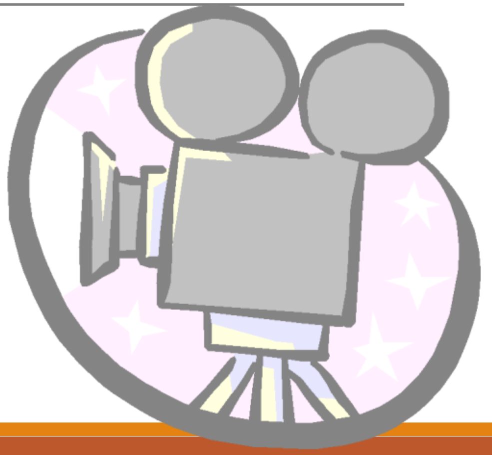
Image Processing

CS-317/CS-341



Outline

- Spatial Filtering
- Spatial Correlation and convolution
- Smoothening Filter
- Sharpening Filter

Spatial Filtering

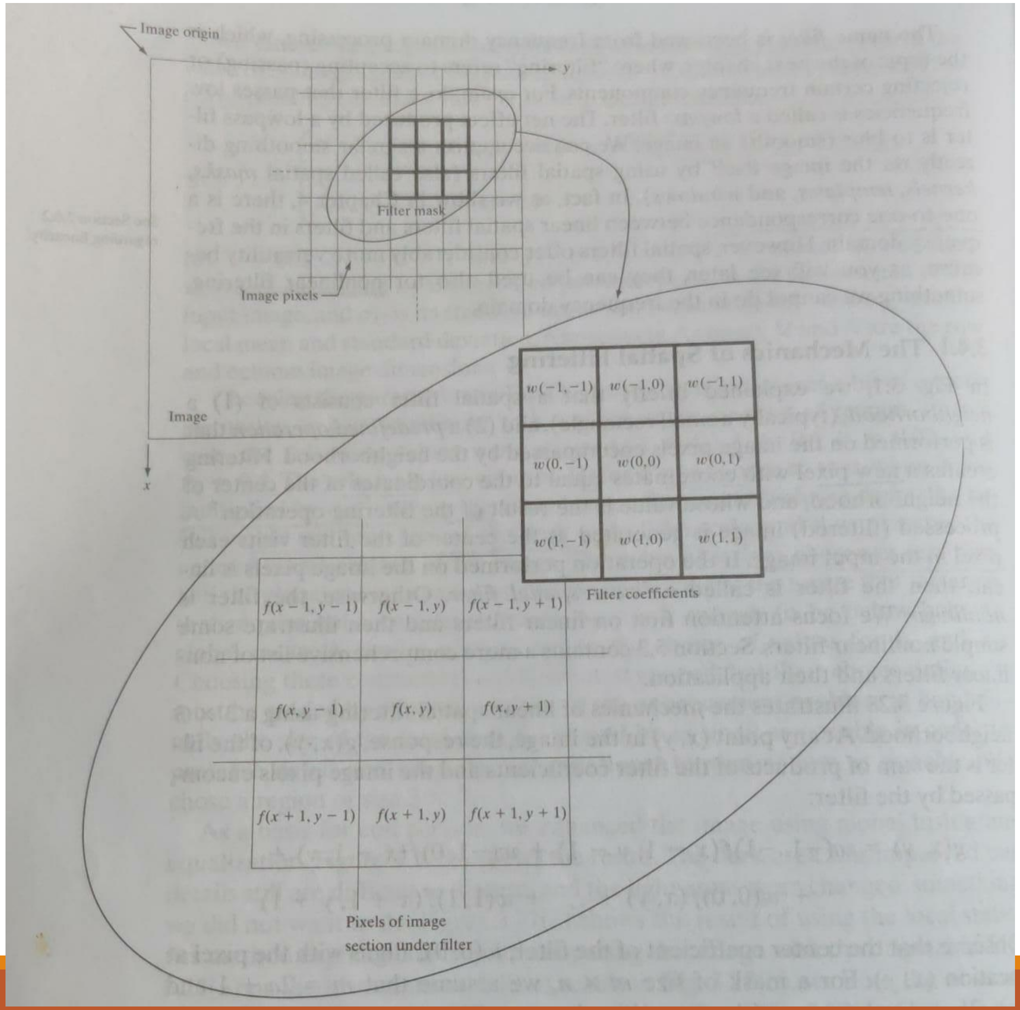
use filter (can also be called as mask/kernel/template or window)

the values in a filter subimage are referred to as coefficients, rather than pixel.

