1-18, Use defination of continuity to determine wether f is continuous at a.

1) $f(x) = 2x+5$
$a=1$
:- $f(a) = 2(1)+5$
$f(a) = 7$
f(x) is continuous at $\boxed{a=1}$

2) $f(x) = 3x+4$
$a=1$
:- $f(a) = 3(1)+4$
$f(a) = 7$
f(x) is continuous at $\boxed{a=1}$

3) $f(x) = x^2-3x+7$
:- $a=4$
$f(a) = 16-12+7$
$f(a) = 11$
f(a) is continuous at $\boxed{a=4}$

4) $f(x) = x^2-5x+6$
$a=4$
:- $f(a) = 16-20+6$
$f(a) = 2$
f(a) is continuous at $\boxed{a=4}$

5) $f(x) = \frac{x^2+4}{x-2}$
$a=3$
$f(a) = \frac{8+4}{3-2}$
$f(a) = 13$
f(a) is continuos at $\boxed{a=3}$

6) $\frac{x^2+6}{x-5}$, $a=6$
:- $\frac{(6)^2+6}{6-5} = \frac{36+6}{1} = 42$
$f(a) \neq 42$
f(a) is continuous at $\boxed{a=6}$

7) $f(x) = \frac{x+5}{x-5}$
$a=5$
:- $f(a) = \frac{5+5}{5-5} = \frac{10}{0}$
f(a) is discontinuous at $\boxed{a=5}$

8) $f(x) = \frac{x+7}{x-7}$
$a=7$
:- $f(a) = \frac{7+7}{7-7} = \frac{14}{0}$
f(a) is discontinuous at $\boxed{a=7}$

9) $f(x) = \frac{x-5}{x+5}$, $a=5$
$f(a) = \frac{5-5}{5+5} = \frac{0}{10} = 0$
f(a) is discontinuous at $\boxed{a=5}$

10) $f(x) = \frac{x-7}{x+7}$
$a=7$
:- $f(a) = \frac{7-7}{7+7} = \frac{0}{14} = 0$
f(a) is discontinuous at $\boxed{a=7}$

11) $f(x) = \frac{x^2+5x}{x^2-5x}$
$a=0$
:- $\frac{x(x+5)}{x(x-5)} = \frac{x+5}{x-5}$
$f(a) = \frac{0+5}{0-5} = \frac{5}{-5} = -1$
f(a) is continuous at $\boxed{a=0}$
Removable discontinuity

12) $f(x) = \frac{x^2+8x}{x^2-8x}$, $a=0$
:- $\frac{x(x+8)}{x(x-8)} = \frac{x+8}{x-8}$
$f(a) = \frac{0+8}{0-8} = \frac{8}{-8} = -1$
f(a) is continuous at $\boxed{a=0}$
Removable discontinuity

13) $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 5 & \text{if } x=2 \end{cases}$

:- $f(x) = \frac{x^2-4}{x-2}$, if $x \neq 2$
$\frac{x^2-4}{x-2} = \frac{x^2-2^2}{x-2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$
$f(x) = x+2$, $x \neq 2$ means $x \to 2$
$f(x) = x+2$, $x \to 2$
$f(2) = 2+2 = 4$
$\boxed{\lim_{x\to 2} f(x) = 4}$

$f(x) = 5$, if $x=2$
$\boxed{f(2) = 5}$
limit is not equal, Hence limit is Doesn't exist Non-Removable Discontinuity at $\boxed{x=2}$

Scanned with CamScanner

14) $f(n) = \begin{cases} \dfrac{n^2-36}{n-6} & \text{if } n \neq 6 \\ 13 & \text{if } n = 6 \end{cases}$ $\quad a = 6$

:- $\dfrac{n^2-36}{n-6} = \dfrac{n^2-6^2}{n-6} = \dfrac{\cancel{(n-6)}(n+6)}{\cancel{(n-6)}} = n+6$

