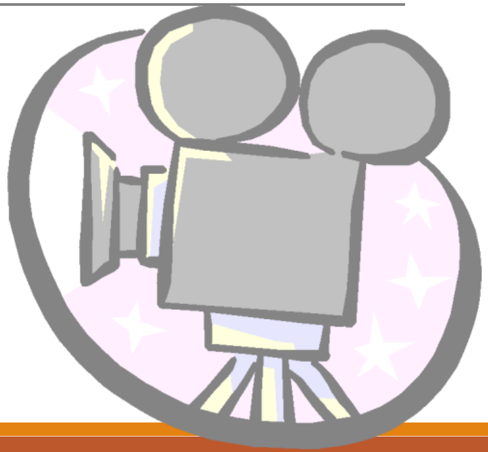
Image Processing

CS-317/CS-341



Outline

- Image Enhancement in the Frequency Domain
  - Smoothing Filters
    - Ideal
    - Butterworth
    - Gaussian

Basic steps for filtering in the frequency domain

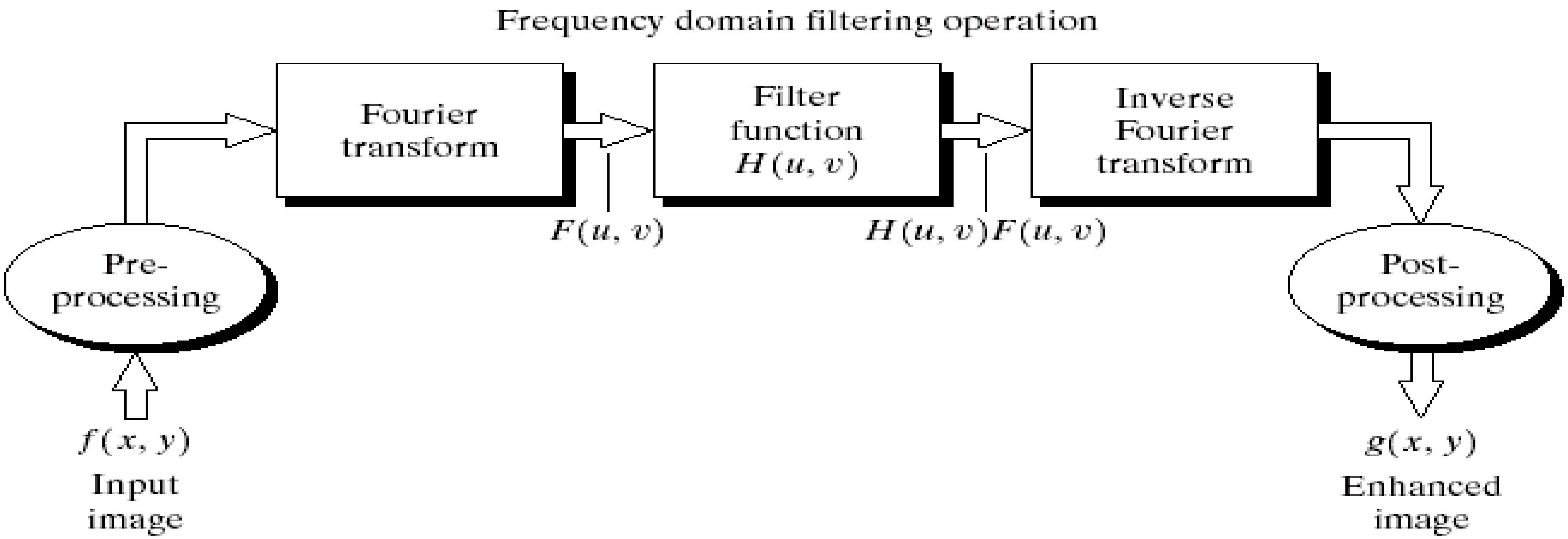


FIGURE 4.5 Basic steps for filtering in the frequency domain.

Basics of filtering in the frequency domain

1. multiply the input image by $(-1)^{x+y}$ to center the transform to u = M/2 and v = N/2 (if M and N are even numbers, then the shifted coordinates will be integers)
2. compute F(u,v), the DFT of the image from (1)
3. multiply F(u,v) by a filter function H(u,v)
4. compute the inverse DFT of the result in (3)
5. obtain the real part of the result in (4)
6. multiply the result in (5) by $(-1)^{x+y}$ to cancel the multiplication of the input image.

Basics of filtering in the frequency domain

- **Low frequencies**

  - Smooth Areas

- **High Frequency**

  - Edge, Texture, noise

- **Low Pass Filter**

  - Pass signal of frequency around DC component(zero frequency)

- **High Pass Filter**

  - Stop signal of frequency around zero frequency.

- **Band Pass Filter**

  - Pass signal having frequency in given range.

Smoothing Frequency-domain filters

- **High Frequency Component in FT:**

edge, sharp transitions (such as noise)

⇩

Smoothing (blurring) is achieved in the frequency domain by attenuation of a specified range of high frequency components in the transform of a given image

- **Basic Filtering Model in Frequency Domain:**

$$G(u,v) = H(u,v)F(u,v)$$

$F(u,v)$: FT of image to be smoothed

$H(u,v)$: Filter Transfer Function

Smoothing Frequency-domain filters: Ideal Lowpass filter

- Cutoff all high frequency components of FT that are at a greater distance than a specified distance D_0 from the origin of the transform.

$$H(u,v) = \begin{cases} 1, & if \;\; D(u,v) \le D_0 \\ 0, & if \;\; D(u,v) > D_0 \end{cases}$$

$D(u,v)$: Distance from (u, v) to the origin of frequency rectangle

Centre of frequency rectangle (M/2, N/2) is origin.

$$D(u,v) = \left[(u - M/2)^2 + (v - N/2)^2 \right]^{1/2}$$

Ideal filter: All frequencies inside the circle of radius D_0 are passed with no attenuation, while all frequencies outside of circle completely attenuated.

The point of transition between H(u,v)=1 and H(u,v)=0 is called the **cutoff frequency.**

