LINEAR PROGRAMMING

1. This time, our immune system is the best defense. With this, a Melagail wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food A contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food B contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₱50 per kg to purchase Food A and ₱70 per kg to purchase Food B. Formulate this problem as a linear programming problem to minimize the cost of such a mixture.

	Vitamin A	**Vitamin C**	**Price**
Food A	2 units/kg	1 unit/kg	Php 50
Food B	1 unit/kg	2 units/kg	Php 70

<u>Define the Variables</u>

Let x be the kilogram of Food A to be mixed in the mixture.
y be the kilogram of Food B to be mixed in the mixture.

<u>Identify the Objective Function</u>

$z = 50x + 70y$

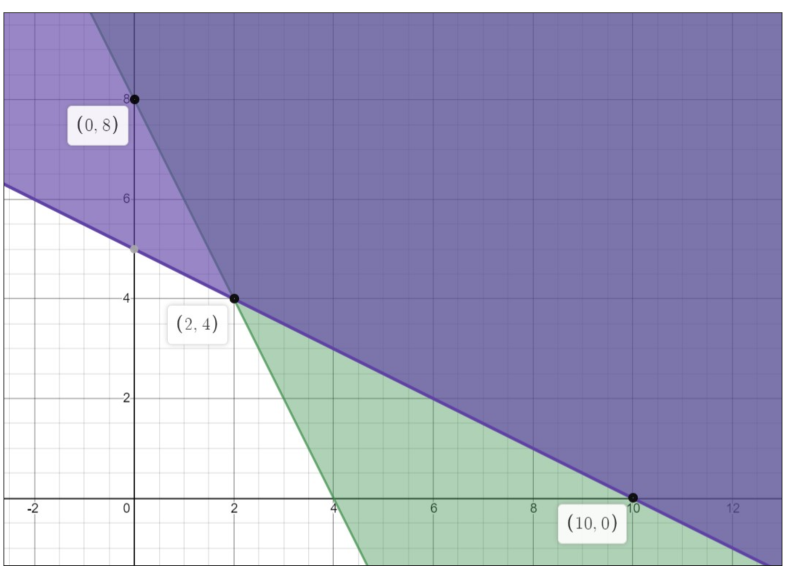
<u>Identify the Constraints</u>

$x \geq 0, y \geq 0$ Non-negativity constraints.

$2x + 1y \geq 8$ The mixture should at least contain 8 units of Vitamin A per kilogram.

$x + 2y \geq 10$ The mixture should at least contain 10 units of Vitamin C per kilogram.

<u>Illustrate Constraints by Graphing</u>



Evaluating the Corner Points and Identify the Optimal Solution for the Objective Function

Corner Points	**Objective Function** **Minimize:** $z=50x+70y$
(0,8)	$z=50(0)+70(8)=560$
(2,4)	$z=50(2)+70(4)=380$
(10,0)	$z=50(10)+70(0)=500$

Conclusion

Thus, the minimum value of z is 380 where x and y values are not equal to 0, and it occurs when x = 2 and y = 4.

Moreover, Melagail will only spend Php 380 per units of the mixture when she mixes 2 units/kilogram of Food A and 4 units/kilogram of Food B to achieve at least 8 units and 10 units of Vitamin A and Vitamin C in the mixture, respectively.

2. A certain dosage of radiation, measured in kilorads, must be given to the tumor near the brain. The dose delivered must be sufficient to kill the malignant cells but the aggregate dose must not exceed established tolerance levels for the brain. Two beams which would deliver radiation exposure to the cells will be used. The goal is to select certain beam durations that would generate the best dosage distribution by minimizing the radiation absorbed by the brain. The data for the radiation therapy is given below:

	Fraction of dose absorbed per second		*Average dosage (in kilorads)*
Area	**Beam 1**	**Beam 2**	
Brain	0.4	0.5	
Spine	0.3	0.1	At most 2.7
Tumor	0.5	0.5	At most 6
Center of Tumor	0.6	0.4	At least 6

Determine the optimal exposure times for beam 1 and beam 2.

