PRACTICAL- 1

Q.1. $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

Q.2 $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

Q.3. $\lim_{x \to \pi/6} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

Q.4. $\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

Q.5. Examine the Continuity of the following function at given points

i $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x \leq \frac{\pi}{2}$

$= \frac{\cos x}{\pi - 2x}$, for $\frac{\pi}{2} < x < \pi$ } at $x = \pi/2$

ii $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \leq x < 6$

$= \frac{x^2-9}{x+3}$ $6 \leq x < 9$ } at $x=3$ & $x=6$.

Scanned with CamScanner

Q.6. Find Value of k, so that the function $f(x)$ is cts at the indicated point

i $f(x) = \dfrac{1-\cos 4x}{x^2}$ $\quad x<0$ $\quad$ at $x=0$

$= k \quad x=0$

ii $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$ $\quad$ at $x=0$

$= k \quad x=0$

iii $f(x) = \dfrac{\sqrt{3}-\tan x}{\pi - 3x}$ $\quad x \neq \dfrac{\pi}{3}$ $\quad$ at $x = \pi/3$

$= k \quad x = \dfrac{\pi}{3}$

Q.7. Discuss the Continuity of the following funct^n which of these funct^n have removable discontinuity ~~Retaliation~~ Redefine function so as to remove the discontinuity?

i $f(x) = \dfrac{k\cos 3x}{x\tan x}$, $x=0$ $\quad$ at $x=0$

$= a \quad x=0$

ii $f(x) = \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}$, $x \neq 0$ $\quad$ at $x=0$

$= \pi/60$, $x=0$

Q.8 If $f(x) = \dfrac{(e^{x^2} - \cos x)}{x^2}$

for $x \neq 0$ is Continuous at $x=0$ find $f(0)$

Q.9 If $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \pi/2$

is Continuous at $x = \pi/2$ find $f(\pi/2)$

Scanned with CamScanner

H.W. $\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

Q.6 $\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$

$\lim_{x\to a}\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$

$\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$

$\frac{1}{3}\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$

$\frac{1}{3}\lim_{x\to a}\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$

$\frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$

$\frac{1}{3}\times\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$

$\frac{1}{3}\times\frac{4\sqrt{a}}{2\sqrt{3}\sqrt{a}}$

$=\frac{2}{3\sqrt{3}}$

2. $\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right)$

$\lim_{y\to 0}\frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$\lim_{y\to 0}\frac{\cancel{y}}{\cancel{y}\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$\frac{1}{\sqrt{a+0}(\sqrt{a+0}+\sqrt{a}}$

$\frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$

$\frac{1}{\sqrt{a}(2\sqrt{a})}=\frac{1}{2a}$

3. $\lim_{x\to\pi/6}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$

By Substituting $x-\frac{\pi}{6}=h$

$x=h+\frac{\pi}{6}$

where $h\to 0$

Scanned with CamScanner

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\,\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

using
$\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\,\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - \cancel{6}\left(\frac{6h+\pi}{\cancel{6}}\right)}$$

$\cos\frac{\pi}{6} = \cos 30^\circ = \sqrt{3}/2$
$\sin\frac{\pi}{6} = \sin 30^\circ = \frac{1}{\sqrt{2}}$

$$\lim_{h\to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h \frac{1}{\sqrt{2}} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{\sqrt{2}}\right)}{\cancel{\pi} - 6h + \cancel{\pi}}$$

$$\lim_{h\to 0} \frac{\cancel{\cos h \cdot \frac{\sqrt{3}}{2}h} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cancel{\cos\frac{\sqrt{3}h}{2}}}{-6h}$$

$$\lim_{h\to 0} \frac{\sin\frac{4h}{2}}{6h}$$

$$\lim_{h\to 0} \frac{\sin 4h}{3 \times 2h}$$

$$\frac{1}{3}\lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$$

4.) $$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$$

By rationalizing Numerator & Denominator both

$$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty} \left[\frac{(\cancel{x^2} + 5 - \cancel{x^2} + 3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\cancel{x^2} + 3 - \cancel{x^2} - 1)(\sqrt{x^2+5} + \sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty} \frac{\cancel{8}^{\,4}(\sqrt{x^2+3} + \sqrt{x^2+1})}{\cancel{2}(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4 \lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit we get,

$= 4$

5.) $$\left.\begin{aligned} f(x) &= \frac{\sin 2x}{\sqrt{1-\cos 2x}}, \quad \text{for } 0 < x \le \pi/2 \\ &= \frac{\cos x}{\pi - 2x}, \quad \text{for } \frac{\pi}{2} < x < \pi \end{aligned}\right\} \text{at } x = \pi/2$$

$$f(\pi/2) = \frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2(\pi/2)}} \qquad \therefore f(\pi/2) = 0$$

f at $x = \pi/2$ define.

Scanned with CamScanner

ii $\lim_{x \to \pi/2^+} f(x) = \lim_{x \to \pi/2^+} \frac{\cos x}{\pi - 2x}$

By Substituting Method

$x - \frac{\pi}{2} = h$

$x = h + \pi/2$

where $h \to 0$

$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(h + \frac{\pi}{2}\right)}$

$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - \cancel{2}\left(\frac{2h + \pi}{2}\right)}$

$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{-2h}$ Usin $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \cdot \sin\frac{\pi}{2}}{-2h}$

$\lim_{h \to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$

$\lim_{h \to 0} \frac{\cancel{-}\sin h}{\cancel{-}2h}$

$\frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h}$

$= \frac{1}{2}$

b $\lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} \frac{\sin 2x}{\sqrt{1 - \cos 2x}}$ Using $\sin 2x = 2 \sin x \cdot \cos x$

$\lim_{x \to \pi/2^-} \frac{2 \sin x \cdot \cos x}{\sqrt{2 \sin^2 x}}$

$\lim_{x \to \pi/2^-} \frac{2 \cancel{\sin x} \cdot \cos x}{\sqrt{2} \cancel{\sin x}}$

$\lim_{x \to \pi/2^-} \frac{2 \cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos x$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore f$ is not ~~Cont~~ Continous at $x = \pi/2$

5.

ii $f(x) = \frac{x^2 - 9}{x - 3}$ $\quad 0 < x < 3$

$= x + 3$ $\quad 3 \leq x \leq 6$

$= \frac{x^2 - 9}{x + 3}$ $\quad 6 \leq x < 9$

at $x = 3$ & $x = 6$

at $x = 3$

Scanned with CamScanner

i $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define

ii $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ L.H.L = R.H.L

f is Continuous at $x=3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{\cancel{27}^{3}}{\cancel{9}_{1}} = 3$

