{473}{300}\right)\right]$$

$$\boxed{W_{rev} = 8191 \text{ KJ}} \longrightarrow$$ Also the energy of the iron block.

Irreversibility of this process

$I = W_{rev} - W_{actual} = 8191\ \text{KJ} - 0 = 8191\ \text{KJ}$

$\boxed{\text{II}^{nd}\ \text{law efficiency}: \ \eta_{II} = 0\%}$

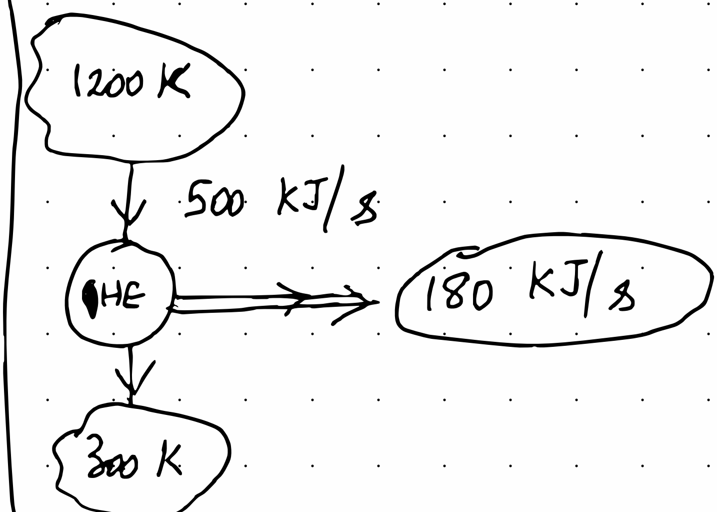
Example 8.3 from C&B

$\dot{W}_{rev} = 500\ \text{KW} \times \left(1 - \frac{300}{1200}\right)$

$= 375\ \text{KW}$

$\dot{I} = 375 - 180 = 195\ \text{KW}$

$\eta_{II} = 100 \times \frac{180}{375}\ \%$



Same iron block (500 kg, $200^\circ C$) is to be used to maintain a house at $200^\circ C$; the outdoor temperature is $5^\circ C$

<u>Determine</u>: Max. amount of heat that can be supplied to the house as the iron block cools to $27^\circ C$;

just allowing the block to cool to $27^\circ C$ will give us,

$$500 \times 450 \times (200 - 27) = \boxed{38925} \text{ kJ of heat.}$$

Max. amount of work that we could get ("exergy") from the iron block in an ambient at $27^\circ C$ is 8191 kJ.

Heat rejected at $27^\circ C$ = $38925 - 8191 = 30734$ kJ

We have 8191 kJ of work which we could give as input to a reversible HP operating between 5°C and 27°C;