Define the Variables

Let x be duration (in seconds) of the radiation exposure of Beam 1.
y be duration (in seconds) of the radiation exposure of Beam 2.

Identify the Objective Function

$z=0.4x+0.5x$

Identify the Constraints

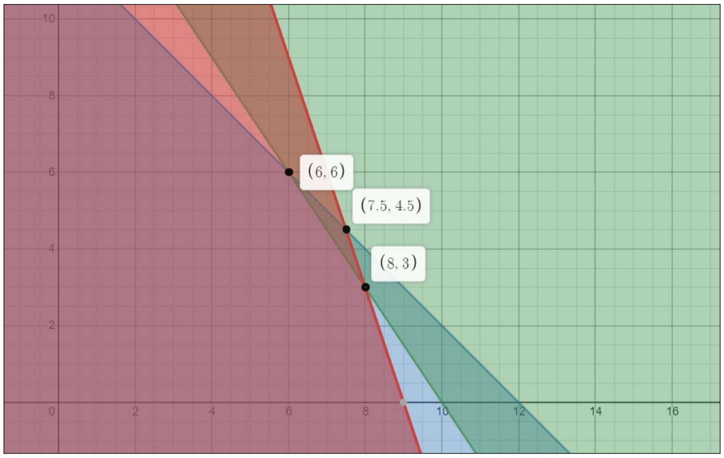
$x>0, y>0$ Non-negativity constraints.

$0.3x+0.1y\leq 2.7$ At most 2.7 kilorads of radiation must be established to kill the malignant cells in the spine.

$0.5x+0.5y\leq 6$ At most 6 kilorads of radiation must be established to kill the malignant cells in the tumor.

$0.6x+0.4y\geq 6$ At least 6 kilorads of radiation must be established to kill the malignant cells in the center of the tumor.

Illustrate Constraints by Graphing



Evaluating the Corner Points and Identify the Optimal Solution for the Objective Function

Corner Points	**Objective Function** **Minimize:** $z=0.4x+0.5x$
(6,6)	$z=0.4(6)+0.5(6)=5.4$
(7.5,4.5)	$z=0.4(7.5)+0.5(4.5)=5.25$
(8,3)	$z=0.4(8)+0.5(3)=4.7$

Conclusion

The minimum value of $z=4.7$, when $x=8$ and $y=3$.

In practical terms, the minimum dosage (in kilorads) to kill the malignant cells in the brain is 4.7. It needs 8 seconds of exposure from Beam A, and 3 seconds from Beam B to achieve this dosage.

3. This pandemic, Abheedette learned to bake while on home quarantine. She also realized that she will be able to make P60.00 profit per tray of banana muffins and P120.00 profit per tray of blueberry muffins. She needs 2 cups of milk and 3 cups of flour to bake a tray of banana muffins. And, baking a tray of blueberry muffins takes 4 cups of milk and 3 cups of flour. She has 16 cups of milk and 15 cups of flour. How many trays of each flavor must be baked to maximize the profit?

Product	*Ingredients per tray (in cups)*		**Profit per tray**
	Milk	**Flour**	
Banana Muffins	2	3	P60.00
Blueberry Muffins	4	3	P120.00

Define the Variables

Let x be the number of trays of banana muffins to be baked.
y be the number of trays of blueberry muffins to be baked.