$f(n) = n+6$, if $n \neq 6$, $n \to 6$

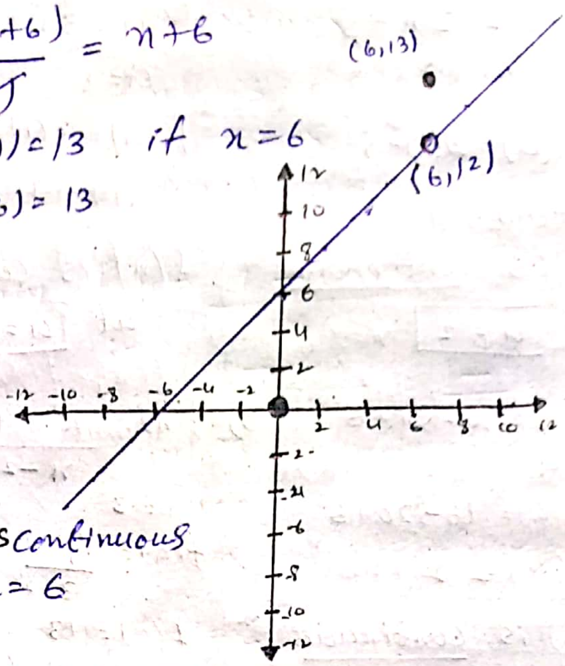
$\lim_{n\to 6} f(n) = n+6$

$\boxed{f(n) = 12}$

Removable discontinuity at $n=6$

$f(n) = 13$ if $n = 6$

$f(6) = 13$



$\lim_{n\to 6} f(n) = n+6 = 12$

$L_1 = 12$

$f(n) = 13$, $n = 6$

$\boxed{f(6) = 13}$ $\quad L_2 = 13$

$f(n)$ is discontinuous at $n = 6$

$L_1 \neq L_2$

15) $f(n) = \begin{cases} n-5, & \text{if } n \leq 0 \\ n^2+n-5, & \text{if } n > 0 \end{cases}$ $\quad a = 0$

:- $f(n)_1 = n-5$, $n \leq 0$

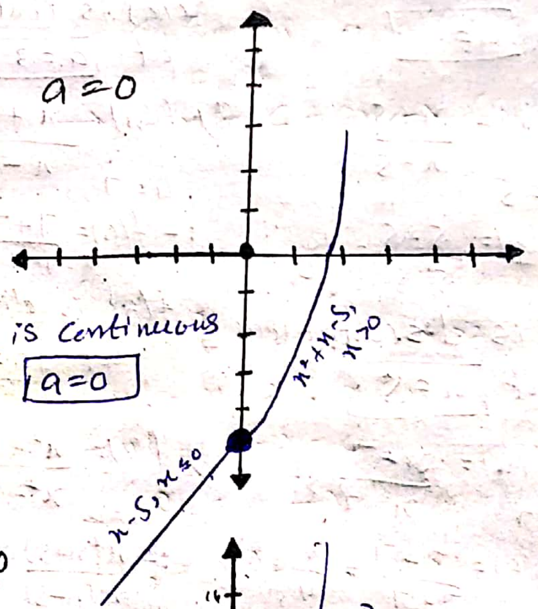
$\boxed{f(0) = -5}$

$f(n)_2 = n^2+n-5$, $n > 0$

$f(0) = 0+0-5$

$\boxed{f(0) = -5}$

$f(n)$ is continuous at $\boxed{a=0}$



16) $f(n) = \begin{cases} n-4, & \text{if } n \leq 0 \\ n^2+n-4, & \text{if } n > 0 \end{cases}$

