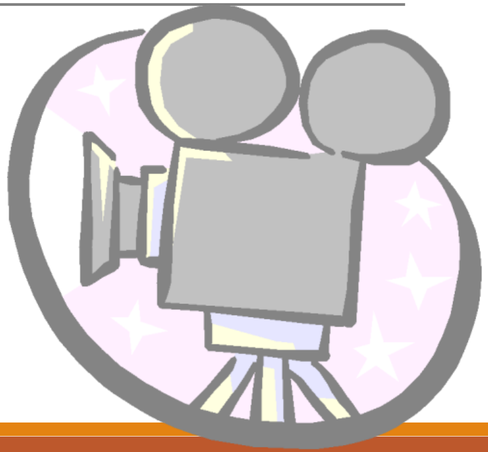
Image Processing

CS-317/CS-341



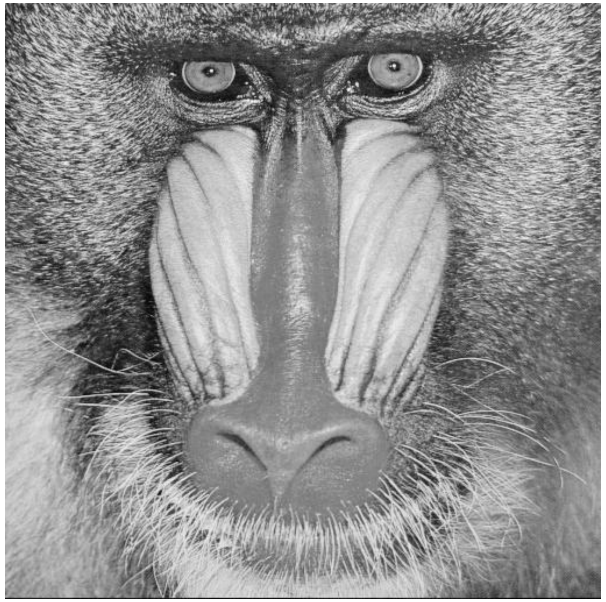
Outline

- Spatial Filtering
    - Sharpening Filter

Sharpening Spatial Filters

to highlight fine detail in an image

or to enhance detail that has been blurred, either in error or as a natural effect of a particular method of image acquisition.



Blurring vs. Sharpening

as we know that blurring can be done in spatial domain by pixel averaging in a neighbors

since averaging is analogous to integration

thus, we can guess that the sharpening must be accomplished by spatial differentiation.

Derivative operator

the strength of the response of a derivative operator is proportional to the degree of discontinuity of the image at the point at which the operator is applied.

thus, image differentiation

- enhances edges and other discontinuities (noise)
- deemphasizes area with slowly varying gray-level values.

First-order derivative

a basic definition of the first-order derivative of a one-dimensional function f(x) is the difference

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$or$$

$$\frac{\partial f}{\partial x} = f(x) - f(x-1)$$

Second-order derivative

similarly, we define the second-order derivative of a one-dimensional function f(x) is the difference

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) - f(x) - f(x) + f(x-1)$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

First and Second-order derivative of f(x,y)

when we consider an image function of two variables, f(x,y), at which time we will dealing with partial derivatives along the two spatial axes.

Gradient operator

$$\nabla \mathrm{f} = \frac{\partial f(x,y)}{\partial x \partial y} = \frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y}$$

Laplacian operator

(linear operator)