2 $\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$

$\therefore$ L.H.L $\neq$ R.H.L

function is not Continuous

6. i $f(x) = \frac{1-\cos 4x}{x^2}$ $x<0$ } at $x=0$

$= k$ $x=0$

Sol^n f is Continuous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$

$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$

$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$\therefore k = 8$

ii $f(x) = (\sec^2 x)^{\cot^2 x}$ $x\neq 0$ } at $x=0$

$= k$ $x=0$

Sol^n $f(x) = (\sec^2 x)^{\cot^2 x}$

Scanned with CamScanner

Using

$\tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

&

$\cot^2 x = \frac{1}{\tan^2 x}$

$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$

We, know, that

$\lim_{x \to 0} (1 + px)^{\frac{1}{px}} = e$

$\therefore = e$

$\therefore k = e.$

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$

$= k \quad x = \pi/3$ $\Big\}$ at $x = \frac{\pi}{3}$

$x - \frac{\pi}{3} = h$

$\therefore x = h + \frac{\pi}{3}$

when $h \to 0$

$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$

$\lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

Using

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\lim_{h \to 0} \frac{\sqrt{3} - \tan\frac{\pi}{3} + \tan h}{\frac{1 - \tan\frac{\pi}{3} \tan h}{\cancel{\pi} - \cancel{\pi} - 3h}}$

$\lim_{h \to 0} \frac{\frac{\sqrt{3}\left(1 - \tan\frac{\pi}{3} \cdot \tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{-3h}$

Using $\tan \pi/3 = \tan 60 = \sqrt{3}$

$\lim_{h \to 0} \frac{\frac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$

$\lim_{h \to 0} \frac{\frac{(\sqrt{3} - 3 \cdot \sqrt{3} \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3} \cdot \tan h}}{-3h}$

$\lim_{h \to 0} \frac{\frac{(\cancel{\sqrt{3}} - 3\tan h - \cancel{\sqrt{3}} - \tan h)}{1 - \sqrt{3} \cdot \tan h}}{-3h}$

$\lim_{h \to 0} \frac{\cancel{-}4 \tan h}{\cancel{-}3h(1 - \sqrt{3} \tan h)}$

TEMPERED

Scanned with CamScanner

$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3}\lim_{h\to 0}\frac{\tan h}{h}\lim_{h\to 0}\frac{1}{(1-\sqrt{3}\tan h)} \qquad \frac{\tan h}{h}=1$$

$$=\frac{4}{3}\,\frac{1}{(1-\sqrt{3}(0))}$$

$$=\frac{4}{3}\left(\frac{1}{1}\right)=\frac{4}{3}$$

7.

i $f(x)=\dfrac{1-\cos 3x}{x\tan x}$ $\quad x\neq 0$

$=9$ $\quad x=0$ $\Big\}$ at $x=0$

$$f(x)=\frac{1-\cos 3x}{x\tan x}$$

$$\lim_{x\to 0}=\frac{2\sin^2 3/2\,x}{x\tan x}$$

$$\lim_{x\to 0}=\frac{2\,\dfrac{\sin^2\frac{3x}{2}}{x^2}\times x^2}{x\cdot\dfrac{\tan x}{x^2}\times x^2}$$

$$=2\lim_{x\to 0}\frac{\left(\frac{3}{2}\right)^2}{1}=\frac{\cancel{2}\times 9}{\cancel{4}\,2}=\frac{9}{2}$$

$$\lim_{x\to 0} f(x)=\frac{9}{2} \qquad g=f(0)$$

$\therefore f$ is not continuous at $x=0$

Redefine function

$$f(x)=\begin{cases}\dfrac{1-\cos 3x}{x\tan x} & x\neq 0\\[2mm] \dfrac{9}{2} & x=0\end{cases}$$

Now $\lim_{x\to 0} f(x)=f(0)$

f has removable discontinuity at $x=0$

7. ii

$f(x)=\dfrac{(e^{3x}-1)\sin x^\circ}{x^2}$ $\quad x\neq 0$

$=\pi/6$ $\quad x=0$ $\Big\}$ at $x=0$

$$\lim_{x\to 0}\frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0}\frac{e^{3x}-1}{x}\lim_{x\to 0}\frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x\to 0}3\cdot\frac{e^{3x}-1}{3x}\lim_{x\to 0}\frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3\lim_{x\to 0}\frac{e^{3x}-1}{3x}\lim_{x\to 0}\frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

Scanned with CamScanner

3^{1} $\log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$

f is Continuous at $x=0$

8. $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is Continuous at $x=0$

$\therefore$ Given, f is continuous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$= \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$\lim_{x\to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$\lim_{x\to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x\to 0} \frac{1 - \cos x}{x^2}$

$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$

$\log e + 2 \lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$

Multiply with 2 on Num & Denominator

$= 1 + 2 \times \frac{1}{4 \cdot 2} = \frac{3}{2} = f(0)$

9. $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

$f(0)$ is Continuous at $x = \pi/2$

$\lim_{x\to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$\lim_{x\to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to \pi/2} \frac{1 + \sin x}{1 - \sin^2 x (\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to \pi/2} \frac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \frac{1}{2(\sqrt{2} + \sqrt{2})}$

$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$

$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$

Scanned with CamScanner

PRACTICAL - 2 (Derivative)

Q.1. Show that the following function defined from R to R are differentiable

i) Cot x ii) Cosec x iii) Sec x

Q.2. If $f(x) = 4x + 1 \quad x \le 2$

$= x^2 + 5 \quad x > 0$ at $x = 2$, then find f is differentiable or not

Q.3. If $f(x) = 4x + 7 \quad x < 3$

$= x^2 + 3x + \quad x \ge 3$ at $x = 3$ then find f is differentiable or not

Q.4 If $f(x) = 8x - 5 \quad x \le 2$

$= 3x^2 - 4x + 7 \quad x < 2$ at $x = 2$ then find f is differentiable or not?

Q.1 i Cot x

$f(x) = \cot x$

$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$

$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$

$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x - a) \tan x \tan a}$

put $x - a = h \quad x = a + h \quad$ as $x \to a, \ h \to 0$

$f(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a) \tan(a+h) \tan a}$

$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$

formula: $\tan(A - B) = \frac{\tan A - \tan B}{1 - \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A - B)(1 + \tan A \cdot \tan B)$

$= \lim_{h \to 0} \frac{\tan(a - a - h) - (1 + \tan a + \tan(a+h))}{h \times \tan(a+h) \tan a}$

$= \lim_{h \to 0} -\frac{\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h) \tan a}$

$= -1 \times \frac{1 \times \tan^2 a}{\tan^2 a}$

$= -\frac{\sec^2 a}{\tan^2 a} = -\frac{1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$

