EL9343 Homework 2

(Due February 9th, 2022)

All problem/exercise numbers are for the third edition of CLRS text book

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1. First use the iteration method to solve the recurrence, draw the recursion tree to analyze.

   $$T(n) = T(\frac{n}{2}) + T(\frac{n}{3}) + n$$

   Then use the substitution method to verify your solution.

2. Use the substitution method to prove that $T(n) = 2T(\frac{n}{2}) + cnlog_2 n$ is $O(n(log_2 n)^2)$.

3. Solving the recurrence:

   $T(n) = 3T(\sqrt[3]{n}) + log_2 n$

   (Hint: Making change of variable)

4. You have three algorithms to a problem and you do not know their efficiency, but fortunately, you find the recurrence formulas for each solution, which are shown as follows:

   A: $T(n) = 3T(\frac{n}{3}) + \theta(n)$

   B: $T(n) = 2T(\frac{9n}{10}) + \theta(n)$

   C: $T(n) = 3T(\frac{n}{3}) + \theta(n^2)$

   Please give the running time of each algorithm (In θ notation), and which of your algorithms is the fastest (You probably can do this without a calculator)?

5. Can the master theorem be applied to recurrence of $T(n) = T(\frac{n}{2}) + n^2 lgn$? Why does it work or not? Provide the asymptotic upper bound for this recurrence.