Knowledge Distillation Using Early Exit LLMs

Experiments on Confidence Scoring

Edvin 24V0074 Sagar 24D0367

CS 769
Optimization in Machine Learning

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Overview

1. Knowledge Distillation
2. KD with early exits
3. Early-Exit with Nested Prediction Sets
4. Fixing overconfidence in Dynamic Neural Networks

Section Overview

Knowledge Distillation

- Introduced in [Hinton et al., 2015]

Large Cumbersome Models are difficult to deploy
need to train smaller models efficiently

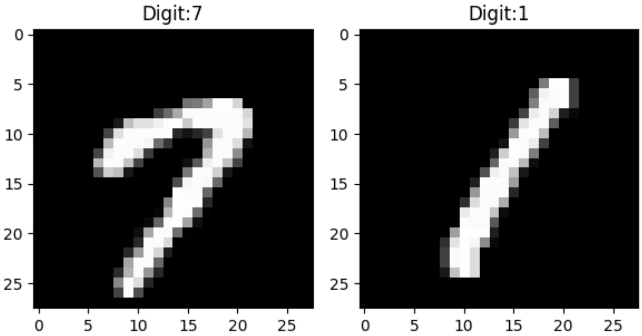
Knowledge Distillation

- Introduced in [Hinton et al., 2015]
- Student - teacher models

Smaller Student model tries to mimic larger teacher model that generalises well

Knowledge Distillation

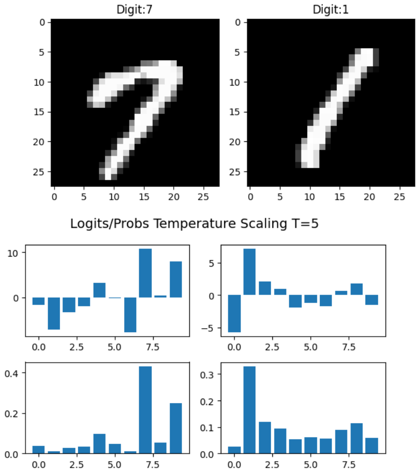
- Introduced in [Hinton et al., 2015]
- Student - teacher models
- Teacher Provides ”Soft Targets”



$$q_i = \frac{exp(z_i/T)}{\sum_j exp(z_i/T)}$$

Knowledge Distillation

- Introduced in [Hinton et al., 2015]
- Student - teacher models
- Teacher Provides "Soft Targets"
- Loss: kl divergence + cross-entropy



Section Overview

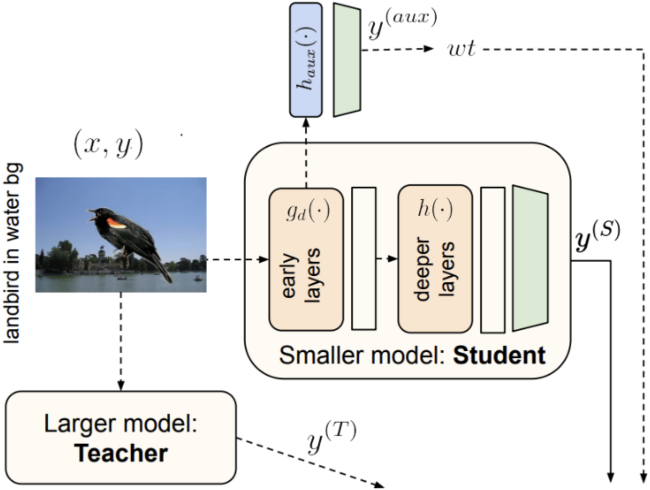
KD with early exits

- Proposed by
  [Tiwari et al., 2024]

