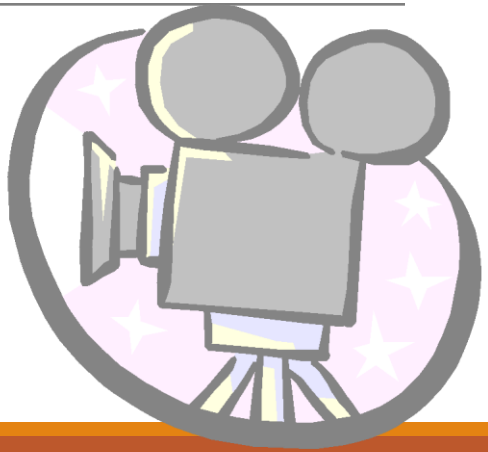
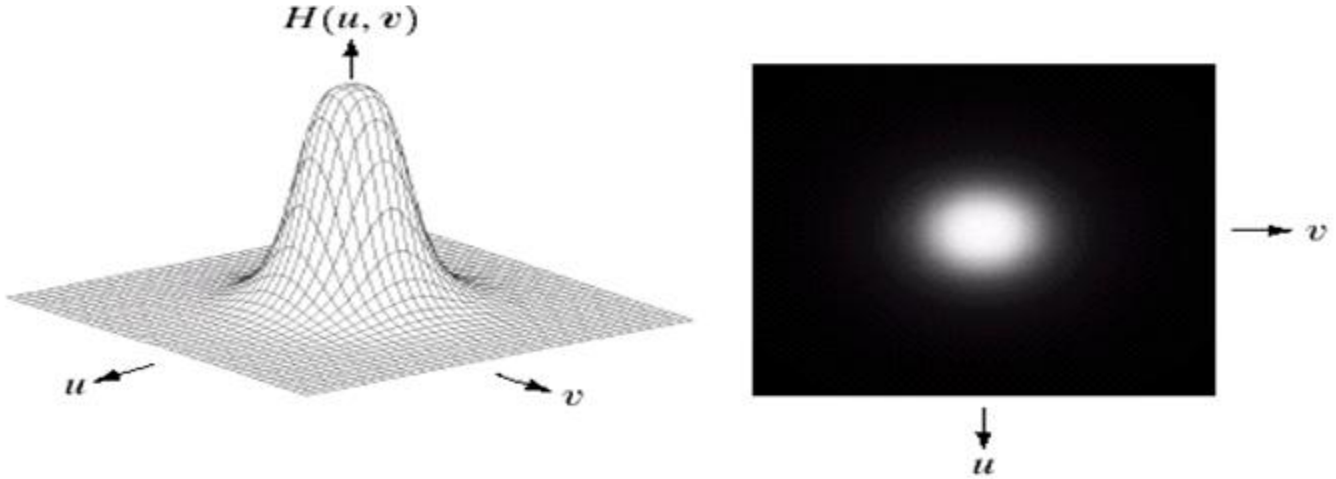
Image Processing

CS-317/CS-341



Outline

- Image Enhancement in the Frequency Domain
  - Smoothing Filters
    - Ideal
    - Butterworth
    - Gaussian
  - Sharpening Filters
    - Ideal
    - Butterworth
    - Gaussian



Smoothing Frequency-domain filters: Ideal Lowpass filter

- Cutoff all high frequency components of FT that are at a greater distance than a specified distance D_0 from the origin of the transform.

$$H(u,v) = \begin{cases} 1, & if \ D(u,v) \le D_0 \\ 0, & if \ D(u,v) > D_0 \end{cases}$$

$D(u,v)$: Distance from (u, v) to the origin of frequency rectangle

Centre of frequency rectangle (M/2, N/2) is origin.

$$D(u,v) = \left[(u - M/2)^2 + (v - N/2)^2 \right]^{1/2}$$

Ideal filter: All frequencies inside the circle of radius D_0 are passed with no attenuation, while all frequencies outside of circle completely attenuated.

The point of transition between H(u,v)=1 and H(u,v)=0 is called the **cutoff frequency.**

Smoothing Frequency-domain filters: Ideal Lowpass filter

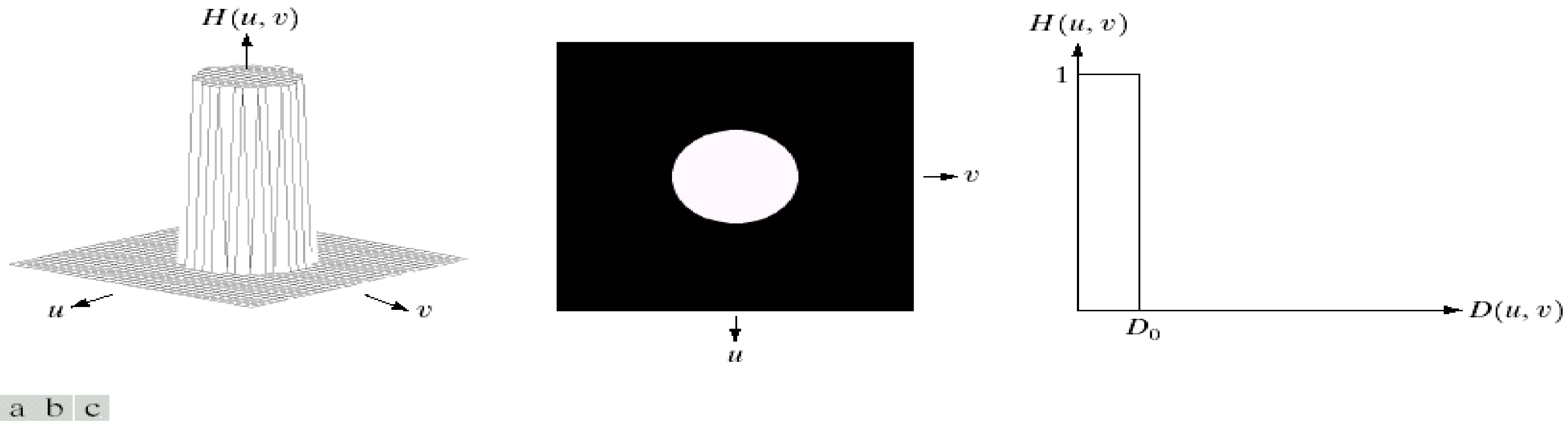


FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Example

- Thus reciprocal nature of $H(u,v)$ and $h(x,y)$ with convolution is responsible for blurring and ringing.