our focus will be on masks of odd sizes, e.g. 3x3, 5x5,…

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Mechanism of linear spatial filtering using 3 * 3 mask



Define a square or rectangular neighborhood and move the center of this area from pixel to pixel.

Convolution

Consider an image of size 6 x 6

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

Convolution

1	-1	-1
-1	1	-1
-1	-1	1

Filter /mask

stride=1

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Dot product

3 -1

6 x 6 image

Convolution

1	-1	-1
-1	1	-1
-1	-1	1

Filter /mask

stride=1

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

Dot product → -1 -3

Convolution

1	-1	-1
-1	1	-1
-1	-1	1

Filter /mask

stride=1

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Dot product

-3 -1

6 x 6 image

Convolution

1	-1	-1
-1	1	-1
-1	-1	1

Filter /mask

If stride=2

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

3 -3

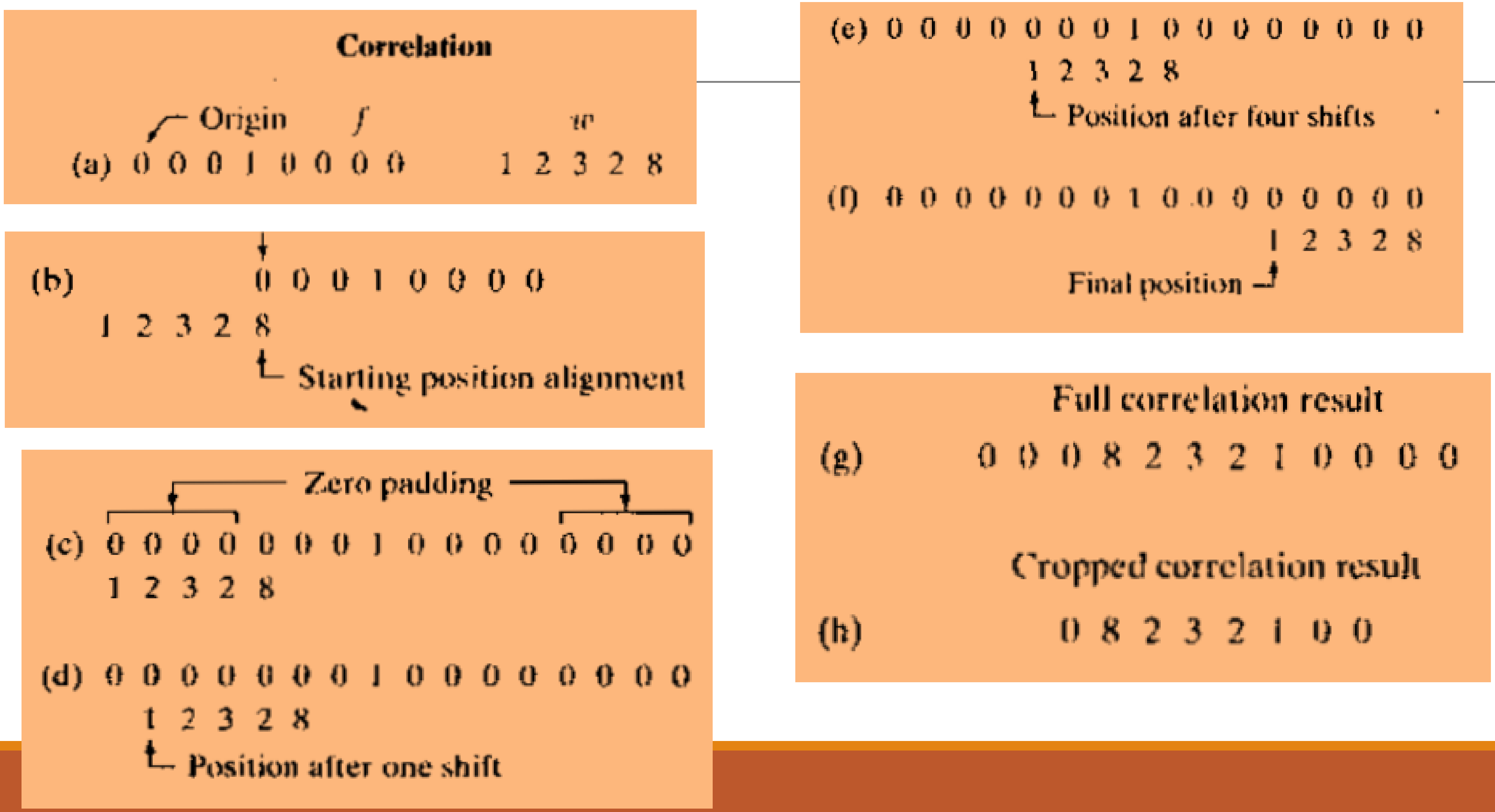
Spatial Correlation and Convolution

There are two closely related concepts that must be understood clearly when performing linear spatial filtering.