KD with early exits

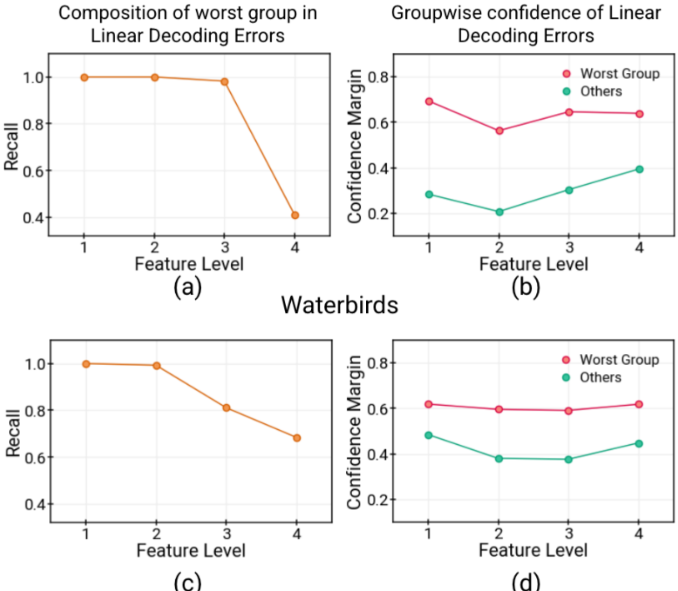
Smaller Models trained via KD rely more on spurious correlations than the teacher model, which leads to Poor performance on group fairness metrics by the student

- Proposed by [Tiwari et al., 2024]
- Student model relies on spurious correlations



KD with early exits

- Proposed by [Tiwari et al., 2024]
- Student model relies on spurious correlations
- Student's early Layers overconfident on hard instances



KD with early exits

- Proposed by [Tiwari et al., 2024]
- Student model relies on spurious correlations
- Student's early Layers overconfident on hard instances
- DEDIER training

$$\mathcal{L}_{student} = \sum_{D_w} (1-\lambda) \cdot l_{ce} + \lambda \cdot \mathbf{wt} \cdot l_{ke}$$

where $\mathbf{wt} = \exp^{\beta.\mathbf{cm}.\alpha}$ and $\mathbf{cm}(\mathbf{p}) = \mathbf{p_{max}} - \max_{\mathbf{p_k} \in \mathbf{p} - \mathbf{p_{max}}} \mathbf{p_k}$

KD with early exits

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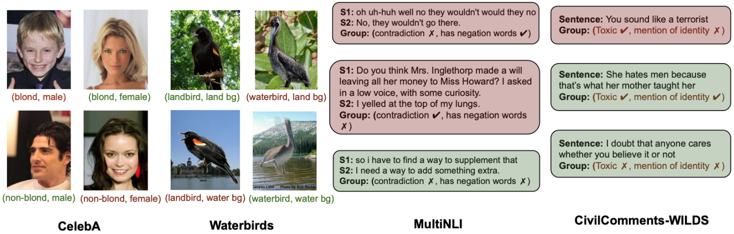
- DEDIER training
- Experiments



KD with early exits

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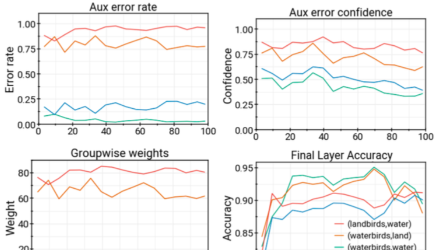
- Experiments

Compared to some other methods

Waterbirds groups	Teacher	DeTT	SimKD	DEDIER
(waterbird, water bg)	94.3	92.6 ± 0.70	89.4 ± 0.06	94.1 ± 0.86
(landbird, land bg)	91.6	90.0 ± 0.06	92.1 ± 0.46	$\boxed{89.8}$ ± 0.46
(waterbird, land bg)	91.7	$\boxed{88.3}$ ± 0.81	$\boxed{71.4}$ ± 2.15	92.1 ± 0.40
(landbird, water bg)	$\boxed{91.4}$	88.8 ± 1.70	84.6 ± 0.79	90.6 ± 0.67
CelebA groups				
(blond, female)	94.3	92.6 ± 1.14	92.2 ± 0.46	92.7 ± 1.48
(non-blond, male)	92.9	92.3 ± 0.58	93.0 ± 0.46	93.2 ± 0.42
(non-blond, female)	92.1	93.0 ± 0.78	93.2 ± 0.35	93.1 ± 0.52
(blond, male)	$\boxed{90.0}$	$\boxed{89.5}$ ± 0.71	$\boxed{89.0}$ ± 0.35	$\boxed{89.6}$ ± 1.96