Smoothing Frequency-domain filters: Ideal Lowpass filter

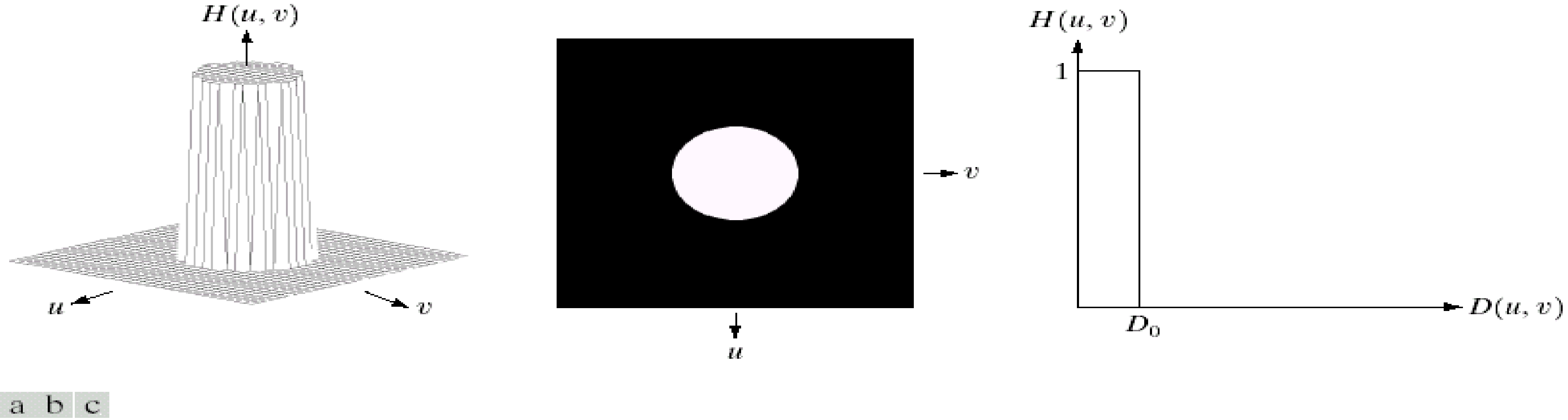


FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Smoothing Frequency-domain filters: Ideal Lowpass filter

- ILPF is compared by studying the behavior as a function of the same cutoff frequency.

- Total power:

Where, $P(u, v)=|F(u, v)|^2$

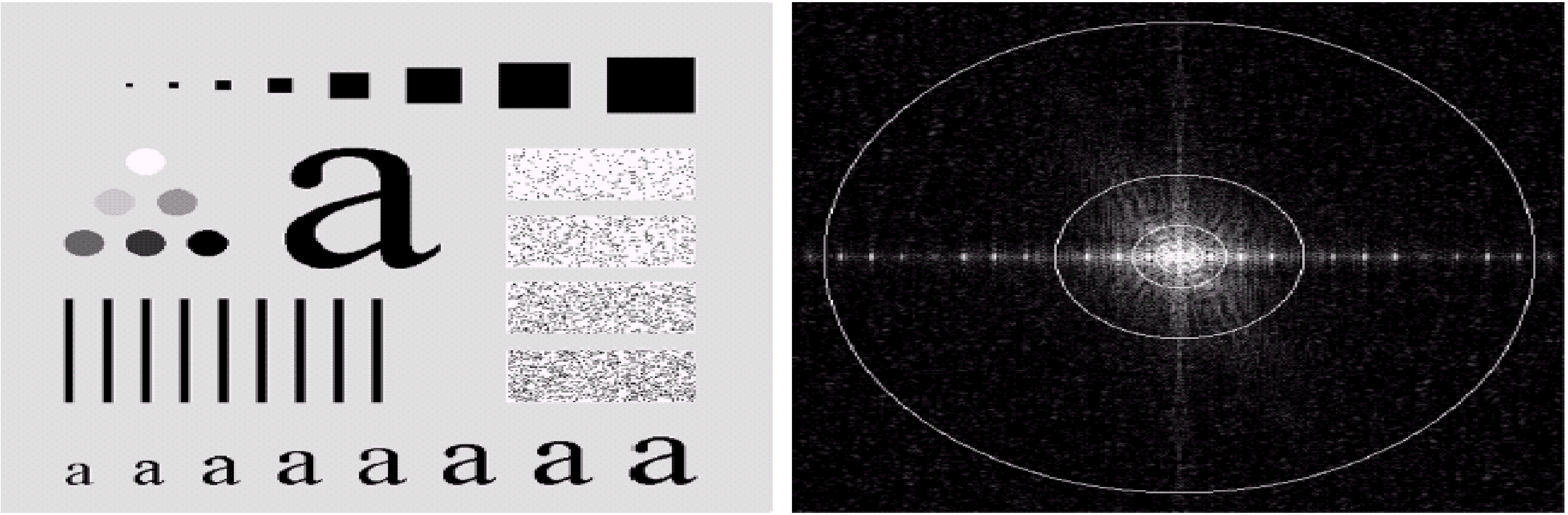
$$P_T = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} P(u,v)$$

- A circle of radius *r* with origin at the centre of the frequency rectangle encloses the percentage power:

$$\alpha = 100\left[\sum_u \sum_v P(u,v) \Big/ P_T \right]$$

- Summation is taken over the values of (u,v) that lie inside the circle or on its boundary.

image power circles



a b

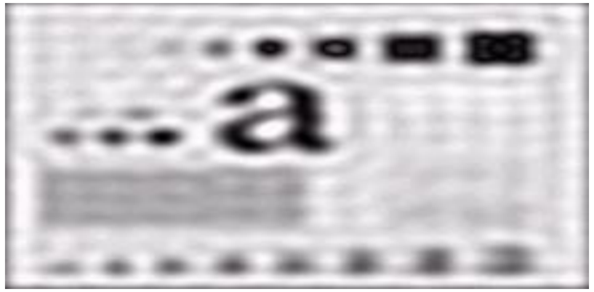
FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

Result of ILPF

a	b	c
d	e	f

Original

Cutoff Freq. at radii 5, power removed 8%



Cutoff Freq. at radii 15, power removed 5.4%

Cutoff Freq. at radii 30, power removed 3.6%

Cutoff Freq. at radii 80, power removed 2%

Cutoff Freq. at radii 230, power removed 0.5%

Result of ILPF

c	d	e

- (b) is useless for all practical purposes, unless objective of blurring is eliminate all fine details, except the blobs representing the largest object.

- Severe blurring in (b) indicate that most of the sharp information detail is contained in 8% of the power removed by the filter.

- As the filter radius is increases, less and less power is removed, resulting less severe blurring.

- (c) –(e) have ringing effect , even though 2% of the total power was removed, it becomes finer in texture as the amount high freq. comp. increases.

Cutoff Freq. at radii 15, power removed 5.4%

Cutoff Freq. at radii 30, power removed 3.6%

Cutoff Freq. at radii 80, power removed 2%

Result of ILPF

c	d	e

- Ringing behavior is characteristic of the ideal filter.

- 99.5% case shows very slight blurring in the noisy squares, but in most part, it is very close to original. Little edge information is contained in upper 0.5% of spectrum.

Original

Cutoff Freq. at radii 230, power removed 0.5%

- Ideal filter is not very practical, it can not be implemented in H/w, however it can be implemented on computer, and use for developing filtering concept.

