SAGAR NEGI SEC-H ROLL NO-10

TUTORIAL-IV

1. $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

Sol:- Here $a = 3$, $b = 2$, $f(n) = n^2$

$c = \log_b a$

$= \log_2 3$

$= 1.58$

$n^c = n^{1.58} < n^2$

$\therefore T.C = \theta(n^2)$

2. $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

Sol:- Here $a = 4$, $b = 2$, $f(n) = n^2$

$c = \log_b a$

$= \log_2 4$

$= \log_2 2^2$

$= 2\log_2 2 = 2$

$n^c = n^2 = n^2(f(n))$

$\therefore T.C = \theta(n^2 \log n)$

3. $T(n) = T\left(\frac{n}{2}\right) + 2^n$

Sol:- Here $a = 1$, $b = 2$, $f(n) = 2^n$

$c = \log_b a$

$= \log_2 1$

$= \log_2 2^0$

$= 0\log_2 2$

$= 0$

$n^c = n^0 = 1 < 2^n$

$\therefore TC = \theta(2^n)$

4. $T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$

Sol:- Here, $a = 2^n$, $b = 2$, $f(n) = n^n$

as 'a' is not constant

$\therefore$ Master's theorem cannot be applied.

5. $T(n) = 16T\left(\frac{n}{4}\right) + n$

Sol:- Here, $a = 16$, $b = 4$, $f(n) = n$

$$c = \log_b a$$
$$= \log_4 16$$
$$= \log_4 4^2$$
$$= 2\log_4 4$$
$$= 2$$

$n^c = n^2 > n$

$\therefore$ T.C $= \Theta(n^2)$

6. $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

Sol:- Here, $a = 2$, $b = 2$, $f(n) = n\log n$

$$c = \log_b a$$
$$= \log_2 2$$
$$= 1$$

$n^c = n^1 < n\log n$

$\therefore$ T.C $= \theta(n\log n)$

7. $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

Sol:- $a = 2, b = 2, f(n) = \frac{n}{\log n}$

$$c = \log_b a$$
$$= \log_2 2$$
$$= 1$$

$$n^c = n > \frac{n}{\log n}$$

$\therefore T.C = \theta(n)$

8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

Sol:- Here $a = 2, b = 4, f(n) = n^{0.51}$

$$c = \log_b a$$
$$= \log_4 2$$
$$= \log_4 (4)^{1/2}$$
$$= \frac{1}{2} \log_4 4$$
$$= \frac{1}{2}$$

$$n^c = n^{1/2} = \sqrt{n} < n^{0.51}$$

$\therefore T.C = \theta(n^{0.51})$

9. $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

Sol:- Here $a = 0.5, b = 2, f(n) = \frac{1}{n}$

as $a < 1$, $\therefore$ Master's theorem cannot be applied.

10. $T(n) = 16T(\frac{n}{4}) + n!$

Sol:- $a = 16, b = 4, f(n) = n!$

$c = \log_b a$
$= \log_4 16$
$= \log_4 4^2$
$= 2 \log_4 4$
$= 2$

$n^c = n^2 < n!$

$\therefore$ T.C $= \theta(n!)$

11. $T(n) = 4T(\frac{n}{2}) + \log n$

Sol:- Here $a = 4$, $b = 2$, $f(n) = \log n$

$c = \log_b a$
$= \log_2 4$
$= \log_2 2^2$
$= 2 \log_2 2$
$= 2$

$n^c = n^2 > \log n$

~~$\therefore$ T.C $= \theta(\log n)$~~

$\therefore$ T.C $= \theta(n^2)$

12. $T(n) = sqrt(n) T(n/2) + \log n$

Sol:- $a = \sqrt{n}$, $b = 2$, $f(n) = \log n$

Master's theorem cannot be applied as a is not constant.

13. $T(n) = 3T\left(\frac{n}{2}\right) + n$

Sol:- $b=3, c=2, f(n) = n$

$c = \log_b a = \log_3 2$

$= 1.58$

$n^c = n^{1.58} > n$

$\therefore T.C = \theta(n^{\log_3 2})$

14. $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

Sol:- $a=3, b=3, f(n) = \sqrt{n}$

$c = \log_b a$

$= \log_3 3$

$= 1$

$n^c = n^1 > \sqrt{n}$

$\therefore T.C = \theta(n)$

15. $T(n) = 4T\left(\frac{n}{2}\right) + cn$

Sol:- $a=4, b=2, f(n) = c * n$

$c = \log_b a$

$= \log_2 4$

$= \log_2 2^2$

$= 2 \log_2 2$

16. $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

Sol:- $a=3, b=4, f(n) = n\log n$

$c = \log_b a$

$= \log_4 3$

$= 0.79$

$n^c = n^{0.79} < n\log n$

$\therefore T.C = \theta(n\log n)$

17. $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

Sol:- $a=3,\ b=3,\ f(n) = \frac{n}{2}$

$c = \log_b a$

$= \log_3 3$

$= 1$

$n^c = n^1 = \frac{n}{2}$

$\therefore TC = \theta(n \log n)$

18. $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

Sol:- $a=6,\ b=3,\ f(n) = n^2 \log n$

$c = \log_b a$

$= \log_3 6$

$= \cancel{\log_3 1.36}$

$= 1.63$

$n^c = n^{1.63} < n^2 \log n$

$\therefore T.C = \theta(n^2 \log n)$

19. $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

Sol:- $a=4,\ b=2,\ f(n) = \frac{n}{\log n}$

$c = \log_b a$

$= \log_2 4$

$= \log_2 2^2$

$= 2 \log_2 2$

$= 2$

$n^c = n^2 > \frac{n}{\log n}$

$\therefore TC = \theta(n^2)$

SNG 20. $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$

Sol:- Master's theorem cannot be applied as $f(n)$ is not increasing function.

21. $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

Sol:- Here $a = 7$, $b = 3$, $f(n) = n^2$

$$c = \log_b a$$
$$= \log_3 7$$
$$= 1.7$$

$$n^c = n^{1.7} < n^2$$

$$\therefore T.C = \theta(n^2)$$

22. $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

Sol:- Master's theorem cannot be applied since regularity condition is violated.