$$COP_{HP} = \frac{T_H}{T_H - T_L}$$

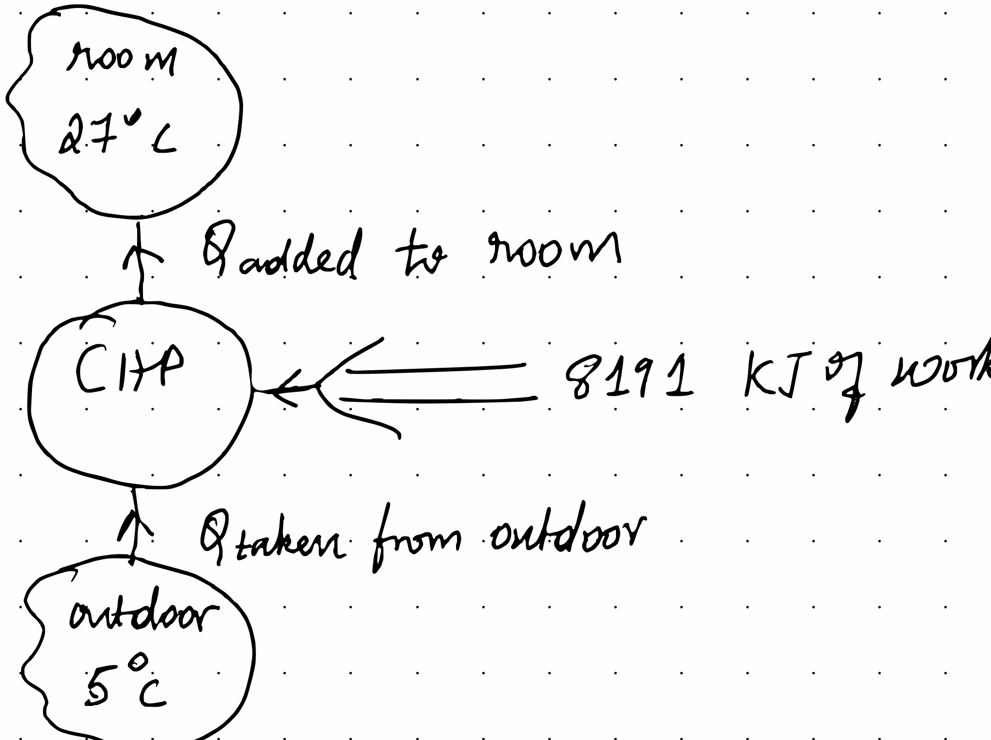
$$= \frac{300}{300 - 278}$$

$$= 13.6$$

$$Q_{added} = COP_{HP} \times W_{in}$$

$$= 13.6 \times 8191 \text{ KJ}$$

$$= 111{,}398 \text{ KJ}$$



Total heat added, $= 111398 + 30734$

$= \underline{142132 \text{ KJ}}$;

direct HT (irreversible)	Isothermal HT (with CHE and CHP)
$Q_{added} = 38925$ KJ	$Q_{added} = 142132$ KJ. more than 3 times the value from direct cooling.
Can this be made better? Yes → find out how? NO → why?	end of lecture 2

... Continued ...

<u>Mod 1: Lecture 2</u> Exergy: definition, formula for control mass and CV

<u>Some useful definitions</u>

$\boxed{\text{Exergy}}$ * is a "<u>property</u>"; work potential of a system in a specified environment;

* represents the maximum amount of "<u>useful work</u>" that can be obtained as the system is brought to equi. with the environment

* The state of environment is referred to as the "Dead state".

* We will consider only heat and work (mechanical) interactions (mixing and chemical rxns. neglected).

Exergy of a fixed mass

1^{st} law $\quad -|\delta Q| - \delta W = dU \quad (*)$; $\delta W = p\,dV$ (boundary work)

$$\delta W = \underbrace{(p-p_0)\,dV}_{\text{useful}} + \underbrace{p_0\,dV}_{\text{not useful}} = \delta W_{b,useful} + p_0\,dV \quad (1)$$

Any heat transfer must be through a $\underbrace{\text{CHE}}_{-\delta Q}$ or $\underbrace{\text{CHP}}_{+\delta Q}$:

$$\delta W_{HE} = \left(1 - \frac{T_0}{T}\right)|\delta Q| = |\delta Q| - |\delta Q|\frac{T_0}{T} \quad ; \quad \text{recall } +|\delta Q| = T|dS|$$

$$\therefore \quad \delta W_{HE} = |\delta Q| - T_0|dS| \Rightarrow \boxed{|\delta Q| = \delta W_{HE} + T_0|dS|} \quad (2)$$

Substituting (1) and (2) in (*)

$$-\delta W_{HE} - T_0|dS| - \delta W_{b,useful} - p_0\,dV = dU$$

$$\delta W_{HE} + \delta W_{b,useful} = -dU - p_0\,dV + T_0\,dS$$

Exergy of a control volume :

$$-|\delta Q| - \delta W_{shaft} = \frac{dU}{dt} + \mathring{m}_{out}\left(h_{out} + \frac{v_{out}^2}{2} + g z_{out}\right) - \mathring{m}_{in}\left(h_{in} + \frac{v_{in}^2}{2} + g z_{in}\right)$$

Considering only steady state (all practical devices can be approx. as steady by proper averaging)

$$-|\delta Q| - \delta W_{shaft} = \mathring{m}\left[h_{out} + \frac{v_{out}^2}{2} + g z_{out} - h_{in} \cdots\right]$$

