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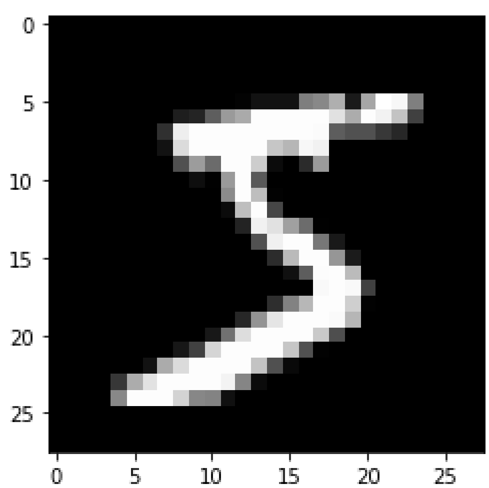
In this problem we will train a neural network from scratch using numpy. In practice, you will never need to do this (you'd just use TensorFlow or PyTorch). But hopefully this will give us a sense of what's happening under the hood.

For training/testing, we will use the standard MNIST benchmark consisting of images of handwritten images.

```
import tensorflow as tf
import matplotlib.pyplot as plt

(x_train, y_train), (x_test, y_test) = tf.keras.datasets.mnist.load_data(path="mnist.r

plt.imshow(x_train[0],cmap='gray');
```



Loading MNIST is the only place where we will use TensorFlow; the rest of the code will be pure numpy.

Let us now set up a few helper functions. We will use sigmoid activations for neurons, the softmax activation for the last layer, and the cross entropy loss.

```
import numpy as np

def sigmoid(x):
  # Numerically stable sigmoid function based on
  # http://timvieira.github.io/blog/post/2014/02/11/exp-normalize-trick/

  x = np.clip(x, -500, 500) # We get an overflow warning without this
```

```
  return np.where(
    x >= 0,
    1 / (1 + np.exp(-x)),
    np.exp(x) / (1 + np.exp(x))
  )

def dsigmoid(x): # Derivative of sigmoid
  return sigmoid(x) * (1 - sigmoid(x))

def softmax(x):
  # Numerically stable softmax based on (same source as sigmoid)
  # http://timvieira.github.io/blog/post/2014/02/11/exp-normalize-trick/
  b = x.max()
  y = np.exp(x - b)
  return y / y.sum()

def cross_entropy_loss(y, yHat):
  return -np.sum(y * np.log(yHat))

def integer_to_one_hot(x, max):
  # x: integer to convert to one hot encoding
  # max: the size of the one hot encoded array
  result = np.zeros(10)
  result[x] = 1
  return result
```

OK, we are now ready to build and train our model. The input is an image of size 28x28, and the output is one of 10 classes. So, first:

Q1. Initialize a 2-hidden layer neural network with 32 neurons in each hidden layer, i.e., your layer sizes should be:

784 -> 32 -> 32 -> 10

If the layer is $n_{in} \times n_{out}$ your layer weights should be initialized by sampling from a normal distribution with mean zero and variance $1/\max(n_{in}, n_{out})$.

```
import math

# Initialize weights of each layer with a normal distribution of mean 0 and
# standard deviation 1/sqrt(n), where n is the number of inputs.
# This means the weighted input will be a random variable itself with mean
# 0 and standard deviation close to 1 (if biases are initialized as 0, standard
# deviation will be exactly 1)

from numpy.random import default_rng

rng = default_rng(80085)

# Q1. Fill initialization code here.
```

```
# ...
size = [784*32, 32*32, 32*10]
b = [32, 32, 10]
mu = [0,0,0]
sigma = [1/math.sqrt(784), 1/math.sqrt(32), 1/math.sqrt(32)]

w1 = np.random.normal(loc= mu[0], scale= sigma[0], size= 784*32).reshape(784,32)
w2 = np.random.normal(loc= mu[1], scale= sigma[1], size= 32*32).reshape(32,32)
w3 = np.random.normal(loc= mu[1], scale= sigma[1], size= 32*10).reshape(32,10)

weights = [w1, w2, w3]
biases = [np.zeros((1,b[i])) for i in range(3)]
```

```
x_train.shape
```

```
(60000, 28, 28)
```

```
y_train[0]
```

```
5
```

Next, we will set up the forward pass. We will implement this by looping over the layers and successively computing the activations of each layer.

Q2. Implement the forward pass for a single sample, and for the entire dataset.