$$\nabla^2 f = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

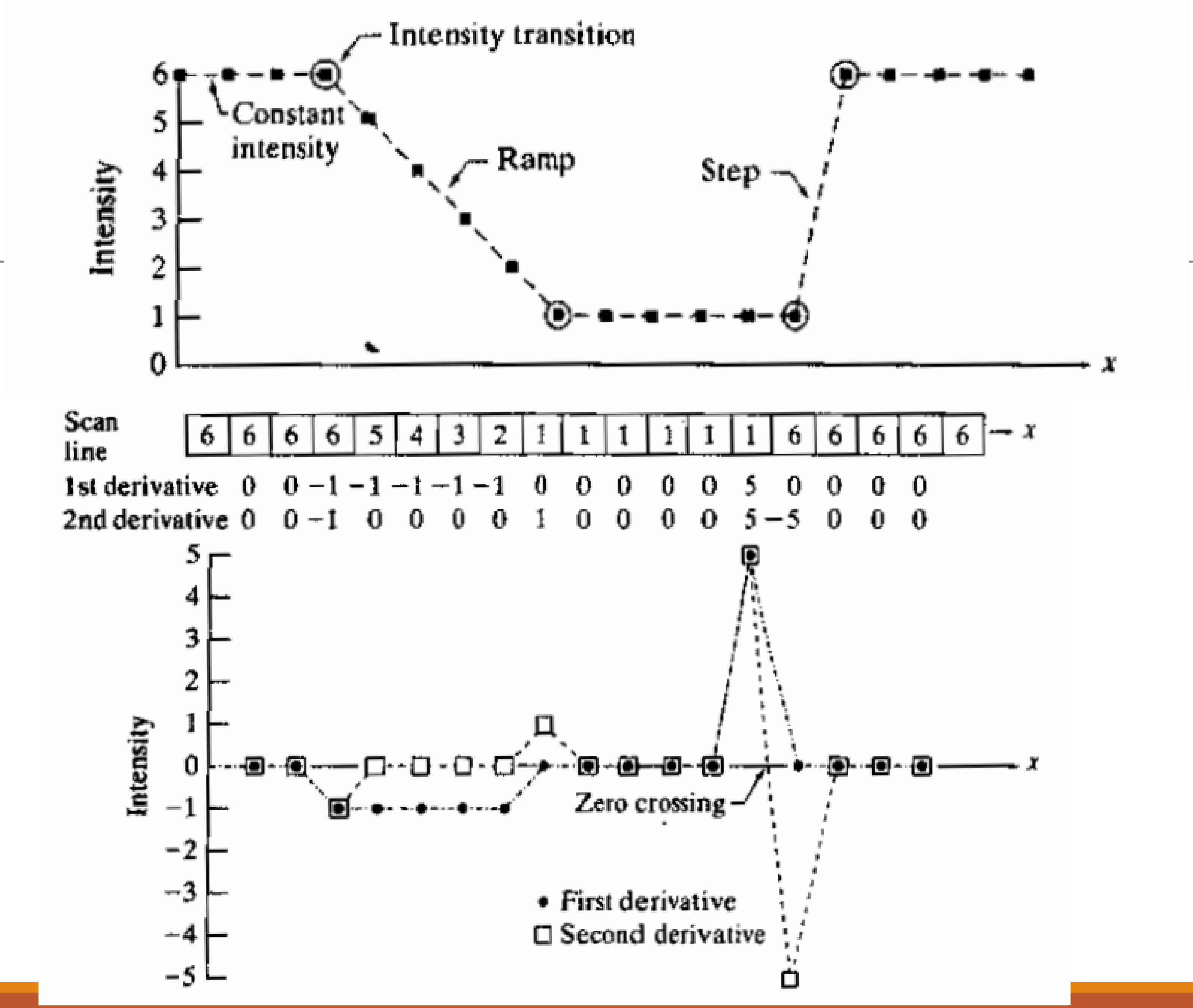
Discrete Form of Laplacian

We have,

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]$$


Intensity transition
Constant intensity
Ramp
Step
Intensity
x
Scan line 6 6 6 6 5 4 3 2 1 1 1 1 1 1 6 6 6 6 6
1st derivative 0 0 −1 −1 −1 −1 −1 0 0 0 0 0 5 0 0 0 0
2nd derivative 0 0 −1 0 0 0 0 1 0 0 0 0 5 −5 0 0 0
Zero crossing
• First derivative
□ Second derivative

Result Laplacian mask

0	1	0
1	−4	1
0	1	0

Laplacian mask implemented an extension of diagonal neighbors

1	1	1
1	−8	1
1	1	1

Other implementation of Laplacian masks

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

give the same result, but we have to keep in mind that when combining (add / subtract) a Laplacian-filtered image with another image.

Effect of Laplacian Operator

as it is a derivative operator,

- it highlights gray-level discontinuities in an image
- it deemphasizes regions with slowly varying gray levels

tends to produce images that have

- grayish edge lines and other discontinuities, all superimposed on a dark,
- featureless background.

Unsharp masking

A process that has been used for many years by printing and publishing industry to sharpen images. The process consists of the following steps