Narrow Filter In Frequency Domain → More severe blurring and ringing

Blurring : Low frequencies are removed

Ringing : Cutoff is too sharp

Objective is to achieve blurring with little or no ringing

Butterworth Lowpass Filter: BLPF

$$H(u,v) = \frac{1}{1+\left[D(u,v)/D_0\right]^{2n}}$$

$D(u,v)$: Distance from (u, v) to the origin of frequency rectangle

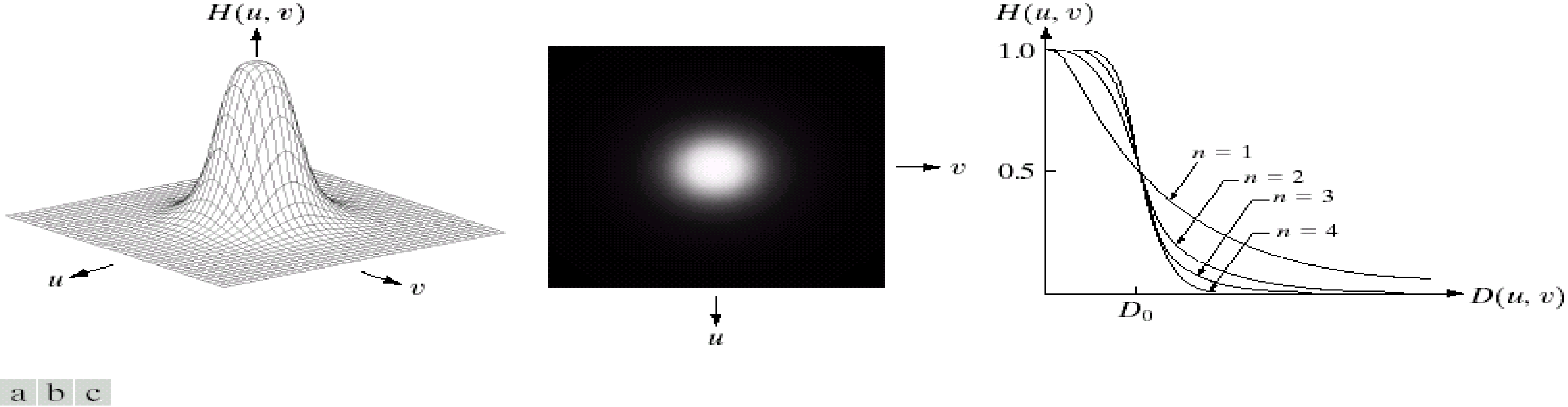


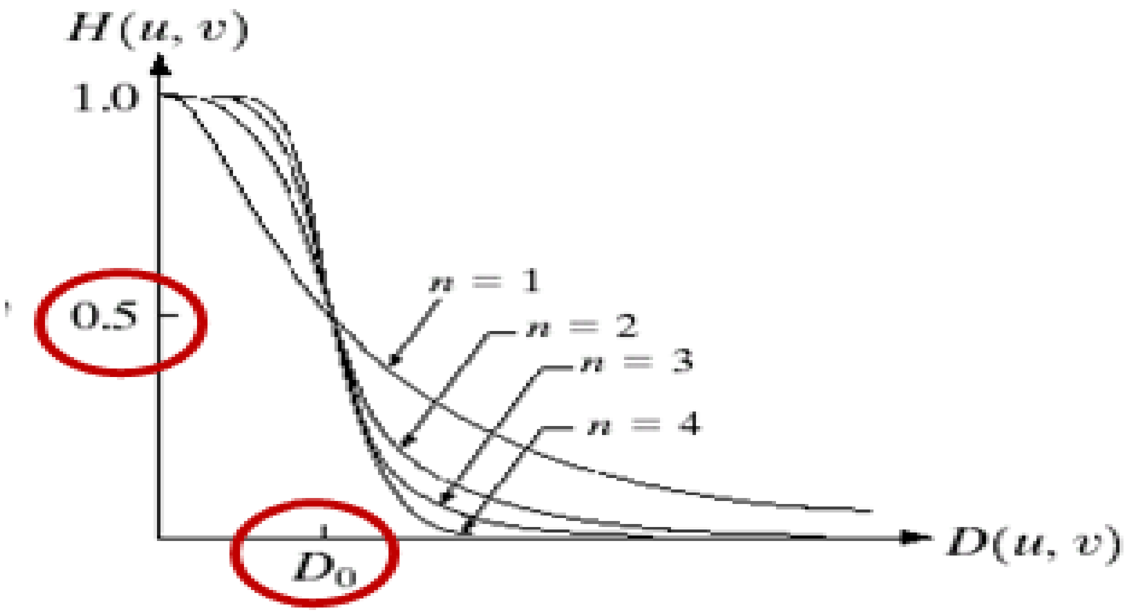
FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Butterworth Lowpass Filter: BLPF

- Unlike the ILPF, the BLPF transfer function does not have a sharp discontinuity that establishes a clear cutoff between passed and filtered frequencies.

- For smooth transfer function, cutoff frequency locus at points for which $H(u,v)$ is down to a certain fraction of its maximum value is customary.