Adaptive to the dataset



Early-Exit with Nested Prediction Sets

- [Jazbec et al., 2024]
  Anytime**V**alid**C**onfidence**S**equences(AVCS)

[Jazbec et al., 2024] propose using **A**nytime**V**alid**C**onfidence **S**equences(AVCS) for uncertainity estimation in Early Exit Networks

Early-Exit with Nested Prediction Sets

- [Jazbec et al., 2024] **A**nytime**V**alid**C**onfidence**S**equences(AVCS)
- AVCS has time uniform and non - asymptotic guarantees

$$\mathcal{C}_t = (l_t, r_t) \subseteq \mathbb{R}$$

$$\mathcal{L}(W_{1:T}, U_{1:T}; \mathcal{D}) = \sum_{n=1}^{N} \frac{1}{T} \sum_{t=1}^{T} \ell(y_n, f(x_n; W_t, U_{1:t}))$$

$$\text{size}(t) := \frac{1}{n_{\text{test}}} \sum_{n=1}^{n_{\text{test}}} |\mathcal{C}_t(x_n)|$$

$$\text{coverage}(t) := \frac{1}{n_{\text{test}}} \sum_{n=1}^{n_{\text{test}}} [y_n \in \mathcal{C}_t(x_n)]$$

$$\mathcal{N}(t) = |\bigcap_{s \leq t} \mathcal{C}_s| / |\mathcal{C}_t|$$

$$\mathbb{P}(\forall t,\, \theta^* \in \mathcal{C}_t) \geq 1 - \alpha$$

Early-Exit with Nested Prediction Sets

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- AVCS has time uniform and non - asymptotic guarantees
- Martingales And Ville's theorem

$$\mathbb{E}_{x_{t+1}}\left[R_{t+1}(\theta^*) \mid x_1, \ldots, x_t\right] = R_t(\theta^*)$$

$$\mathbb{P}(\exists t : R_t(\theta^*) \geq 1/\alpha) \leq \alpha$$

$$\mathcal{C}_t := \{\theta : R_t(\theta) \leq 1/\alpha\}$$

Early-Exit with Nested Prediction Sets

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  Anytime**V**alid**C**onfidence**S**equences(AVCS)
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- Predictive-likelihood ratio

Idea is to create a sequence of martingales for each exit layer and then use Ville's theorem to construct AVCS.

$$R_t^*(y) = \prod_{l=1}^{t} \frac{p_l(y|x^*, \mathcal{D})}{p(y|x^*, W_l)}, W_l \sim p(W_l|D^*)$$

Early-Exit with Nested Prediction Sets

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- Predictive-likelihood ratio
- AVCS for Regression

$$y \sim \mathcal{N}\left(y; h_t(x)^T W_t, \sigma_t^2\right),$$

$$W_t \sim \mathcal{N}(W_t; \hat{W}_t, \sigma_{w,t}^2 \mathbb{I}_H)$$

$$p(W_t|\mathcal{D}) = \mathcal{N}(W_t; \mu_t, \Sigma_t),$$

$$p_t(y|x^*, \mathcal{D}) = \mathcal{N}(y; h_t(x^*)^T \mu_t, v_* + \sigma_t^2)$$

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- AVCS for Regression
- AVCS for Classification

$$p(y|\pi_t) = \mathrm{Cat}(y|\pi_t),$$

$$p(\pi_t|x^*, \mathcal{D}) = \mathrm{Dir}(\pi_t|\alpha_t(x^*; \mathcal{D}))$$

$$p_t(y = y|x^*, \mathcal{D}) = \int p(y = y|\pi_t)\, p(\pi_t|x^*, \mathcal{D})\, d\pi_t$$