:- $f(n) = n-4$, $n \leq 0$

$\boxed{f(0) = -4}$

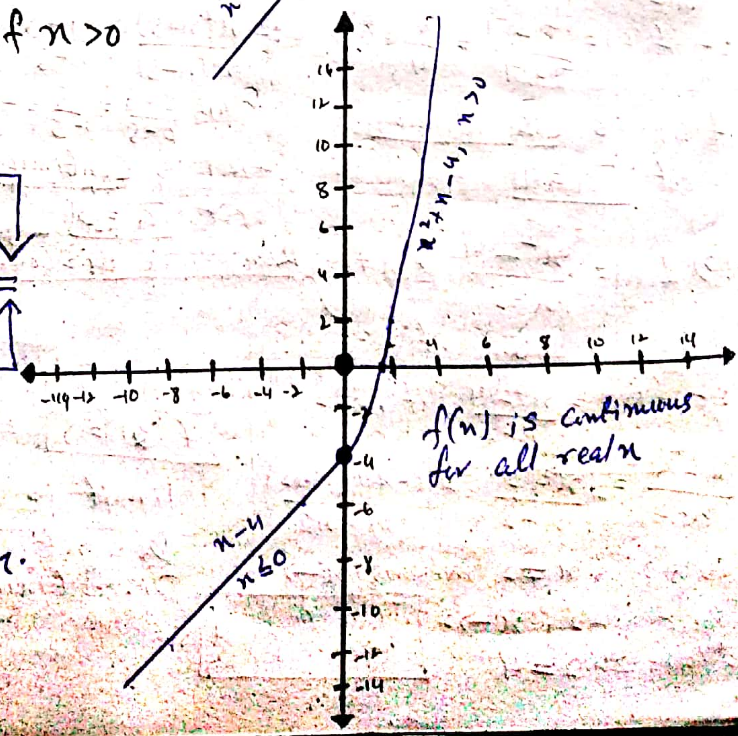
$f(n) = n^2+n-4$, $n > 0$

$f(0) = 0^2+0-4$

$\boxed{f(0) = -4}$

$\lim f(n)_1 = f(n)_2$

Limit exist, $f(n)$ is Continuous for all real n.

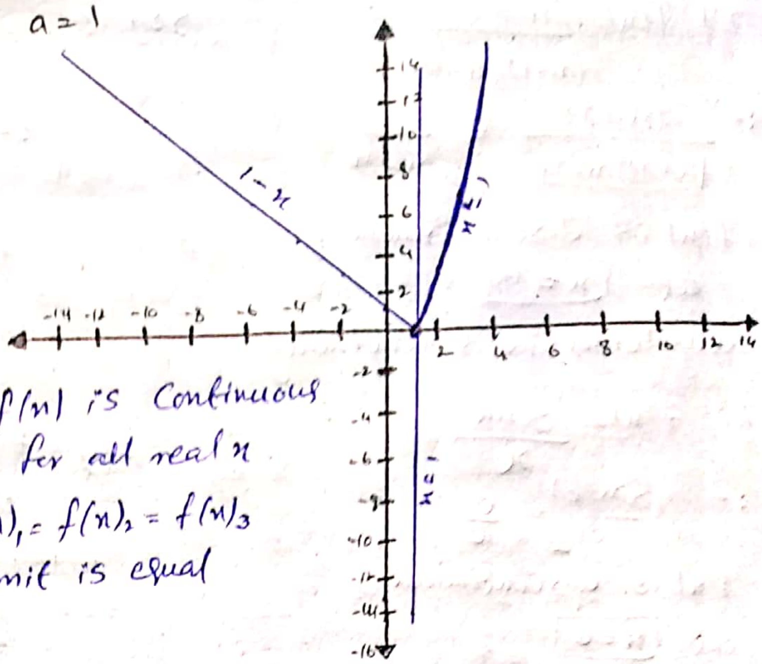


$f(n)$ is continuous for all real n

Scanned with CamScanner

17) $f(x) = \begin{cases} 1-x & \text{if } x<1 \\ 0 & \text{if } x=1 \\ x^2-1 & \text{if } x>1 \end{cases}$

$a = 1$



:- x is exactly equal to 1,
So the y is equal to 0.