$= -\operatorname{cosec}^2 a$

$Df(a) = -\cos^2 a$

f is differentiable $\forall a \in R$

Scanned with CamScanner

2. Cosec x

$f(x) = \text{Cosec } x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\text{Cosec } x - \text{Cosec } a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\sin x - 1/\sin a}{x - a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a) \sin a \sin x}$$

put $x - a = h$ $\quad x = a + h$ $\quad$ as $x \to a$ $\quad h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a) \sin a \cdot \sin(a+h}$$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a) \sin a \cdot \sin(a+h}$$

formula

$$\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2 \cos\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times 1/2 \times \frac{2 \cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \frac{-1/2 \times 2 \cos\left(\frac{2a}{2}\right)}{\sin(a+0}$$

$$= \frac{-\cos a}{\sin^2 a} = -\text{Cota Cosec } a$$

3. Sec x

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec(x) - \sec(a)}{x - a}$$

$$= \lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a) \cos a \cos x}$$

put $x - a = h$
$x = a + h$

as $x \to a$ $\quad h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

Formula :

$$-2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{-2 \sin\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{\cos a \cos(a+h) \times -h/2}$$

$$= -1/2 \times -2 \frac{\sin a}{\cos a \times \cos a}$$

$$= \tan a \sec a$$

Q.2 Sol^n

L.H.D

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x+1 - (4 \times 2 + 1)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$$Df(2^-) = 4$$

Scanned with CamScanner

R.H.D :

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 2+2 = 4$$

$Df(2^+) = 4$ R.H.D = L.H.D

f is differentiable at $x=2$

Q3 Soln R.H.D :

$$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\cdot 3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}} = 3+6 = 9$$

$Df(3^+) = 9$

L.H.D $= Df(3^-)$

$$= \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3} = \lim_{x\to 3^-} \frac{4x-12}{x-3} = \lim_{x\to 3^-} \frac{4(x-3)}{x-3}$$

$Df(3^+) = 4$

R.H.D $\neq$ L.H.D

f is not differentiable at $x=3$

Q.4 Soln

$f(2) = 8\times 2 - 5 = 16-5 = 1$

R.H.D : $$Df(2^+) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x+7-11}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x(x-2)+2(x-2)}{(x-2)}$$

$$= \lim_{x\to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$$

$$= 3\times 2+2 = 8$$

$Df(2^+) = 8$

L.H.D : $$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-5-11}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$Df(2^-) = 8$

$\therefore$ L.H.D = R.H.D

f is differentiable at $x=3$

Scanned with CamScanner

PRACTICAL - 3
(Application of Derivative)

Q.1) Find the intervals in which funct^n is increasing or decreasing

(i) $f(x) = x^3 - 5x - 11$

(ii) $f(x) = x^2 - 4x$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$

(iv) $f(x) = x^3 - 27x + 5$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

Q.2) Find the intervals in which function is concave upwards.

i) $Y = 3x^2 - 2x^3$

ii) $Y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $Y = x^3 - 27x + 5$

iv) $Y = 69 - 24x - 9x^2 + 2x^3$

v) $Y = 2x^3 + x^2 - 20x + 4$

Sol^n

Q.1.

1) $f(x) = x^3 - 5x - 11$

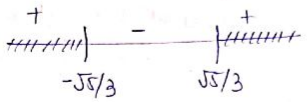
$f'(x) = 3x^2 - 5$

$\because f$ is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5}/3)(x + \sqrt{5}/3) > 0$



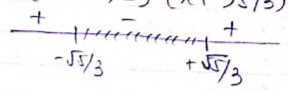
$x \in (-\infty, -\sqrt{5}/3) \cup (\sqrt{5}/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5}/3)(x + \sqrt{5}/3) < 0$



$x \in (-\sqrt{5}/3, \sqrt{5}/3)$

2. $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x - 2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and f is ~~decre~~ decreasing iff $f'(x) < 0$

$\because 2x - 4 < 0$

$2(x - 2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

3. $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

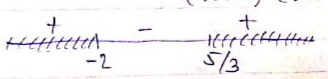
$2(3x^2 + x - 10) > 0$

$3x^2 + x - 10 > 0$

$3x^2 + 6x - 5x - 10 > 0$

$3x(x + 2) - 5(x + 2) > 0$

$(x + 2)(3x - 5) > 0$



$x \in (-\infty, -2) \cup (5/3, \infty)$

Scanned with CamScanner

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x^2 + 6x - 5x - 10 < 0$

$3x(x+2) - 5(x+2) < 0$

$(x+2)(3x-5) < 0$

$+ \quad - \quad +$; -2, $5/3$

$x \in (-2, 5/3)$

4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$(x-3)(x+3) > 0$

$+ \quad - \quad +$

$x \in (-\infty, -3) \cup (3, \infty)$

& f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$

$+ \quad - \quad +$; -3, 3

$\therefore x \in (-3, 3)$

5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 4x - x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x-4)(x+1) > 0$

$+ \quad - \quad +$; -1, 4

$x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4) < 0$

$(x-4)(x+1) < 0$

$+ \quad - \quad +$; -1, 4

$\therefore x \in (-1, 4)$

Q.2 (1) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is Concave upward if $f''(x) > 0$

$(6 - 12x) > 0$

$12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\because f''(x) > 0$

$\therefore x \in (1/2, \infty)$

(2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is Concave upward if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

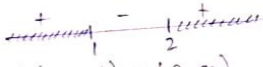
$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$x(x-2) - 1(x-2) > 0$

$(x-2)(x-1) > 0$

Scanned with CamScanner



$x \in (-\infty, 1) \cup (2, \infty)$

3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is Concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$x > 0$

$x \in (0, \infty)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is Concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0$

$\therefore x > 3/2 \quad \therefore x \in (3/2, \infty)$

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is Concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$12x + 2 > 0$

$12(x + 2/12) > 0$

$x + 1/6 > 0$

$x > -1/6$

$f''(x) \not> 0 \quad \therefore$ There exists interval $(-1/6, \infty)$

A
21/12/19

Practical -4

Topic: Application of Derivative & Newton's Method.

Q.1 Find maximum & minimum values of following functions

i $f(x) = x^2 + \frac{16}{x^2}$

ii $f(x) = 3 - 5x^3 + 3x^5$

iii $f(x) = x^3 - 3x^2 + 1$ in $[-1/2, 4]$

iv $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q.2 Find the root of the following equation by Newton's Method (Take 4 iteration only) Correct upto 4 decimal.

i $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

ii $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

i $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now Consider,

$f'(x) = 0$

Scanned with CamScanner

$\therefore 2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + \frac{96}{16} = 2+6 = 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$\cancel{f(2)}$ $f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + \frac{16}{4}$

$= 4 + 4 = 8$

$f''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + \frac{96}{16} = 2+6 = 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore$ function reaches minimum value at $x = 2$, $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

Consider,

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

$\therefore f$ has minimum at $x = 1$

$f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore f$ has maximum at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 8 - 3 = 5$

$\therefore f$ has the maximum value 5 at $x = -1$ & has the minimum value 1 at $x = 1$

Scanned with CamScanner

iii $f(x) = x^3 - 3x^2 + 1$ in $[-1/2, 4]$

$f'(x) = 3x^2 - 6x$

Consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore f$ has maximum value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12 = -3$