Blurring and Ringing by ILPF

- Ringing behavior is characteristic of the ideal low pass filter.
- Filtering process is related to convolution process as:

$$G(u,v) = H(u,v)F(u,v)$$

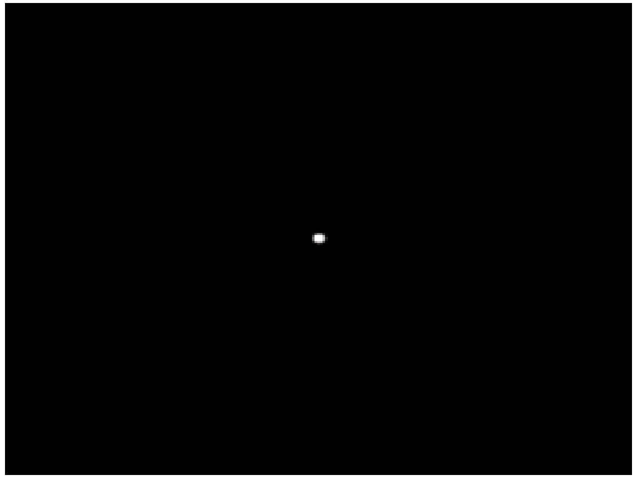
- $H(u,v)$ is filter function and F and G are FT of original image $f(x,y)$ and filtered / blurred image $g(x,y)$.
- Equivalent processes in spatial domain is convolution:

$$g(x,y) = h(x,y) \otimes f(x,y)$$

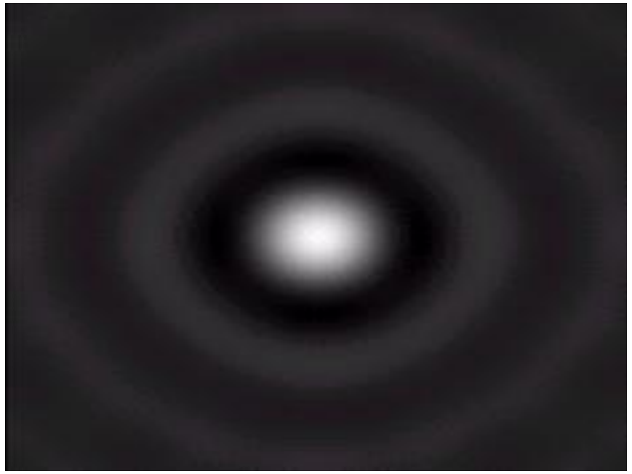
$h(x,y)$: inverse FT of filter transfer function $H(u,v)$.

Example

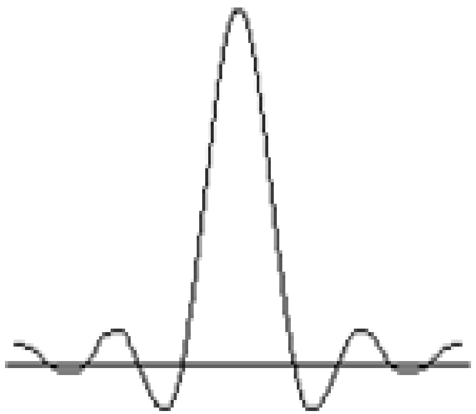
a	b	c



FD ILPF of radius 5



Spatial filter corresponding (a)



Horizontal central scan of (b)

$h(x,y)$ has two component :

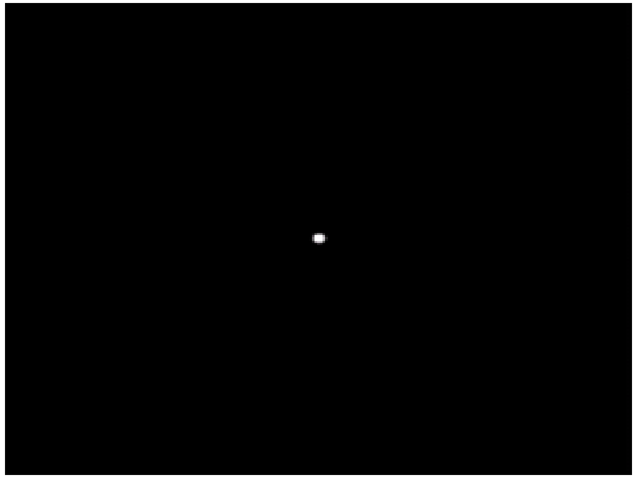
i. Dominant central at the origin : Responsible for blurring

ii. Concentric, circular components about the center component:

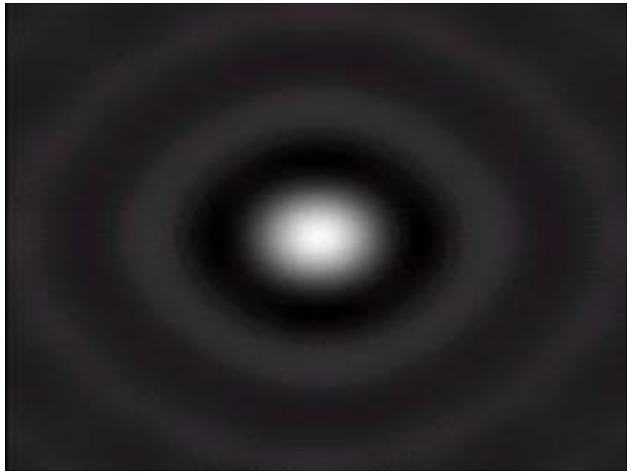
Responsible for ringing

Example

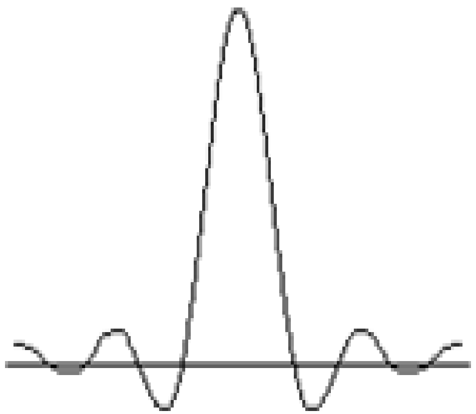
a	b	c



FD ILPF of radius 5



Spatial filter corresponding (a)



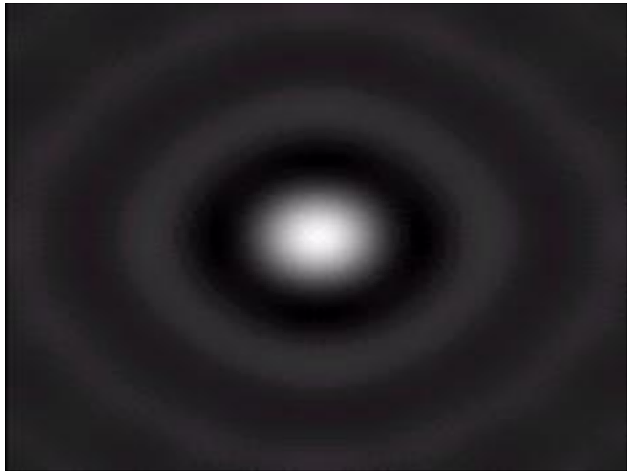
Horizontal central scan of (b)

- Radius of central component is inversely proportional to the cutoff frequency of ideal filter.
- Number of circle per unit distance from the origin is inversely proportional to the cutoff frequency of ideal filter.