$$= \frac{\alpha_{t,y}}{\sum_{y' \in \mathcal{Y}} \alpha_{t,y'}}$$

Early-Exit with Nested Prediction Sets

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- AVCS has time uniform and non - asymptotic guarantees
- Martingales And Ville's theorem
- Predictive-likelihood ratio
- AVCS for Regression
- AVCS for Classification
- Post-Hoc Implementation

$$a_t(x) = \text{ReLU}(x, \tau_t)$$

Evaluations

$$\mathcal{C}_t = (l_t, r_t) \subseteq \mathbb{R}$$

$$\mathcal{L}(W_{1:T}, U_{1:T}; \mathcal{D}) =$$

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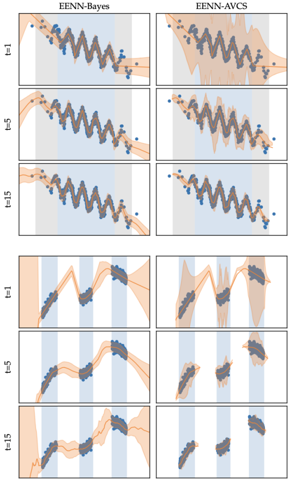

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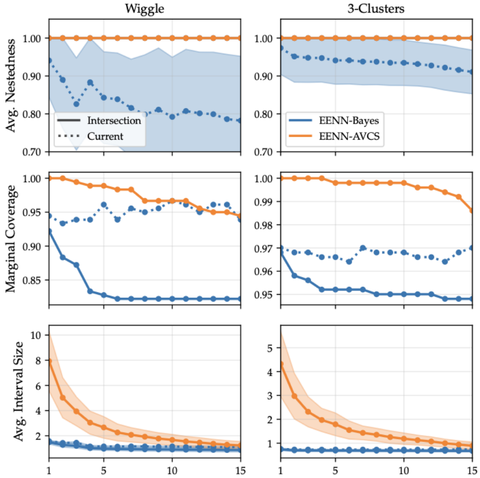

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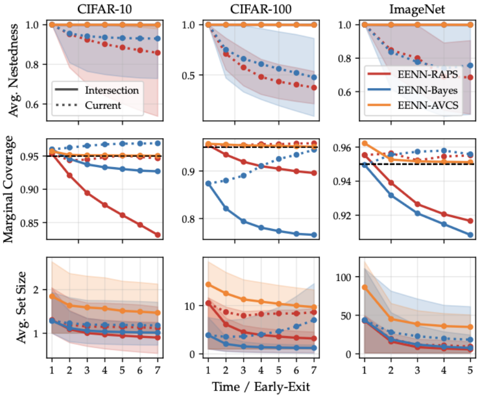


Section Overview

Fixing overconfidence in Dynamic Neural Networks

[Meronen et al., 2023]

- To decrease computational cost, we do not want to run network for more layers that required for the specific task
- To be able to know where to stop, the model needs good uncertainty estimates
- The paper aims to improve uncertainty estimates for a model

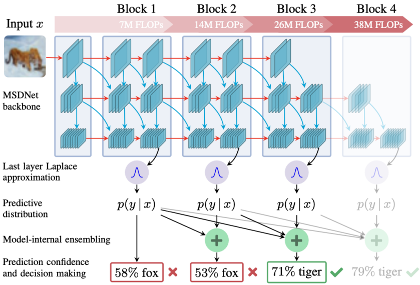


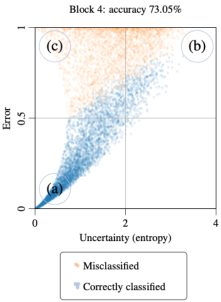
Figure: Increasing depth of dynamic neural network