1. Blur the original image.

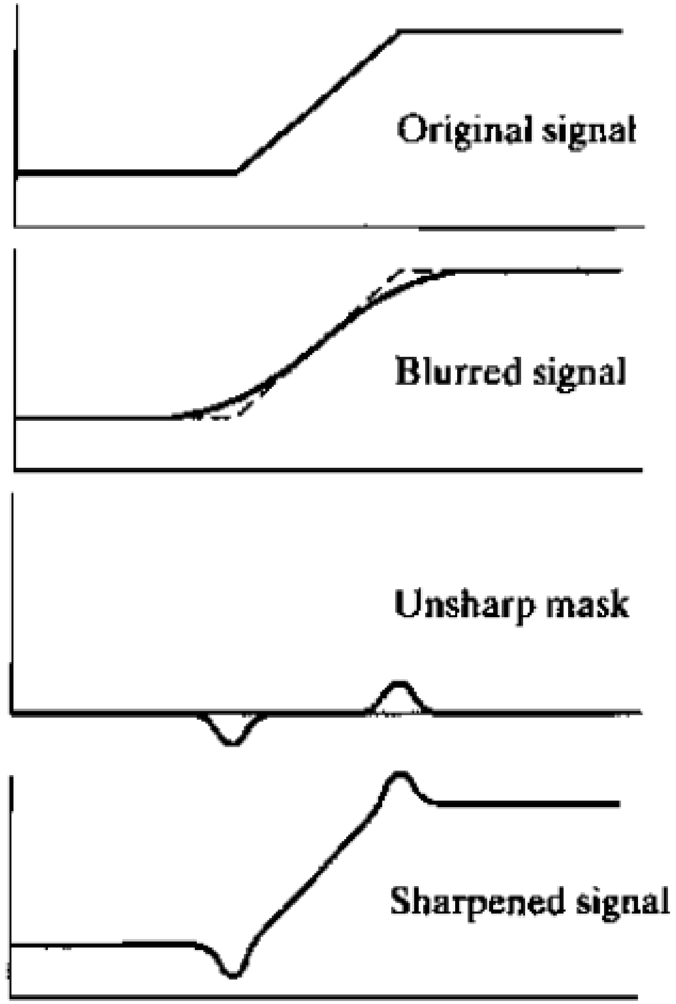
2. Subtract the blurred image from the original (resulting difference is called mask).

3. Add mask to the original.

Unsharp masking

a
b
c
d

FIGURE : 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for refere-nce. (c) Unsharp mask. (d) Sharp-ened signal, obtained by adding (c) to (a).



Unsharp masking

Let $\bar{f}(x, y)$ denote the blurred image, the unsharp masking is expressed as:

$$g_{mask}(x, y) = f(x, y) - \bar{f}(x, y)$$

mask= original image – blurred image

to subtract a blurred version of an image produces sharpening output image.

Then we add a weighted portion of the mask back to the original image:

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

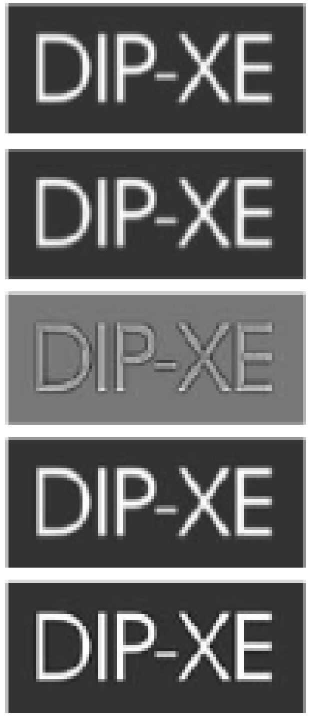
Here, we included a weight k (k$\geq$ 0), for generality.

When k=1, we have **unsharp masking**.

When k>1, the Process is referred to as **highboost filtering**.

Choosing k<1 de-emphasizes the contribution of the unsharp mask.

