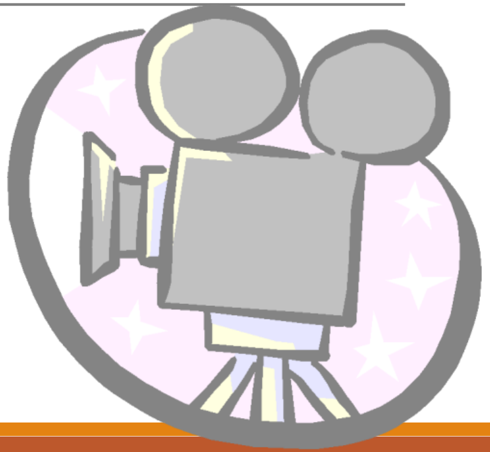
Image Processing

CS-317/CS-341



Outline

- Image Enhancement in the Frequency Domain
  - Sharpening Filters
- Image Restoration

Laplacian in the Frequency domain

The Laplacian in the Frequency domain can be implemented using the filter

$$H(u, v) = -4\pi^2(u^2 + v^2)$$

or, with respect to the center of the frequency rectangle, using the filter

$$H(u, v) = -4\pi^2\left[(u - M/2)^2 + (v - N/2)^2\right]$$

$$= -4\pi^2 D^2(u, v)$$

M N

Where, D(u,v) is the distance function

Laplacian in the Frequency domain: Image Enhancement

The Laplacian image is obtained as:

$$\nabla^2 f(x, y) = \Im^{-1}\left\{H(u, v)F(u, v)\right\} \quad =IFT\{H(u,v)F(u,v)\}\}$$

Where, F(u, v) is DFT of f(x, y), Now image enhancement is achieved using the equation (in spatial domain):

$$g(x, y) = f(x, y) + c\nabla^2 f(x, y)$$

Here, c=-1, because H(u, v) is negative.

Laplacian in the Frequency domain: Image Enhancement

In frequency domain

$$
\begin{aligned}
g(x, y) &= \Im^{-1}\{F(u, v) - H(u, v)F(u, v)\} \\
&= \Im^{-1}\{[1 - H(u, v)]F(u, v)\} \\
&= \Im^{-1}\{[1 + 4\pi^2 D^2(u, v)]F(u, v)\}
\end{aligned}
$$