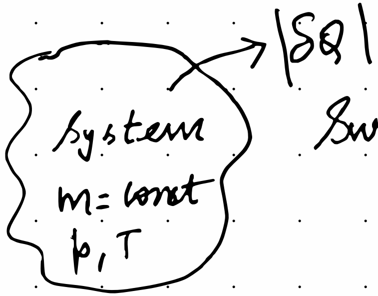
recall $|\delta Q| = \delta W_{HE} + T_0 |ds|$

$$\therefore \; -\delta W_{HE} - \delta W_{shaft} = T_0 |dS| + \mathring{m}\left(h_{out} + \cdots\cdots\right)$$

$$\Delta \psi = \frac{-\delta W_{HE} - \delta W_{shaft}}{\mathring{m}} = T_0(s_{out} - s_{in}) + (h_{out} - h_{in}) + \left(\frac{v_{out}^2 - v_{in}^2}{2}\right) + g(z_{out} - z_{in})$$

Exergy of a control mass

(1) work producing device ('pdv' kind of boundary work)
(2) subject to general heat loss



Surroundings (p_0, T_0)

Aim: derive a formula for exergy (calculate maximum amount of useful work)

useful work, $\delta W_{useful} = \delta W_b + \delta W_{HE}$

δW_b: (boundary work pdv kind)

δW_{HE}: heat engine work

✓ $\delta W_b = p\,dV = \underbrace{(p-p_0)\,dV}_{\text{useful ✓}} + \underbrace{p_0\,dV}_{\text{atmospheric displacement}}$

(not useful in work producing devices)

$$\delta W_{HE} = \left(1 - \frac{T_0}{T}\right)|\delta Q| = |\delta Q| - \left\{|\delta Q|\frac{T_0}{T}\right\}$$

$ds = \frac{\delta Q}{T}$ (reversible, ∴ equality holds)

$\Rightarrow \boxed{\delta Q = T\,dS} \longrightarrow$ II[nd] law applied to rev. process:

✓ $\delta W_{HE} = |\delta Q| + \left\{T\,dS\,\frac{T_0}{T}\right\} = |\delta Q| + T_0\,dS$ ✓ (*)

I[st] law ~ $-|\delta Q| - \delta W = dU$;

$$-\delta W_{HE} + T_0 dS - (p-p_0)dV - p_0 dV = dU$$

<u>defn of exergy</u> maximum useful work

$$\delta W_{HE} + (p-p_0)dV = -dU - p_0 dV + T_0 dS$$

$$\text{Exergy } X_1 = \int_{1\to 0} \left[\delta W_{HE} + (p-p_0)dV\right] = -\int_1^0 dU - \int_1^0 p_0 dV + T_0 \int_1^0 dS$$

$$\text{Exergy } X_1 = -\left[U_0 - U_1\right] - p_0\left[V_0 - V_1\right] + T_0\left[S_0 - S_1\right]$$

$$\boxed{X_1 = (U_1 - U_0) + p_0(V_1 - V_0) - T_0(S_1 - S_0)}$$

$$\boxed{X = (U-U_0) + p_0(V-V_0) - T_0(S-S_0) + mgz + \frac{mV^2}{2}}$$

Change in exergy when the system goes from state $1\to 2$

$$\Delta X_{1\to 2} = X_2 - X_1 = (U_2-U_1) + p_0(V_2-V_1) - T_0(S_2-S_1) + \cdots$$

(1) X is a state function

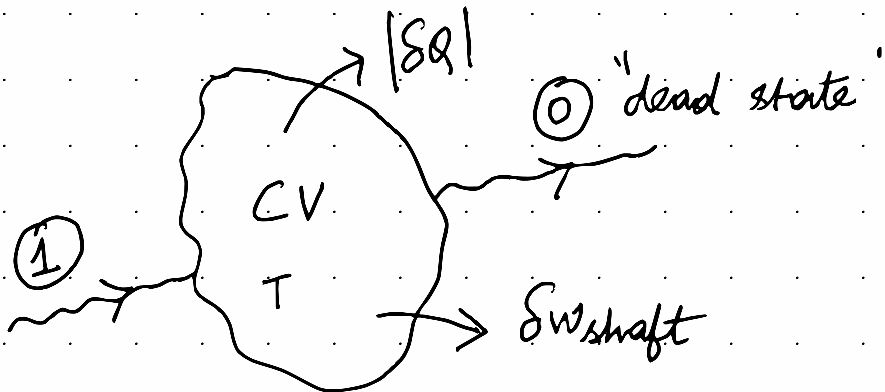
(2) X is extensive

$$\boxed{\frac{X}{m} = \phi = (u-u_0) + p_0(v-v_0) - T_0(s-s_0) + \cdots}$$

"specific exergy"

