Let $\mathcal{M}$ be a Riemannian manifold. If you have a coordinate system, say $\xi = (x^1, \dots, x^n)$ centered at p in $\mathcal{M}$, then a basis for the tangent space $T_p\mathcal{M}$ is given by $\{\frac{\partial}{\partial x^1}, \dots, \frac{\partial}{\partial x^n}\}$, which we simply write as $\{\partial_1, \dots, \partial_n\}$. The Basis Theorem (following the book *Semi-Riemannian Geometry* by Barrett O'Neill) states that any vector v tangent to $\mathcal{M}$ at p can be written as

$$v = \sum_{i=1}^{n} v(x^i)\partial_i.$$

For your example, let $\xi = (x, y, z)$ be the natural coordinate system in $\mathbb{R}^3$ with $\{\partial_x, \partial_y, \partial_z\}$ as the associated tangent basis, and let $\eta = (a, \phi, h)$ be the natural cylindrical coordinate system in $\mathbb{R}^3 \setminus H$, where H is the hyperplane $x \geq 0, y = 0$. Now, define $x = a\cos\phi$, $y = a\sin\phi$, and $z = h$. Notice that

$$\left\{\frac{\partial}{\partial a}, \frac{\partial}{\partial \phi}, \frac{\partial}{\partial h}\right\} \overset{\text{notation}}{=} \{\partial_a, \partial_\phi, \partial_h\}$$

is composed of tangent vectors, to which we can apply the Basis Theorem:

$$\begin{aligned}\partial_a &= \partial_a(x)\partial_x + \partial_a(y)\partial_y + \partial_a(z)\partial_h \\ &= \cos(\phi)\partial_x + \sin(\phi)\partial_y,\end{aligned}$$

$$\begin{aligned}\partial_\phi &= \partial_\phi(x)\partial_x + \partial_\phi(y)\partial_y + \partial_\phi(z)\partial_h \\ &= -a\sin(\phi)\partial_x + a\cos(\phi)\partial_y,\end{aligned}$$

and

$$\begin{aligned}\partial_h &= \partial_h(x)\partial_x + \partial_h(y)\partial_y + \partial_h(z)\partial_h \\ &= \partial_h.\end{aligned}$$

Now, recall that every metric g is a $(0, 2)$-type tensor, which in particular implies its local expression is a linear combination of (tensor) products of one forms (whose coefficients are exactly the components of the metric in the associated coordinate system). When evaluated at a point, it's an inner product, which when evaluated at two vectors returns a real number. To sum it up, if ξ is the aforementioned coordinate system, we can write

$$g = \sum g_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j,$$

where $\{\mathrm{d}x^1, \dots, \mathrm{d}x^n\}$ is the local frame for the contangent space, determined by the frame $\{\partial_1, \dots, \partial_n\}$, and $g_{ij} = g(\partial_i, \partial_j)$. Thus, the line element is

$$\mathrm{d}s^2 = \sum g_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j$$

For your example, from $\{\partial_a, \partial_\phi, \partial_h\}$, we have $\{\mathrm{d}a, \mathrm{d}\phi, \mathrm{d}h\}$ as the cotangent basis. Set $a = y^1$, $\phi = y^2$, and $h = y^3$. The line element in the η coordinate system is written as

$$\mathrm{d}s^2 = \sum_{i,j=1}^{3} g_{ij}\mathrm{d}y^i \otimes \mathrm{d}y^j.$$

The last step is to compute the g_{ij}:

$$\begin{aligned}g_{11} &= g(\partial_a, \partial_a) = \cos^2\phi + \sin^2\phi = 1, \\ g_{22} &= g(\partial_\phi, \partial_\phi) = a^2\sin^2\phi + a^2\cos^2\phi = a^2, \\ g_{33} &= g(\partial_h, \partial_h) = 1,\end{aligned}$$

while $g_{ij} = 0$ for $i \neq j$, because $g(\partial_a, \partial_\phi) = -a\sin\phi\cos\phi + a\sin\phi\cos\phi = 0$ and $g(., \partial_h) = 0$ as other vectors have 0 as a component in the ∂_h direction.

Finally, we obtain

$$\mathrm{d}s^2 = g_{11}\ \mathrm{d}a \otimes \mathrm{d}a + g_{22}\ \mathrm{d}\phi \otimes \mathrm{d}\phi + g_{33}\mathrm{d}h \otimes \mathrm{d}h = \mathrm{d}a^2 + a^2\mathrm{d}\phi^2 + \mathrm{d}h^2,$$

as desired.