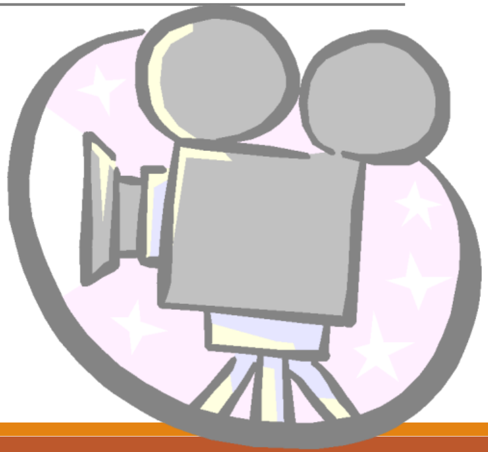
# Image Processing

## CS-317/CS-341



# Outline

- Adaptive Filters
- Band-pass and band-reject filters
- Notch Filters

# Model of the Image Degradation

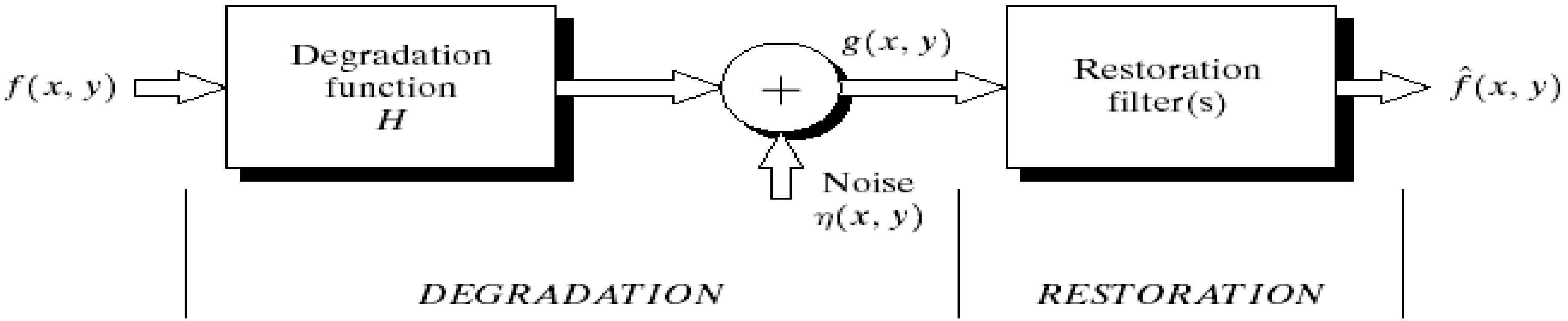



**FIGURE 5.1** A model of the image degradation/restoration process.

# Adaptive Filters

- Filter behavior changes based on statistical characteristics of the image inside the filter region.

**Adaptive, local noise reduction filter**

- Simplest statistical measures of a random variable are its mean and variance

- Mean gives a measure of average gray level inside the filter region, and the variance gives a measure of average contrast in the filter region.

# Adaptive Filter

Filter operate on a local region $S_{xy}$. The response of the filter at the point (x, y) is to be based on four points:

a) g(x, y), the value of the noisy image at (x, y).

b) $\sigma_{\eta}^{2}$ the variance of the noise corrupting f(x, y) to form g(x, y)

c) $m_L$, the local mean of the pixel in $S_{xy}$. ,

d) $\sigma_{L}^{2}$ The local variance of the pixel in $S_{xy}$.

## - Requirement for adaptive filter

1. If noise variance, $\sigma_{\eta}^{2}$ , is zero, the filter should return simply the value *g*(x,y). This is the trivial, zero-noise case in which *g*(x,y) is equal to *f*(x,y).