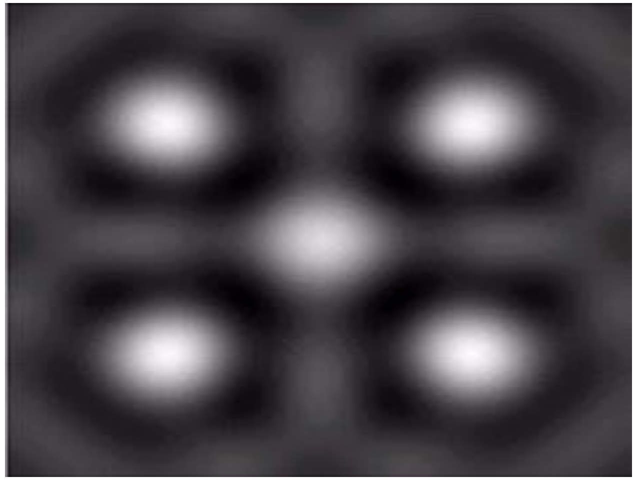
Example

a	b	c

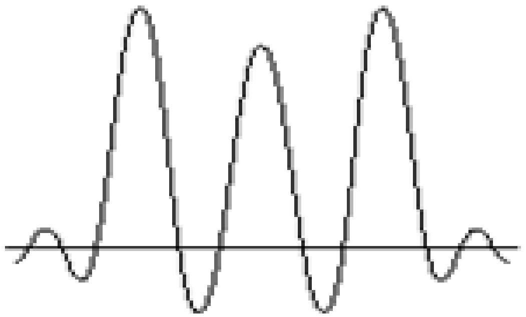
5 impulse in spatial domain
simulating the values of 5 pixel



Spatial filter



Convolution of (a) with (b)



Diagonal scan line of (c)

Example

- Thus reciprocal nature of $H(u,v)$ and $h(x,y)$ with convolution is responsible for blurring and ringing.

Narrow Filter In Frequency Domain $\Rightarrow$ More severe blurring and ringing

Blurring : Low frequencies are removed

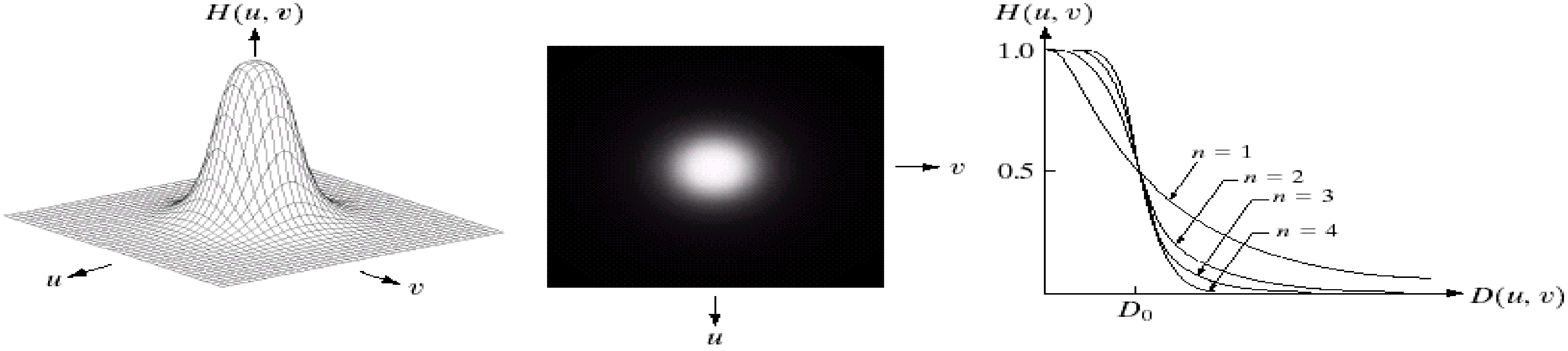
Ringing : Cutoff is too sharp

Objective is to achieve blurring with little or no ringing

Butterworth Lowpass Filter: BLPF

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$

$D(u,v)$: Distance from (u, v) to the origin of frequency rectangle



a b c

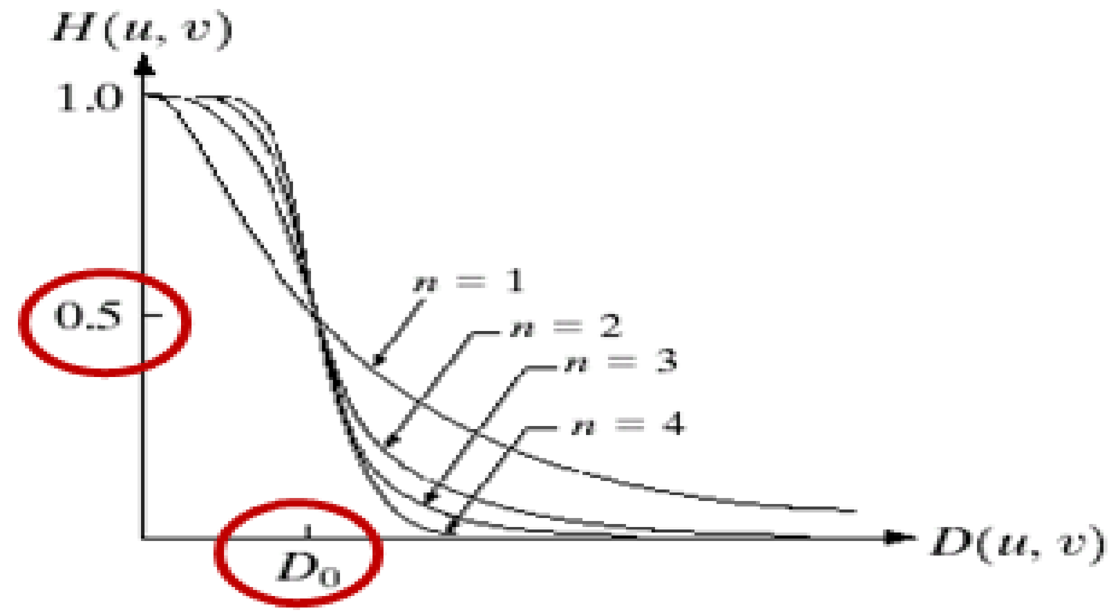
FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Butterworth Lowpass Filter: BLPF

- Unlike the ILPF, the BLPF transfer function does not have a sharp discontinuity that establishes a clear cutoff between passed and filtered frequencies.

- For smooth transfer function, cutoff frequency locus at points for which $H(u,v)$ is down to a certain fraction of its maximum value is customary.