$\therefore f$ has maximum value 1 or $x = 0$ and f has minimum value -3 at $x = 2$

iv $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

$f'(x) = 6x^2 - 6x - 12$

Consider,

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6 = 18 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 12(2) + 1$

$= 16 - 12 - 24 + 1 = -19$

$f''(-1) = 12(-1) - 6$

$= -12 - 6 = -18 < 0$

$\therefore f$ has maximum value, at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1 = 8$

$\therefore f$ has maximum value 8 at $x = -1$ & f has minimum value -19 at $x = 2$

Scanned with CamScanner

2(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f'(x_0) = 0 - 0 - 55$
$= -55$

① $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$f(x_0) = 0 - 0 - 0 + 9.5$
$= 9.5$

$x_1 = 0 - \frac{9.5}{-55}$

$x_1 = 0 + \frac{9.5}{55}$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$
$= -0.0828$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$
$= -55.9467$

② $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.1727 - \left(\frac{-0.0828}{-55.9467}\right)$

$= 0.1727 - \frac{0.0828}{55.9467}$

$x_2 = 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$
$= 0.0010$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$
$= -55.9393$

③ $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.1712 - \left(\frac{0.0010}{-55.9392}\right)$

$= 0.1712 + \frac{0.0010}{55.9392}$

$= 0.1712$

$f(x_3) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$
$= 0.0010$

$f'(x_3) = 3(0.1712)^2 - \cancel{55(0.1712)}\ 6(0.01712) - 55$
$= -55.9392$

④ $x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.1712 - \frac{0.0010}{(-55.9392)}$

$= 0.1712 + \frac{0.0010}{55.9392}$

$= 0.1711$

ii) $f(x) = x^3 - 4x - 9$ [2, 3]

$f'(x) = 3x^2 - 4$

$\cancel{f(x2) = 3(2)^2 - 4(2) = 3\times4 - 4\times2 = 12 - 8 =}$

$f(2) = 2^3 - 4(2) - 9 = 8 - 8 - 9 = -9$

$f(3) = (3)^3 - 4(3) - 9 = 27 - 12 - 9 = 6$

Let $x_0 = 3$ be the initial approximation.

By Newton Methods.

① For x_1

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{6}{23}$ $\quad \therefore f(x_0) = 3$

$= \frac{69 - 6}{23} = \frac{63}{23} = 2.7391$

$\therefore x_1 = 2.739$

Scanned with CamScanner

For x_2

② $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_1 = 2.7391$

$f(x_1) = (2.7391)^3 - 4(2.7391) - 9 = 0.5941$

$f'(x_1) = 3(2.7391)^2 - 4 = 18.5080$

$\therefore x_2 = 2.739 - \frac{0.5941}{18.5080}$

$x_2 = 2.7070$

For x_3

③ $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$x_2 = 2.7070$

$f(x_2) = (2.7070)^3 - 4(2.7070) - 9 = 0.0084$

$f'(x_2) = 3(2.7070)^2 - 4 = 17.9835$

$\therefore x_3 = 2.7070 - \frac{0.0084}{17.9835}$

$x_3 = 2.7065$

For x_4

④ $x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$x_3 = 2.7065$

$f(x_3) = (2.7065)^3 - 4(2.7065) - 9 = -0.0005$

$f'(x_3) = 3(2.7065)^2 - 4 = 55.4764$

$\therefore x_4 = 2.7065 - \left(\frac{-0.0005}{55.4764}\right)$

$= 2.7065 + \frac{0.0005}{55.4764}$

$= 2.7065$

iii $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17 = 1 - 1.8 - 10 + 17 = 7.8$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17 = 8 - 7.2 - 20 + 17 = -2.2$

let $x_0 = 2$, be the initial approximation By Newton's Method.

for x_1

① $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_0 = 2$

$f(x_0) = -2.2$

$f'(x_0) = 3(2)^2 - 3.6(2) - 10 = -5.2$

$\therefore x_1 = 2 - \frac{(-2.2)}{(-5.2)}$

$x_1 = 2 - \frac{2.2}{5.2} = 1.5769$

for x_2

② $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_1 = 1.5769$

$f(x_1) = (1.5769)^3 - 1.8(1.5769)^2 - 10(1.5769) + 17 = 0.7256$

$f'(x_1) = 3(1.5769)^2 - 3.6(1.5769) - 10 = -8.2520$

$x_2 = 1.5769 - \frac{0.7256}{(-8.2520)} = 1.5769 + \frac{0.7256}{8.2520}$

$x_2 = 1.6588$

Scanned with CamScanner

(3) For x_3

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_2 = 1.6588$

$f(x_2) = (1.6588)^3 - 1.8(1.6588)^2 - 10(1.6588) + 17 = 0.0234$

$f'(x_2) = 3(1.6588)^2 - 3.6(1.6588) - 10 = -7.7168$

$$x_3 = 1.6588 - \frac{0.0234}{(-7.7168)}$$

$$x_3 = 1.6588 + \frac{0.0234}{7.7168} = 1.6618$$

(4) For x_4

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$x_3 = 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17 = 0.0003$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10 = -7.6977$

$$x_4 = 1.6618 + \frac{0.0003}{7.6977}$$

$$x_4 = 1.6618$$

PRACTICAL - 5

Integration

1) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

2) $\int (4e^{3x} + 1)\,dx$

3) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

4) $\int x^3 + \frac{3x+4}{\sqrt{x}}\,dx$

5) $\int t^7 \sin(2t^4)\,dx$

6) $\int \sqrt{x}\,(x^2-1)\,dx$

7) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

8) $\int \frac{\cos x}{3\sqrt{\sin^2 x}}$

9) $\int e^{\cos^2 x} \sin 2x\,dx$

10) $\int \frac{x^2-2x}{(x^3-3x^2+1)}\,dx$

Scanned with CamScanner

Sol^n 1. $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$$I = \int \frac{dx}{\sqrt{x^2+2x-3}}$$

$$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$$

Comparing with $\int \frac{dx}{\sqrt{x^2-a^2}}$ $= x^2 = (x+1)^2$

$$I = \log|x+\sqrt{x^2-a^2}| + c$$

$$= \log\left|x+1+\sqrt{(x+1)^2-4}\right| + c$$

2. $\int (4e^{3x}+1)\,dx$

$$I = \int (4e^{3x}+1)\,dx$$

$$= \int 4e^{3x}dx + \int 1\,dx$$

$$= \frac{4e^{3x}}{3} + x + c$$

3. $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

$$I = \int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$$

$$= 2\int x^2 dx - 3\int \sin x\,dx + 5\int \sqrt{x}\,dx$$

$$= \frac{2}{3}x^3 - 3\cos x + 5\frac{x^{3/2}}{3/2} + c$$

$$= \frac{2}{3}x^3 - 3\cos x + \frac{10}{3}x^{3/2} + c$$

4. $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$$I = \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$$

$$\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$$

$$\int \left(\frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\right)dx$$

$$\int x^{5/2}\,dx + 3\int x^{-1/2}\,dx + 4\int x^{-1/2}\,dx$$

$$= \frac{2}{7}x^{7/2} + 2x^{3/2} + 8\sqrt{x} + c$$

5. $\int t^7 \sin(2t^4)\,dx$

$$I = \int t^7 \sin(2t^4)\,dx$$

let $t^4 = x$

$4t^3\,dt = dx$

$$= \frac{1}{4}\int 4t^3 \cdot t^4 \sin(2t^4)\,dt$$

$$= \frac{1}{4}(x \cdot \sin(2x)\,dx$$

$$= \frac{1}{4}\left[x\int \sin 2x - \int\left[\int \sin 2x \cdot \frac{d\,x}{dx}x\right]\right]$$

$$= \frac{1}{4}\left[-x\frac{\cos 2x}{2} + \frac{1}{2}\int \cos 2x \cdot 1\right]$$

