SAGAR NEGI SEC-H ROLL No:- 10

Tutorial - 6

1. What do you mean by minimum spanning tree? what are the applications of MST?

Sol:- A minimum spanning tree is a spanning tree in which the sum of the weight of the edges is as minimum as possible, but that does not form a cycle.

Applications of MST:

(i) Design of networks such as computer networks, telecommunication networks, water supply networks, etc.

(ii) Cluster Analysis.

(iii) Image registration and segmentation.

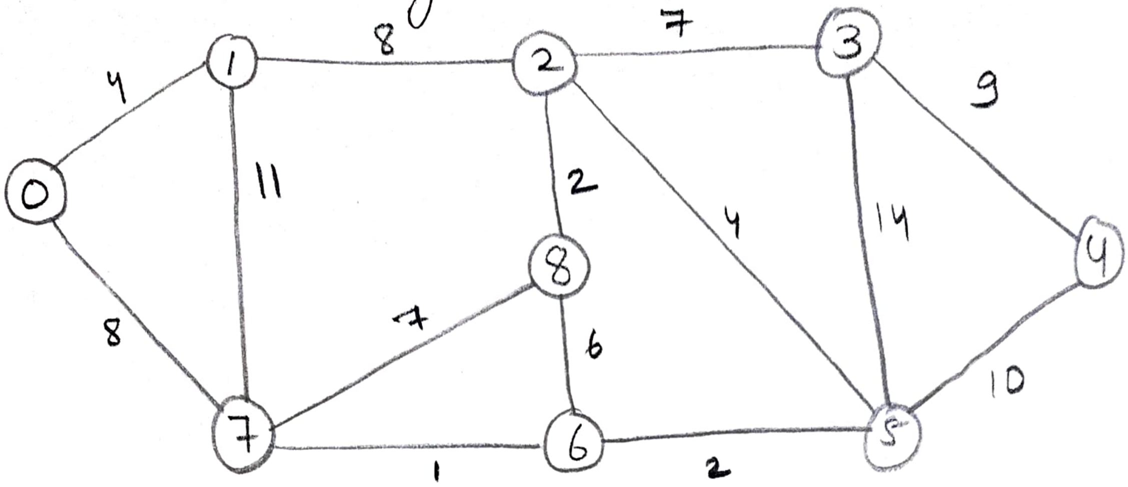
(iv) Handwriting recognition of mathematical expressions.

2. Analyse the time and space complexity of Prim, Kruskal, Dijkstra and Bellman ford algorithm.

Sol:-

Algorithm	Time Complexity	Space complexity
Prims Algorithm	$O(V^2)$	~~$O(E \log V)$~~ $O(V+E)$
Kruskal Algorithm	$O(E \log V)$	$O(\log(E))$
Dijkstra Algorithm	$O(V^2)$	$O(V^2)$
Bellman Ford Algorithm	$O(VE)$	$O(E)$

