EL9343 Homework 01

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Question 01

Prove the *Transpose Symmetry* property, i.e., $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$.

Solution

To prove that $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$

Let us consider the Left hand Side –
If $f(n) = O(g(n))$, then for that condition to satisfy, based on the definition of big-Oh notation,
There exists a constant c, and $\Rightarrow f(n) \leq c.\, g(n)$
And for c to be positive $(c > 0) \Rightarrow g(n) \geq \frac{1}{c}.\, f(n)$

Upon inspecting the equation $[g(n) \geq \frac{1}{c}.\, f(n)]$, we can notice that it closely aligns with the definition of Omega notation (Ω),
Which implies that $g(n) = \Omega(f(n))$

Therefore, upon evaluation, **it is proved that** $f(n) = O(g(n))$ **if and only if** $g(n) = \Omega(f(n))$

Question 02

Problem 3-1 in CLRS textbook.

Asymptotic behavior of polynomials
where $a_d > 0$, be a degree-d polynomial in n, and let k be a constant. Use the definitions of the asymptotic notations to prove the following properties.

2.a

If $k \geq d$, then $p(n) = O(n^k)$.

Solution

Let $c_1 = \frac{a_d}{2}, c_2 = \frac{3.a_d}{2}, 0 \leq c_1.\, n^d \leq p(n) \leq c_2.\, n^d$ for all $n \geq n_0$ $\quad \ldots.(1)$
For a given function $g(n)$, denote $O(g(n))$ as the set of functions given by –
$O(g(n)) = \{f(n) :$ there exists positive constants c and n_0 such that $0 \leq f(n) \leq c.\, g(n)$ for all $n \geq n_0\}$

From the given statement, $f(n) = p(n)$ and $g(n) = n^k$, where k is a constant.
Given $k \geq d \Rightarrow n^d \leq n^k \Rightarrow c_2 . n^d \leq c_2 . n^k \quad .... (2)$
From equations (1) and (2), $0 \leq p(n) \leq c_2 . n^d \leq c_2 . n^k$ for all $n \geq n_0$
Hence, $p(n) = O(n^k)$

2.b

If $k \leq d$, then $p(n) = \Omega(n^k)$.

Solution

For a given function $g(n)$, denote $\Omega(g(n))$ as the set of functions given by –
$\Omega(g(n)) = \{ f(n)$: there exists positive constants c and n_0 such that $0 \leq c . g(n) \leq f(n)$ for all $n \geq n_0 \}$
From the given statement, $f(n) = p(n)$ and $g(n) = n^k$, where k is a constant.
Given $k \leq d \Rightarrow n^k \leq n^d \Rightarrow c_1 . n^k \leq c_1 . n^d \quad .... (3)$
From equations (1) and (3), $0 \leq c_1 . n^k \leq c_1 . n^d \leq p(n)$ for all $n \geq n_0$
Hence, $p(n) = \Omega(n^k)$

2.c

If $k = d$, then $p(n) = \Theta(n^k)$.

Solution

For a given function $g(n)$, denote $\Theta(g(n))$ as the set of functions given by –
$\Theta(g(n)) = \{ f(n)$: there exists positive constants c_1, c_2 and n_0 such that
$0 \leq c_1 . g(n) \leq f(n) \leq c_2 . g(n)$ for all $n \geq n_0 \}$
From the given statement, $f(n) = p(n)$ and $g(n) = n^k$, where k is a constant.
From equation (1),
$0 \leq c_1 . n^d \leq p(n) \leq c_2 . n^d$ for all $n \geq n_0$
Hence, $p(n) = \Theta(n^k)$

2.d

If $k > d$, then $p(n) = o(n^k)$.

Solution

For a given function $g(n)$, denote $o(g(n))$ as the set of functions given by –
$o(g(n)) = \{ f(n)$: for any positive constants $c > 0$, there exist a constant $n_0 > 0$ such that $0 \leq f(n) < c . g(n)$ for all $n \geq n_0 \}$
From the given statement, $f(n) = p(n)$ and $g(n) = n^k$, where k is a constant.
Given $k > d \Rightarrow n^d < n^k \Rightarrow c_2 . n^d < c_2 . n^k \quad .... (4)$
From equations (1) and (4), $0 \leq p(n) \leq c_2 . n^d < c_2 . n^k$ for all $n \geq n_0$
Hence, $p(n) = o(n^k)$

2.e

If $k < d$, then $p(n) = \omega(n^k)$.

Solution

For a given function $g(n)$, denote $\omega(g(n))$ as the set of functions given by –
$\omega(g(n)) = \{f(n)$: for any positive constants $c > 0$, there exist a constant $n_0 > 0$ such that $0 \le c.\, g(n) < f(n)$ for all $n \ge n_0\}$
From the given statement, $f(n) = p(n)$ and $g(n) = n^k$, where k is a constant.
Given $k < d \Rightarrow n^k < n^d \Rightarrow c_1.\, n^k < c_1.\, n^d \quad \ldots.(5)$
From equations (1) and (5), $0 \le c_1.\, n^k < c_1.\, n^d \le p(n)$ for all $n \ge n_0$
Hence, $p(n) = \omega(n^k)$

Question 03

Problem 3-2 in CLRS textbook.

Relative asymptotic growths
Indicate, for each pair of expressions (A,B) in the table below, whether A is O, o, Ω, ω, or Θ of B. Assume that $k \ge 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

Solution

A	B	O	o	Ω	ω	Θ
$lg^k n$	n^ϵ	Yes	Yes	No	No	No
n^k	c^n	Yes	Yes	No	No	No
$\sqrt{n}$	n^{sinn}	No	No	No	No	No
2^n	$2^{n/2}$	No	No	Yes	Yes	Yes
n^{lgc}	c^{lgn}	Yes	No	Yes	No	Yes
$lg(n!)$	$lg(n^n)$	Yes	No	Yes	No	Yes

Question 04

For each algorithm write down all the possible formulas that could be associated with it.

Solution

For the sake of understanding, I will tabulate them

Formula	A1	A2	A3	A4	A5
a	No	Yes	Yes	Yes	Yes
b	No	No	Yes	Yes	No

Formula	A1	A2	A3	A4	A5
c	Yes	No	Yes	Yes	Yes
d	Yes	No	No	Yes	Yes
e	No	No	Yes	No	No
f	No	No	Yes	Yes	No
g	No	No	Yes	Yes	No

Question 05

For the following algorithm: Show what is printed by the following algorithm when called with MAXIMUM(A, 1, 5) where A = [10, 8, 6, 4, 2]? Where the function PRINT simple prints its arguments in some appropriate manner.

Solution

The final output will be —

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8 10
6 10
2 4
4 10
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