Scanned with CamScanner

$$I = \frac{-1}{2}\int \frac{-2}{x^3}\sin\left(\frac{1}{x^2}\right)dx$$

$$= \frac{-1}{2}\int \sin t$$

$$= \frac{-1}{2}(-\cos t) + c$$

$$= \frac{1}{2}\cos t + c$$

Resubstitution $t = 1/x^2$

$$I = \frac{1}{2}\cos\left(\frac{1}{x^2}\right) + c$$

8) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

$$I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$$

let $\sin x = t$

$\cos x\, dx = dt$

$$I = \int \frac{dt}{\sqrt[3]{t^2}}$$

$$= \int \frac{dt}{t^{2/3}}$$

$$= \int t^{-2/3}\,dt$$

$$= 3t^{1/3} + c$$

$$= 3(\sin x)^{1/3} + c$$

$$= 3\sqrt[3]{\sin x} + c$$

$$= \frac{1}{4}\left[-\frac{\cos 2x}{2} + \frac{1}{4}\sin 2x\right] + c$$

$$= -\frac{1}{8}x\cos 2x + \frac{1}{16}\sin 2x + c$$

$$= -\frac{1}{8}t^4\cos 2(t^4) + \frac{1}{16}\sin(2t^4) + c$$

6. $\int \sqrt{x}\,(x^2 - 1)\,dx$

$$I = \int \sqrt{x}\,(x^2 - 1)\,dx$$

$$\int (\sqrt{x}\,x^2 - \sqrt{x})\,dx$$

$$\int (x^{5/2} - \sqrt{x})\,dx$$

$$= \frac{2}{7}x^{7/2} - \frac{2}{3}x^{3/2} + c$$

7. $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$$I = \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$$

let $\frac{1}{x^2} = t$

$x^{-2} = t$

$-\frac{2}{x^3}\,dx = dt$

9) $\int e^{\cos^2 x} \sin 2x \, dx$

$I = \int e^{\cos^2 x} \sin 2x \, dx$

let $\cos^2 x = t$

$-2\cos x \cdot \sin x \, dx = dt$

$-\sin 2x \, dx = dt$

$I = \int -\sin 2x \, e^{\cos^2 x} \cdot dx$

$= -\int e^t \, dt$

$= e^t + C$

Resubstituting $t = \cos^2 x$

$I = -e^{\cos^2 x} + C$

10) $\int \frac{x^2 - 2x}{(x^3 - 3x^2 + 1)} \, dx$

$I = \int \frac{x^2 - 2x}{(x^3 - 3x^2 + 1)} \, dx$

let $x^3 - 3x^2 + 1 = t$

$3(x^2 - 2x) \, dx = dt$

$(x^2 - 2x) \, dx = dt/3$

04/01/2020

$I = \int \frac{1}{t} \, dt$

$= \frac{1}{3} \int \frac{dt}{t}$

$= \frac{1}{3} \log t + C$

$= \frac{1}{3} \log(x^3 - 3x^2 + 1) + C$

Application of Integration & Numerical Integration

Q.1 Find the length of the following curve

$x = t \sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$.

$x = t \sin t$ $\quad$ $y = 1 - \cos t$

$\frac{dx}{dt} = 1 - \cos t$ $\quad$ $\frac{dy}{dt} = \sin t$

$L = \int_0^{2\pi} \sqrt{(1 - \cos t + \sin t)^2 + (\sin t)^2}$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1} \, dt$

$= \int_0^{2\pi} \sqrt{2(1 - \cos t)} \, dt$

$= \int_0^{2\pi} \sqrt{4 \sin^2 t/2} \, dt$

$= 2 \int_0^{2\pi} \sin t/2 \, dt$

$= 2 [-\cos t/2]_0^{2\pi} = 2[-\cos 2\pi + \cos 0]$

$= 4$

Scanned with CamScanner

2 $y = \sqrt{4-x^2}$ $\quad x \in [-2, 2]$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$y = \sqrt{4-x^2}$$

$$\therefore \frac{dy}{dx} = 2^2 \int_0^2 1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2 dx$$

$$= 2\int_0^2 \sqrt{\frac{1+x^2}{4-x^2}}\, dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\left[\sin^{-1}(x/2)\right]_0^2 = 2\pi$$

3. $y = x^{3/2}$ in $[0, 4]$

$$f'(x) = 3/2\, x^{1/2}$$

$$[f'(x)]^2 = 9/4\, x$$

$$L = \int_0^4 \sqrt{1 + [f'(x)]^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$$

$$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$$

$$= \frac{1}{2}\int_0^4 \frac{(4+9x)^{1/2+1}}{\sqrt{1/2+1}}\, dx$$

$$= \frac{1}{27}\left[[4+0]^{3/2} - [4+36]^{3/2}\right]$$

$$= \frac{1}{27}\left[(4)^{3/2} - (40)^{3/2}\right]$$

4. $x = 3\sin t \quad y = 3\cos t$

$$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$$

$$L = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$L = \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{9}\, dt = 3\int_0^{2\pi} dt$$

$$= 3[t]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

Scanned with CamScanner

5. $x = \frac{1}{6} y^3 + \frac{1}{2y}$ $\quad y \in [1, 2]$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4 - 1}{2y^2}$$

$$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right] = \frac{17}{12}$$

Q.2. Using Simpson's Rule solve the following

(1) $3\int_0^2 e^{x^2}\, dx$ with $n=4$

Soln

$a=0$, $b=2$, $n=4$

$$h = \frac{2-0}{4}$$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981
	y_0	y_1	y_2	y_3	y_4

By Simpson's Rule

$$\int_0^2 e^{x^2}\, dx = \frac{0.5}{3}\left[(1 + 54.5981) + 4(1.2840 + 9.4877) + 2(2.7182 + 54.5981)\right]$$

$$= \frac{0.5}{3}\left[55.5981 + 43.0868 + 114.63266\right]$$

$$= 1.1279$$

(2) $\int_0^4 x^2\, dx$

$$h = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2 = \frac{1}{3}\left[(0+16) + 4(1+9) + 2(4)\right]$$

$$= 21.333$$

(3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n=6$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$6\pi/18$	$8\pi/18$
y	0	0.4166	0.58	0.70	0.8087	0.8722	0.99

$$\int_0^{\pi/3} \sqrt{\sin x} = \frac{\pi}{4} \times 12.1163$$

$$= 0.7049$$

Scanned with CamScanner

PRACTICAL- 7

Differential Equation

Q.1. Solve the following differential equation.