3. Apply Kruskal and Prims Algorithm on graph to compute MST and its weight.

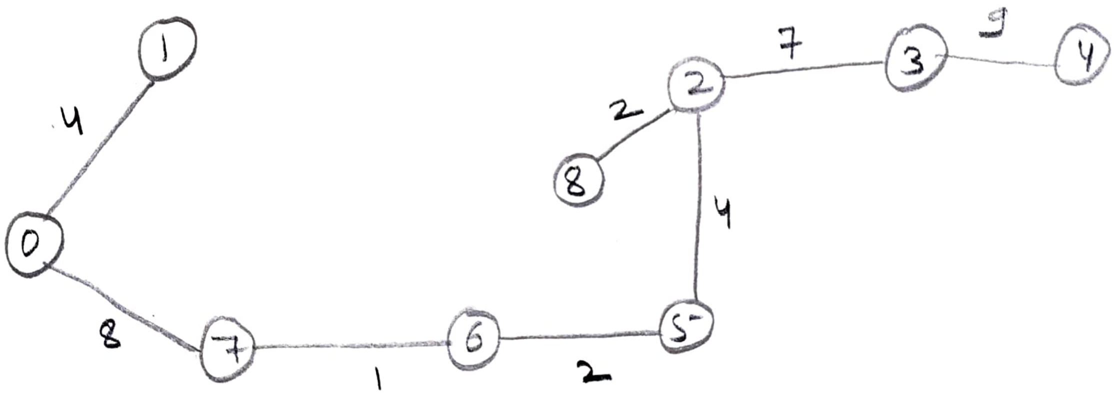


Sol:- Prims Algorithm

If $(w(u,v) < v.key)$

$v.key = w(u,v)$

Vertex	0	1	2	3	4	5	6	7	8
Key	0	4	4	7	9	2	1	8	2



Weight of minimum spanning tree $= 0+4+4+7+9+2+1+8+2$

$= 37$

Kruskal Algorithm

$(6, 7) = 1$ ✓

$(2, 8) = 2$ ✓

$(6, 5) = 2$ ✓

$(0, 1) = 4$ ✓

$(2, 5) = 4$ ✓

$(8, 6) = 6$ ✗

$(7, 8) = 7$ ✗

$(2, 3) = 7$ ✓

$(0, 7) = 8$ ✓

$(1, 2) = 8$ ✗

$(3, 4) = 9$ ✓

$(4, 5) = 10$ ✗

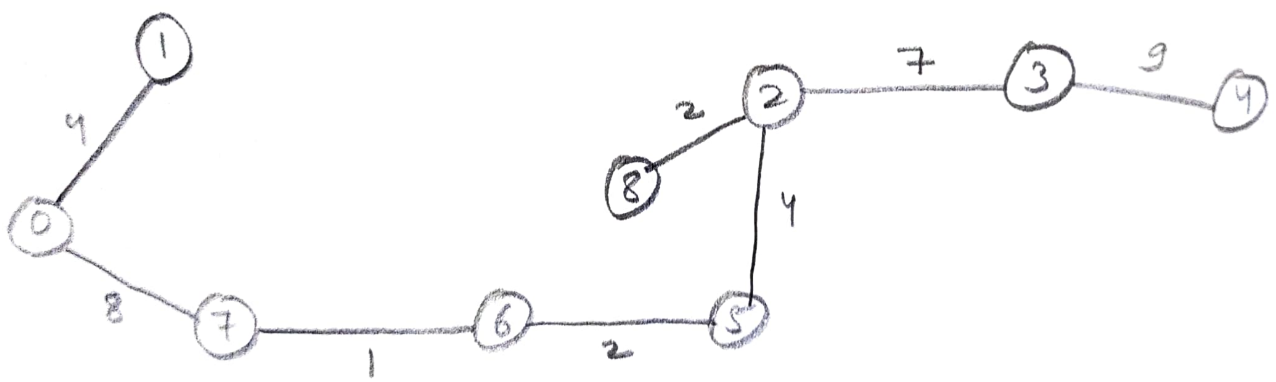
$(1, 7) = 11$ ✗

$(3, 5) = 14$ ✗

Minimum spanning tree

weight $= 1+2+2+4+4+7+8+9$

$= 37$



4. Given a directed weighted graph. You are also given the shortest path from source vertex 's' to a destination vertex 't'. Does the shortest path remain same in modified graph in following cases?

→ If weight of every edge is increased by 10 units.

→ If weight of every edge is multiplied by 10 units.

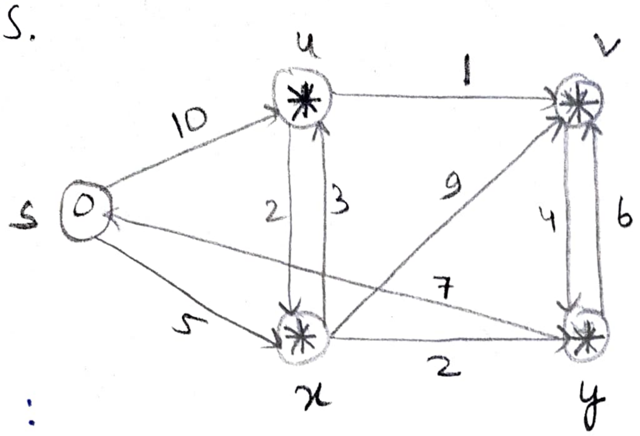
Sol:- If weight of every edge is increased by 10 units, the shortest path may change. The reason is there may be different number of edges in different paths from s to t.