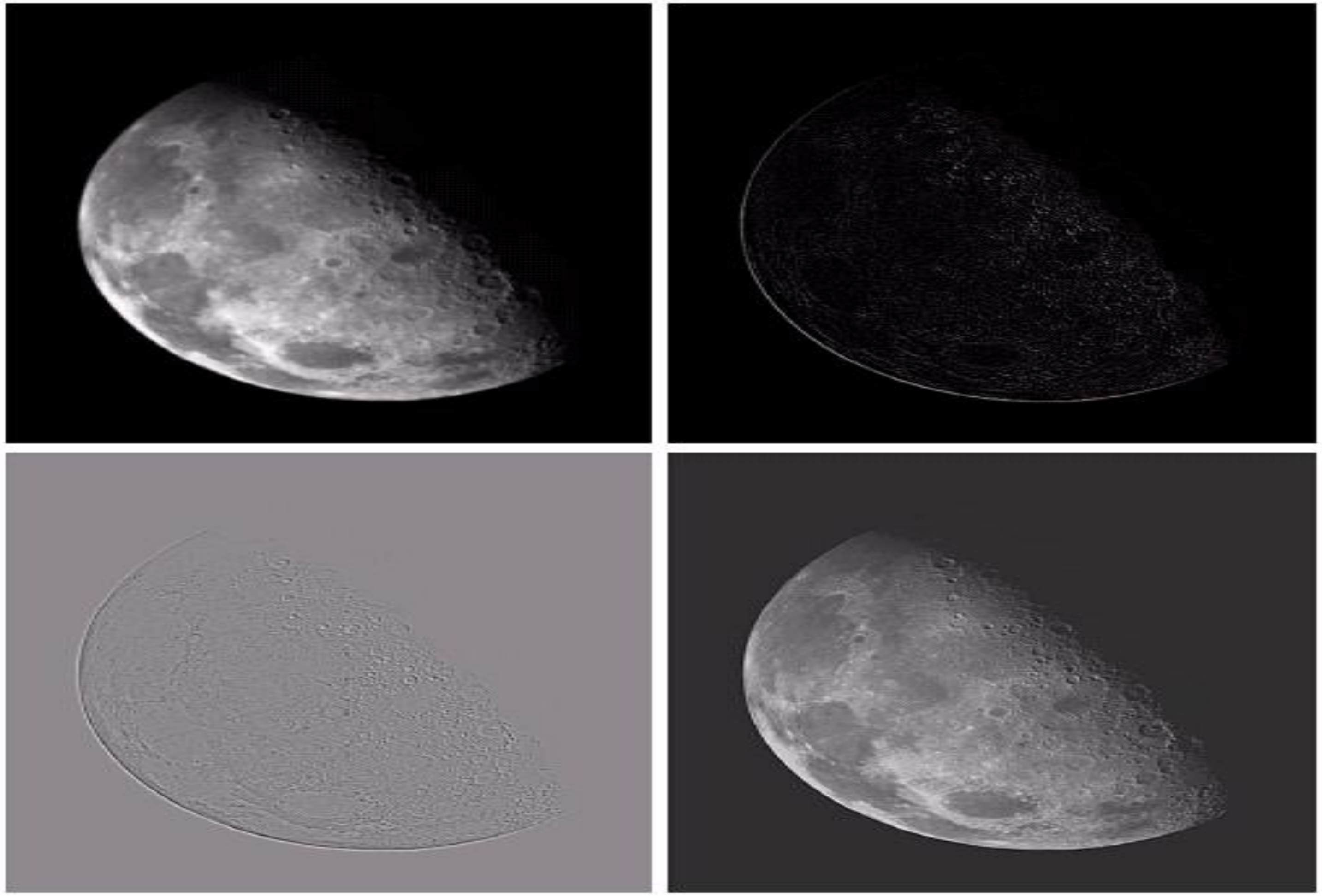
As,

$$H(u, v) = -4\pi^2(u^2 + v^2)$$

Example: Laplacian filtered image

a b
c d

FIGURE 4.28
(a) Image of the North Pole of the moon.
(b) Laplacian filtered image.
(c) Laplacian image scaled.
(d) Image enhanced by using Eq. (4.4-12).
(Original image courtesy of NASA.)



Unsharp masking

The unsharp masking is expressed as:

$$g_{mask}(x,y) = f(x,y) - f_{lp}(x,y)$$

$$With,\ f_{lp} = \nabla^2 f(x,y) = \Im^{-1}\{H_{LP}(u,v)F(u,v)\}$$

Here, $H_{LP}(u,v)$ is a low pass filter and F(u,v) is DFT of f(x, y),

Then we add a weighted portion of the mask back to the original image:

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

Here, we included a weight k (k≥ 0), for generality.

When k=1, we have **unsharp masking**.

When k>1, the Process is referred to as **highboost filtering**.

$$Lowpass$$

$$g(x,y) = IFT\{(1 + k*(1 - H_{LP}(u,v))F(u,v)\}$$

$$Highpass,\ filter$$

$$g(x,y) = IFT\{[1 + k*H_{HP}(u,v)]F(u,v)\}$$

$$General..Formula$$

$$g(x,y) = IFT\{[k_1 + k_2 * H_{HP}(u,v)]F(u,v)\}$$

Where, $k_1 >= 0$, offsets from origin
k2>=0 , contribute to High frequency filtering

Homomorphic Filtering

The illumination and reflectance model can be used to develop a frequency domain procedure for improving the appearance of an image by simultaneous intensity range compression and contrast enhancement.

An image f(x, y) can be expressed as the product of illumination i(x, y) and reflectance r(x, y) components as:

$$f(x, y) = i(x, y)r(x, y)$$

This equation can not be used directly to operate on the frequency components of illumination and reflectance because Fourier Transformation of a product is not the product of the transformations

$$\Im[f(x, y)] \neq \Im[i(x, y)]\,\Im[r(x, y)]$$

Homomorphic Filtering

Suppose we define,

$Z(x, y) = \ln f(x, y)$

$Z(x, y) = \ln i(x, y) + \ln r(x, y)$

Now, in frequency domain FT of z(x, y)

$FT(z(x, y) = FT(\ln i(x, y) + \ln r(x, y))$

$FT(z(x, y) = F(\ln i(x, y)) + FT(\ln r(x, y))$

$z(u, v) = F_i(u, v) + F_r(u, v)$

where $F_i(u, v)$ and $F_r(u, v)$ are the Fourier transforms of $\ln i(x, y)$ and $\ln r(x, y)$, respectively.

Now image is in Frequency domain, so we can apply filter

Homomorphic Filtering

We can filter $Z(u, v)$ using a filter $H(u, v)$ so that

$$S(u, v) = H(u, v)Z(u, v)$$

$$= H(u, v)F_i(u, v) + H(u, v)F_r(u, v)$$

The filtered image in the spatial domain is

$$s(x, y) = \Im^{-1}\left\{S(u, v)\right\}$$

$$= \Im^{-1}\left\{H(u, v)F_i(u, v)\right\} + \Im^{-1}\left\{H(u, v)F_r(u, v)\right\}$$

Where, $\Im^{-1}$ is the inverse FT.

