

## Principal component Analysis

Given the data in the table, reduce the dimension from 2 to 1 using principal component analysis

Feature	Example 1	Example 2	Example 3	Example 4
$X_1$	4	8	13	7
$X_2$	11	4	8.5	14

$$\lambda_1 = 30.38$$

$$\lambda_2 = 6.61$$

$$e_1 = \begin{bmatrix} -0.5574 \\ -0.8303 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0.8303 \\ 0.5574 \end{bmatrix}$$

$$\text{Mean of } X_1 = 8$$

$$\text{Mean of } X_2 = 8.5$$

Mean centered matrix

$$X_{\text{centered}} = \begin{bmatrix} 4-8 & 8-8 & 13-8 & 7-8 \\ 11-8.5 & 4-8.5 & 8.5-8.5 & 14-8.5 \end{bmatrix} = \begin{bmatrix} -4 & 0 & 5 & -1 \\ 2.5 & -4.5 & 0 & 5.5 \end{bmatrix}$$

Use first eigenvector as it has largest eigenvalue

$$Z = e_1^T X_{\text{centered}}$$

$$= \begin{bmatrix} -0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} -4 & 0 & 5 & -1 \\ 2.5 & -4.5 & 0 & 5.5 \end{bmatrix}$$

$$Z_1 = 0.15385$$

$$Z_2 = 3.736$$

$$Z_3 = 0.11905$$

$$Z_4 = -4.009$$

Final reduced 1D data

$$Z = \begin{bmatrix} 0.15385 & 3.736 & 0.11905 & -4.009 \end{bmatrix}$$