Ex → let shortest path is 15 of 5 edges. let there be another path with 2 edges and total weight 25. The weight of shortest path is increased by 5 * 10 and becomes 15 + 50. The weight of other path is increased by 2 * 10 and becomes 25 + 20. So the shortest path changes to other path with weight 45.

→ If we multiply all edge weights by 10, the shortest path doesn't change. The reason is weight of all paths from s to t get multiplied by some amount. The number of edge doesn't matter. It is like changing unit of weights.

5. Apply Dijkstra and Bellman algorithm on graph given on right side to compute shortest path to all nodes from node S.

Sol:-



Dijkstra :

S	u	v	x	y
0	~~∞~~	~~∞~~	∞	∞
0	~~10~~	5	~~∞~~	~~∞~~
0	8	5	~~14~~	7
0	8	5	~~13~~	7
0	8	5	9	7

Answer:

S → 0

u → 8

v → 5

x → 9

y → 7

Bellmon ford

s	u	v	x	y
0	$\cancel{\infty}$	$\cancel{\infty}$	$\cancel{\infty}$	$\cancel{\infty}$
0	$\cancel{10}$	9	5	$\cancel{13}$
0	8	9	5	7

Answer:

$S \rightarrow 0$

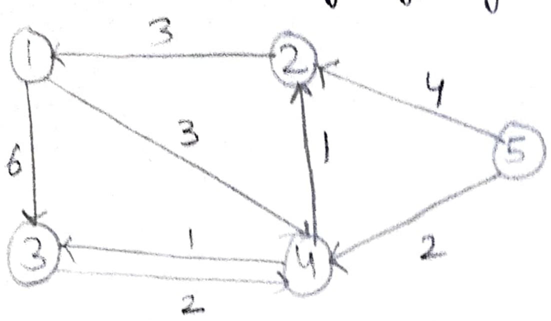
$u \rightarrow 8$

$v \rightarrow 9$

$x \rightarrow 5$

$y \rightarrow 7$

6. Apply all pair shortest path algorithm - Floyd Warshall and also analyse the time and space complexity of algorithm.



Sol:-

$$D^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & \infty \end{bmatrix} \end{array}$$

$$D^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 4 & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 3 & 2 & \infty \end{bmatrix} \end{array}$$

$(1,2) = \infty$
$1 \rightarrow 4 \rightarrow 2$
$\Rightarrow 3+1 = 4$

$(2,3) = \infty$
$2 \rightarrow 1 \rightarrow 3$
$\Rightarrow 3+6 = 9$

$(2,4) = \infty$
$2 \rightarrow 1 \rightarrow 4$
$\Rightarrow 3+3 = 6$

$(3,1) = \infty$
$3 \rightarrow 4 \rightarrow 2 \rightarrow 1$
$\Rightarrow 2+1+3 = 6$

$(3,2) = \infty$
$3 \rightarrow 4 \rightarrow 2$
$\Rightarrow 2+1 = 3$

$(4,1) = \infty$
$4 \rightarrow 2 \rightarrow 1$
$\Rightarrow 1+3 = 4$

$(5,1) = \infty$
$5 \rightarrow 2 \rightarrow 1$
$\Rightarrow 4+3 = 7$

$(5,3) = \infty$
$5 \rightarrow 4 \rightarrow 3$
$\Rightarrow 2+1 = 3$

$$D^3 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 4 & 5 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 6 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

$(1,3) = 6$

$1 \to 4 \to 3$

$\Rightarrow 3+2 = 5$

$(2,3) = 9$

$2 \to 1 \to 4 \to 3$

$\Rightarrow 3+3+1 = 7$

$(5,1) = 7$

$5 \to 4 \to 2 \to 1$

$\Rightarrow 2+1+3 = 6$

$(5,2) = 4$

$5 \to 4 \to 2$

$\Rightarrow 2+1 = 3$