a: $f(x) = 1-x, \ x<1$

$f(1) = 1-1 = 0$

$\boxed{f(1) = 0}$ → defined

b: $f(x) = 0, \ x=1$

$\boxed{f(1) = 0}$ → defined

c: $f(x) = x^2-1, \ x>1$

$f(1) = (1)^2-1$

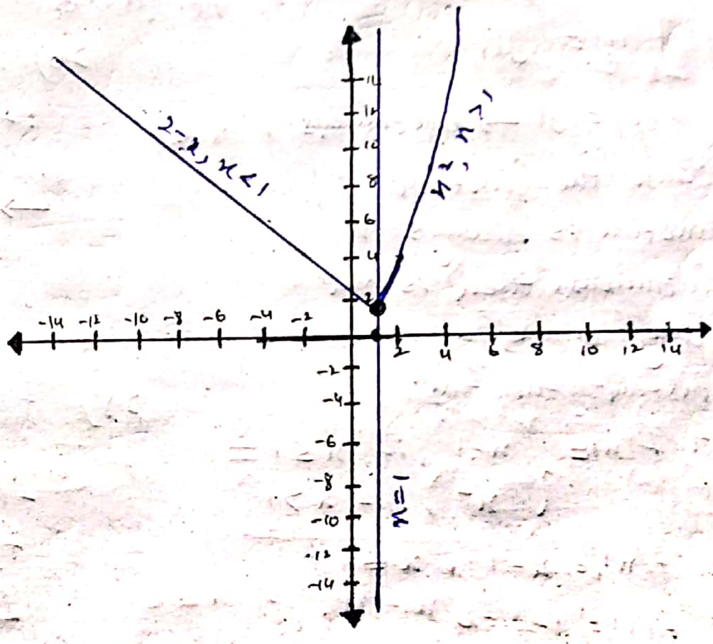
$f(1) = 1-1 = 0$

$\boxed{f(1) = 0}$ → defined

f(x) is Continuous for all real x

$f(x)_1 = f(x)_2 = f(x)_3$
Limit is equal

18) $f(x) = \begin{cases} 2-x, & \text{if } x<1 \\ 1, & \text{if } x=1 \\ x^2, & \text{if } x>1 \end{cases}$



:-

a: $f(x) = 2-x, \ x<1$

$f(1) = 2-1 = 1$

$\boxed{f(1) = 1}$ → defined

b: $f(x) = 1, \ x=1$

$\boxed{f(1) = 1}$ → defined

c: $f(x) = x^2, \ x>1$

$f(1) = (1)^2 = 1$

$\boxed{f(1) = 1}$ → defined

18–34, determine for what numbers, if any, the given function is discontinuous.

19) $f(x) = x^2+4x-6$

:- $\dfrac{-4 \pm \sqrt{(4)^2-4(1)(-6)}}{2(1)}$

$x = \dfrac{-4 \pm \sqrt{16+24}}{2} = \dfrac{-4 \pm \sqrt{40}}{2}$

$x = \dfrac{-4 \pm 6.32}{2} \rightarrow \dfrac{2.32}{2}$ and $\dfrac{-10.32}{2}$

$\boxed{x = 1.16}$ OR $\boxed{x = -5.16}$

x is Continuous for all real x.

20) $f(x) = x^2+8x-10$

:- $\dfrac{-8 \pm \sqrt{(8)^2-4(1)(-10)}}{2(1)}$

$x = \dfrac{-8 \pm \sqrt{64+40}}{2} = \dfrac{-8 \pm \sqrt{104}}{2}$

$x = \dfrac{-8 \pm 10.19}{2}$

$x = \dfrac{-18.19}{2}, \quad x = \dfrac{2.19}{2}$

$\boxed{x = -9.095}$ $\boxed{x = 1.095}$

f(x) is Continuous for all real values x.