- $H(u,v)$ = 0.5 when $D(u,v) = D_0$



Example

ILPF

FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

BLPF

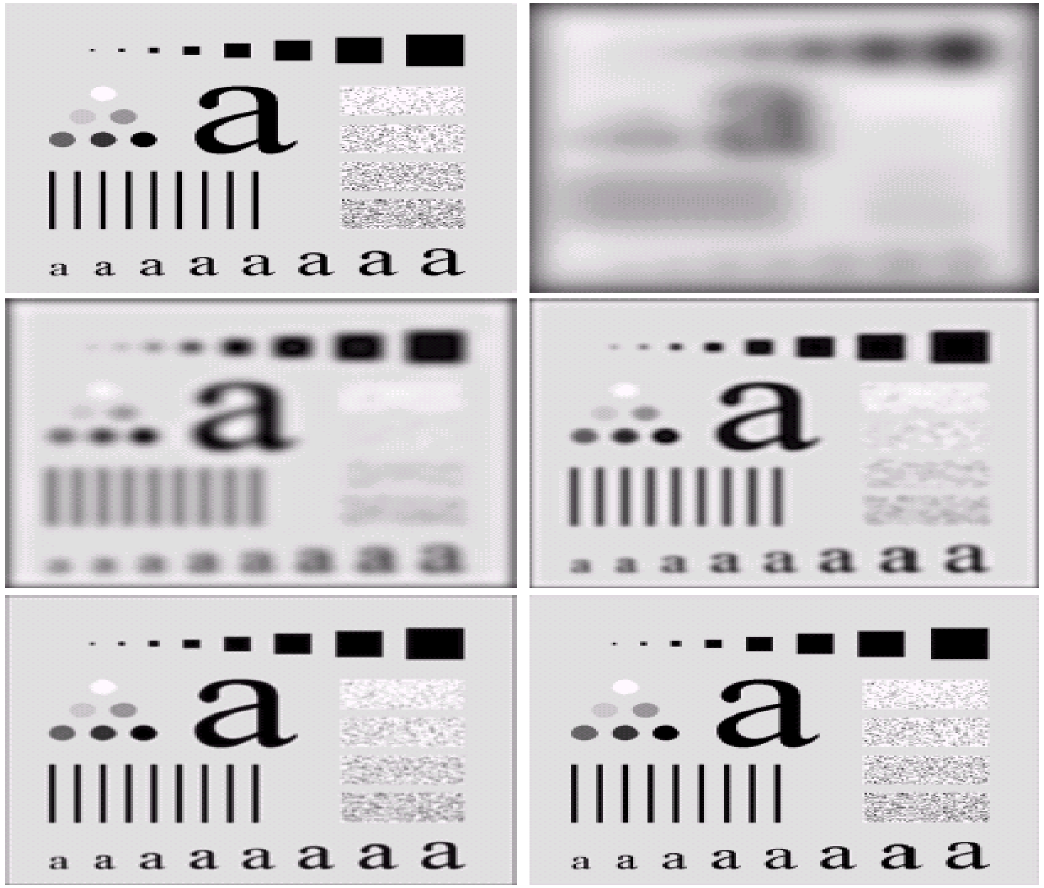
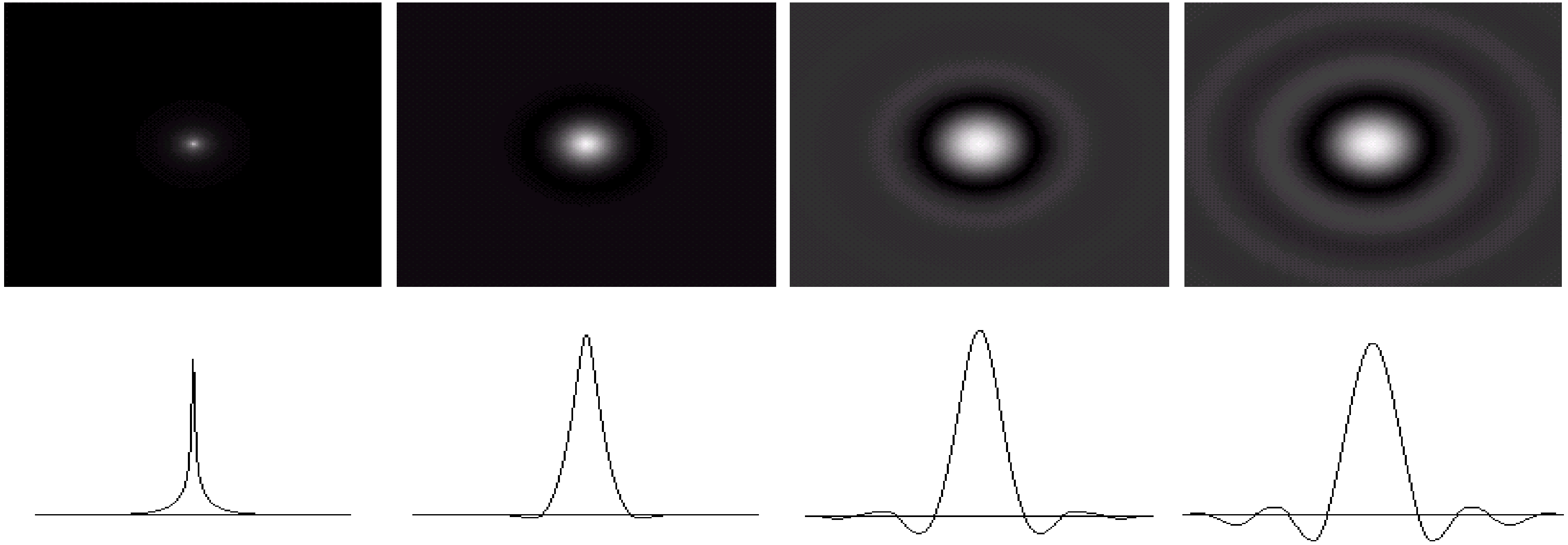


FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Spatial representation of BLPFs



a b c d

FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Spatial representation of BLPFs

- BLPF of order 1 has neither ringing nor negative value.
- Order 2 have mild ringing and small negative value but less than ILPF.
- Ringing in BLPF becomes significant for higher – order filters.
- BLPF of order 20 already have characteristics of the ILPF.

a b c d

FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Gaussian Lowpass Filter: GLPF

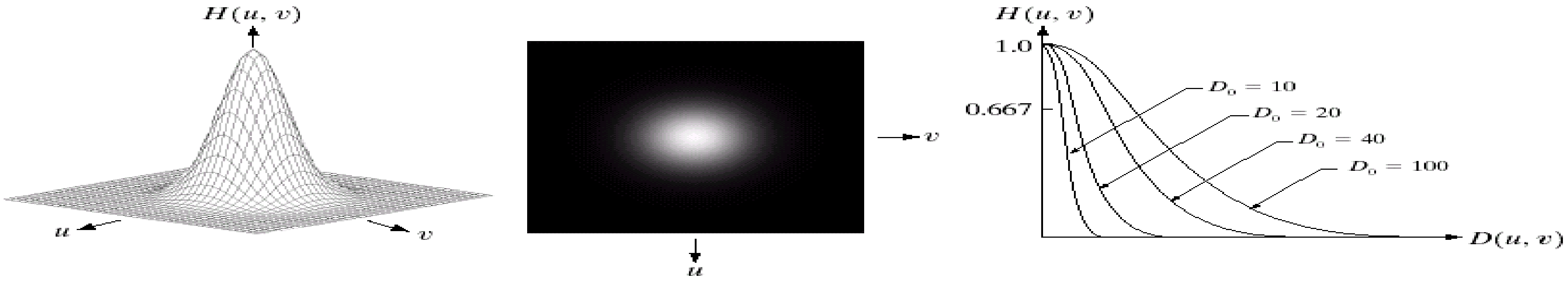
- Filter transfer function: $$H(u,v) = e^{-D^2(u,v)/2\sigma^2};$$

σ : Measure of spread of Gaussian curve

$D(u,v)$: Distance from (u, v) to the origin of frequency rectangle

- Taking $\sigma = D_0$ cutoff frequency: $$H(u,v) = e^{-D^2(u,v)/2D_0^2};$$

- Inverse FT of Gaussian low pass filter is Gaussian. Spatial GLPF will have no ringing.