Right now, your network weights should be random, so doing a forward pass with the data should not give you any meaningful information. Therefore, in the last line, when you calculate test accuracy, it should be somewhere around 1/10 (i.e., a random guess).

```
def feed_forward_sample(sample, y):
  """ Forward pass through the neural network.
    Inputs:
      sample: 1D numpy array. The input sample (an MNIST digit).
      label: An integer from 0 to 9.

    Returns: the cross entropy loss andmost likely class of "sample"
  """
  # Q2. Fill code here.
  # ...
  L1 = sigmoid(np.dot(sample, weights[0]) + biases[0])
  L2 = sigmoid(np.dot(L1, weights[1]) + biases[1])
  output = softmax(np.dot(L2, weights[2]) + biases[2])
  one_hot_guess = integer_to_one_hot(np.argmax(output), 10)

  y = integer_to_one_hot(y, 10)
  loss = cross_entropy_loss(y, output)
  return loss, one_hot_guess
```

```
  return loss, one_hot_guess


def feed_forward_dataset(x, y):
  losses = np.empty(x.shape[0])
  one_hot_guesses = np.empty((x.shape[0], 10))


  # ...
  # Q2. Fill code here to calculate losses, one_hot_guesses
  # ...
  x = x.reshape(-1, 784)
  for i in range(len(x)):
    l, h = feed_forward_sample(x[i], y[i])
    losses[i] = 1
    one_hot_guesses[i] = h

  y_one_hot = np.zeros((y.size, 10))
  y_one_hot[np.arange(y.size), y] = 1

  correct_guesses = np.sum(y_one_hot * one_hot_guesses)
  correct_guess_percent = format((correct_guesses / y.shape[0]) * 100, ".2f")

  print("\nAverage loss:", np.round(np.average(losses), decimals=2))
  print("Accuracy (# of correct guesses):", correct_guesses, "/", y.shape[0], "(", co

def feed_forward_training_data():
  print("Feeding forward all training data...")
  feed_forward_dataset(x_train, y_train)
  print("")

def feed_forward_test_data():
  print("Feeding forward all test data...")
  feed_forward_dataset(x_test, y_test)
  print("")

feed_forward_test_data()
```

```
Feeding forward all test data...

Average loss: 1.0
Accuracy (# of correct guesses): 1132.0 / 10000 ( 11.32 %)
```

OK, now we will implement the backward pass using backpropagation. We will keep it simple and just do training sample-by-sample (no minibatching, no randomness).

Q3: Compute the gradient of all the weights and biases by backpropagating derivatives all the way from the output to the first layer.

```
def train_one_sample(sample, y, learning_rate=0.003):
```

```
    a = sample.flatten()

    # We will store each layer's activations to calculate gradient
    activations = []

    # Forward pass

    # Q3. This should be the same as what you did in feed_forward_sample above.
    # ...
    A1 = np.dot(a,weights[0]) + biases[0]
    Z1 = sigmoid(A1)

    A2 = np.dot(Z1,weights[1])+biases[1]
    Z2 = sigmoid(A2)

    A3 = np.dot(Z2,weights[2])+biases[2]
    Z3 = softmax(A3)

    one_hot_guess = integer_to_one_hot(np.argmax(Z3),10)
    y = integer_to_one_hot(y,10)
    loss = cross_entropy_loss(y,Z3)

    activations.append(Z1)
    activations.append(Z2)
    activations.append(Z3)

    # Backward pass

    # Q3. Implement backpropagation by backward-stepping gradients through each layer.
    # You may need to be careful to make sure your Jacobian matrices are the right shape
    # At the end, you should get two vectors: weight_gradients and bias_gradients.
    # ...
    dZ3 = -1 * (one_hot_guess - (one_hot_guess*Z3))
    Z2 = Z2.reshape(1,32)
    dZ3 = dZ3.reshape(1,10)
    dW3 = np.dot(Z2.T,dZ3)
    dB3 = dZ3

    dZ2 = dsigmoid(Z2)*np.dot(dZ3,Z3.T)
    A1 = A1.reshape(1,32)
    dZ2 = dZ2.reshape(1,32)
    dW2 = np.dot(A1.T,dZ2)
    dB2 = dZ2

    dZ1 = dsigmoid(Z1)*np.dot(dZ2,Z2.T)
    a = a.reshape(1,784)
    dZ1 = dZ1.reshape(1,32)
    dW1 = np.dot(a.T,dZ1)
    dB1 = dZ1

    weight_gradients = [dW1,dW2,dW3]
```

```
  bias_gradients = [dB1,dB2,dB3]
  for i in range(0,3):
    # Update weights & biases based on your calculated gradient
    weights[i] -= weight_gradients[i] * learning_rate
    biases[i] -= bias_gradients[i].flatten() * learning_rate
```

Finally, train for 3 epochs by looping over the entire training dataset 3 times.

Q4. Train your model for 3 epochs.

```
def train_one_epoch(learning_rate=0.003):
  print("Training for one epoch over the training dataset...")

  # Q4. Write the training loop over the epoch here.
  # ...
  for i in range(len(x_train)):
    train_one_sample(x_train[i],y_train[i])
  print("Finished training.\n")


feed_forward_test_data()

def test_and_train():
  train_one_epoch()
  feed_forward_test_data()

for i in range(3):
  print('Epoch ', i)
  test_and_train()
```

```
Feeding forward all test data...

Average loss: 1.0
Accuracy (# of correct guesses): 1010.0 / 10000 ( 10.10 %)

Epoch  0
Training for one epoch over the training dataset...
Finished training.

Feeding forward all test data...

Average loss: 1.0
Accuracy (# of correct guesses): 1010.0 / 10000 ( 10.10 %)

Epoch  1
Training for one epoch over the training dataset...
Finished training.

Feeding forward all test data...

Average loss: 1.0
```

```
Accuracy (# of correct guesses): 1010.0 / 10000 ( 10.10 %)

Epoch  2
Training for one epoch over the training dataset...
Finished training.

Feeding forward all test data...

Average loss: 1.0
Accuracy (# of correct guesses): 1010.0 / 10000 ( 10.10 %)
```

That's it!

Your code is probably very time- and memory-inefficient; that's ok. There is a ton of optimization under the hood in professional deep learning frameworks which we won't get into.

If everything is working well, you should be able to raise the accuracy from ~10% to ~85% accuracy after 3 epochs.