- $H(u,v) = 0.5$ when $D(u,v) = D_0$



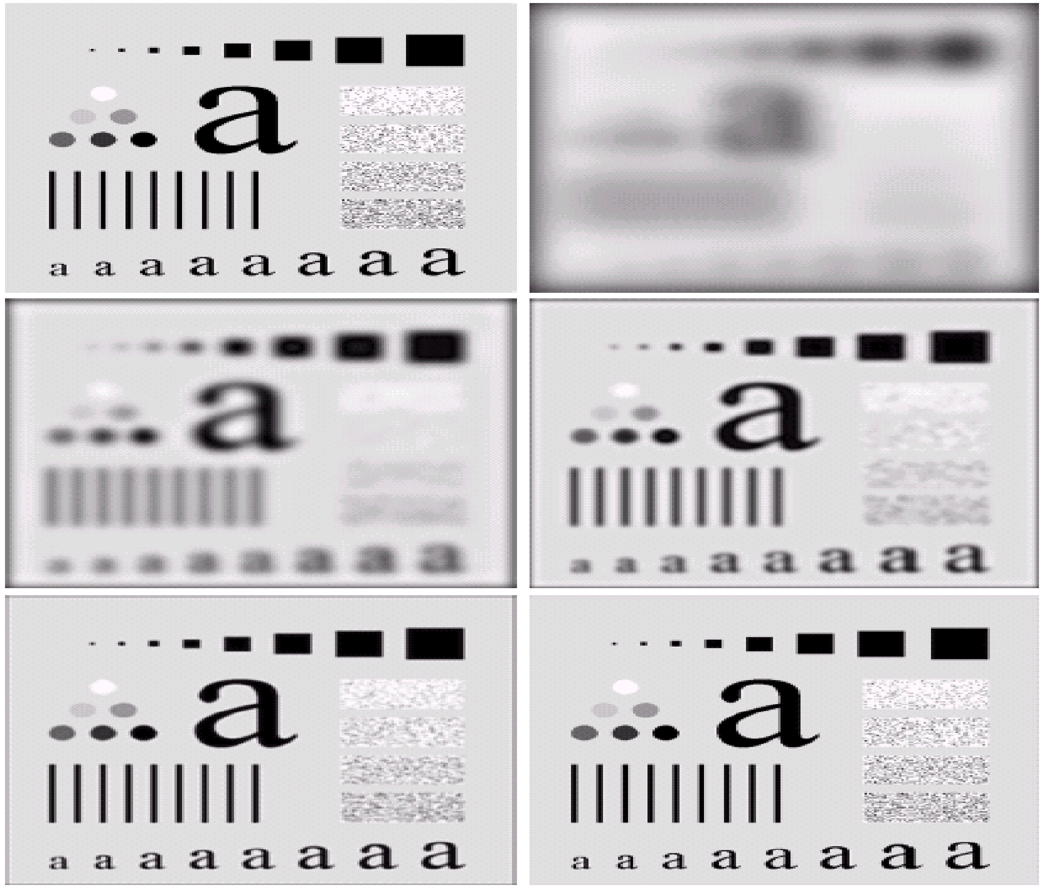
Example

ILPF

a b
c d
e f

FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

BLPF



a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Spatial representation of BLPFs

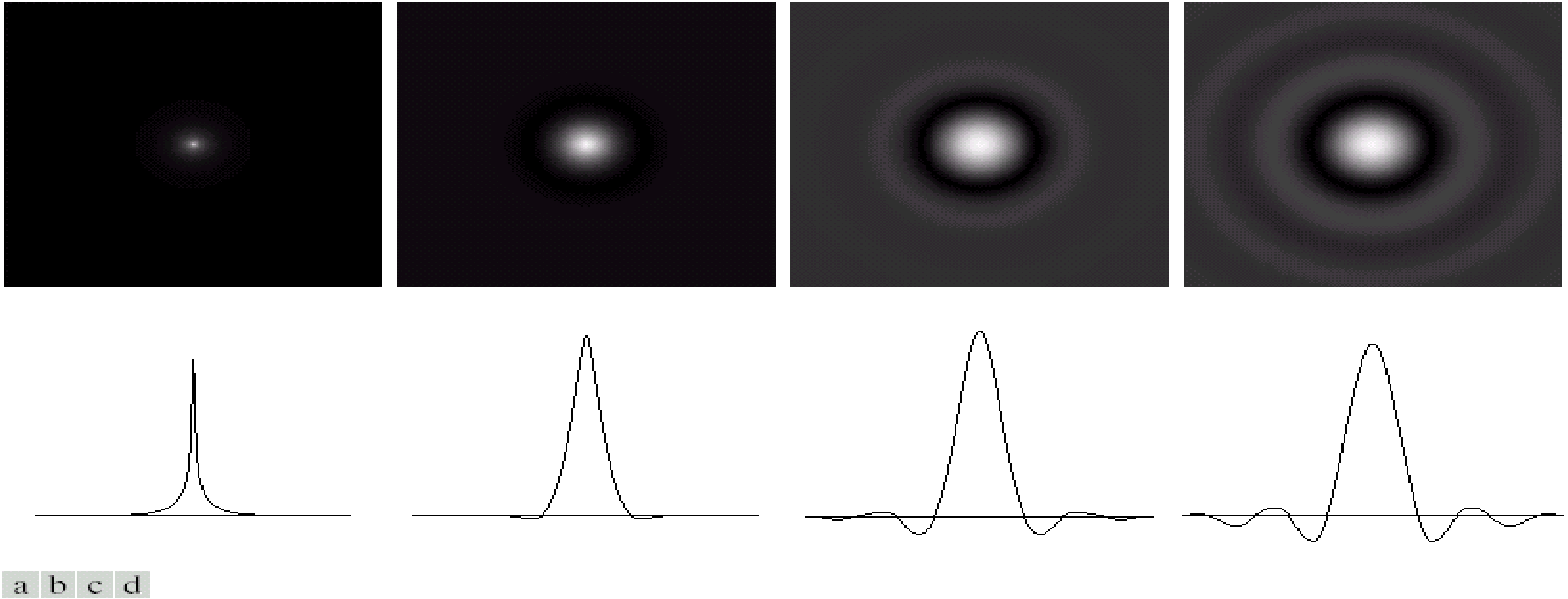


FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Spatial representation of BLPFs

- BLPF of order 1 has neither ringing nor negative value.
- Order 2 have mild ringing and small negative value but less than ILPF.
- Ringing in BLPF becomes significant for higher – order filters.
- BLPF of order 20 already have characteristics of the ILPF.

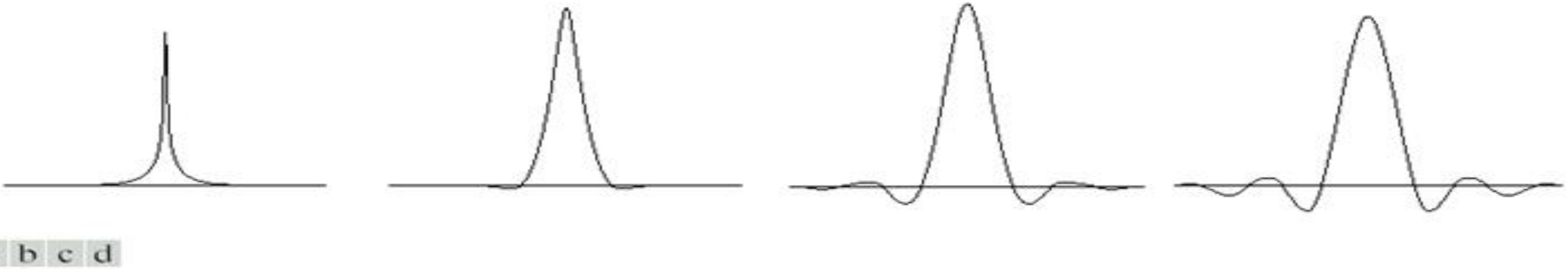


FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Gaussian Lowpass Filter: GLPF

