7. (a) The CDF of the time T it takes a bank teller to serve a customer is defined by 7

$$F_T(t) = \begin{cases} 0, & t < 2 \\ A(t-2), & 2 \le t < 6 \\ 1, & t \ge 6 \end{cases}$$

i. What is the value of A?
ii. With the above value of A, what is $P[T > 4]$?
iii. With the above value of A, what is $P[3 \le T \le 5]$?
iv. Find the density function of this CDF.
v. Find the expected value, if it exists.
vi. Find the variance, if it exists.
vii. Find second order factorial moment, if it exists.

(b) Suppose a random sample of size n is drawn from the Bernoulli distribution. 7

What is the maximum likelihood estimate of p, the success of probability?

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4. (a) Assume that we roll two dice and define three events A, B, and C, where A is the event that the first die is odd, B is the event that the second die is odd, and C is the event that the sum is odd. Show that these events are pairwise independent but the three are not independent. 7

(b) Ken was watching some people play poker, and he wanted to model the PMF of the random variable N that denotes the number of plays up to and including the play in which his friend Joe won a game. He conjectured that if p is the probability that Joe wins any game and the games are independent, then the answer the following: 7

i. Give PMF of N.
ii. Find CDF of N.
iii. Find the expected value of N.
iv. Find the variance of N.
v. Find the moment generating function of N.

5. Consider the following set of the data:

2, 3, 5, 5, 6, 7, 7, 7, 8, 9, 9, 10, 10, 11, 12, 14, 14, 16, 18, 18, 22, 24, 26, 28, 28, 32, 45, 50, and 55.

i. Plot the box and whiskers diagram of the set. 3
ii. Determine if there are any outliers in the data set. 3
iii. Plot the histogram of the data set 2
iv. Plot the frequency polygon of data set. 2
v. Determine the skewness of the data. 4

6. (a) Data were collected for a random variable Y as function of another random variable X. The recorded (x, y) pairs are as follows: 7

(3, 2), (5, 3), (6, 4), (8, 6), (9, 5), (11, 8)

i. Plot the scatter diagram for these data.
ii. Find the linear regression line of y on x that best fits these data.
iii. Estimate the value of y when $x = 15$.

(b) Assume that the random variable Y is estimated from the random variable X by the following linear function of X: 7

$$\hat{Y} = aX + b$$

Determine the values of a and b that minimize the mean squared error.

REFNO070213

Roll No. 23419CMP026

M.Sc. in COMPUTER SCIENCE SEMESTER I EXAMINATION 2023-24

CS - 201 : Probability and Statistics

Time : Three hours **Max. Marks : 70**

(WRITE YOUR ROLL NO. AT THE TOP IMMEDIATELY ON THE RECEIPT OF THIS QUESTION PAPER)

Note: Attempt total **Five** question including **compulsory question 1**. The figures in right margin indicate the marks.

1. (a) Give Kolmogorov axiomatic definition of probability. 2

(b) Define conditional probability. 2

(c) Discuss concept of statistical independence of events. 2

(d) Discuss the concept of total probability. 2

(e) Define random variable. What do you understand by continuous and discrete random variable? Give one example of each. 3

(f) Define probability mass function (PMF), probability density function (pdf) 3

2. (a) Define Bernoulli trial and give associated random variable. Find the expected value, variance, moment generating function, and factorial moment of second order. 8

(b) A coin is tossed 10 times. Given that there are 6 heads in the 10 tosses, what is the expected number of heads in the first 5 tosses? 2

(c) Four fair coins are tossed. If the outcomes are assumed to be independent, find the PMF of the number of heads obtained. 4

3. (a) Probability of the events A, B, and $A \cap B$ are 0.6, 0.3 and 0.2 respectively. Find the probability of event $\overline{A} \cap \overline{B}$. 2

(b) Suppose a random variable X has a probability density function (pdf) given by: 9

$$f_X(x) = \begin{cases} k(x-3)(1-x), & 1 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

i. Find the value of k that makes this function a pdf.

ii. Find mean and variance of X.

iii. Find the probability of events $(-1 \le X \le 1)$, $(1 \le X \le 3)$, and $(0 \le X < \infty)$

(c) A random variable Y is defined using random variable X as $Y = aX + b$, where a and b are constant. Show that $E[Y] = aE[X] + b$, E[.] stands for expectation. 3

P.T.O.