- Correlation
- Convolution

- Correlation is the process of moving a filter mask over the image and computing sum of products at each location.
- The mechanism of convolution is same, except that the filter is first rotated by 180^o.

Spatial Correlation Example: consider a 1D function f with mask w



Spatial Convoution Example: consider a 1D function f with mask w

Convolution

a

Origin f w rotated 180°

0 0 0 1 0 0 0 0 8 2 3 2 1

b

0 0 0 1 0 0 0 0

8 2 3 2 1

c

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0

8 2 3 2 1

d

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0

8 2 3 2 1

e

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0

8 2 3 2 1

f

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0

8 2 3 2 1

g

Full convolution result

0 0 0 1 2 3 2 8 0 0 0 0

h

Cropped convolution result

0 1 2 3 2 8 0 0

Correlation for 2 D function f(x,y)

Origin $f(x, y)$

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$w(x, y)$

$$\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix}$$

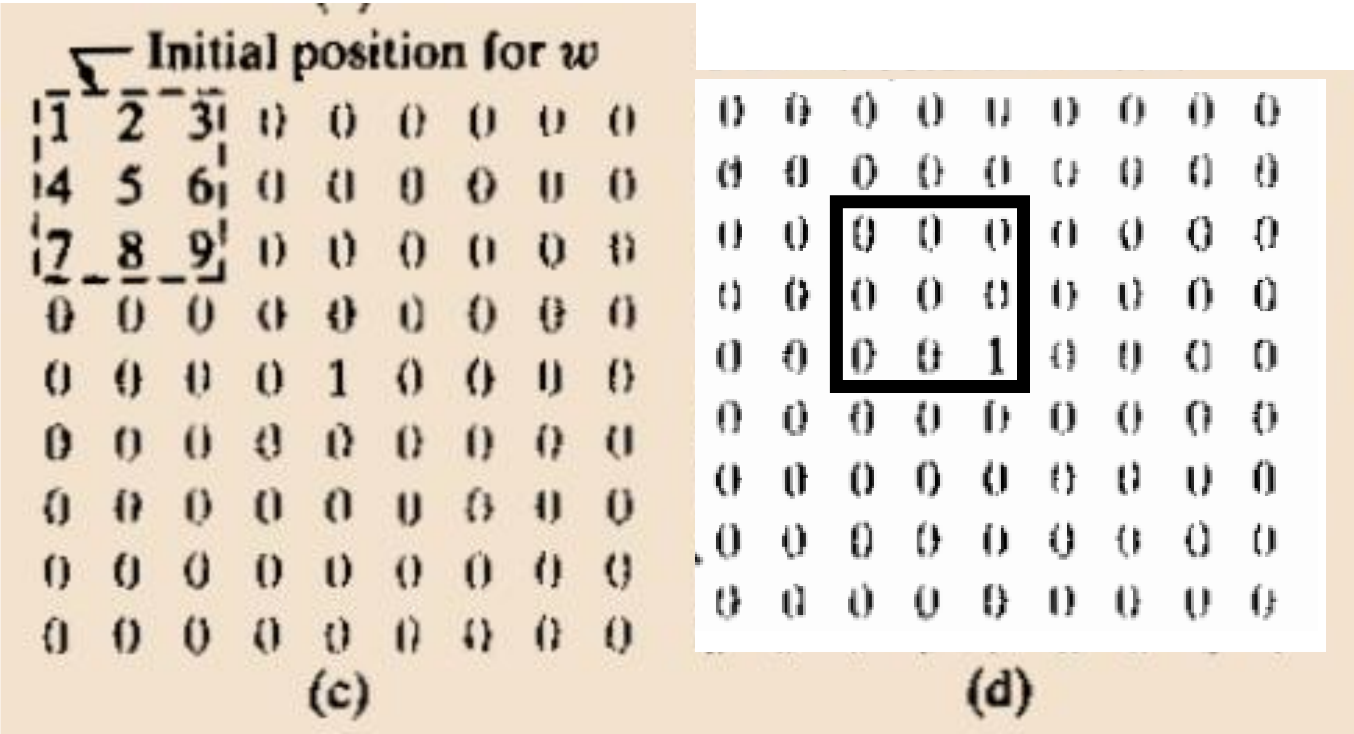
(a)