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Example

ILPF **BLPF** **GLPF**

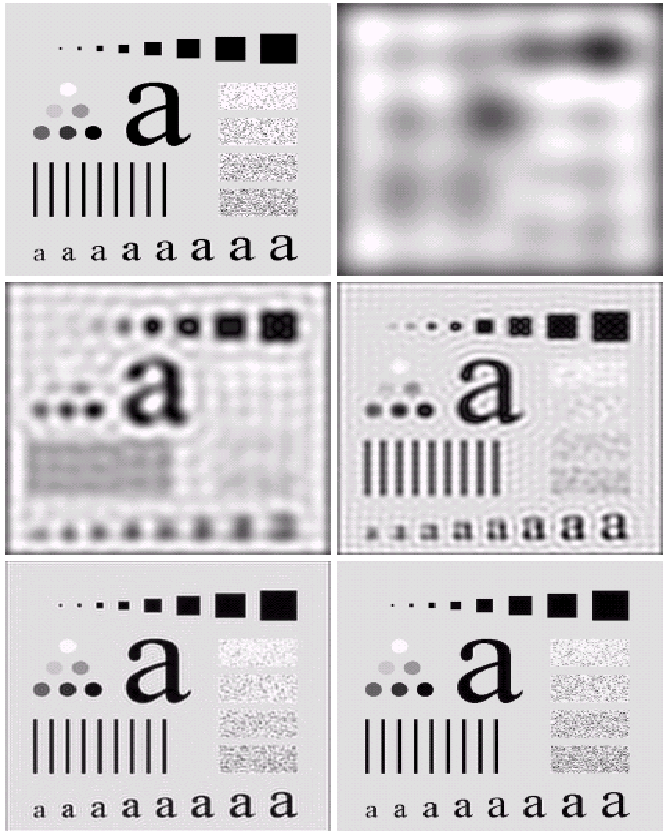


FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

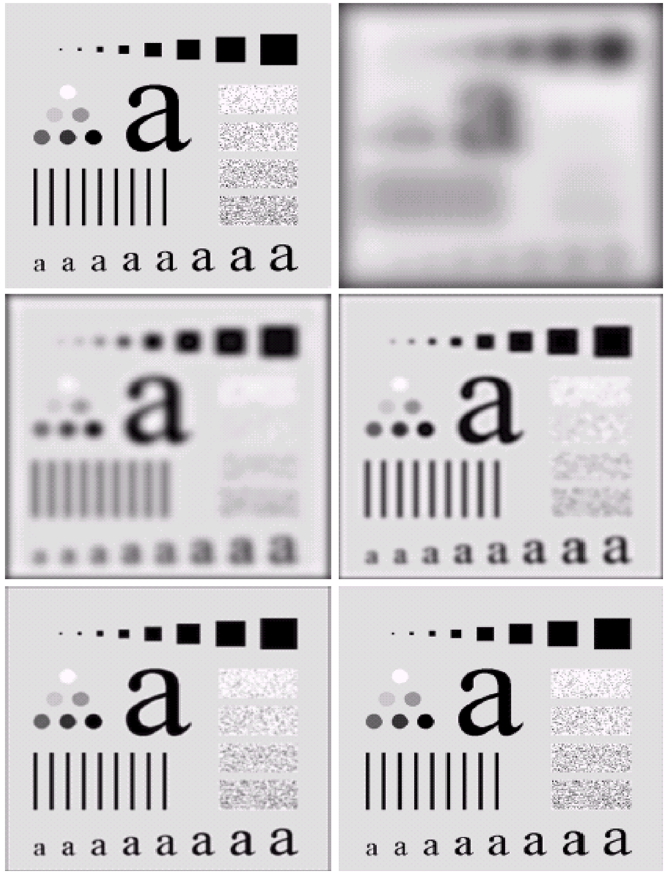


FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

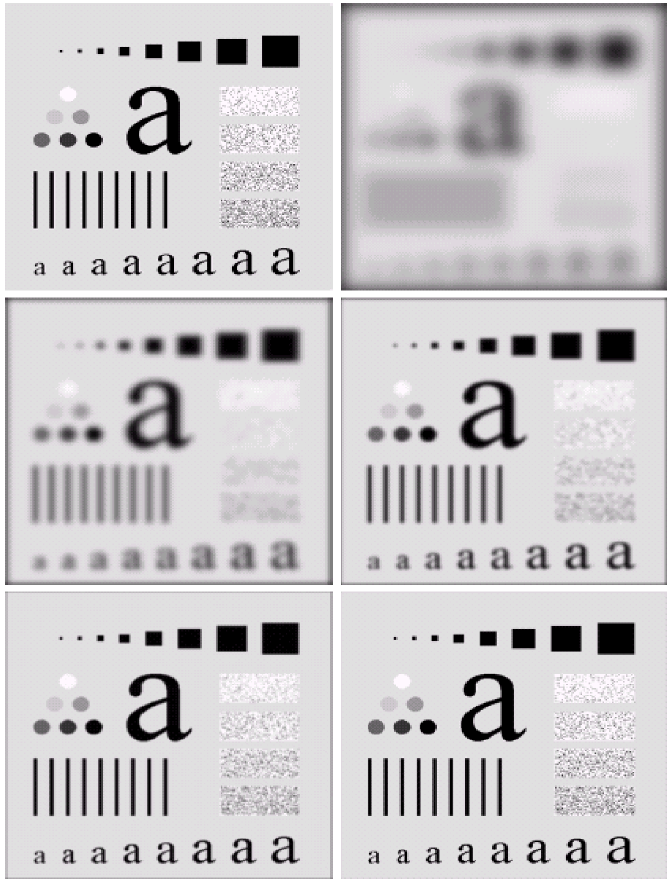


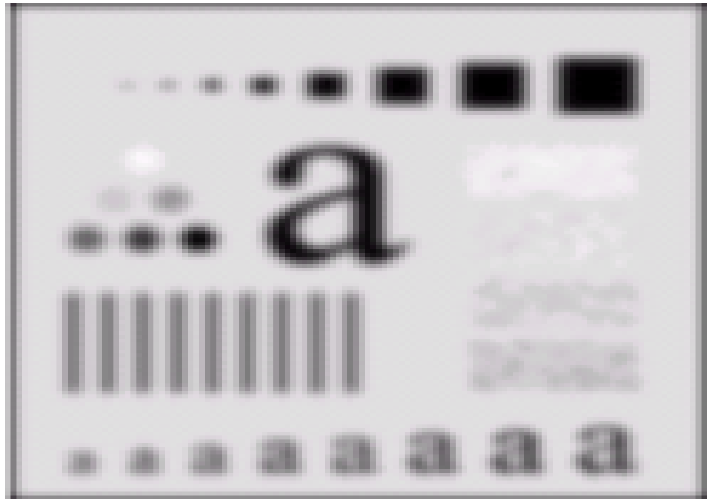
FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

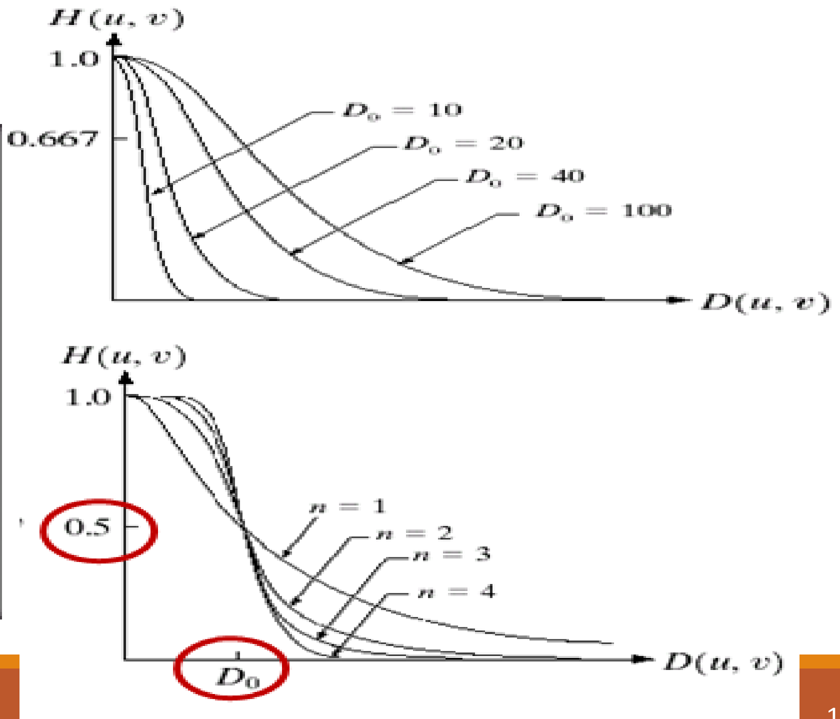
BLPF vs. GLPF

- In case of BLPF smooth transition in blurring as a function of increasing cutoff frequency.
- The GLPF does did not achieve as much smoothing as the BLPF of order 2 of same cutoff frequency.

BLPF (c)

GLPF (c)