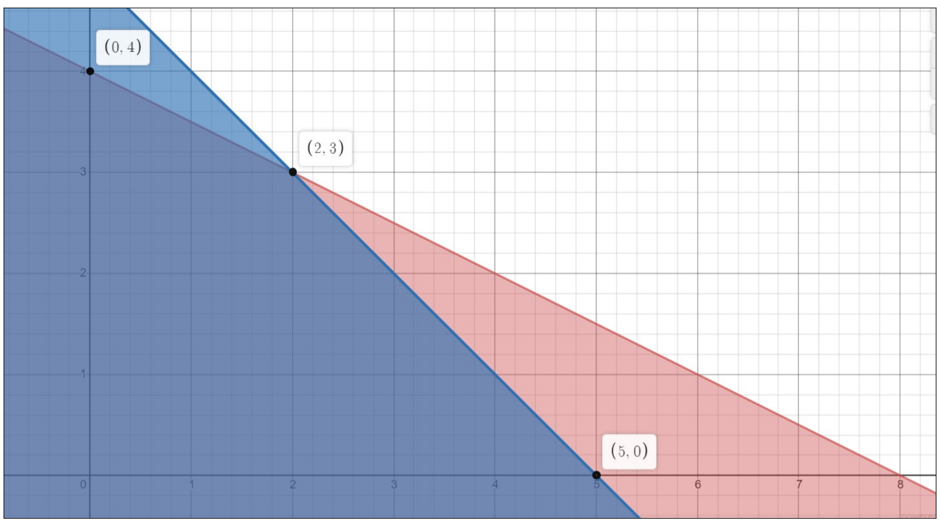
Identify the Objective Function

$z=60x+120y$

Identify the Constraints

$x\geq 0, y\geq 0$	Non-negativity constraints.
$2x+4y\leq 16$	Per tray of Banana muffins and Blueberry muffins need 2 and 4 cups of milk, respectively. Abheedette has only 16 cups of milk.
$3x+3y\leq 15$	Per tray of Banana muffins and Blueberry muffins need 4 and 3 cups of flour, respectively. Abheedette has only 15 cups of flour.

Illustrate Constraints by Graphing



Evaluating the Corner Points and Identify the Optimal Solution for the Objective Function

Corner Points	**Objective Function** **Maximize:** $z=60x+120y$
(0, 4)	$z=60(0)+120(4)=480$
(2, 3)	$z=60(2)+120(3)=480$
(5, 0)	$z=60(5)+120(0)=300$

Conclusion

The maximum value of z is 480. It occurs when x = 0 and y = 4 or when x = 2 and y = 3.

To achieve the maximum profit of P480, Abheedette should either make no Banana muffins, and 4 trays of Blueberry muffins or 2 trays of Banana muffins and 3 Blueberry muffins from 16 cups of milk and 15 cups of flour.

4. Axis is an aspiring freshman at Saint Louis University. He realizes that “all study and no play make Axis a dull boy.” As a result, Axis wants to apportion his available time of about 10 hours a day between study and play. He estimates that play is twice as much fun as study. He also wants to study at least as much as he plays. However, Axis realizes that if he is going to get all his homework done, he cannot play more than 4 hours a day. With the use of Linear Programming (LP), help Axis allocate his time to maximize his pleasure from both study and play.

<u>Define the Variables</u>

Let x be time (in hours) to study in a day.
y be time (in hours) to play in a day