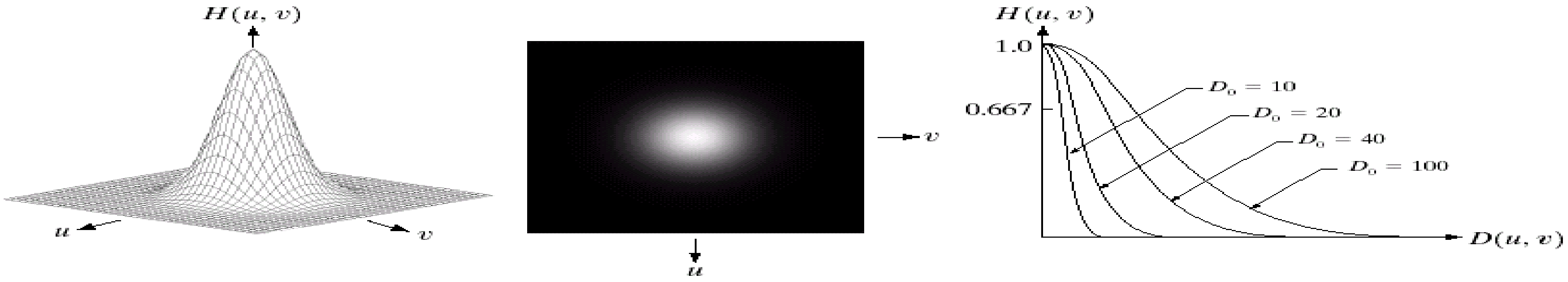
- Filter transfer function: $$H(u,v) = e^{-D^2(u,v)/2\sigma^2};$$

σ : Measure of spread of Gaussian curve

$D(u,v)$: Distance from (u, v) to the origin of frequency rectangle

- Taking $\sigma = D_0$ cutoff frequency: $$H(u,v) = e^{-D^2(u,v)/2D_0^2};$$

- Inverse FT of Gaussian low pass filter is Gaussian. Spatial GLPF will have no ringing.