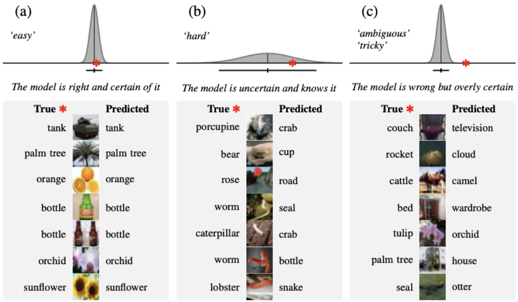
Background

- Investigates image classification under budget restrictions
  $\mathcal{D}_{\text{train}} = \{(\mathbf{x}_i, \mathbf{y}_i)\}_{i=1}^{n_{\text{train}}}$
- Budget B (FLOPs) must be distributed across a batch for highest possible accuracy
- With n_{block} intermediate classifiers, the predictive distribution:
  $p_k(\hat{\mathbf{y}}_i \mid \mathbf{x}_i),\ k = 1, 2, \ldots, n_{\text{block}}$
- Feature representation on the last linear layer
  $\phi_{i,k} = f_k(\mathbf{x}_i)$ with parameters $\theta_k = \{\mathbf{W}_k, \mathbf{b}_k\}$
- Prediction of layer k:
  $p_k(\hat{\mathbf{y}}_i \mid \mathbf{x}_i) = \text{softmax}(\hat{\mathbf{z}}_{i,k}),\ \text{where } \hat{\mathbf{z}}_{i,k} = \mathbf{W}_k \phi_{i,k} + \mathbf{b}_k$

Aleatoric and epistemic uncertainty

- Aleatoric uncertainty is related to randomness intrinsic to the task at hand and cannot be reduced.
- Epistemic uncertainty is related to our knowledge of the task and can be reduced by learning more about the task $\rightarrow$ more data





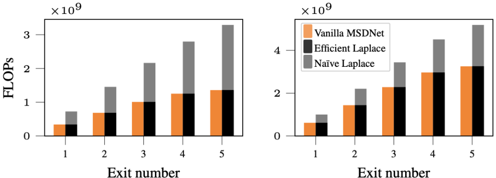
Bayesian treatment of parameters

$$p(\boldsymbol{\theta} \mid \mathcal{D}_{\text{train}}) = \frac{p(\mathcal{D}_{\text{train}} \mid \boldsymbol{\theta})\, p(\boldsymbol{\theta})}{\int_{\boldsymbol{\theta}} p(\mathcal{D}_{\text{train}}, \boldsymbol{\theta})\, d\boldsymbol{\theta}} = \frac{[\text{likelihood}] \times [\text{prior}]}{[\text{model evidence}]}$$

- Posterior distribution over the model parameters is intractable in deep learning
- Laplace approximation (second order Taylor expansion)
- MAP estimate can be found by maximising the unnormalised posterior:

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \log p(\mathcal{D}_{\text{train}} \mid \boldsymbol{\theta}) + \log p(\boldsymbol{\theta})$$

$$p(\boldsymbol{\theta} \mid \mathcal{D}_{\text{train}}) \approx \mathcal{N}(\hat{\boldsymbol{\theta}}, \mathbf{H}^{-1})$$



Method

- Last layer Laplace approximation for each intermediate classifier of the DNN
- Final prediction:

$$\hat{\mathbf{y}}_i = \frac{1}{n_{\mathrm{MC}}} \sum_{l=1}^{n_{\mathrm{MC}}} \mathrm{softmax}(\hat{\mathbf{z}}_i^{(l)})$$

- Laplace implementation has cost:

$$\mathrm{FLOPs}_{\mathrm{efficient}} = 2cn_{\mathrm{MC}} + 2p^2 + 5p + 2$$

  (c: number of classes, p: feature dimensionality, n: number of MC samples)
- Gaussian distribution:

$$p(\hat{\mathbf{z}}_i \mid \mathbf{x}_i) = \mathcal{N}(\hat{\mathbf{W}}_{\mathrm{MAP}}^{\top} \hat{\phi}_i,\ (\hat{\phi}_i^{\top} \mathbf{V} \hat{\phi}_i)\mathbf{U})$$