By defining

$$i'(x, y) = \Im^{-1}\left\{H(u, v)F_i(u, v)\right\}$$

and

$$r'(x, y) = \Im^{-1}\left\{H(u, v)F_r(u, v)\right\}$$

we can express Eq. (4.9-23) in the form

$$s(x, y) = i'(x, y) + r'(x, y)$$

Finally, because $z(x, y)$ was formed by taking the natural logarithm of the input image, we reverse the process by taking the exponential of the filtered result to form the output image:

$$
\begin{aligned}
g(x, y) &= e^{s(x,y)} \\
&= e^{i'(x,y)} e^{r'(x,y)} \\
&= i_0(x, y) r_0(x, y)
\end{aligned}
$$

where

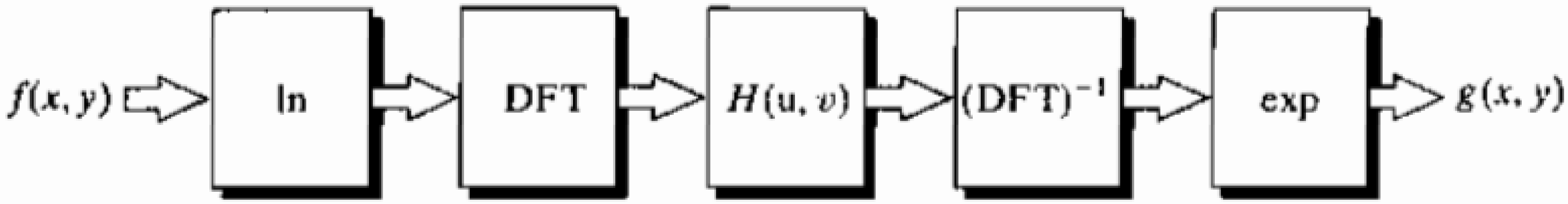
$$i_0(x, y) = e^{i'(x,y)}$$

and

$$r_0(x, y) = e^{r'(x,y)}$$

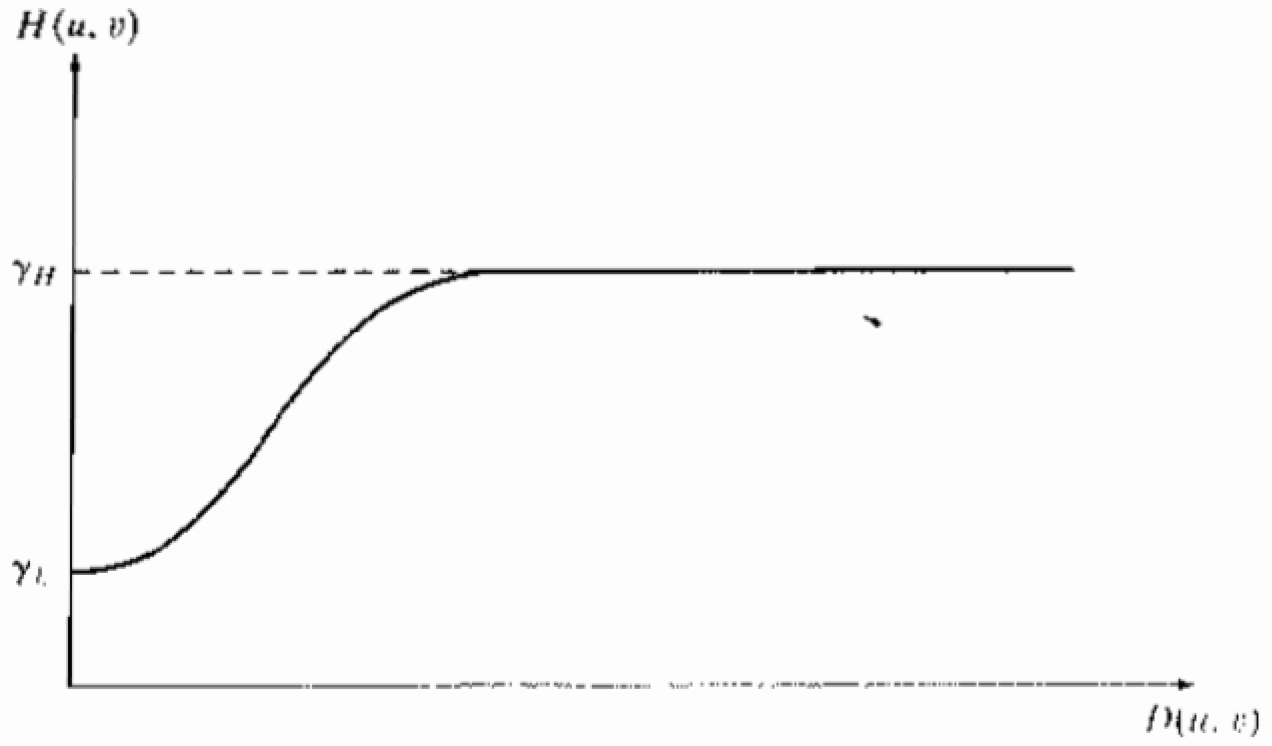
are the illumination and reflectance components of the output (processed) image.

Steps of Homomorphic Filtering



y_L<1, it compresses low frequency components Y_H>1, contrast enhancement

$$H(u, v) = (\gamma_H - \gamma_L)\left[1 - e^{-c[D^2(u,v)/D_0^2]}\right] + \gamma_L$$



Suggested Readings

- **Digital Image Processing by Rafel Gonzalez, Richard Woods, Pearson Education India, 2017.**
- **Fundamental of Digital image processing by A. K Jain, Pearson Education India, 2015.**

Thank you