Padded f

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

(b)

Correlation for 2 D function f(x,y)



(c) (d)

Correlation for 2 D function f(x,y)

Initial position for w

1	2	3	0	0	0	0	0	0
4	5	6	0	0	0	0	0	0
7	8	9	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(c)

Full correlation result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	9	8	7	0	0	0
0	0	0	6	5	4	0	0	0
0	0	0	3	2	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(d)

Cropped correlation result

0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

(e)

Correlation for 2 D function f(x,y)

Initial position for w

1	2	3	0	0	0	0	0	0
4	5	6	0	0	0	0	0	0
7	8	9	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(c)

Full correlation result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	9	8	7	0	0	0
0	0	0	6	5	4	0	0	0
0	0	0	3	2	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(d)

Cropped correlation result

0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

(e)

Spatial Filtering Process

simply move the filter mask from point to point in an image.

at each point (x,y), the response of the filter at that point is calculated using a predefined relationship.

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn}$$

$$= \sum_{i=i}^{mn} w_i z_i$$

Linear Filtering

Linear Filtering of an image f of size MxN filter mask of size mxn is given by the expression

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\,f(x+s,y+t)$$

where a = (m-1)/2 and b = (n-1)/2

To generate a complete filtered image this equation must be applied for x = 0, 1, 2, … , M-1 and y = 0, 1, 2, … , N-1

Smoothing Spatial Filters

used for blurring and for noise reduction

blurring is used in preprocessing steps, such as

- removal of small details from an image prior to object extraction
- bridging of small gaps in lines or curves

noise reduction can be accomplished by blurring with a linear filter and also by a nonlinear filter

Smoothing Linear Filters

output is simply the average of the pixels contained in the neighborhood of the filter mask.

called averaging filters or lowpass filters.

Smoothing Linear Filters

replacing the value of every pixel in an image by the average of the gray levels in the neighborhood will reduce the “sharp” transitions in gray levels.

sharp transitions

- random noise in the image
- edges of objects in the image

thus, smoothing can reduce noises (desirable) and blur edges (undesirable)

3x3 Smoothing Linear Filters

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

box filter

weighted average

the center is the most important and other pixels are inversely weighted as a function of their distance from the center of the mask

Weighted average filter

the basic strategy behind weighting the center point the highest and then reducing the value of the coefficients as a function of increasing distance from the origin is simply **an attempt to reduce blurring in the smoothing process**.