① $x\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $\quad Q(x) = \frac{e^x}{x}$

$IF = e^{\int \frac{1}{x}dx}$

$= x$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int e^x\,dx + c$

$xy = e^x + c$

② $e^x\frac{dy}{dx} + 2e^{xy} = 1$

Sol^n $\frac{dy}{dx} + \frac{2e^x}{e^x}y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \quad Q(x) = e^{-x}$

$\int P(x)\,dx$

$IF = \cancel{\int e^{2x}dx}\ e^{\int 2dx}$

$= e^{2x}$

$Y(IF) = \int Q(x)(IF)\,dx + c$

$= \int e^{-x}e^{2x}\,dx + c$

$= \int e^x\,dx + c$

$y \cdot e^{2x} = e^x + c$

3 $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

Sol^n $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x) \quad Q(x) = \frac{\cos x}{x^2}$

$IF = \cancel{\int e^{\int P dx}}\ e^{\int P(x)dx}$

$= e^{\int \frac{2}{x}dx}$

$= x^2$

$Y(IF) = \int Q(x)(IF)\,dx + c$

$= \int \frac{\cos x}{x^2} \cdot x^2\,dx + c$

$= \int \cos x + c$

$\therefore x^2y = \sin x + c$

Scanned with CamScanner

(4) If $\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

Sol^n $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$P(x) = 3/x \qquad Q(x) = \sin x / x^3$

$P(x) = \int 3/x \, dx$

$= x^3$

$IF = e^{\int P(x) dx}$

$= x^3$

$Y(IF) = \int Q(x) (IF) dx + c$

$= \int \frac{\sin x}{x^3} \cdot x^3 \, dx + c$

$= \int \sin x + c$

$x^3 y = -\cos x + c$

$\therefore e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \qquad Q(x) = \frac{2x}{e^{2x}} = 2x e^{-2x}$

$(IF) = e^{\int P(x) dx}$

$= e^{\int 2 dx}$

$= e^{2x}$

$Y(IF) = \int Q(x) (IF) dx + c$

$= \int 2x e^{-2x} e^{2x} + c$

$y e^{2x} = \int 2x + c = x^2 + c$

6 $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \tan x \, dy$

$\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 x}{\tan y} dy$

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 x}{\tan y} dy$

$\log |\tan x| = -\log |\tan y| + c$

$\log |\tan x + \tan y| = c$

$\tan x + \tan y = e^c$

7. $\frac{dy}{dx} = \sin^2 (x - y + 1)$

put $x - y + 1 = v$

Scanned with CamScanner

$x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v \, dv = \int dx$

$\tan v = x + c$

$\tan \cancel{(x+y-1)} (x - y + 1) = x + c$

8. $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

Soln

Put $2x + 3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$= \frac{1}{3}\left(\frac{dv}{x} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v - 1 + 2v + 4}{v+2}$

$= \frac{3v+3}{v+2}$

$= 3\frac{(v+1)}{v+2}$

$\int \frac{v+2}{v+1} dv = 3dx$

$\int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = \int 3dx$

$v + \log|v| = 3x + c$

$2x + 3y + \log|2x + 3y + 1| = 3x + c$

$3y = x - \log|2x + 3y + 1| + c$

18/01/2020

Scanned with CamScanner

Practical- 8

Euler's Method

Q.1. $\frac{dy}{dx} = y + e^x - 2$, $y(0) = 2$, $h = 0.5$, find $y(2) = ?$

Sol^n $f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

Q.2 $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$, $h = 0.2$ find ~~(x=1)~~ $y(1) = ?$

$x_0 = 0$, $y_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

Q.3 $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$, $h = 0.2$ find $y(1) = ?$

Sol^n $x_0 = 0$, $y(0) = 1$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7694	1.5051
5	1	1.5051		

$\therefore y(1) = 1.5051$

Scanned with CamScanner

Q.4. $\frac{dy}{dx} = 3x^2+1$ $y(1)=2$ find $y(2)$ $h=0.5$ & $h=0.25$

Soln $y_0=2$ $x_0=1$ $h=0.5$

(1)

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

(2) $y_0=2$ $x_0=1$ $h=0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9966
4	2	299.9966		

$y(2) = 299.9966$

Q.5 $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1)=1$, $h=0.2$

Soln $x_0=1$ $y_0=1$ $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

11/01/2020

Practical - 9

Limits & Partial order derivatives.

Q1. Evaluate the following limits.

1) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3x+y^2-1}{xy+5}$

$= \lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3x+y^2-1}{xy+5}$

Apply limit

$= \dfrac{(-4)^3-3(-4)+(-1)^2-1}{(-4)(-1)+5}$

$= \dfrac{-64+12+1-1}{4+5} = \dfrac{-52}{9}$

2) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

$= \lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

Apply limit

$= \dfrac{(0+1)((2)^2+(0)^2-4(2))}{2+3(0)}$

$= \dfrac{1(4+0-8)}{2}$

$= \dfrac{4-8}{2} = \dfrac{-\cancel{4}\,2}{\cancel{2}} = -2$

iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2y^2z}$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

Apply limit

$= \dfrac{(1)^2-(1)^2(1)^2}{(1)^3-(1)^2(1)(1)}$

$= \dfrac{1-1}{1-1} = \dfrac{0}{0}$

$\therefore$ limit does not exist.

Q2 Find f_x, f_y for each of the following f

i $f(x,y) = xye^{x^2+y^2}$

$f_x = y(1.e^{x^2+y^2}) + xy(e^{x^2+y^2}.2x)$

$= ye^{x^2+y^2} + 2x^2ye^{x^2+y^2}$

$f_y = x(1.e^{x^2+y^2}) + xy(e^{x^2+y^2}.2y)$

$= xe^{x^2+y^2} + 2xy^2e^{x^2+y^2}$

$\therefore f_x = ye^{x^2+y^2} + 2x^2ye^{x^2+y^2}$

$f_y = xe^{x^2+y^2} + 2xy^2e^{x^2+y^2}$

Scanned with CamScanner

ii) $f(x,y) = e^x \cos y$

$f_x = \cos y \, e^x$

$f_y = e^x \cdot -\sin y$

$\therefore f_y = -\sin y \, e^x$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = y^2 3x^2 - 6xy \, 2x + 0 + 0$

$= 3x^2y^2 - 6xy$

$f_y = x^3 \, 2y - 3x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

3 Using definition find values of f_x, f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

$f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$

According to ~~given~~ given $(a,b) = (0,0)$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{2h - 0}{h} = 2$