$$\mathbf{V}^{-1} \otimes \mathbf{U}^{-1} = \mathbf{H}^{-1}$$

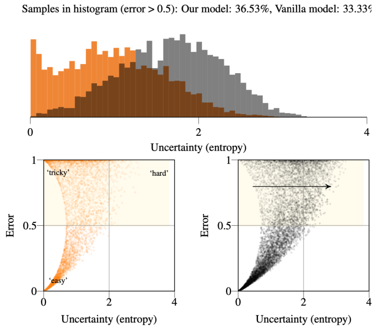
- Samples:

$$\hat{\mathbf{z}}_i^{(l)} = \hat{\mathbf{W}}_{\mathrm{MAP}}^{\top} \hat{\phi}_i + (\hat{\phi}_i^{\top} \mathbf{V} \hat{\phi}_i)^{\frac{1}{2}} (\mathbf{L}\mathbf{g}^{(l)})$$

  $\mathbf{g}^{(l)} \sim \mathcal{N}(0, \mathbf{I})$ and $\mathbf{L}$ is the Cholesky factor of $\mathbf{U}$
- Temperature scaling is recommended for well-calibrated predictions

Uncertainty

- The uncertainty should be high for the model to be able to recognize these samples as ‘tricky’, and continue their evaluation to the next block.
- For paper model these samples have a high uncertainty, while the vanilla MSDNet is overconfident.



(a) Vanilla MSDNet (ACC: 69.08%) (b) Our model (ACC: 70.90%)

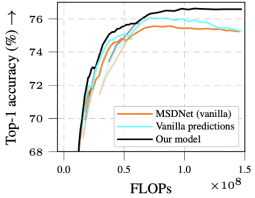
Ensamble prediction

$$p_k^{\text{ens}}(\hat{\mathbf{y}}_i \mid \mathbf{x}_i) = \frac{1}{\sum_{l=1}^{k} w_l} \sum_{m=1}^{k} w_m \, p_m(\hat{\mathbf{y}}_i \mid \mathbf{x}_i)$$

Weights w are the computational costs in FLOPs up to classifier m

- Early exiting decisions based on model predicted confidence (referred to as MIE)
- Thresholds for exiting are calculated on the validation set. Different for every layer (not included in paper)

Results



	(n_{train}, d, c, n_{batch})	CIFAR-100 (50000, 3072, 100, 64)			
		Top-1 ACC ↑	Top-5 ACC ↑	NLPD ↓	ECE ↓
Small	MSDNet (vanilla)	69.25	90.48	1.498	0.182
	+ Laplace T_{opt} σ_{opt}	69.06 −0.19	90.58 +0.10	1.208 −0.289	0.073 −0.109
	+ MIE	**69.97** +0.72	90.88 +0.40	1.218 −0.280	0.080 −0.102
	+ MIE Laplace T_{opt} σ_{opt}	69.84 +0.59	**91.09** +0.61	**1.133** −0.364	**0.017** −0.165
Medium	MSDNet (vanilla)	74.12	91.94	1.549	0.190
	+ Laplace T_{opt} σ_{opt}	73.92 −0.20	92.01 +0.06	1.070 −0.479	0.083 −0.107
	+ MIE	**75.03** +0.91	92.97 +1.03	1.011 −0.538	0.050 −0.140
	+ MIE Laplace T_{opt} σ_{opt}	74.99 +0.86	**93.23** +1.29	**0.944** −0.605	**0.026** −0.164
Large	MSDNet (vanilla)	75.36	92.78	1.475	0.178
	+ Laplace T_{opt} σ_{opt}	75.32 −0.05	92.83 +0.05	0.996 −0.479	0.075 −0.103
	+ MIE	76.32 +0.95	93.50 +0.72	0.949 −0.525	0.061 −0.117
	+ MIE Laplace T_{opt} σ_{opt}	**76.34** +0.98	**93.84** +1.05	**0.885** −0.590	**0.025** −0.152

Figure: Enter Caption

- Improving confidence margins in teacher model makes for more efficient knowledge transfer in early exit training
- Implementing confidence margins in student model can increase accuracy

For our use

- Improving confidence margins in teacher model makes for more efficient knowledge transfer in early exit training
- Implementing confidence margins in student model can increase accuracy

References

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Thank You