General form : smoothing mask

filter of size mxn (m and n odd)

$$g(x,y)=\frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\,f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

summation of all coefficient of the mask

a	b
c	d
e	f

Example

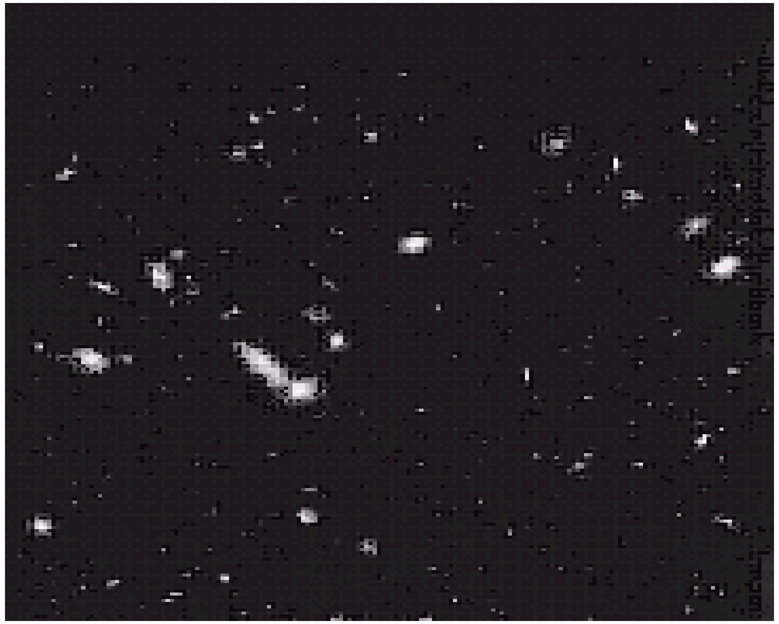
a) original image 500x500 pixel

b) - f) results of smoothing with square averaging filter masks of size n = 3, 5, 9, 15 and 35, respectively.

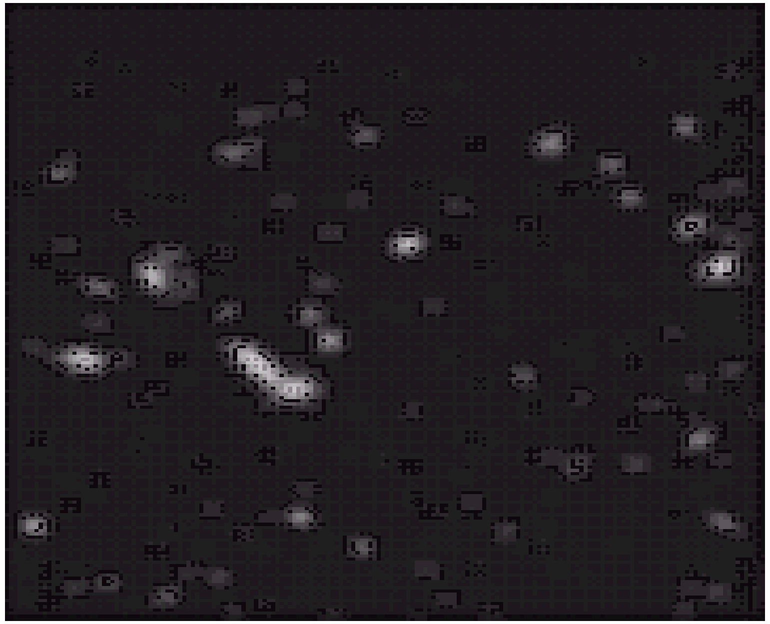
Note:

- big mask is used to eliminate small objects from an image.
- the size of the mask establishes the relative size of the objects that will be blended with the background.

