Let M be a smooth Riemannian Manifold. If you have a coordinate system, say $\xi = (x^1, ..., x^n)$ at p in M, then a basis for the tangent space T_pM is given by $\{\frac{\partial}{\partial x^1}, \ldots, \frac{\partial}{\partial x^n}\}$, where we simply write it as $\{\partial_1, \ldots, \partial_n\}$. The basis theorem (I'm following the book of Semi-Riemannian Geometry, by Barrett O'Neill) says that, any vector v tangent to M at p can be written as

$$v = \sum_{i=1}^{n} v(x^i)\partial_i.$$

For your example, we have $\xi = (x, y, z)$ as the natural coordinate system in $\mathbb{R}^3$ with $\{\partial_x, \partial_y, \partial_z\}$ associated tangent basis and we let $\eta = (a, \phi, h)$ be the natural cylindrical coordinate system in $\mathbb{R}^3 - H$, where H is the hyperplane $x \geq 0, y = 0$. Now we can define $x = acos\phi$, $y = asin\phi$ and $z = h$. Notice that

$$\left\{\frac{\partial}{\partial a}, \frac{\partial}{\partial \phi}, \frac{\partial}{\partial h}\right\} \overset{notation}{=} \{\partial_a, \partial_\phi, \partial_h\}$$

is composed by tangent vectors, where we can apply The Basis Theorem:

$$\begin{aligned}
\partial_a &= \partial_a(x)\partial_x + \partial_a(y)\partial_y + \partial_a(z)\partial_h &(1)\\
&= \partial_\phi(a\cos\phi)\partial_x + \partial_\phi(a\sin\phi)\partial_y + \partial_\phi(z)\partial_h &(2)\\
&= \cos\phi\partial_x + \sin\phi\partial_y, &(3)
\end{aligned}$$

$$\begin{aligned}
\partial_\phi &= \partial_\phi(x)\partial_x + \partial_\phi(y)\partial_y + \partial_\phi(z)\partial_h &(4)\\
&= \partial_\phi(a\cos\phi)\partial_x + \partial_\phi(a\sin\phi)\partial_y + \partial_\phi(z)\partial_h &(5)\\
&= -a\sin\phi\partial_x + a\cos\phi\partial_y &(6)
\end{aligned}$$

and

$$\begin{aligned}
\partial_h &= \partial_h(x)\partial_x + \partial_h(y)\partial_y + \partial_h(z)\partial_h &(7)\\
&= \partial_h(a\cos\phi)\partial_x + \partial_h(a\sin\phi)\partial_y + \partial_h(z)\partial_h &(8)\\
&= \partial_h. &(9)
\end{aligned}$$

Now, as every metric g is a $(0, 2)$ type tensor, that is, it's written in a basis with two one forms and it's fed with two vector fields, it returns (when applied to a point in the Manifold) a real value. If ξ is that coordinate system given, then we have a basis for the cotangent space $\{dx^1, \ldots, dx^n\}$. We can write it as

$$g = \sum g_{ij} dx^i \otimes dx^j, \tag{10}$$

where $g_{ij} = g(\partial_i, \partial_j)$ and so our line element is

$$q = ds^2 = \sum g_{ij} dx^i dx^j.$$

For your example, from $\{\partial_a, \partial_\phi, \partial_h\}$ we have $\{da, d\phi, dh\}$ as the cotangent basis associated. Set $a = y^1, \phi = y^2$ and $h = y^3$ and the line element above on the η coordinate system can be written as

$$ds^2 = \sum_{i,j=1}^{3} g_{ij} dy^i dy^j$$

We are almost done! The last thing needed are those g_{ij}.

$$\begin{aligned}
g_{11} &= g(\partial_a, \partial_a) = \cos^2\phi + \sin^2\phi = 1 &(11)\\
g_{22} &= g(\partial_\phi, \partial_\phi) = a^2\sin^2\phi + a^2cos^2\phi = a^2 &(12)\\
g_{33} &= g(\partial_h, \partial_h) = 1 &(13)
\end{aligned}$$

while $g_{ij} = 0$ for $i \neq j$, because $g(\partial_a, \partial_\phi) = -a\sin\phi\cos\phi + a\sin\phi\cos\phi = 0$ and $g(., \partial_h) = 0$ because the other vectors have 0 as component in the ∂_h direction.

Finally, we obtain

$$ds^2 = g_{11}dada + g_{22}d\phi d\phi + g_{33}dhdh = da^2 + a^2d\phi^2 + dh^2, \tag{14}$$

as desired.