a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Example

ILPF

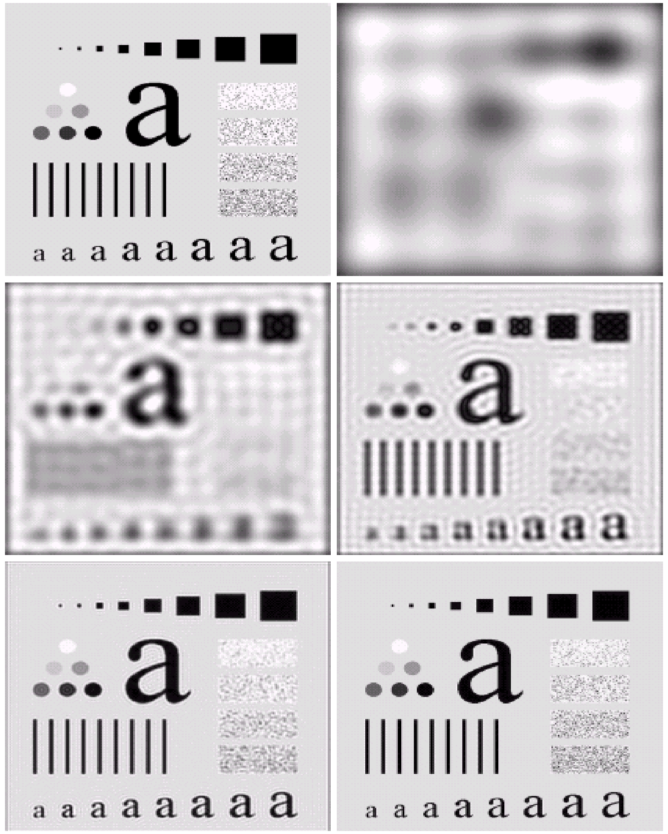


FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

BLPF

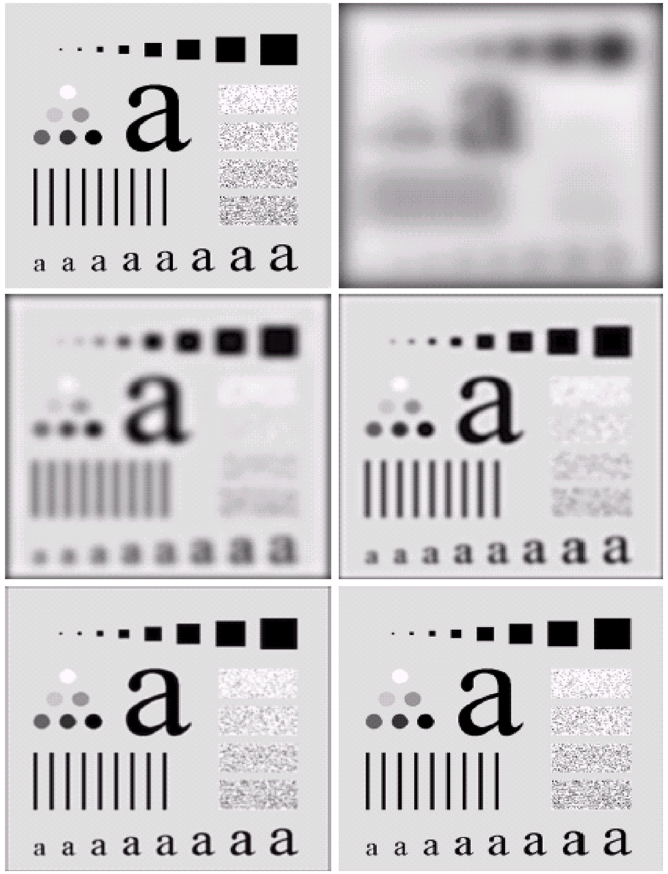


FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

GLPF

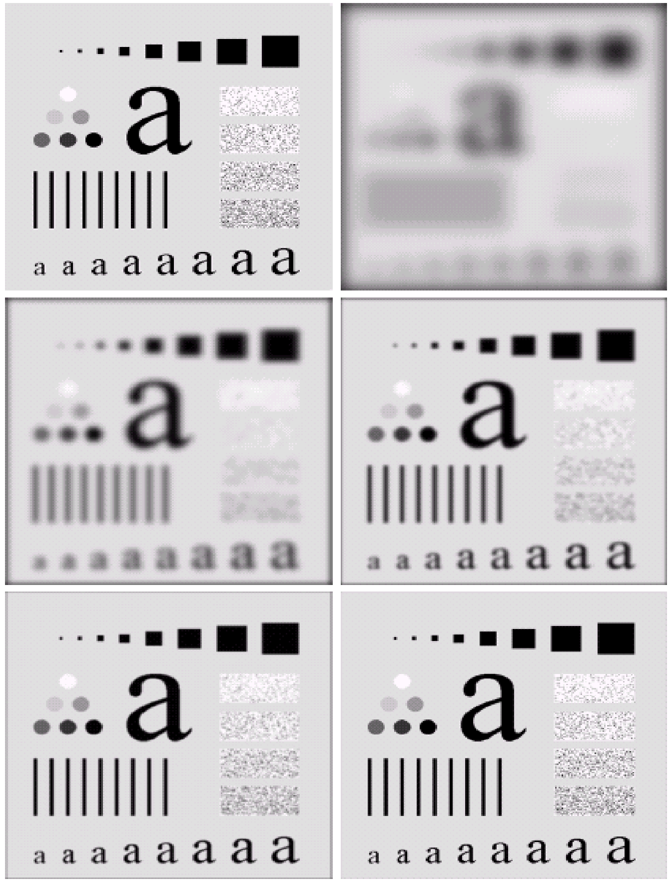


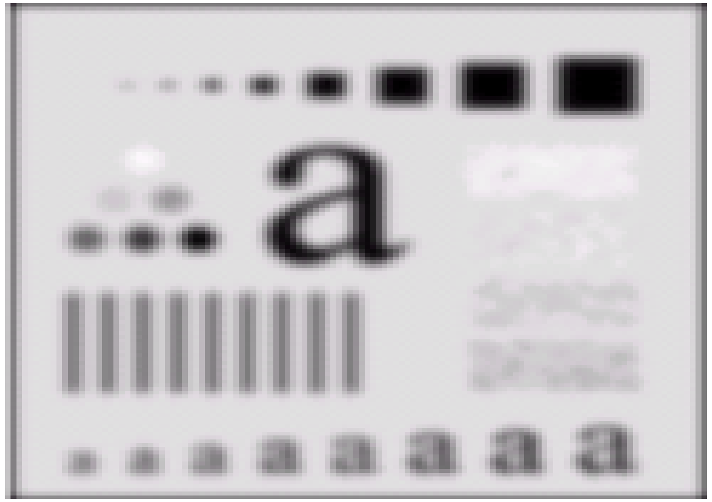
FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

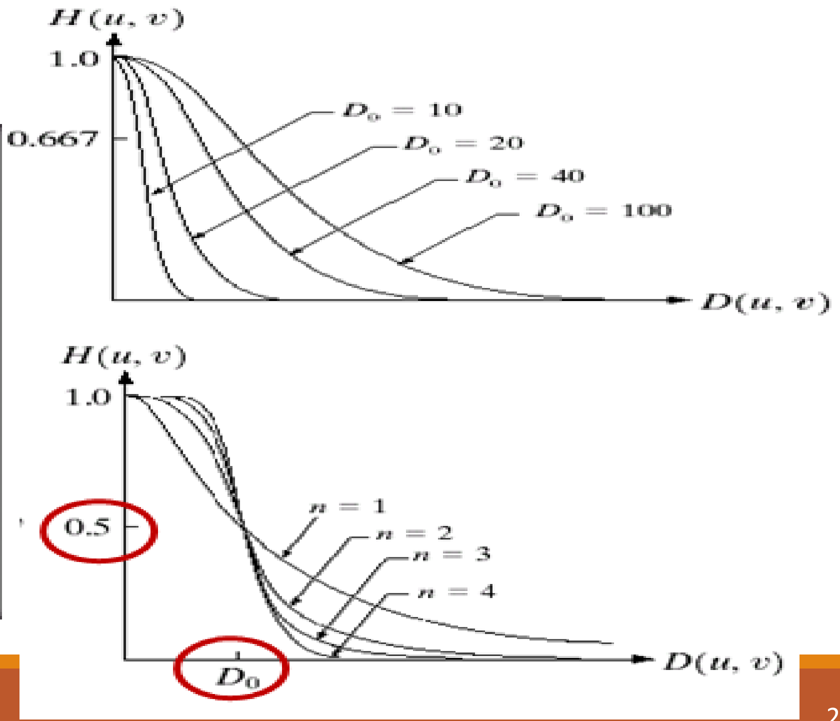
BLPF vs. GLPF

- In case of BLPF smooth transition in blurring as a function of increasing cutoff frequency.
- The GLPF does did not achieve as much smoothing as the BLPF of order 2 of same cutoff frequency.

BLPF (c)

GLPF (c)