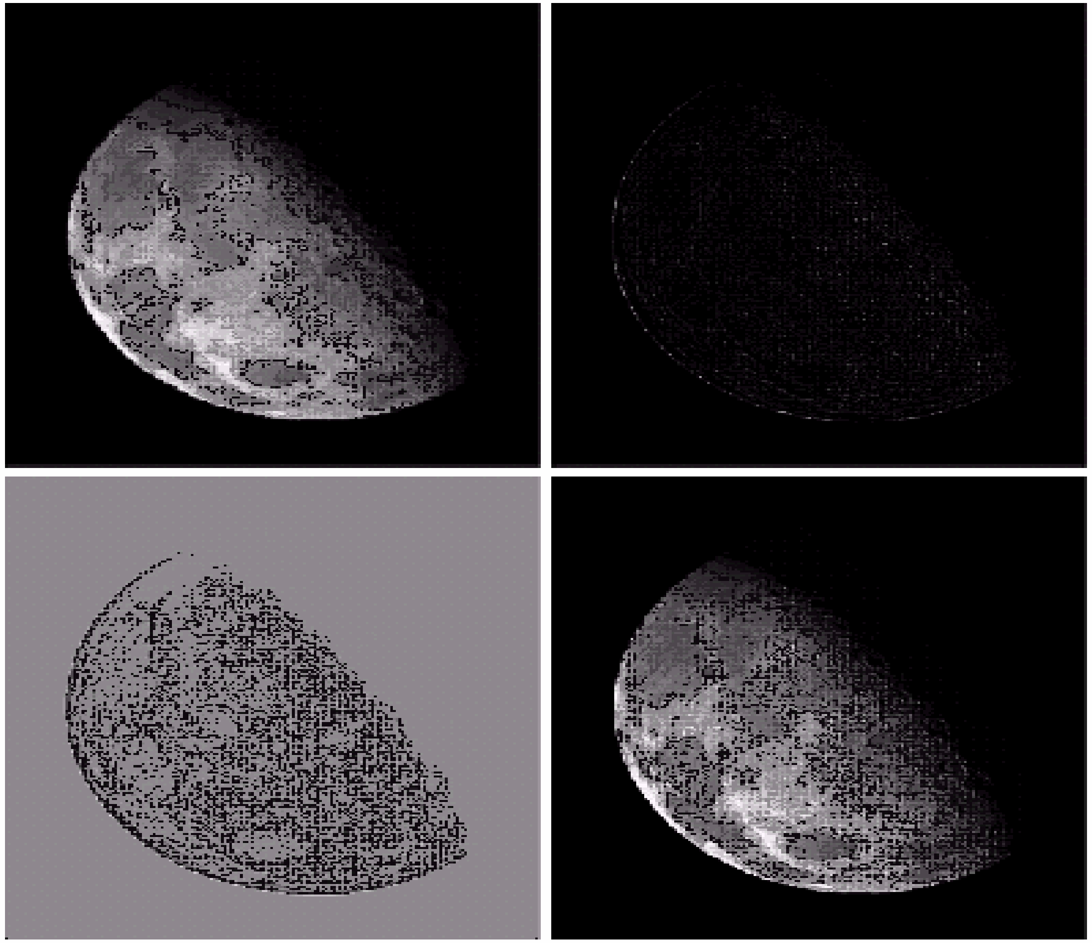
Example



a
b
c
d
e

FIGURE
(a) Original image.
(b) Result of blurring with a Gaussian filter.
(c) Unsharp mask. (d) Result of using unsharp masking.
(e) Result of using highboost filtering.

Example



a). image of the North pole of the moon

b). Laplacian-filtered image with

1	1	1
1	-8	1
1	1	1

c). Laplacian image scaled for display purposes [0, L-1]

d). image enhanced by addition with original image

Gradient Operator

first order derivatives are implemented using the magnitude of the gradient.

For a function f(x,y), the gradient of f at coordinates (x, y) is defined as

$$\nabla \mathrm{f} = grad(f) = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

Gradient Operator

$$\nabla \mathrm{f} = grad(f) = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

the magnitude (length) of the gradient of vector (∇f) is denoted as M(x,y) or mag(∇f)

$$|\nabla f| = mag(\nabla \mathrm{f}) = [G_x^2 + G_y^2]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

commonly approx.

$$|\nabla f| \approx |G_x| + |G_y|$$

the magnitude becomes nonlinear

Gradient Mask

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$f(x-1,y-1)$	$f(x-1,y)$	$f(x-1,y+1)$
$f(x,y-1)$	$f(x,y)$	$f(x,y+1)$
$f(x+1,y-1)$	$f(x+1,y)$	$f(x+1,y+1)$

simplest approximation, 2x2

$$\nabla \mathrm{f} = grad(f) = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$G_x = \frac{\partial f}{\partial x} = f(x+1, y) - f(x, y) = z_8 - z_5$$

$$G_y = \frac{\partial f}{\partial y} = f(x, y+1) - f(x, y) = z_6 - z_5$$

-1	0
1	0

G_x

-1	1
0	0

G_y

Gradient Mask

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$f(x-1,y-1)$	$f(x-1,y)$	$f(x-1,y+1)$
$f(x,y-1)$	$f(x,y)$	$f(x,y+1)$
$f(x+1,y-1)$	$f(x+1,y)$	$f(x+1,y+1)$

simplest approximation, 2x2

$$G_x = (z_8 - z_5) \quad \text{and} \quad G_y = (z_6 - z_5)$$

$$\nabla f = [G_x^2 + G_y^2]^{1/2} = [(z_8 - z_5)^2 + (z_6 - z_5)^2]^{1/2}$$

$$\nabla f \approx |z_8 - z_5| + |z_6 - z_5|$$

Cross-gradient operators

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Another definition proposed by Roberts in the early development of image processing use cross difference, called Roberts cross-gradient operators, 2x2

$$G_x = (z_9 - z_5) \quad \text{and} \quad G_y = (z_8 - z_6)$$

$$\nabla f = [G_x^2 + G_y^2]^{1/2} = [(z_9 - z_5)^2 + (z_8 - z_6)^2]^{1/2}$$

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

G_x

−1	0
0	1

G_y

0	−1
1	0

Gradient Mask

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Sobel operators, 3x3

$$G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

$$\nabla f \approx |G_x| + |G_y|$$

the weight value 2 is to achieve smoothing by giving more important to the center point