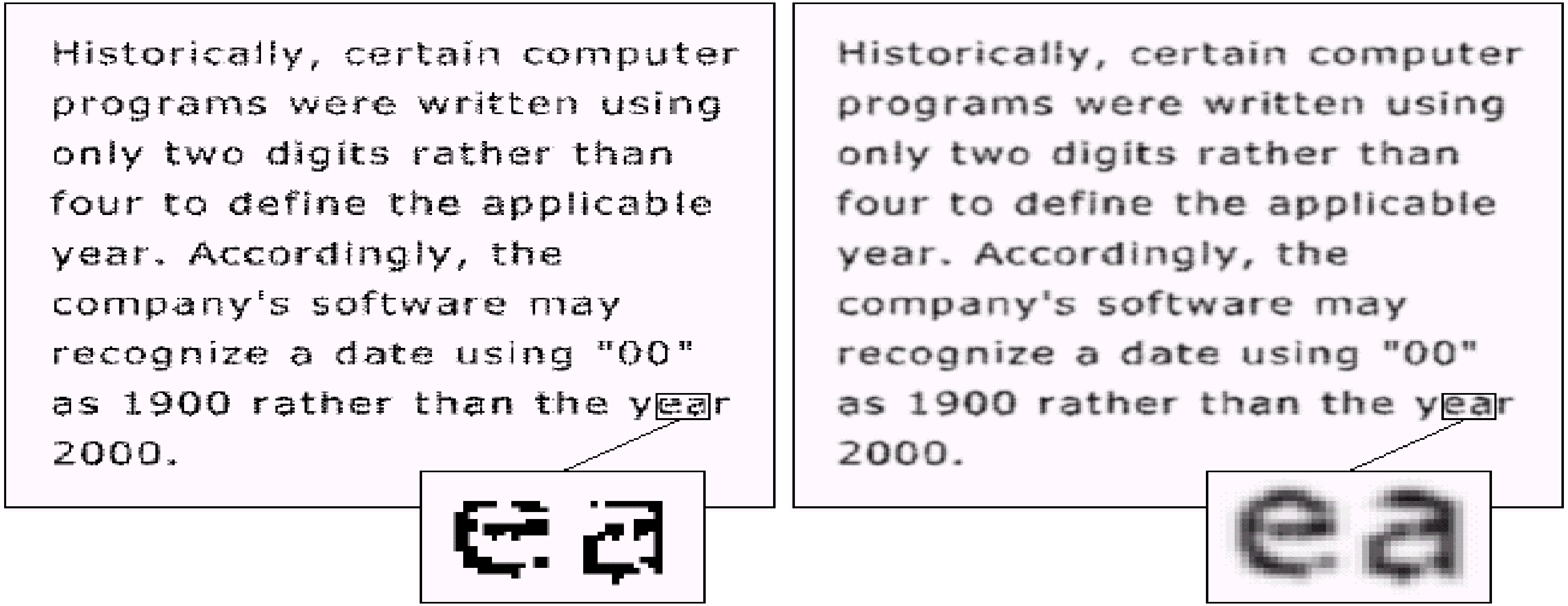


Example

Bridge small gaps in the input image by blurring

a b

FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



GLPF with D_0 =80

Example

Unsharp masking: Printing and Cosmetic industry

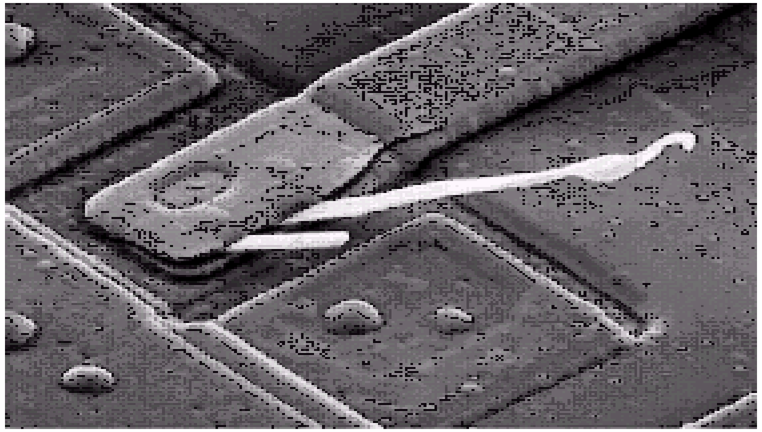
Produce a smoother, softer-looking result from a sharp original.

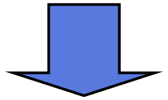
a b c

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

Notch filter

- this filter is to force the F(0,0) to 0,which is the average value of an image (dc component of the spectrum)
- the output has prominent edges
- in reality the average of the displayed image can't be zero as it needs to have negative gray levels. the output image needs to scale the gray level





$$H(u,v) = \begin{cases} 0 & \text{if } (u,v) = (M/2, N/2) \\ 1 & \text{otherwise} \end{cases}$$

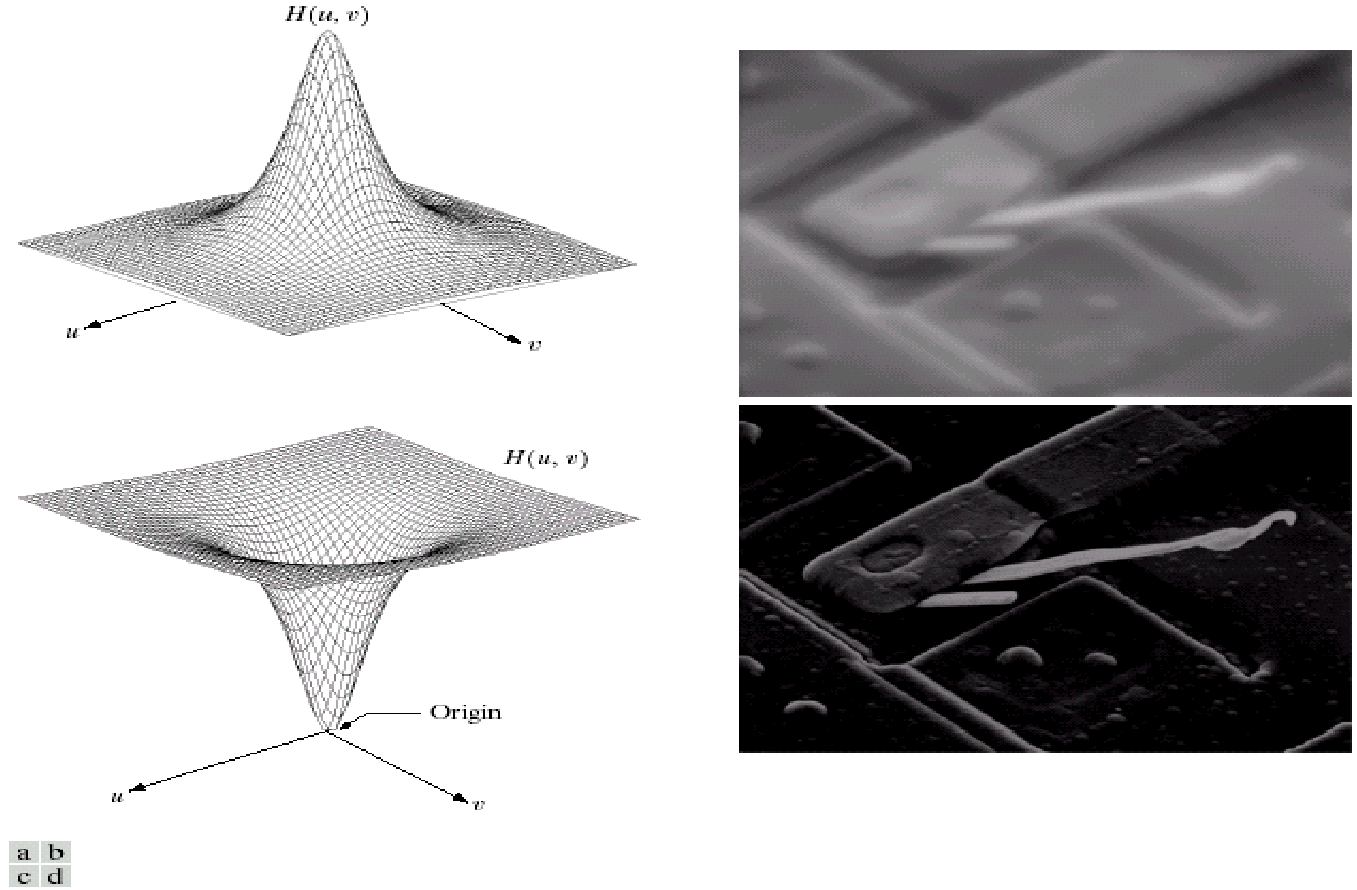


FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

Low pass filter

High pass filter

Comparison between filtering in spatial and frequency domains

- **Filtering in the frequency domain**

  - Significant degree of intuitiveness regarding how to specify filters.

  - More computational efficiency for a large window size.

  - FD can be viewed as a "laboratory" in which we take advantage of the correspondence between frequency content and image appearance.

  - Some enhancement method is extremely difficult / impossible to formulate in spatial domain but almost trivial in frequency domain.

  - From frequency domain we can get the spatial domain but almost trivial in frequency domain.

- **Filtering in the spatial domain**

  - We often specify small spatial mask that attempt to capture the essence of the full filter function

Sharpening Frequency Domain Filter:

Highpass Filter: Perform the reverse operation of the low pass filter.