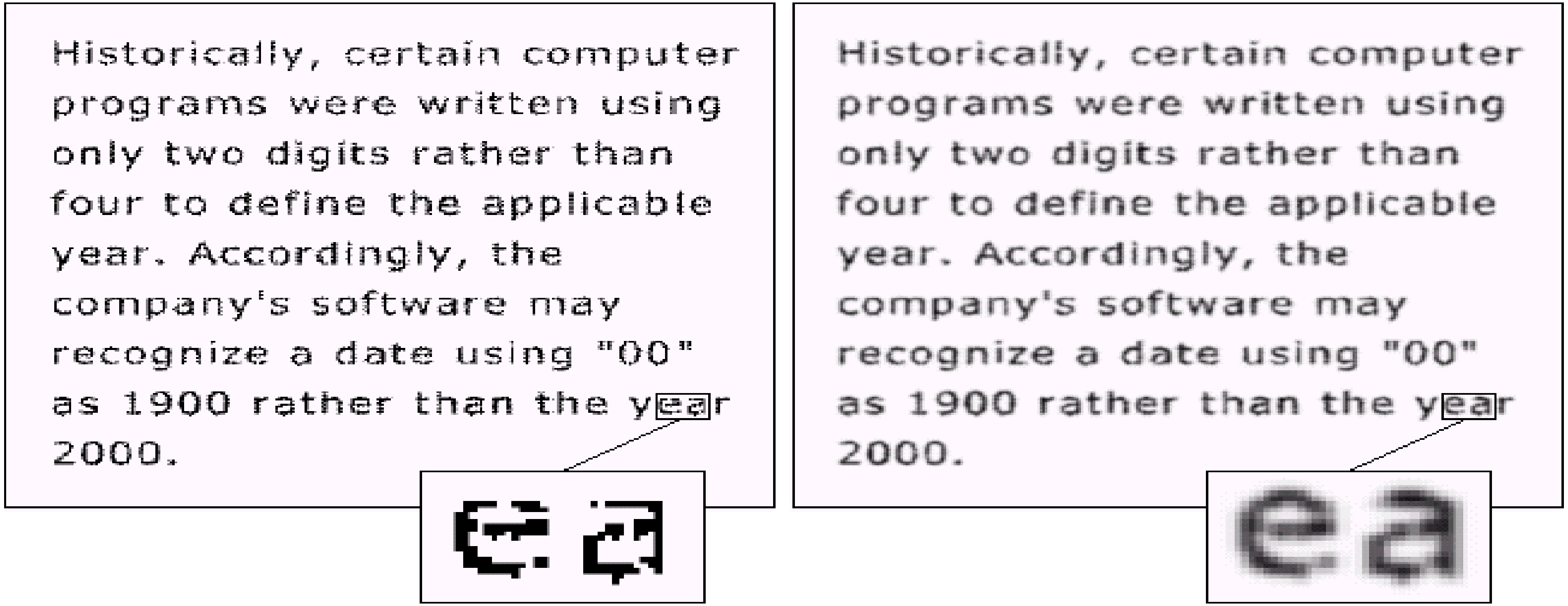


Example

Bridge small gaps in the input image by blurring

a b

FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



GLPF with D_0 =80

Example

Unsharp masking: Printing and Cosmetic industry

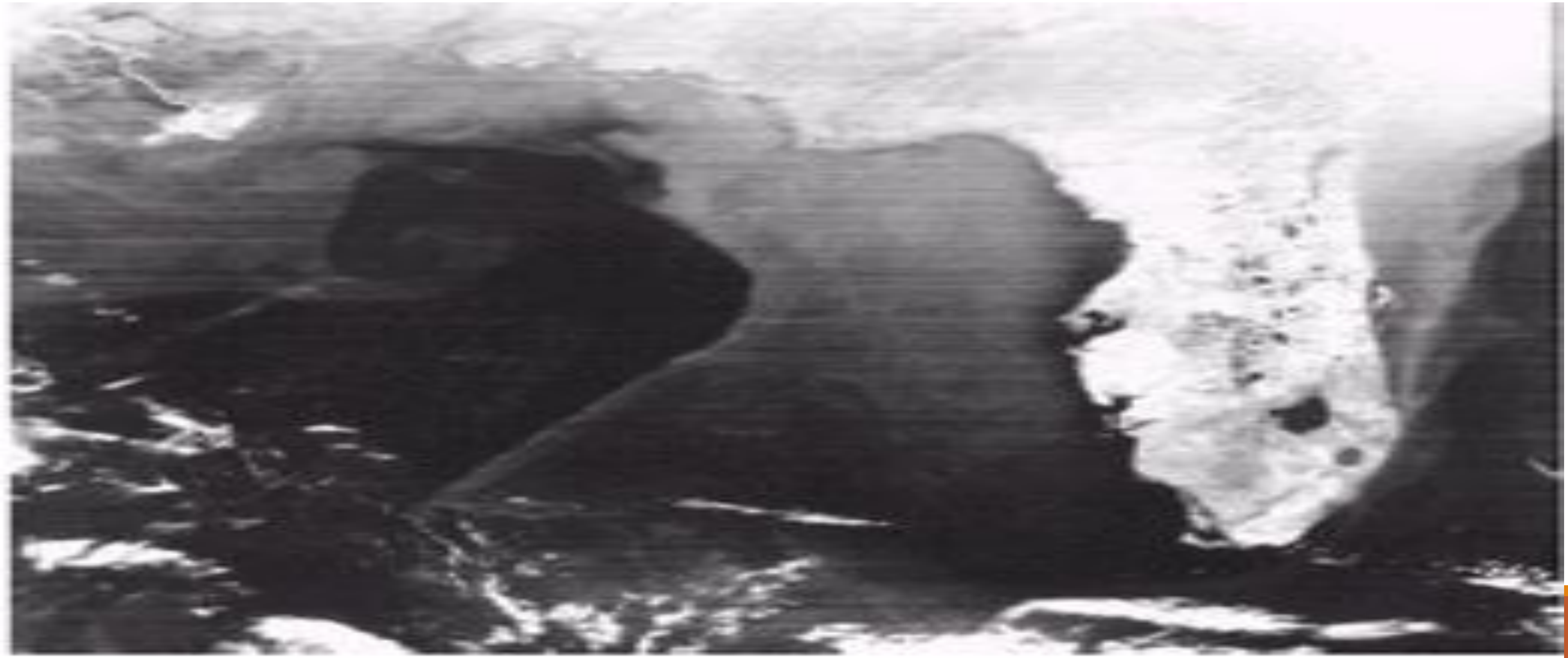
Produce a smoother, softer-looking result from a sharp original.

a b c

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

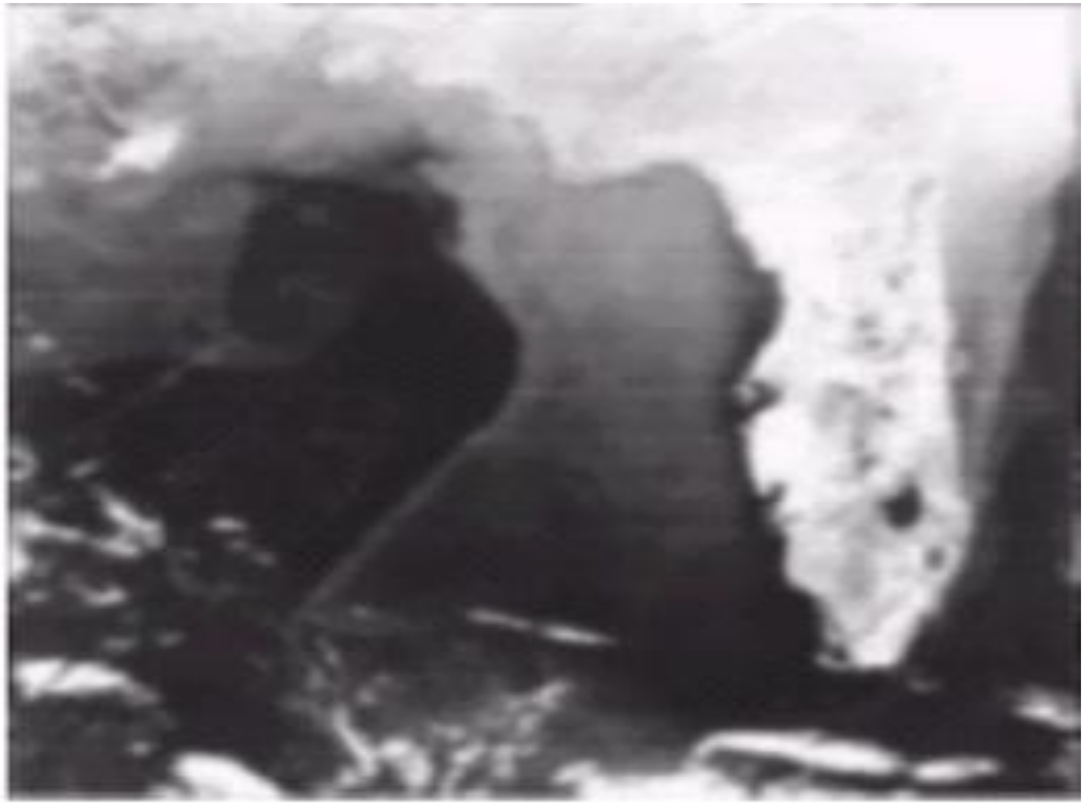
Example

- High Resolution Radiometer Image of gulf of Mexico (dark) and Florida (light)
- Prominent scan line along the direction in which scene is being scanned.



Example

- Low pass filter is crude but simple to reduce the effect of these scan line.



GLPF, D_0 = 30

GLPF, D_0 = 10

Suggested Readings

- **Digital Image Processing by Rafel Gonzalez, Richard Woods, Pearson Education India, 2017.**
- **Fundamental of Digital image processing by A. K Jain, Pearson Education India, 2015.**

Thank you