$f_y(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{0 - 0}{h} = 0$

$\therefore f_x = 2, \; f_y = 0$

04. Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$

i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$\therefore f_{xx} = \frac{d^2f}{dx^2} \qquad f_{yy} = \frac{d^2f}{dy^2}$

~~f_{xx}~~ Appling $\frac{u}{v}$ rule

$f_x = \frac{x^2(0-y) - (y^2 - xy)2x}{x^4}$

$= \frac{-x^2y - 2xy^2 + 2x^2y}{x^4}$

$\therefore f_x = \frac{x^2y - 2xy^2}{x^4}$

$\therefore f_{xx} = \frac{x^4(2xy - 2y^2) - (x^2y - 2xy^2)(4x^3)}{x^8}$

$= \frac{2x^5y - 2x^4y^2 - (4x^5y - 8x^4y^2)}{x^8}$

Scanned with CamScanner

$$= \frac{2x^5y - 2x^4y^2 - (4x^5y - 8x^4y^2)}{x^8}$$

$$= \frac{2x^5y - 2x^4y^2 - 4x^5y + 8x^4y^2}{x^8}$$

$$= \frac{-2x^5y + 6x^4y^2}{x^8}$$

$$= \frac{6x^4y^2 - 2x^5y}{x^8}$$

$$f_{xx} = \frac{6y^2 - 2xy}{x^4}$$

$$f_y = \frac{1}{x^2}(2y - x)$$

$$\therefore f_y = \frac{2y - x}{x^2}$$

$$\therefore f_{yy} = \frac{1}{x^2}\,2 = \frac{2}{x^2}$$

$$f_{xy} = \frac{2y - x}{x^2}$$

$$= \frac{x^2(-1) - (2y - x)(2x)}{x^4}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$= \frac{x^2 - 4xy}{x^4}$$

$$= \frac{x - 4y}{x^3}$$

$$\therefore f_{xy} = \frac{x - 4y}{x^3}$$

$$f_{yx} = \frac{x^2y - 2xy^2}{x^4}$$

$$= \frac{1}{x^4}(x^2 - 4xy) = \frac{1}{x^4}(x^2 - 4xy)$$

$$= \frac{x^2 - 4xy}{x^4}$$

$$= \frac{x - 4y}{x^3} = f_{xy}$$

$$\therefore f_{yx} = f_{xy}$$

Hence, Verified

2. $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f_x = 3x^2 + 6xy^2 - \frac{1}{x^2+1}\,2x$$

$$f_y = 0 + 6x^2y - 0$$

$$f_{xx} = 6x + 6y^2 - \left(\frac{x^2+1\,(2) - 2x\,(2x)}{(x^2+1)^2}\right)$$

$$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \rightarrow (1)$$

$$f_{yy} = 6x^2 \rightarrow (2)$$

Scanned with CamScanner

$$f_{xy} = \left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 0 + 12xy - 0$$
$$= 12xy$$

$$f_{yx} = 6x^2y$$
$$= 12xy$$

$$\therefore f_{xy} = f_{yx}$$

Hence Verified

iii $f(x,y) = \sin(xy) + e^{x+y}$

$$f_x = \cos xy \cdot y + e^{x+y}$$

$$f_y = \cos xy \cdot x + e^{x+y}$$

$$f_{xx} = -\sin xy \; y^2 + e^{x+y}$$

$$f_{yy} = -\sin xy \; x^2 + e^{x+y}$$

$$f_{xy} = -\sin(xy)\, yx + \cos xy + e^{x+y}$$
$$f_{yx} = -\sin(xy)\, yx + \cos xy + e^{x+y}$$

$\therefore f_{xy} = f_{yx}$ Hence Verified

5 Find the linearization of $f(x,y)$ at given point

i $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$$f(1,1) = \sqrt{1+1} = \sqrt{2}$$

$$f_x(1,1) = (x^2+y^2)^{1/2}$$

$$= \frac{1}{2\sqrt{x^2+y^2}} 2x + 0$$

$$= \frac{1}{\cancel{2}\sqrt{2}} \times \cancel{2} = \frac{1}{\sqrt{2}}$$

$$f_y(1,1) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y$$

$$= \frac{1}{\cancel{2}\sqrt{2}\,\cancel{2}\sqrt{2}} \cancel{2}y$$

$$= \frac{1}{\sqrt{2}}$$

$$L(x,y) = \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \sqrt{2} + \frac{x-1}{\sqrt{2}} + \frac{y-1}{\sqrt{2}}$$

$$= \sqrt{2} + \frac{x-1+y-1}{\sqrt{2}}$$

$$= \frac{\cancel{2}+x+y\cancel{-2}}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$$

Scanned with CamScanner

ii) $f(x,y) = 1 - x + y \sin x$ at $(\pi/2, 0)$

$$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2} + 0 \cdot \sin\frac{\pi}{2}$$
$$= 1 - \frac{\pi}{2}$$

$$f_x = 0 - 1 + \cos x$$
$$= \cos x - 1$$
$$0 - 1 = -1$$

$$f_y = 0 - 0 + \sin x$$
$$= \sin\frac{\pi}{2} = 1$$

$$L(x,y) = 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y - 0)$$

$$= 1 - \cancel{\frac{\pi}{2}} - x + \cancel{\frac{\pi}{2}} + y$$

$$= 1 - x + y$$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$$f(1,1) = 0$$

25/01/2020

$$f_x = \frac{1}{x} + 0$$
$$= \frac{1}{1} = 1$$

$$f_y = \frac{1}{1} = 1$$

$$L(x,y) = 0 + (1)(x-1) + 1(y-1)$$
$$= x - 1 + y - 1$$
$$x + y - 2$$

Q1 Find the directional derivative of the following funct^n at given points & in the direction of given Vector.

i $f(x,y) = x + 2y - 3$ $\quad a = (1, -1)$ $\quad u = 3i - j$

Here, $u = 3i - j$ is not a unit Vector.