__HW__ use the formula for X to calculate "max. work" from 500 kg Iron block at 200°C in 27°C ambient.

Mod 1 Lecture 2 : part 3 — Exergy of a flowing stream



(1) Work producing device (shaft work)
(2) subject to general heat loss

Exergy of stream (1) = maximum useful work = $\delta W_{shaft} + \delta W_{HE}$ ✓

$$\delta W_{HE} = \left(1 - \frac{T_0}{T}\right)|\delta Q| = |\delta Q| - \frac{T_0}{T}|\delta Q| ;$$

II nd law : $\delta Q = T ds$; $\boxed{\delta W_{HE} = |\delta Q| + T_0 ds}$;

(*) <u>1st law</u> $-|\delta \dot{Q}| - \delta \dot{W}_{shaft} = \frac{dU}{dt} + \sum_{outlets} \dot{m}\left(h + \frac{v^2}{2} + gz\right)$

$$- \sum_{inlets} \dot{m}\left(h + \frac{v^2}{2} + gz\right)$$

(1) assuming steady state $\left(\sum \dot{m}_{in} = \sum \dot{m}_{out},\ dU/dt = 0 \right)$

(2) noting that the conditions at the outlet must be "dead state."

$$- |\delta \dot{Q}| - \delta \dot{W}_{shaft} = \dot{m}\left[\left(h_0 + \frac{v_0^2}{2} + gz_0\right) - \left(h_1 + \frac{v_1^2}{2} + gz_1\right)\right]$$

$$- \delta \dot{W}_{HE} + T_0 \frac{dS}{dt} - \delta \dot{W}_{shaft} = \dot{m}\left[\left(h_0 + \dots\right) - \left(h_1 + \dots\right)\right]$$

$$+ \delta \dot{W}_{HE} + \delta \dot{W}_{shaft} = \dot{m}\left[\left(h_1 - h_0\right) + \left(\frac{v_1^2 - v_0^2}{2}\right) + g\left(z_1 - z_0\right)\right]$$

$$+ T_0 \frac{dS}{dt}$$

$$\frac{\mathring{\delta W}_{HE} + \mathring{\delta W}_{shaft}}{\mathring{m}} = (h_1 - h_0) + \left(\frac{v_1^2 - v_0^2}{2}\right) + g(z_1 - z_0) + \frac{T_0}{\mathring{m}}\frac{ds}{dt}$$

$$\frac{\mathring{\delta W}_{HE} + \mathring{\delta W}_{shaft}}{\mathring{m}} = (h_1 - h_0) + \left(\frac{v_1^2 - v_0^2}{2}\right) + g(z_1 - z_0) + T_0(s_0 - s_1)$$

$\psi_1 \equiv$ exergy of a flowing stream

$$= (h_1 - h_0) + \left(\frac{v_1^2 - \cancel{v_0^2}}{2}\right) + g(z_1 - \cancel{z_0}) - T_0(s_1 - s_0)$$

$\boxed{\text{for dead state, } v_0 = 0,\ z_0 = 0;}$

$\phi \equiv$ exergy of control mass

$$= (u - u_0) - T_0(s - s_0) + p_0(v - v_0) + \frac{V^2}{2} + gz$$

* $\psi \equiv$ exergy of flowing stream $= (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$

hint: "think about flow work"

Example 8.7 (C & B) Work potential of compressed air in a tank

$$\boxed{\begin{matrix} 200\,m^3 \\ 1\ MPa \\ 300\,K \end{matrix}} \Rightarrow \quad \begin{matrix} 300\,K \\ 100\,KPa \end{matrix}$$