21) $f(x) = \dfrac{x+1}{(x+1)(x-4)}$

:- $\dfrac{\cancel{(x+1)}}{\cancel{(x+1)}(x-4)} = \dfrac{1}{x-4}$

f(x) is discontinuous at $\boxed{x=4}$

Non-Removable discontinuity

22) $f(x) = \dfrac{x+2}{(x+2)(x-5)}$

:- $\dfrac{\cancel{(x+2)}}{\cancel{(x+2)}(x-5)} = \dfrac{1}{x-5}$

f(x) is discontinuous at $\boxed{x=5}$

Non-Removable discontinuity

23) $f(x) = \dfrac{\sin x}{x}$

:- $\dfrac{\sin(0)}{0} = \dfrac{0}{0}$

f(x) is discontinuous at $\boxed{x=0}$

Non-Removable Discontinuity

24) $f(x) = \dfrac{1-\cos x}{x}$

:- $\dfrac{1-\cos(0)}{0} = \dfrac{1-1}{0} = \dfrac{0}{0}$

f(x) is discontinuous at $\boxed{x=0}$

Non-Removable discontinuity

25) $f(x) = \pi$

:- $f(0) = \pi \rightarrow$ continuous

because the constant function is always in continuous, there is not any break in a constant function.

26) $f(x) = c$

:- Continuous for all real value x

$\rightarrow f(x) = c + 0x \Leftrightarrow f(x) = c$

27) $f(x) = \begin{cases} x-1, & \text{if } x \le 1 \\ x^2, & \text{if } x > 1 \end{cases}$

:- $f(x)_1 = x-1, \; x \le 1$

$f(1)_1 = 1-1 = 0$

$\boxed{f(1)_1 = 0} \rightarrow$ defined.

$f(x)_2 = x^2, \; x > 1$

$f(1)_2 = (1)^2$

$\boxed{f(1)_2 = 1}$

$\boxed{f(x)_1 \neq f(x)_2}$

Limit is not equal.

f(x) is discontinuous at $\boxed{x=1}$

Non-Removable Discontinuity

28) $f(x) = \begin{cases} x-2, & \text{if } x \le 2 \\ x^2-1, & \text{if } x > 2 \end{cases}$

$f(x)_1 = x-2, \; x \le 2$

$f(x)_1 = 2-2 = 0$

$\boxed{f(2)_1 = 0} \rightarrow$ defined

$f(x)_2 = x^2-1, \; x > 2$

$f(2)_2 = (2)^2 - 1 = 4-1$

$\boxed{f(2)_2 = 3} \rightarrow$ defined

$\boxed{f(x)_1 \neq f(x)_2}$

Limit is not equal.

f(x) is discontinuous at $\boxed{x=2}$

Non-Removable Discontinuity.

29) $f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & \text{if } x \neq 1 \\ 2, & \text{if } x = 1 \end{cases}$

:- $\dfrac{x^2-1}{x-1} = \dfrac{x^2-1^2}{x-1} = \dfrac{(x-1)(x+1)}{\cancel{(x-1)}}$

$= \boxed{x+1}$

$f(x)_1 = x+1 \quad , \quad f(x) = 2$

$\lim\limits_{x\to 1} f(x)_1 = x+1$

$\boxed{f(1)_1 = 2} \to$ defined

$\lim\limits_{x\to 1} f(x)_2 = 2$

$\boxed{f(1) = 2} \to$ defined

$\boxed{\lim f(x)_1 = f(x)_2}$

Limit is equal.
f(x) is Continuous.
Removable discontinuity at $\boxed{x=1}$

30) $f(x) = \begin{cases} \dfrac{x^2-9}{x-3}, & \text{if } x \neq 3 \\ 6, & \text{if } x = 3 \end{cases}$

:- $\dfrac{x^2-9}{x-3} = \dfrac{(x-3)(x+3)}{\cancel{(x-3)}} = x+3$

$\boxed{f(x)_1 = x+3}$

$f(x)_2 = 6$

$\lim\limits_{x\to 3} f(x)_1 = x+3$

$f(3) = 3+3 = 6$

$\boxed{f(3) = 6} \to$ defined

$\lim\limits_{x\to 3} f(x)_2 = 6 + 0x$

$f(3) = 6 + 0(3)$

$\boxed{f(3) = 6} \to$ defined

$\boxed{\lim\limits_{x\to 3} f(x)_1 = f(x)_2}$

Limit is equal, f(x) is Continuous.
Removable Discontinuity at $\boxed{x=3}$

31) $f(x) = \begin{cases} x+6 & \text{if } x \leq 0 \\ 6 & \text{if } 0 < x \leq 2 \\ x^2+1 & \text{if } x > 2 \end{cases}$