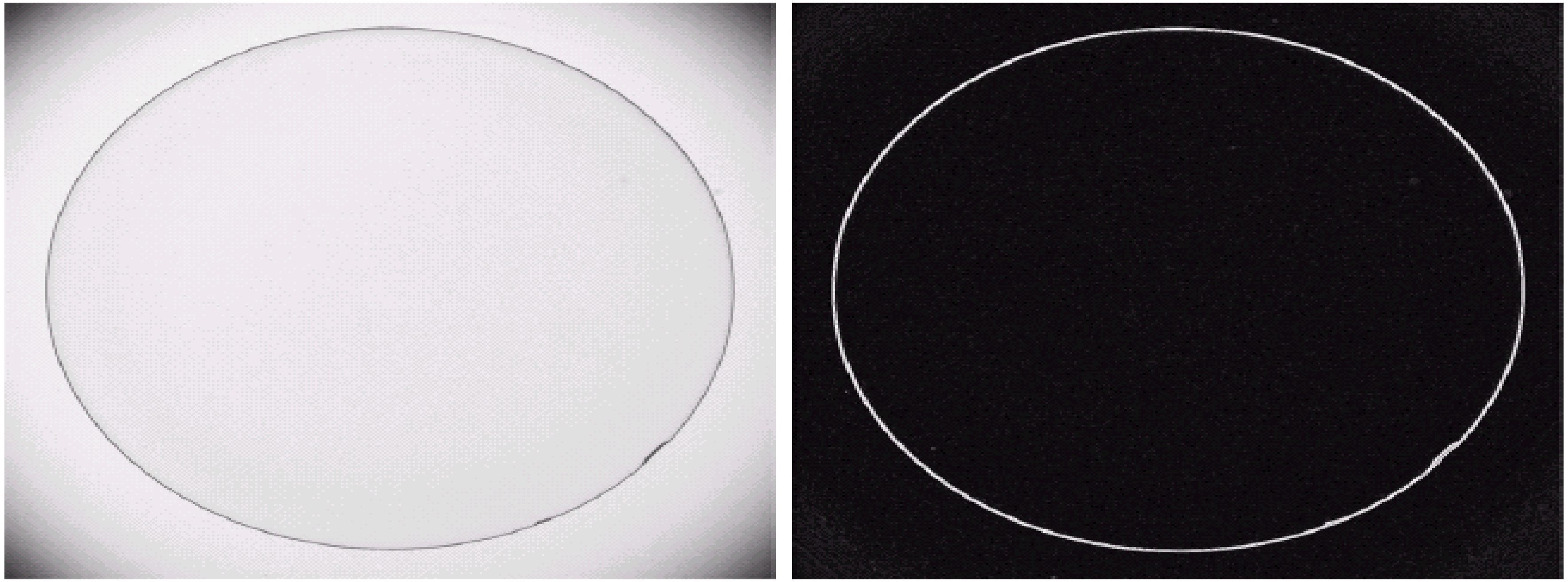
−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Note

the summation of coefficients in all masks equals 0, indicating that they would give a response of 0 in an area of constant gray level.