$$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9 + 1} = \sqrt{10}$$

~~$f(a+hu) = f(1,-1) + h($~~

Unit Vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3, -1)$

$$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$$

$$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$= f\left(1 + \frac{3}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right)$$

$$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$$

$$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$$

$$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$$

Scanned with CamScanner

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-4} + \cancel{h}/\sqrt{10} + \cancel{4}}{\cancel{h}}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

ii $f(x) = y^2 - 4x + 1 \qquad a = (3,4) \qquad u = i + 5j$

Here $u = i + 5j$ is not a unit vector

$$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$$

$$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} \cancel{+5} \cancel{-5}}{h}$$

$$\frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

iii $2x + 3y \qquad a = (1,2), \; u = (3i + 4j)$

Here $u = 3i + 4j$ is not a unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

$$f(a) = f(1,2) = 2(1) + 3(2) = 8$$

$$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$$

$$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$$

Scanned with CamScanner

$$f(a+hu) = 2\left(1+\frac{3h}{5}\right)+3\left(2+\frac{4h}{5}\right)$$

$$= 2+\frac{6h}{5}+6+\frac{12h}{5}$$

$$= \frac{18h}{5}+8$$

$$D_u f(a) = \lim_{h\to 0} \frac{\frac{18h}{5} \cancel{+8-8}}{h}$$

$$= \frac{18}{5}$$

Q.2. Find gradient vector for the following funct^n at given point

i) $f(x,y) = x^y + y^x$, $a=(1,1)$

$$f_x = y.x^{y-1} + y^x \log y$$

$$f_y = x^y \log x + xy^{x-1}$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= (yx^{y-1} + y^2 \log y,\ x^y \log x + xy^{x-1})$$

$$f(1,1) = (1+0,\ 1+0)$$

$$= (1,1)$$

ii) $f(x,y) = (\tan^{-1}x).y^2$ $\quad a=(1,-1)$

$$f_x = \frac{1}{1+x^2}.y^2$$

$$f_y = 2y\tan^{-1}x$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= \left(\frac{y^2}{1+x^2},\ 2y\tan^{-1}x\right)$$

$$f(1,-1) = \left(\frac{1}{2},\ \tan^{-1}(1)(-2)\right)$$

$$= \left(\frac{1}{2},\ \frac{\pi}{4}(-2)\right)$$

$$= \left(\frac{1}{2},\ \frac{-\pi}{2}\right)$$

iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a=(1,-1,0)$

$$f_x = yz - e^{x+y+z}$$

$$f_y = xz - e^{x+y+z}$$

$$f_z = xy - e^{x+y+z}$$

$$\nabla f(x,y,z) = f_x, f_y, f_z$$

$$= yz - e^{x+y+z},\ xz - e^{x+y+z},\ xy - e^{x+y+z}$$

$$f(1,-1,0) = \left((-1)(0) - e^{1+(-1)+0},\ (1)(0) - e^{1+(-1)+0},\ (1)(-1) - e^{1+(-1)+0}\right)$$

$$= (0-e^0,\ 0-e^0,\ -1-e^0)$$

$$= (-1,\ -1,\ -2)$$

Q.3. Find the equation of tangent & normal to each of the following using curves at given points.

i) $x^2\cos y + e^{xy} = 2$ at $(1,0)$

Scanned with CamScanner

$f_x = \cos y \; 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1, 0) = \quad \therefore x_0 = 1, \quad y_0 = 0$

eq^n of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \; 2(1) + e^0 \; 0$
$= 1(2) + 0$
$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$
$= 0 + 1 \cdot 1$
$= 1$

$2(x-1) + 1(y-0) = 0$
$2x - 2 + y = 0$
$2x + y - 2 = 0 \rightarrow$ It is the required eq^n of tangent

eq^n of Normal

$= ax + by + c = 0$
$bx + ay + d = 0$

$1(x) + 2(y) + d = 0$
$\therefore 1 + 2y + d = 0 \quad$ at $(1, 0)$
$= 1 + 2(0) + d = 0$
$d + 1 = 0$
$\therefore d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0 \quad$ at $(2, -2)$

$f_x = 2x + 0 - 2 + 0 + 0$
$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$
$= 2y + 3$

$(x_0, y_0) = (2, -2) \quad \therefore x_0 = 2, \; y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

eq^n of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x - 2) + (-1(y + 2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0 \rightarrow$ It is required eq^n of tangent

eq^n of Normal

$= ax + by + c = 0$
$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$
$-x + 2y + d = 0 \quad$ at $(2, -2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore d = 6$

Scanned with CamScanner

Q.4 Find the eqⁿ of tangent & normal line to each of the following surface

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0) \therefore x_0 = 2,\ y_0 = 1,\ z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

eqⁿ of tangent

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0$ → This is required eqⁿ of tangent

Eqⁿ of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

ii $3xyz - x - y + z = -4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $x_0 = 1,\ y_0 = -1,\ z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eqⁿ of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ → This is required eqⁿ of tangent

Eqⁿ of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Scanned with CamScanner

Q.5 Find the local maxima & minima for the following fn

i $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2 \rightarrow (1)$

$f_y = 0$

$2y - 3x - 4 = 0$
$2y - 3x = 4 \rightarrow (2)$

Multiply eqⁿ 1 with 2

$4x - 2y = -4$
$2y - 3x = 4$
$x = 0$

Substitute value of x in eqⁿ (1)

$2(0) - y = -2$
$-y = -2$ $\quad y = 2$

∴ Critical points are $(0, 2)$

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$
$= 6(2) - (-3)^2$
$= 12 - 9$
$= 3 > 0$

∴ f has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$
$= -4$

ii $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$
$f_y = 3x^2 - 2y$

$f_x = 0$

$8x^3 + 6xy = 0$
$2x(4x^2 + 3y) = 0$
$\cancel{4x^2 + 3y = 0}$ $4x^2 + 3y = 0 \rightarrow (1)$

$f_y = 0$

$3x^2 - 2y = 0 \rightarrow (2)$

Multiply eqⁿ (1) with 3
(2) with 4

$12x^2 + 9y = 0$
$-12x^2 - 8y = 0$
$17y = 0$ $\quad y = 0$

Substitute value of y in eqn ①

$$4x^2 + 3(0) = 0$$
$$4x^2 = 0$$
$$x = 0$$

Critical point is $(0, 0)$

$$r = f_{xx} = 24x^2 + 6x$$
$$t = f_{yy} = 0 - 2 = -2$$
$$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$$

r at $(0,0)$

$$= 24(0) + 6(0) = 0$$
$$\therefore r = 0$$
$$rt - s^2 = 0(-2) - (s)^2$$
$$= 0 - 0 = 0$$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

$f(x,y)$ at $(0,0)$

$$\cancel{2x}\ 2(0)^4 + 3(0)^2(0) - (0)^2$$
$$= 0 + 0 - 0$$
$$= 0$$

iii $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$$f_x = 2x + 2$$
$$f_y = -2y + 8$$

$f_x = 0 \quad \therefore \quad 2x + 2 = 0$

$$x = \frac{-2}{2} = -1 \quad \therefore x = -1$$

$f_y = 0 \qquad -2y + 8 = 0$

$$y = \frac{-8}{-2} = 4$$
$$\therefore y = 4$$

$\therefore$ Critical point is $(-1, 4)$

$$r = f_{xx} = 2$$
$$t = f_{yy} = -2$$
$$s = f_{xy} = 0$$

$$r > 0$$

$$rt - s^2 = 2(-2) - (0)^2$$
$$= -4 - 0$$
$$= -4 < 0$$

$f(x,y)$ at $(-1, 4)$

$$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$$
$$= 1 + 16 - 2 + 32 - 70$$
$$= 17 + 30 - 70$$
$$= 37 - 70 = 33$$

01/02/2020

Scanned with CamScanner