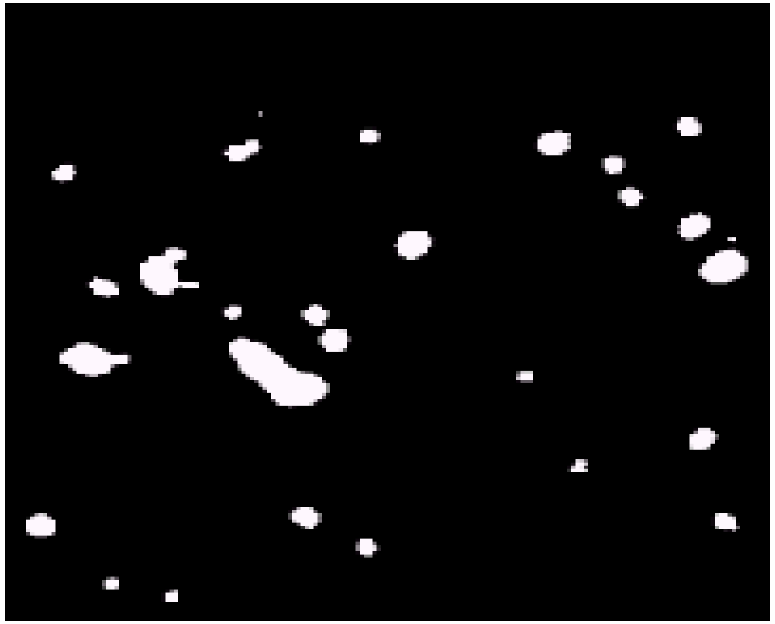
Example



original image



result after smoothing with 15x15 averaging mask



result of thresholding

we can see that the result after smoothing and thresholding, the remains are the largest and brightest objects in the image.

Order-Statistics Filters (Nonlinear Filters)

the response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter

example

- median filter : $R = \text{median}\{z_k \mid k = 1,2,\ldots,n \times n\}$
- max filter : $R = \max\{z_k \mid k = 1,2,\ldots,n \times n\}$
- min filter : $R = \min\{z_k \mid k = 1,2,\ldots,n \times n\}$

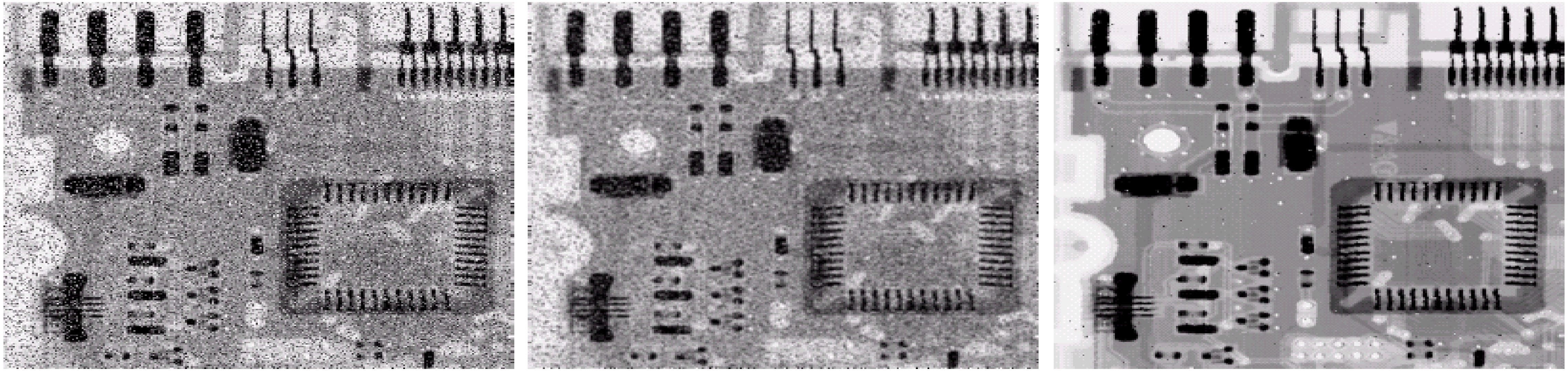
note: n x n is the size of the mask

Median Filters

replaces the value of a pixel by the median of the gray levels in the neighborhood of that pixel (the original value of the pixel is included in the computation of the median)

quite popular because for certain types of random noise (impulse noise ⇨ salt and pepper noise) , they provide excellent noise-reduction capabilities, with considering less blurring than linear smoothing filters of similar size.

Example : Median Filters



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Suggested Readings

- **Digital Image Processing by Rafel Gonzalez, Richard Woods, Pearson Education India, 2017.**

- **Fundamental of Digital image processing by A. K Jain, Pearson Education India, 2015.**

Thank you