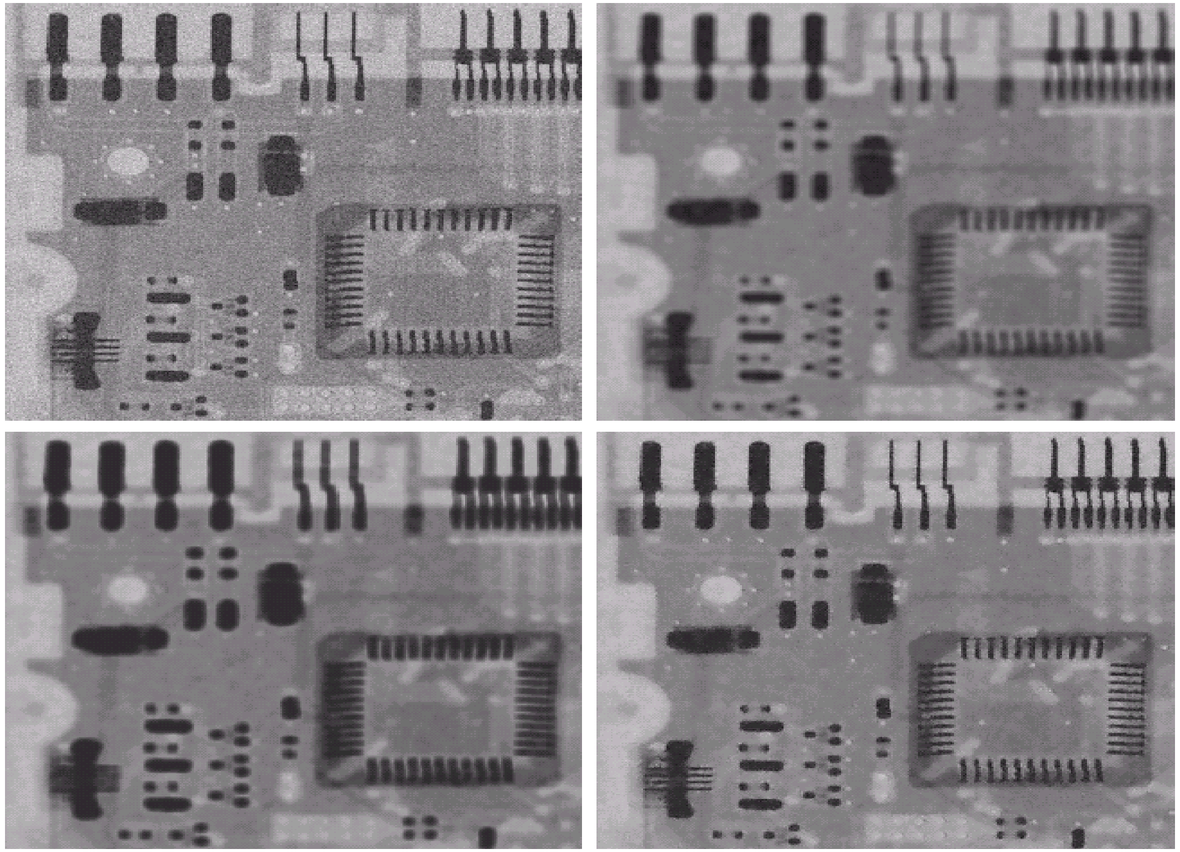
2. If local variance, $\sigma_{L}^{2}$ is high relative to noise variance $\sigma_{\eta}^{2}$ , the filter should return a value close to *g*(x,y). High local variance typically is associated with edges, and these should be preserved.

3. If two variances are equal, we want the filter to return the arithmetic mean value of the pixels in filter region.

# Adaptive Filters

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

- Quantity that needs to be know or estimated is the variance of the overall noise, $\sigma_\eta^2$



a b
c d

**FIGURE 5.13** (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size $7 \times 7$.

# Adaptive Median Filter

- The median filter performs well as long as the spatial density of the impulse noise is not large (as a rule of thumb, $P_a$ and $P_b$ less than 0.2).

$Z_{min}$ = minimum gray level value in filter region, $W_{mn}$.

$Z_{max}$ = maximum gray level value in $W_{mn}$.

$Z_{med}$ = median of gray level $W_{mn}$.

$Z_{xy}$ = Gray level at coordinate (x,y).

$W_{max}$= maximum allowed size of $W_{mn}$.

Adaptive median filtering algorithm:

Level A: $A1 = Z_{med} - Z_{min}$; $A2 = Z_{med} - Z_{max}$

if A1 >0 AND A2<0, Go to level *B*

Else increase the window size

If window size <= $W_{max}$ repeat level A

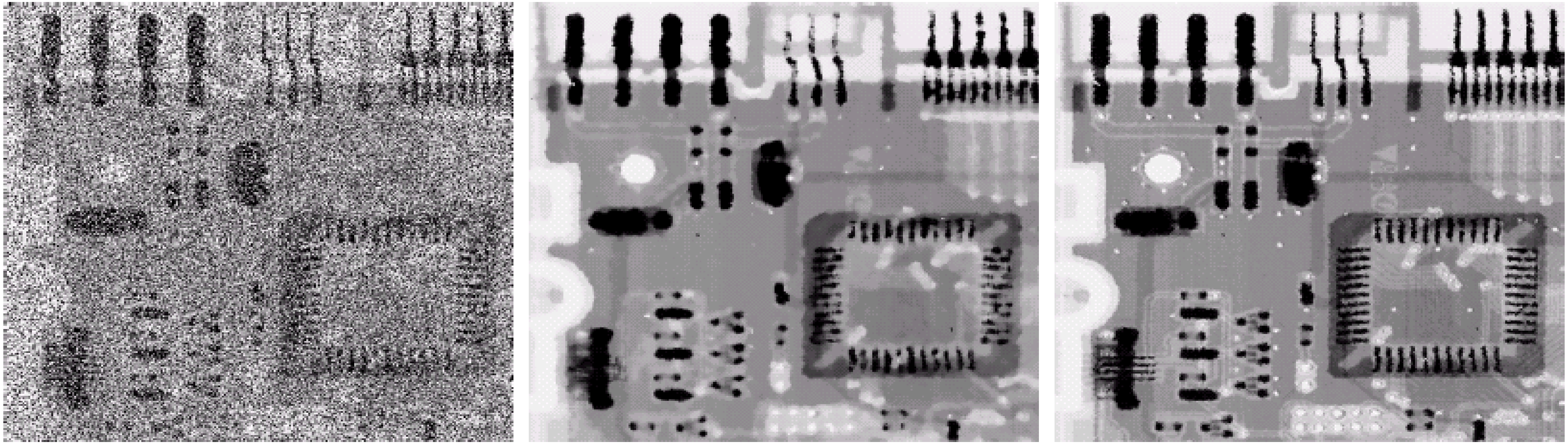
Else output $z_{xy}$

Level B: $B1 = Z_{xy} - Z_{min}$; $B2 = Z_{xy} - Z_{max}$

if B1 >0 AND B2<0, output $z_{xy}$

Else output $z_{med}$.

# Adaptive Median Filtering



a b c

**FIGURE 5.14** (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a $7 \times 7$ median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

# Periodic Noise

- Periodic noise is sinusoidal at multiple of a specific frequency and periodic in nature.

- It can be removed by Band-pass, Band reject and Notch filter.

# Periodic Noise reduction by frequency domain Filtering

Band Pass filter: The objective of band pass filter is to pass limited range frequency while leaving the other.

Band Reject filter: The objective of band reject filter is to attenuate limited range frequency while leaving the other.

# Periodic Noise reduction by frequency domain Filtering

## Ideal Band Rejected Filter

$$H(u,v)=\begin{cases}1, & if\ \ D(u,v)<D_0-\dfrac{W}{2}\\ 0, & if\ \ D_0-\dfrac{W}{2}\le D(u,v)\le D_0+\dfrac{W}{2}\\ 1, & if\ \ D(u,v)\ge D_0+\dfrac{W}{2}\end{cases}$$

## Butterworth Band Rejected Filter

$$H(u,v)=\left\{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}\right\}^{-1}$$

## Gaussian Band Rejected Filter

$$H(u,v)=1-\exp\left\{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2\right\}$$

$D(u,v)$ is the distance from the origin, $W$ is the width of the band,
$D_o$ is the radial center.