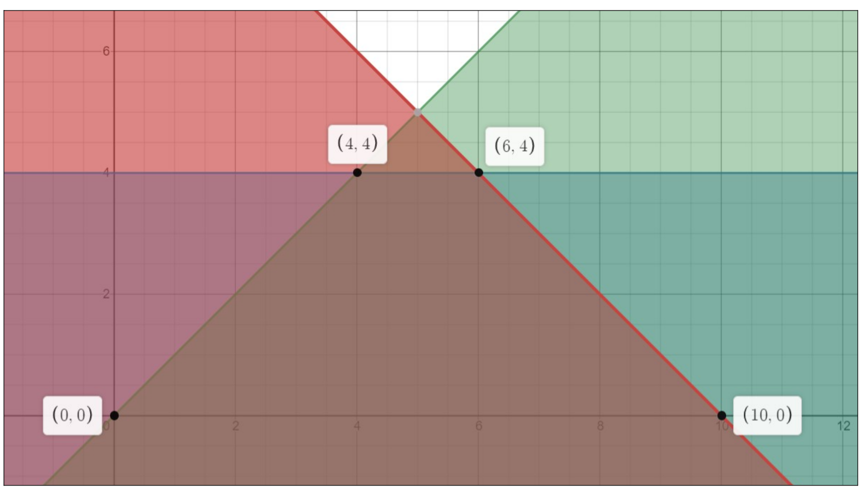
<u>Identify the Objective Function</u>

$z = x + 2y$

<u>Identify the Constraints</u>

$x \geq 0, y \geq 0$	Non-negativity constraints.
$x + y \leq 10$	Axis’ available time is about 10 hours a day between study and play.
$y \leq 4$	Axis cannot play more than 4 hours a day if he is going to get all his homework done for the day.
$x \geq y$	Axis wants to study at least as much as he plays.

<u>Illustrate Constraints by Graphing</u>



<u>Evaluating the Corner Points and Identify the Optimal Solution for the Objective Function</u>

Corner Points	Objective Function Maximize: $z = x + 2y$
(0,0)	$z = 0 + 2(0) = 0$
(4,4)	$z = 4 + 2(4) = 12$
(6,4)	$z = 6 + 2(4) = 14$
(10,0)	$z = 10 + 2(0) = 0$

Conclusion

The maximum value of z is 14. It occurs when $x=6$ and $y=4$.

In practical terms, given that Axis needs to study at least as much as he plays, and cannot play for more than 4 hours to accomplish his academic responsibilities, Axis can achieve 14 units of pleasure if he will allocate his study time for 6 hours and for 4 hours in his study time.

5. A Company produces precision medical diagnostic equipment at two factories. Three medical centers have placed orders for this month's production output. The table below shows what the cost would be for shipping each unit from each factory to each of these customers. Also shown are the number of units that will be produced at each factory and the number of units ordered by each customer.

   A decision now needs to be made about the shipping plan for how many units to ship from each factory to each customer. Formulate a linear programming model for this problem.

To/From	*Unit Shipping Cost*			**Output**
	Customer 1	**Customer 2**	**Customer 3**	
Factory 1	$600	$800	$700	400 units
Factory 2	$400	$900	$600	500 units
Order Size	300 units	200 units	400 units	

Define the Variables

Let x_1 be the number of units shipped from Factory 1 to Customer 1
x_2 be the number of units shipped from Factory 1 to Customer 2
x_3 be the number of units shipped from Factory 1 to Customer 3
y_1 be the number of units shipped from Factory 2 to Customer 1
y_2 be the number of units shipped from Factory 2 to Customer 2
y_3 be the number of units shipped from Factory 2 to Customer 3

Identify the Objective Function

$z=600x_1+800x_2+700x_3+400y_1+900y_2+600y_3$

Identify the Constraints

$x\geq 0, y\geq 0$ Non-negativity constraints.
$x_1+x_2+x_3=400$ Factory 1 has a maximum production capacity of 500 units.
$y_1+y_2+y_3=500$ Factory 2 has a maximum production capacity of 500 units.

$x_1+y_1=300$ 300 units must be shipped to Customer 1.
$x_2+y_2=200$ 200 units must be shipped to Customer 2.
$x_3+y_3=400$ 400 units must be shipped to Customer 3.

Illustrate Constraints by Graphing

Evaluating the Corner Points and Identify the Optimal Solution for the Objective Function

Corner Points	**Objective Function** **Minimum Cost:** $z=¿$

Conclusion

The minimum value of z is $540,000.00. It occurs when x = 0, x = 200, x=200, x = 300, x=0, x =200.

Therefore, the least possible shipping fee of the units produced from each factory to the customers is $540,000.00. To achieve this, Factory one should produce 200 units for Customer 2 and 200 units for Customer 3. Then, Factory two should produce 300 units for Customer 1, and 200 units for Customer 3.