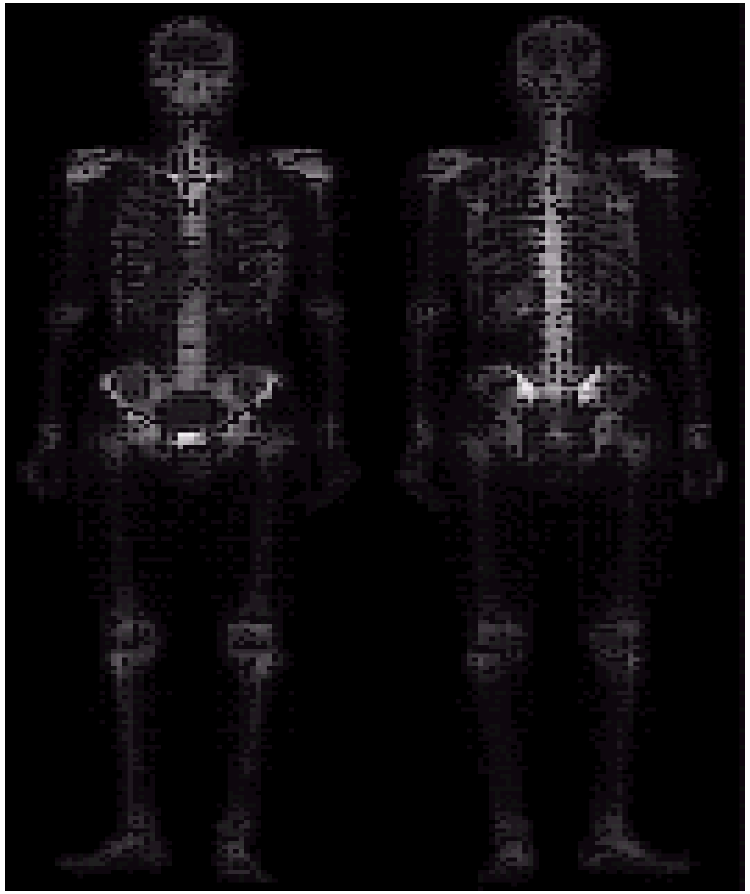
Example



a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

Example of Combining Spatial Enhancement Methods



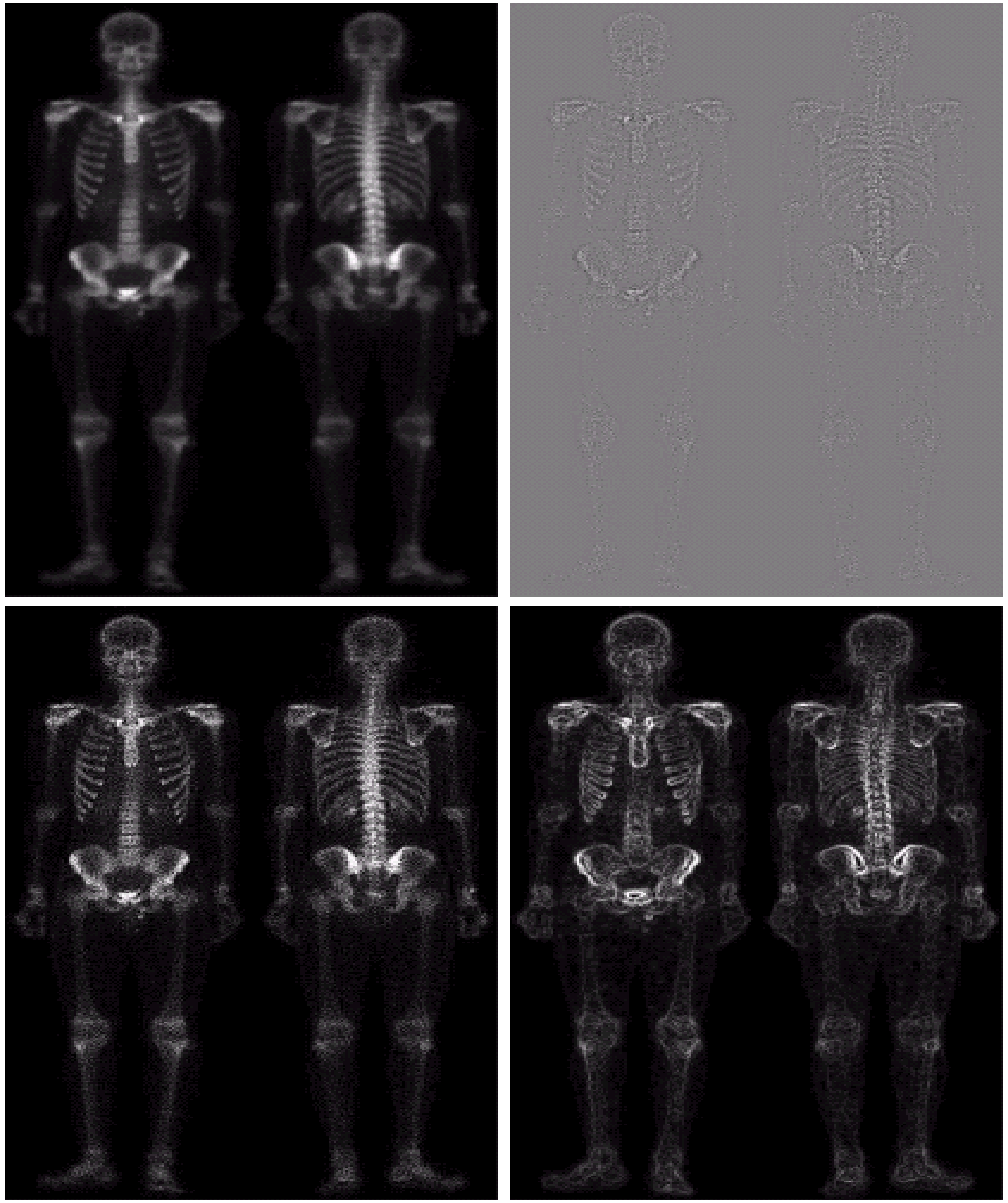
want to sharpen the original image and bring out more skeletal detail.

problems: narrow dynamic range of gray level and high noise content makes the image difficult to enhance