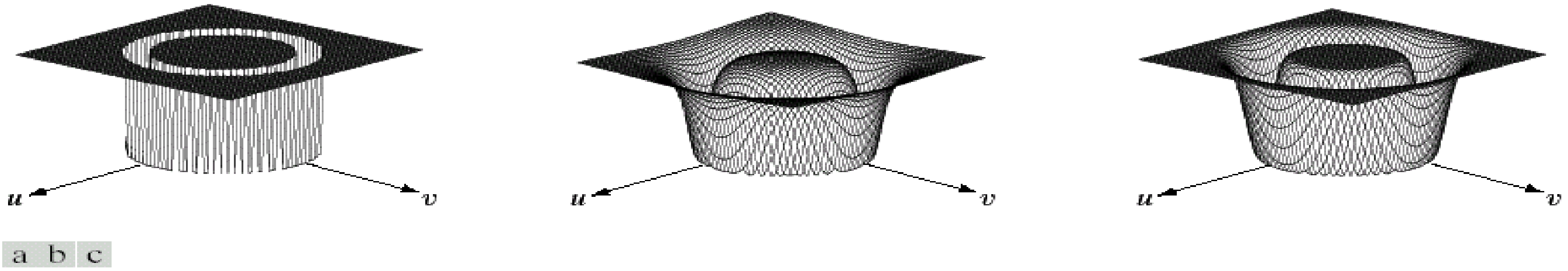
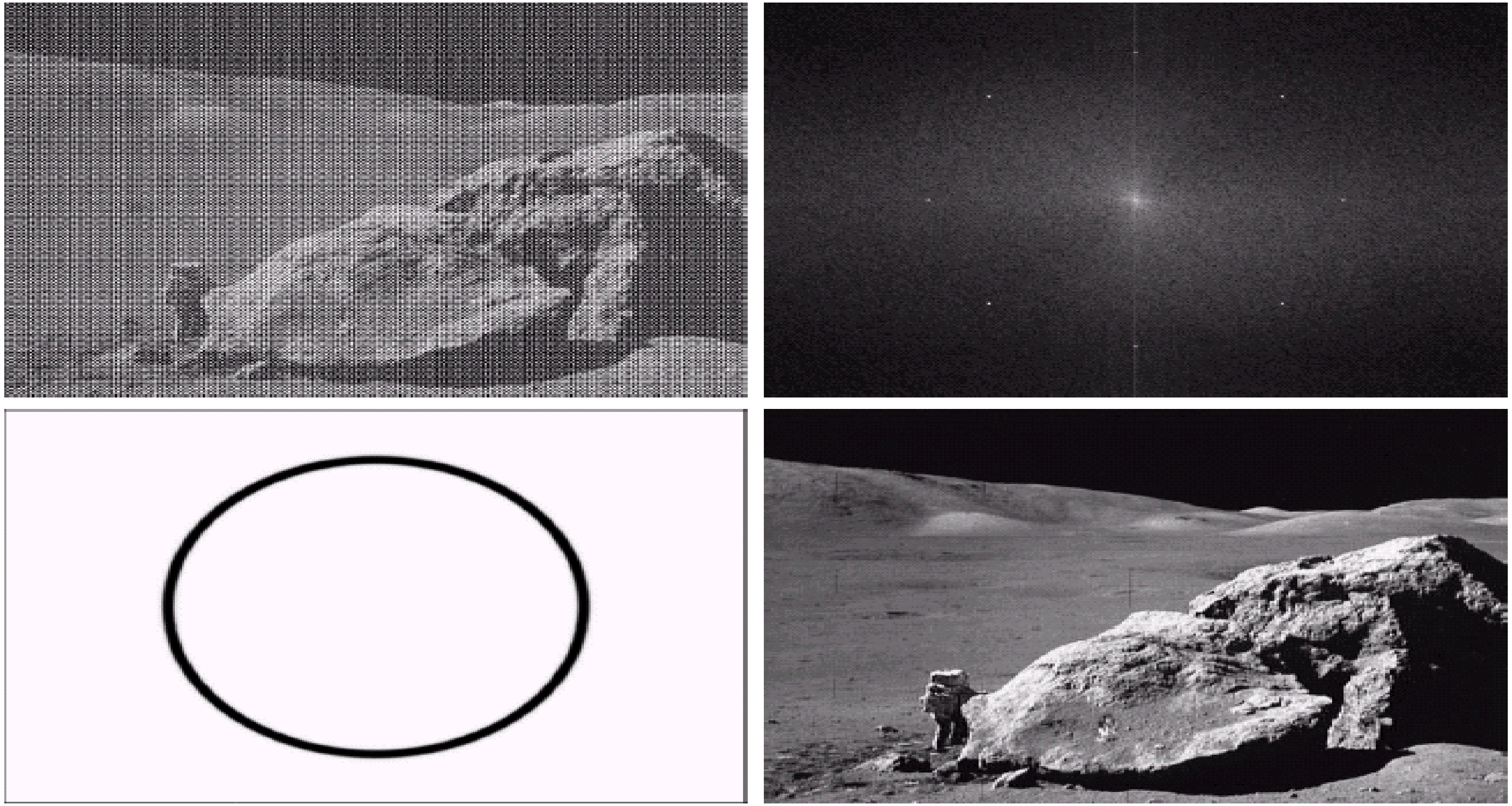



**FIGURE 5.15** From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

# Periodic Noise reduction by frequency domain Filtering



a b
c d

**FIGURE 5.16** (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

- Image is heavily corrupted by sinusoidal noise of various frequency.
- Noise components lie on an approximate circle about the origin of the transform. Circularly symmetric band rejected filter is good choice.