& determine the max. useful work

$$X_1 = m\phi_1 = m\left[\cancelto{0}{(u_1 - u_0)} - T_0(s_1 - s_0) + p_0(v_1 - v_0)\right]$$

$$= m\left[- T_0(s_1 - s_0) + p_0(v_1 - v_0)\right]$$

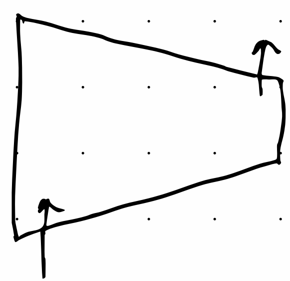
$$p_0(v_1 - v_0) = p_0\left[\frac{RT_1}{p_1} - \frac{RT_0}{p_0}\right] = RT_0\left[\frac{p_0}{p_1} - 1\right]; \ (T_1 = T_0)$$

$$T_0(s_1 - s_0) = -RT_0 \ln\left(\frac{p_1}{p_0}\right); \quad (\because T_1 = T_0)$$

$$X_1 = m\phi_1 = (2323\ kg)(120.76\ KJ/kg) \simeq 281\ MJ.$$

Example 8-8 (C & B)

$T_0 = 20^\circ C$
95 kPa



determine

(1) Change in exergy of the stream $(\psi_2 - \psi_1)$

(2) minimum work input.

$$\Delta\psi = \psi_2 - \psi_1 = (h_2 - h_1) - T_0(s_2 - s_1)$$

$P_1 = 0.14$ MPa ; $T_1 = -10^\circ C$; $h_1 = 246.37$ kJ/kg ; $s_1 = 0.9724$ kJ/kg·K

$P_2 = 0.8$ MPa ; $T_2 = 50^\circ C$; $h_2 = 286.71$ kJ/kg ; $s_2 = 0.9803$ kJ/kg·K

$$\boxed{\Delta\psi = \psi_2 - \psi_1 = 38 \text{ kJ/kg}}$$

$$\boxed{W_{in,min} = 38 \text{ kJ/kg}}$$

Mod-1 Lecture 3 Exergy balance for CM and CV

Control mass

$$X = (U - U_0) + p_0(V - V_0) - T_0(S - S_0)$$

$$dX = dU + p_0 dV - T_0 dS$$

IInd law

$$dS = \frac{\delta Q}{T} + dS_{gen}$$

$$dX = dU + p_0 dV - T_0 \frac{\delta Q}{T} - T_0 dS_{gen}$$

Ist law

$$\delta Q - \delta W = dU$$

$$dX = \delta Q - \delta W + p_0 dV - T_0 \frac{\delta Q}{T} - T_0 dS_{gen}$$

$$\boxed{dX = \delta Q\left(1 - \frac{T_0}{T}\right) - \delta W + p_0 dV - T_0 dS_{gen}}$$

$$dX = \sum_i \delta Q_i \left[1 - \frac{T_0}{T_i}\right] - \delta W + p_0 dV - T_0 dS_{gen} \checkmark$$

isolated system $\delta Q = 0$; $\delta W = 0$; $p_0 dV = 0$

$$dX = -T_0 dS_{gen} \leq 0 \quad \because \ dS_{gen} \geq 0$$

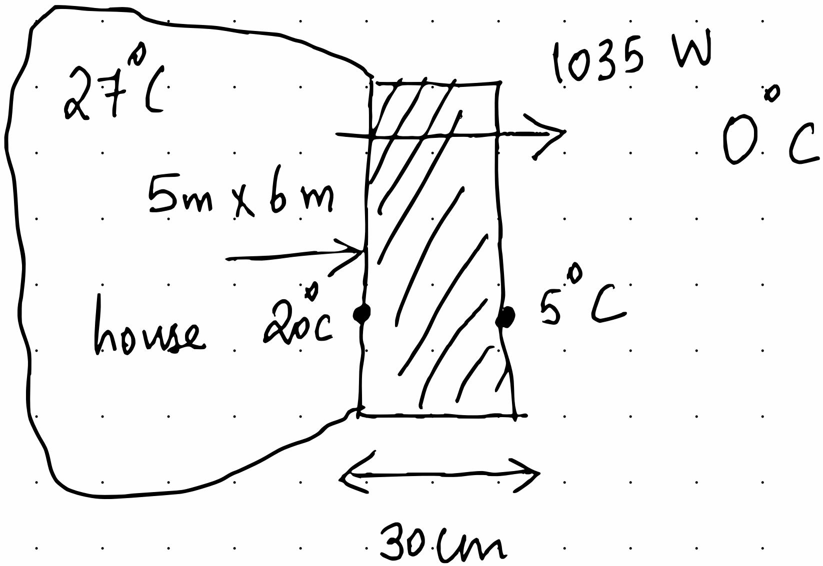
principle of decrease of exergy $\equiv$ principle of increase of entropy.