Example of Combining Spatial Enhancement Methods

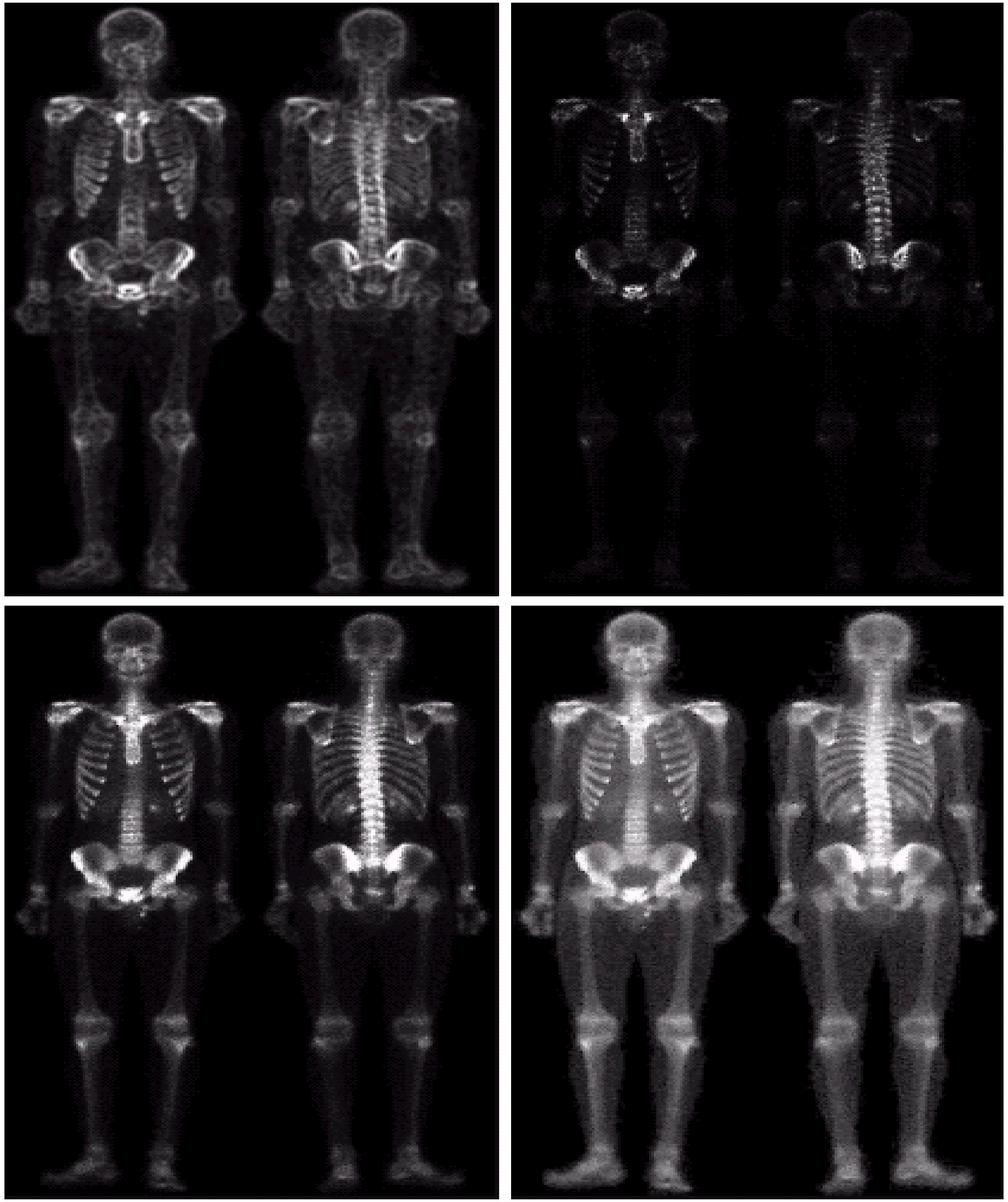
solve :

1. Laplacian to highlight fine detail
2. gradient to enhance prominent edges
3. gray-level transformation to increase the dynamic range of gray levels



a	b
c	d

FIGURE 3.46
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).



e	f
g	h

FIGURE 3.46 *(Continued)* (e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

Suggested Readings

- Digital Image Processing by Rafel Gonzalez, Richard Woods, Pearson Education India, 2017.
- Fundamental of Digital image processing by A. K Jain, Pearson Education India, 2015.

Thank you

Correct the effect of featureless background

easily by adding the original and Laplacian image.

be careful with the Laplacian filter used

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) \\ f(x,y) + \nabla^2 f(x,y) \end{cases}$$

if the center coefficient of the Laplacian mask is negative

if the center coefficient of the Laplacian mask is positive

Mask of Laplacian + addition

to simply the computation, we can create a mask which do both operations, Laplacian Filter and Addition the original image.