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

$H_{lp}(u,v)$: Transfer function of the corresponding low pass filter

When low pass filter cutoff the frequencies, the high pass filter passes them and vice versa.

Sharpening Frequency Domain Filter:

Ideal highpass filter

$$H(u,v)=\begin{cases}0 & \text{if } D(u,v)\le D_0\\ 1 & \text{if } D(u,v)>D_0\end{cases}$$

Butterworth highpass filter

$$H(u,v)=\frac{1}{1+[D_0/D(u,v)]^{2n}}$$

Gaussian highpass filter

$$H(u,v)=1-e^{-D^2(u,v)/2D_0^2}$$

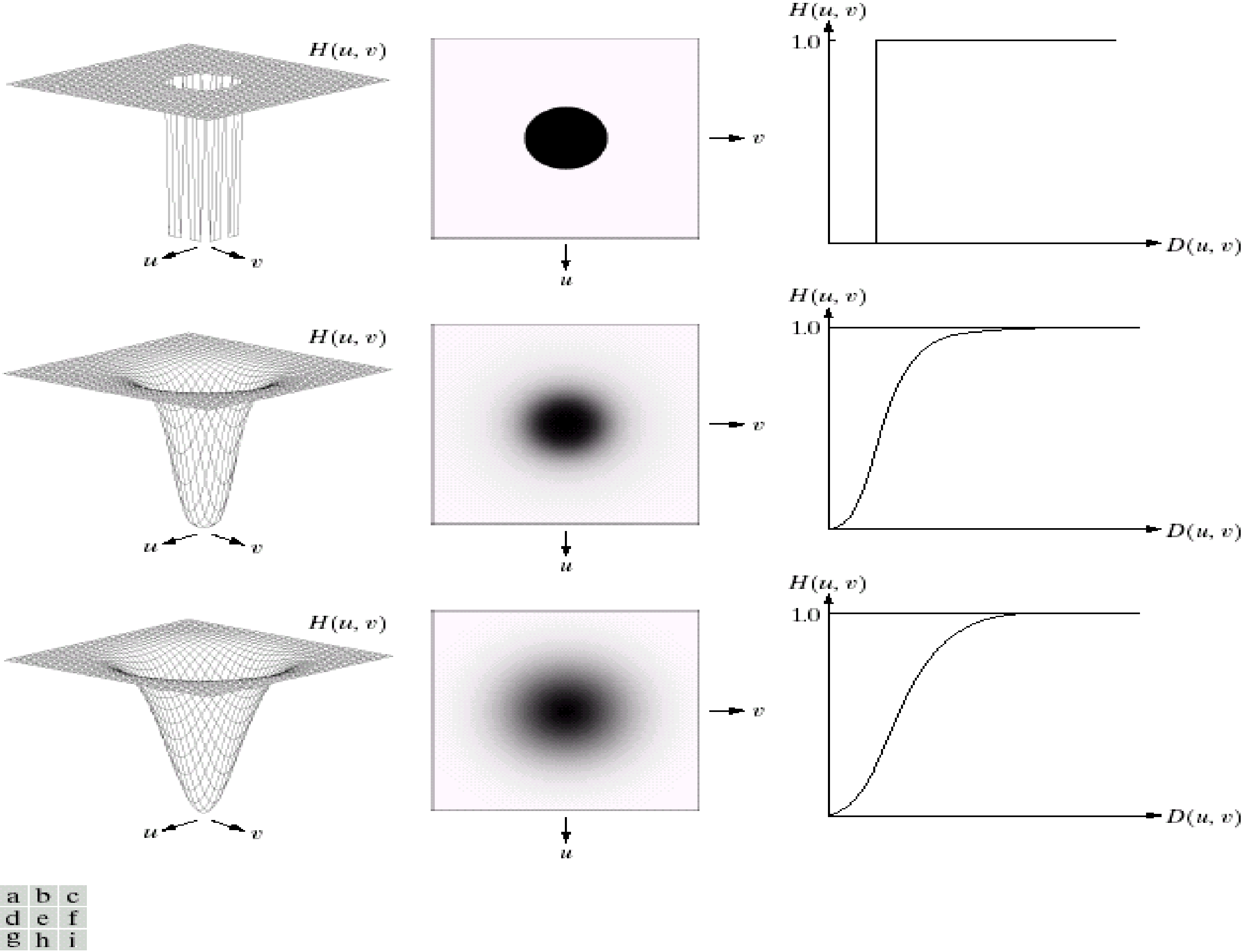


FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Spatial representation of Ideal, Butterworth and Gaussian highpass filters