# Periodic Noise reduction by frequency domain Filtering

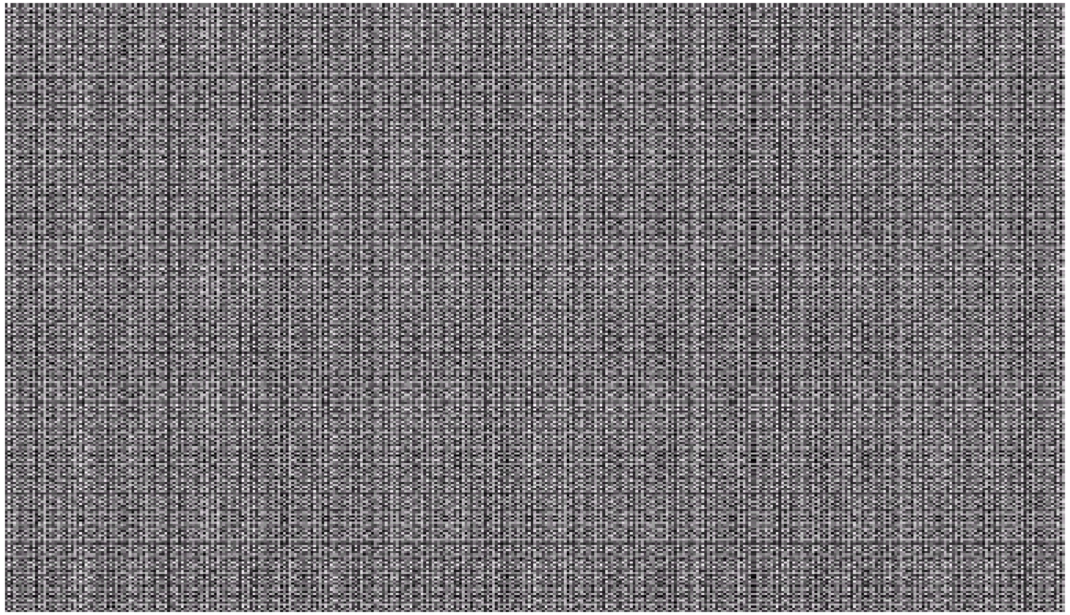
## Band Pass Filter: passes frequency within a particular range

Opposite to band reject filter

$$H_{bp}(u,v) = 1 - H_{br}(u,v)$$

- Band pass filtering generally removes the too much image detail.
- Band pass filtering is quite useful in isolating the effect on an image of selected frequency band.

**FIGURE 5.17** Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



# Notch Filters

**A notch filter rejects (or passes) frequencies in predefined neighborhoods about a center frequency. It is sepecial form of band reject filter.**

## Ideal Notch

$$\boxed{H(u,v)=\begin{cases}0, & if\ \ D_1(u,v)\le D_0(u,v)\ \text{ or } D_2(u,v)\le D_0(u,v)\\ 1, & \text{Otherwise}\end{cases}}$$

$$D_1(u,v)=\left[\left(u-M/2-u_0\right)^2+\left(v-N/2-v_0\right)^2\right]^{1/2}$$

$$D_2(u,v)=\left[\left(u-M/2+u_0\right)^2+\left(v-N/2+v_0\right)^2\right]^{1/2}$$

Assume that center is shifted to the point (*M/2, N/2*)

Instead of removing the entire range of frequency,
It removes only selected frequency component