Mask of Laplacian + addition

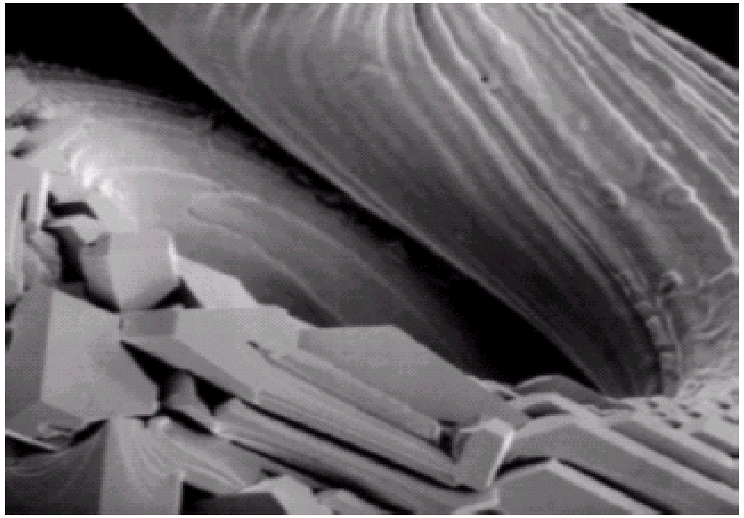
$$\begin{aligned} g(x,y) &= f(x,y) - [f(x+1,y) + f(x-1,y) \\ &\quad + f(x,y+1) + f(x,y-1) - 4f(x,y)] \\ &= 5f(x,y) - [f(x+1,y) + f(x-1,y) \\ &\quad + f(x,y+1) + f(x,y-1)] \end{aligned}$$

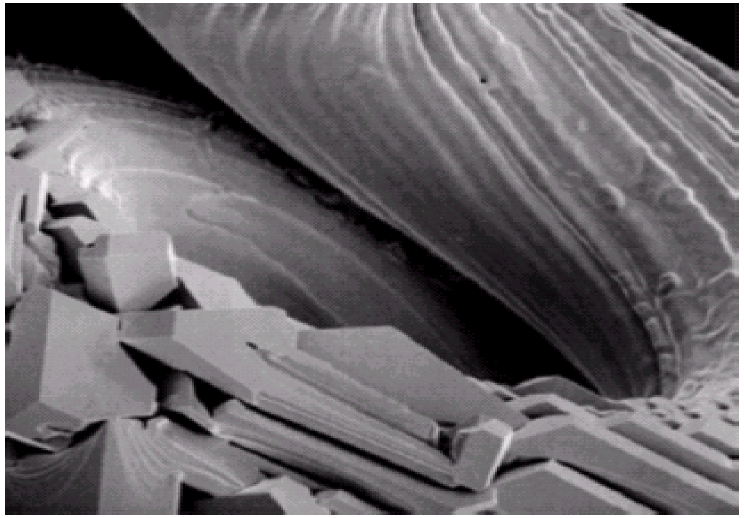
0	-1	0
-1	5	-1
0	-1	0

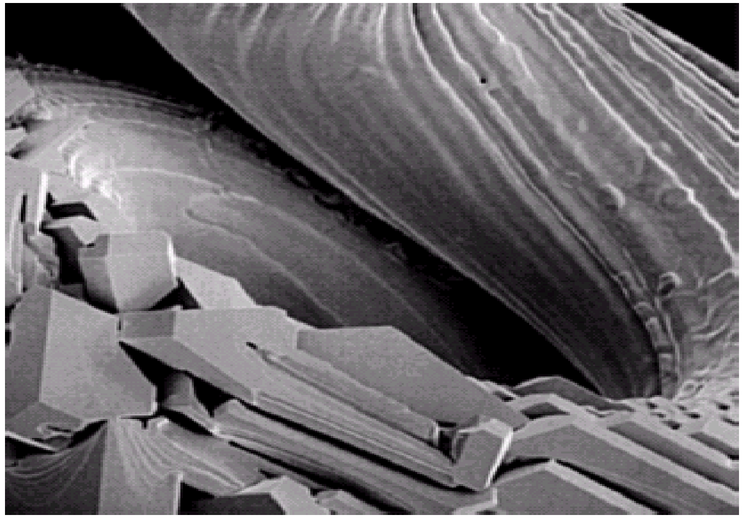
Example

0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1







a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Note

$$g(x,y)=\begin{cases} f(x,y)-\nabla^2 f(x,y) \\ f(x,y)+\nabla^2 f(x,y) \end{cases}$$

$$\begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 5 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 4 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 9 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline 0 & -1 & 0 \\ \hline -1 & 8 & -1 \\ \hline 0 & -1 & 0 \\ \hline \end{array}$$