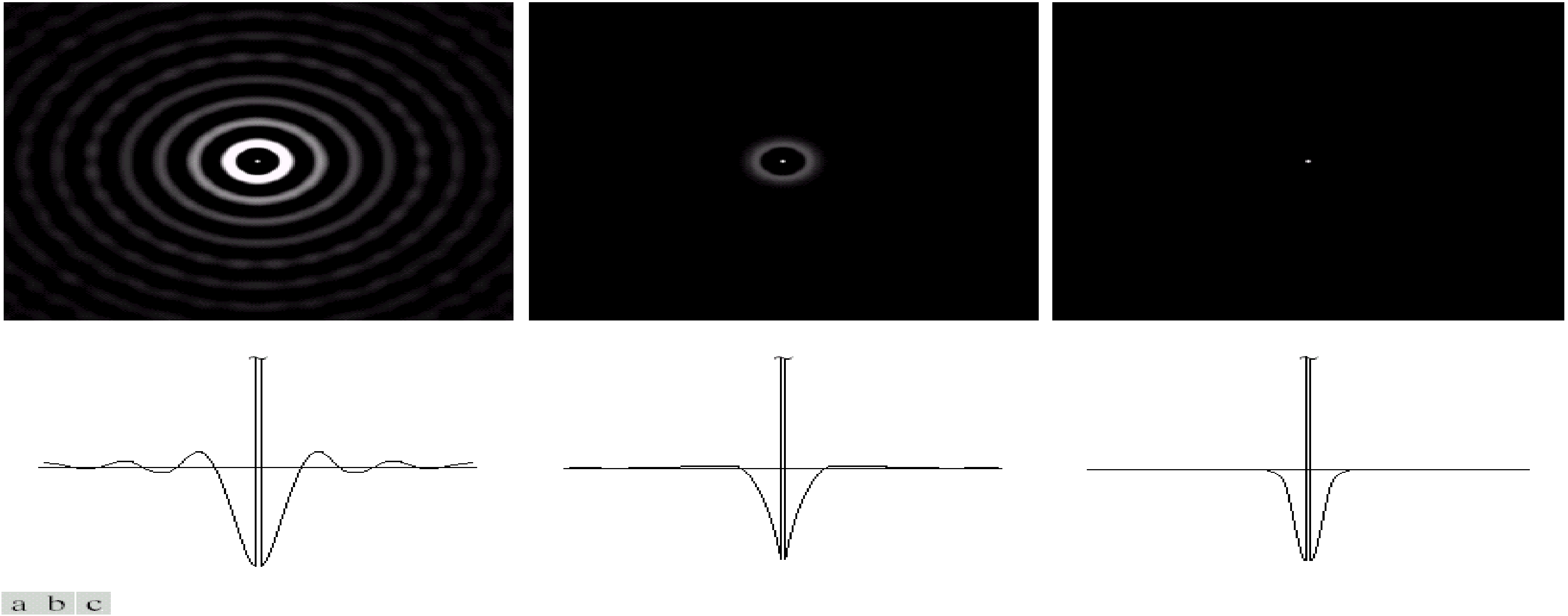
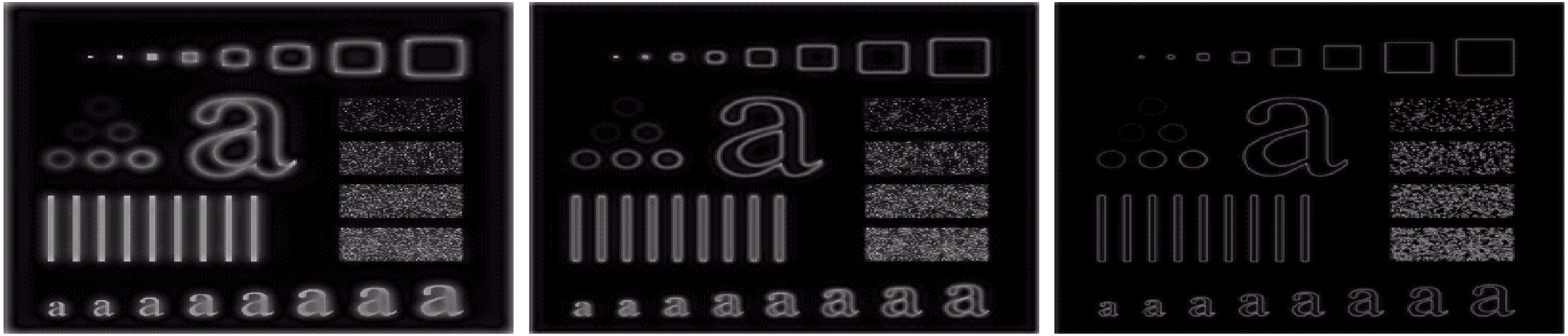


FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

Example: result of IHPF, BHPF

a b c

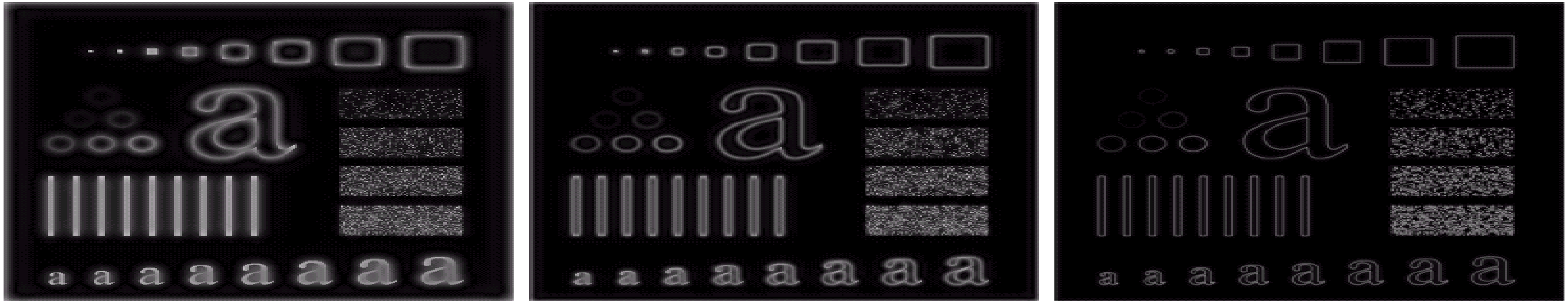
FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).



a b c

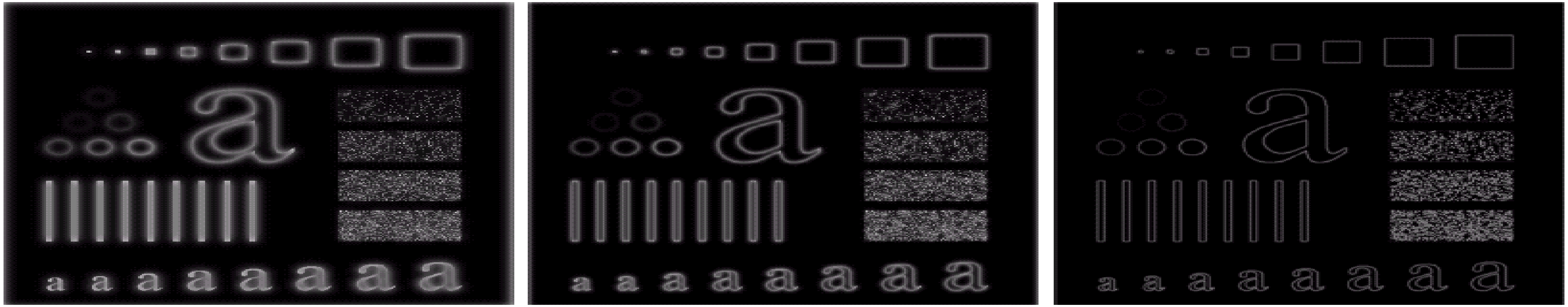
FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

Example: result of BHPF,GHPF



a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.



a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

Example: result of IHPF, BHPF, GHPF

Increasing order of smoothness

IHPF → BHPF → GHPF

Suggested Readings

- **Digital Image Processing by Rafel Gonzalez, Richard Woods, Pearson Education India, 2017.**

- **Fundamental of Digital image processing by A. K Jain, Pearson Education India, 2015.**

Thank you