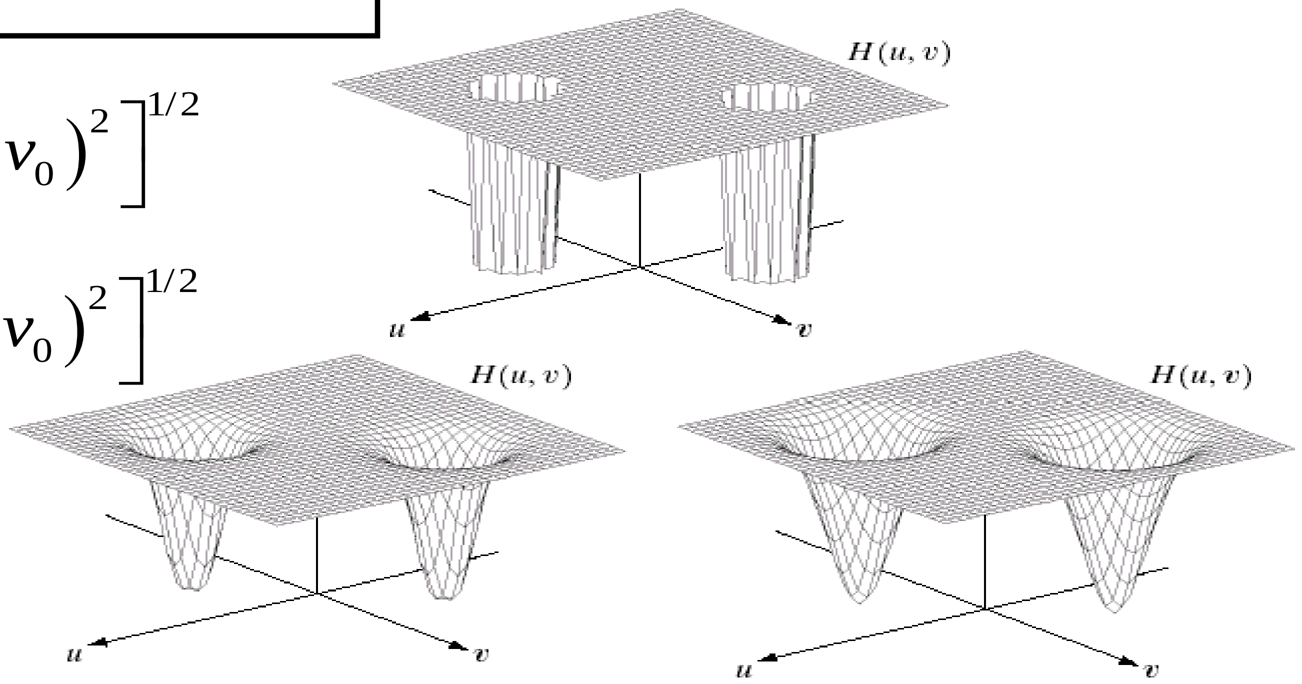



**FIGURE 5.18** Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

# Notch Filters

## Butterworth Notch Reject filter

$$H(u,v)=\left\{1+\left[\frac{D_0^2}{D_1(u,v)D_2(u,v)}\right]^n\right\}^{-1}$$

## Gaussian Notch Reject filter

$$H(u,v)=1-\exp\left\{-\frac{1}{2}\left[\frac{D_1(u,v)D_2(u,v)}{D_0^2}\right]\right\}$$