Control volume

$$dX = \sum_i \delta Q_i \left[1 - \frac{T_0}{T_i}\right] - \delta W + p_0 dV - T_0 dS_{gen} + \sum m_{in} \psi_{in} - \sum m_{out} \psi_{out}$$

"exergy destroyed" is either 0 or positive; $X_{des} = T_0 dS_{gen}$

Example 8-10 (C & B)



Determine

(1) Rate of $X_{destruction}$ in the wall

(2) rate of total exergy destruction associated with the HT process.

(1) $$dX = \sum_i \delta Q_i \left[1 - \frac{T_0}{T_i}\right] - \delta W + p_0 dV - T_0 dS_{gen}$$

$$dX = \sum_i \delta \dot{Q}_i \left[1 - \frac{T_0}{T_i}\right] - \delta W + p_0 dV - X_{destroyed}$$

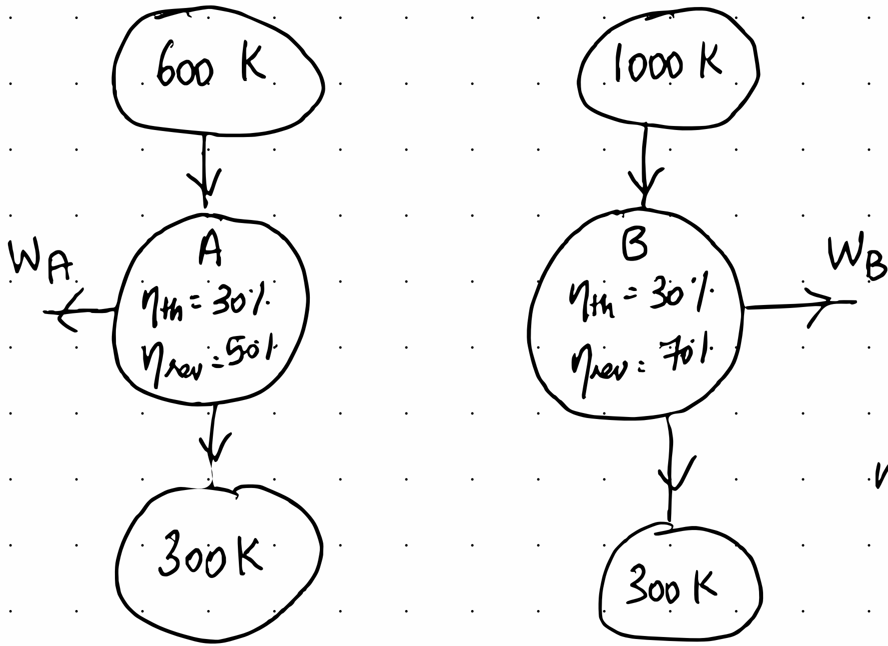
$$\dot{X}_{destroyed} = 1035\left(1 - \frac{273}{293}\right) - 1035\left(1 - \frac{273}{278}\right)$$

$$\boxed{\dot{X}_{destroyed} = 52\ W}$$

$$\dot{X}_{destroyed,\,total} = 1035\left(1 - \frac{273}{300}\right) = 93\ W.$$

HW Start from II^{nd} law, $ds = \frac{\delta Q}{T} + dS_{gen}$, Calculate dS_{gen} and then Calculate $\dot{X}_{destroyed}$ by using $\dot{X}_{destroyed} = \int T_0\, d\dot{S}_{gen}$

Mod 1: Lecture 3 - part 2 - II^{nd} law efficiency



Are A and B equally well performing?

No, because ...

$$\eta_{\text{II},A} = \frac{0.3}{0.5} = 60\%$$

$$\eta_{\text{II},B} = \frac{0.3}{0.7} = 43\%$$

$$\eta_{rev,A} = 1 - \frac{300}{600} = \underline{\underline{50\%}} \; ; \quad \eta_{rev,B} = 1 - \frac{300}{1000} = \underline{\underline{70\%}} \; ;$$

$$\eta_{\text{II}} = \frac{W_u}{W_{rev}} \text{ (work producing device)} \; ; \quad \eta_{\text{II}} = \frac{W_{rev}}{W_u} \text{ (consuming)}$$

Consider a work producing device: (CM)

$$\eta_{II} = \frac{W_u}{W_{rev}} = \frac{\text{Exergy expended} - \text{Exergy destroyed}}{\text{Exergy expended}}$$

$$\eta_{II} = \frac{\text{Exergy recovered}}{\text{Energy expended}} = 1 - \frac{X_{destroyed}}{X_{expended}};$$

Cooling systems:

$$\eta_{II} = \frac{COP}{COP_{rev}}$$ (refrigerators, AC & HPs)

what about steady flow devices?

...

II^{nd} law efficiency for steady flow devices:

Turbine: $$\eta_{\mathrm{II}} = \frac{W_{out}}{\psi_1 - \psi_2} = \frac{h_1 - h_2}{\psi_1 - \psi_2} = \frac{W_{out}}{W_{rev,out}}$$

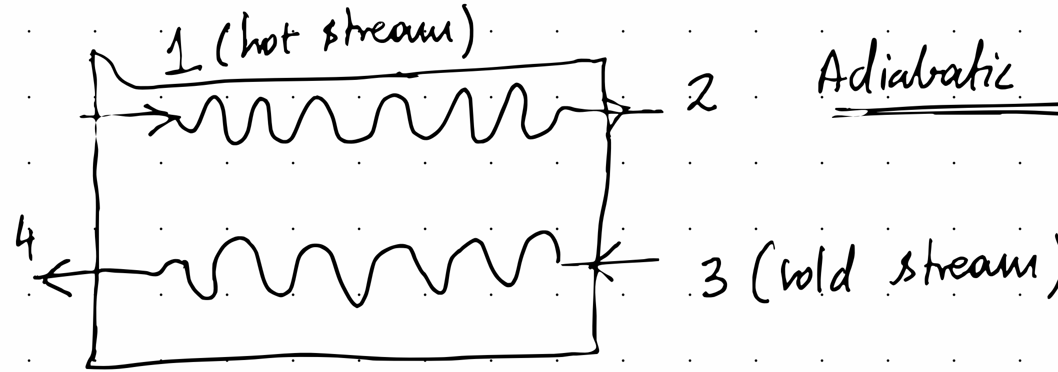
(or) $$\eta_{\mathrm{II}} = 1 - \frac{T_0 S_{gen}}{\psi_1 - \psi_2};$$

Compressor: $$\eta_{\mathrm{II}} = \frac{W_{rev,in}}{W_{act,in}} = \frac{\psi_2 - \psi_1}{h_2 - h_1} = 1 - \frac{T_0 S_{gen}}{h_2 - h_1};$$

Passive devices: heat exchangers, nozzles and diffusers

o o o

HX:



Adiabatic HX.

$$\eta_{II,HX} = \frac{\dot{m}_{cold}(\psi_4 - \psi_3)}{\dot{m}_{hot}(\psi_1 - \psi_2)} = 1 - \frac{T_0 \dot{S}_{gen}}{\dot{m}_{hot}(\psi_1 - \psi_2)};$$

non-adiabatic HX

$$\eta_{II,HX} = \frac{\dot{m}_{cold}(\psi_4 - \psi_3) + \dot{Q}_{loss}(1 - T_0/T_b)}{\dot{m}_{hot}(\psi_1 - \psi_2)}$$

$$= 1 - \frac{T_0 \dot{S}_{gen}}{\dot{m}_{hot}(\psi_1 - \psi_2)};$$