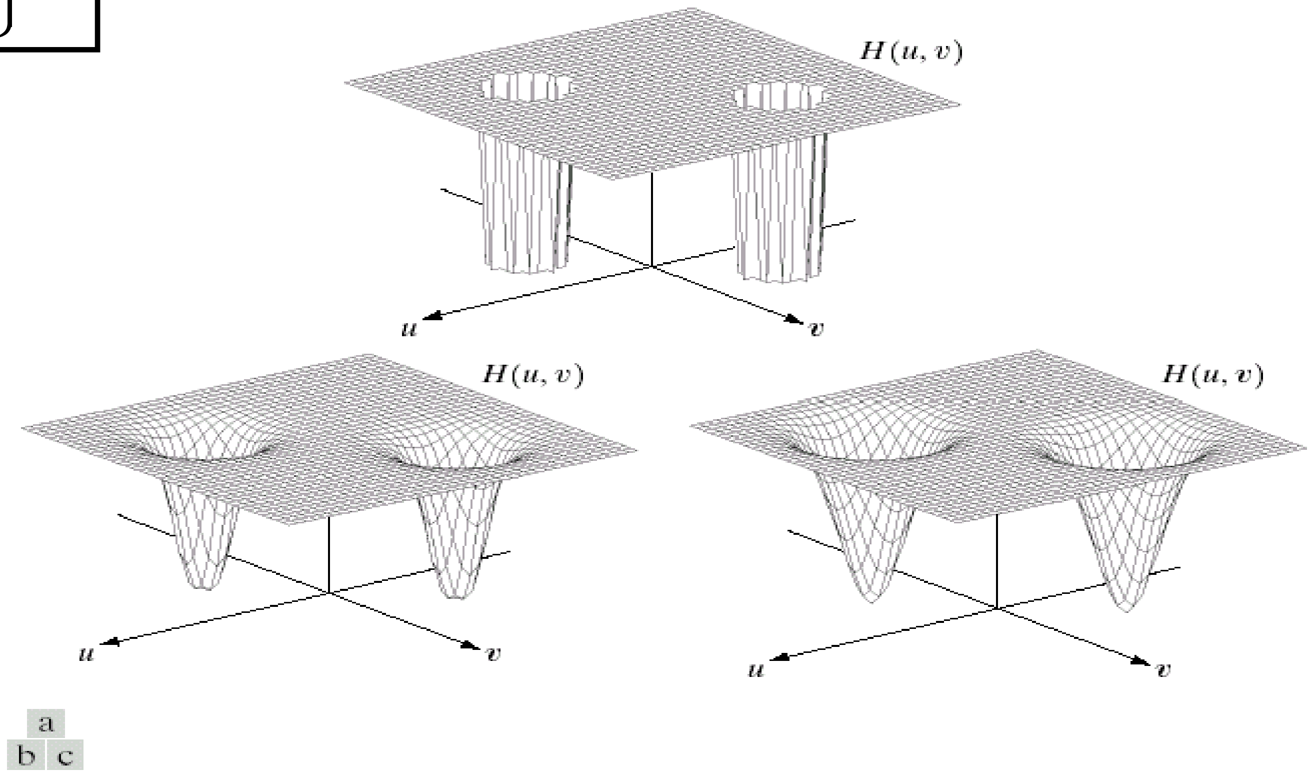



**FIGURE 5.18** Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

# Notch Filters

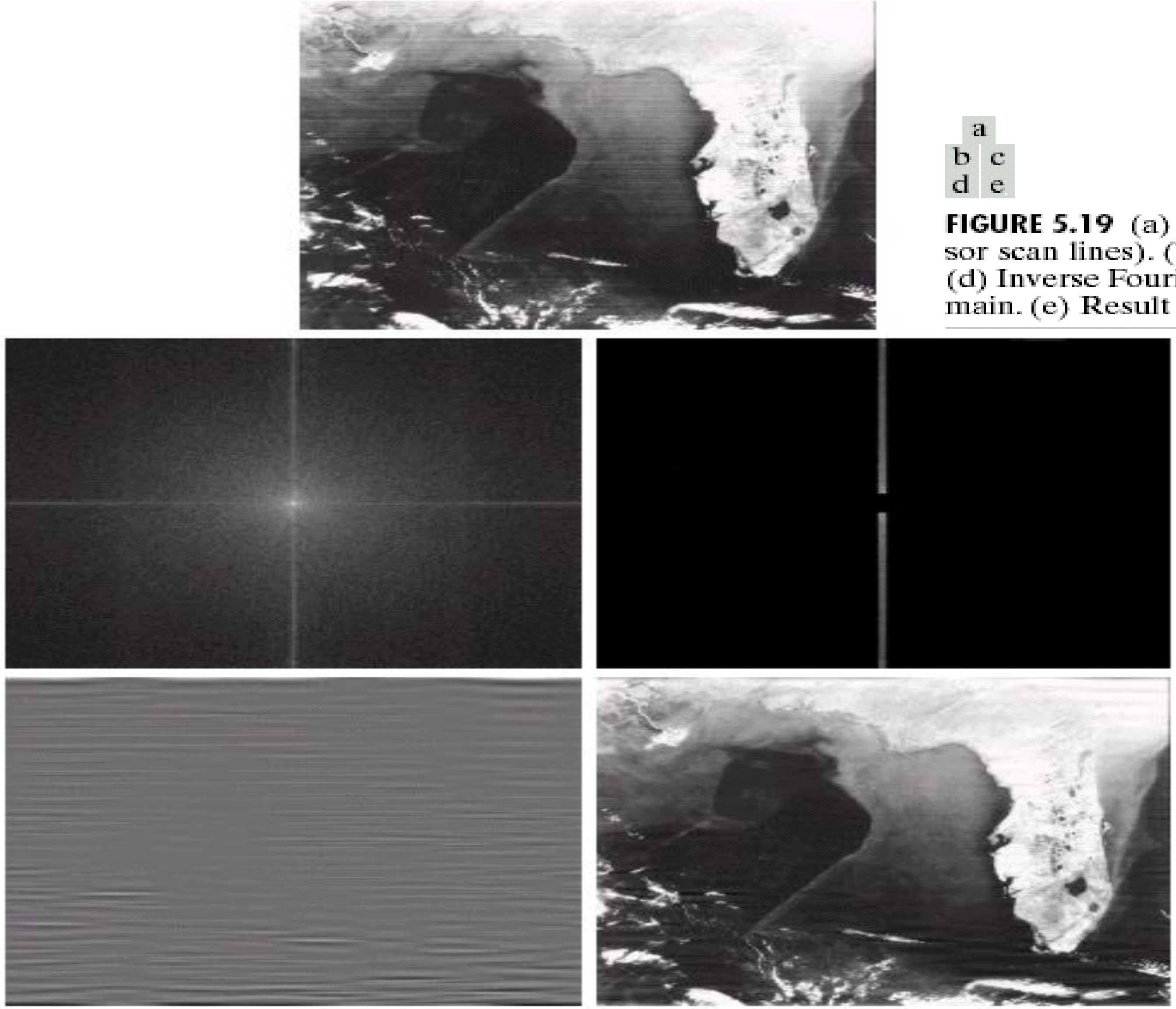



**FIGURE 5.19** (a) Satellite image of Florida and the Gulf of Mexico (note horizontal sensor scan lines). (b) Spectrum of (a). (c) Notch pass filter shown superimposed on (b). (d) Inverse Fourier transform of filtered image, showing noise pattern in the spatial domain. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

# Notch Filters

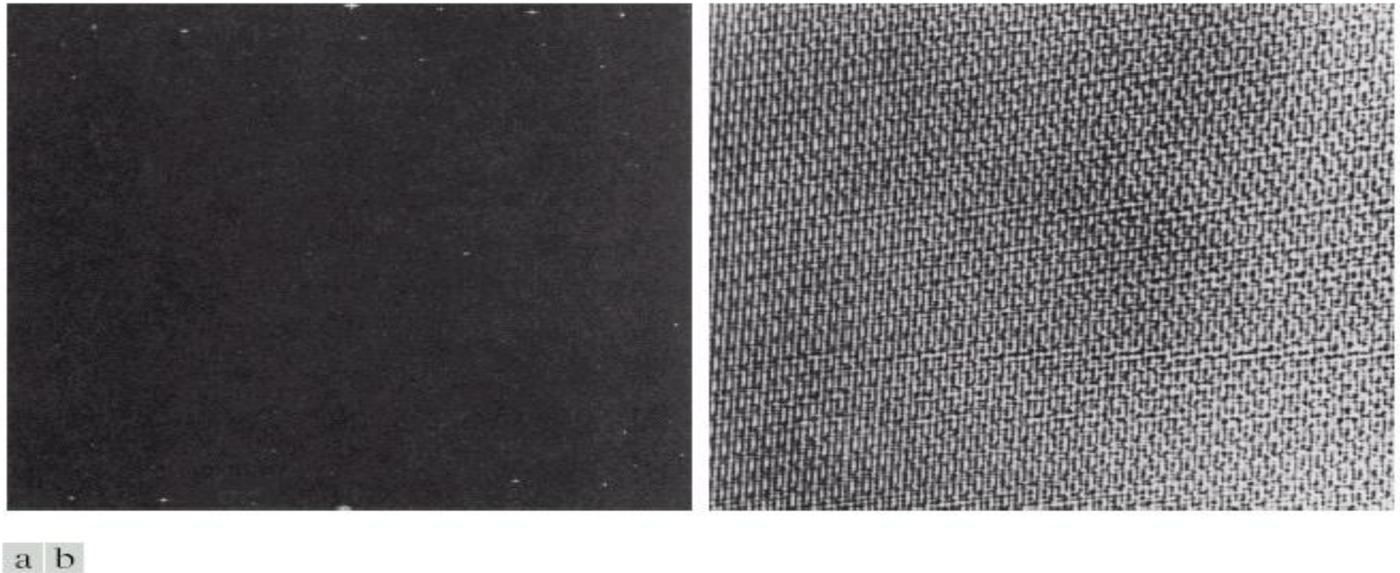




**FIGURE 5.22** (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

# Suggested Readings

- **Digital Image Processing by Rafel Gonzalez, Richard Woods, Pearson Education India, 2017.**

- **Fundamental of Digital image processing by A. K Jain, Pearson Education India, 2015.**

# Thank you