:- $f(x)_1 = x+6 \quad , \; x \leq 0$

$f(x)_2 = 6 \quad , \; 0 < x \leq 2$

$f(x)_3 = x^2+1 \quad , \; x > 2$

① put $x=0$ in $f(x)_1$ and $f(x)_2$, then Compare them

② put $x=2$ in $f(x)_2$ and $f(x)_3$, then Compare them

① $f(x)_1 = x+6$

$f(0)_1 = 0+6 = 6$

$\boxed{f(0)_1 = 6}$

$f(x)_2 = 6 + 0x$

$f(0)_2 = 6$

$\boxed{f(0)_2 = 6}$

$\lim\limits_{x\to 0} \boxed{f(x)_1 = f(x)_2}$

② $f(x)_2 = 6 + 0x$

$f(2)_2 = 6 + 0(2) = 6$

$\boxed{f(2)_2 = 6}$

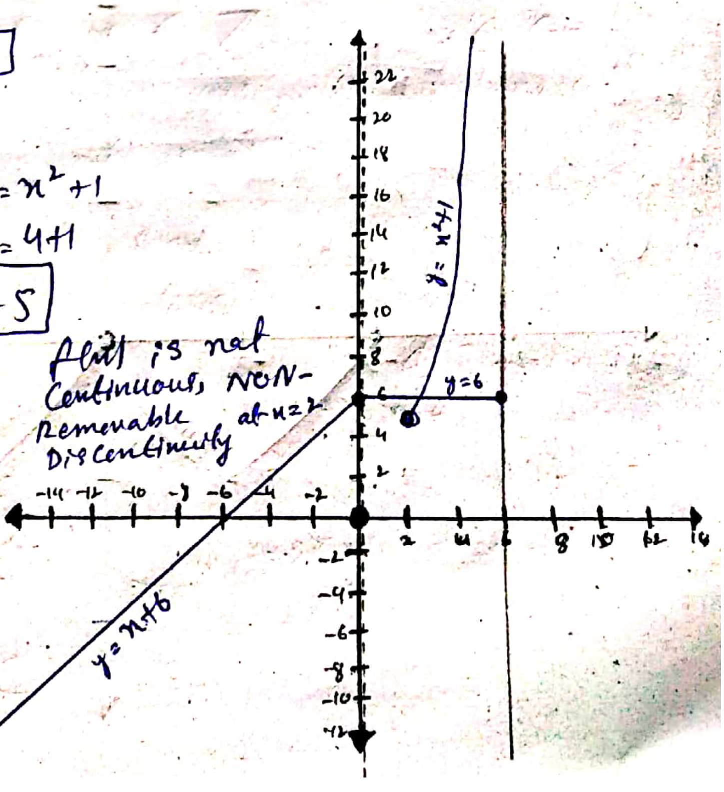
$f(x)_3 = x^2+1$

$f(2)_3 = 4+1$

$\boxed{f(2)_3 = 5}$

$\boxed{\lim\limits_{x\to 2} f(x)_2 \neq f(x)_3}$

f(x) is not Continuous, NON-Removable DisContinuity at $x=2$



32) $f(x) = \begin{cases} x+7 & \text{if } x \le 0 \\ 7 & \text{if } 0 < x \le 3 \\ x^2-1 & \text{if } x > 3 \end{cases}$

:- $f(x)_1 = x+7,\ x \le 0$

$f(x)_2 = 7,\ 0 < x \le 3$

$f(x)_3 = x^2-1,\ x > 3$

$\lim_{x \to 0} f(x)_1 = x+7,$

$\boxed{f(0)_1 = 7}$ → defined

$f(x)_2 = 7$

$\boxed{f(0)_2 = 7}$ → defined

$\lim \boxed{f(x)_1 = f(x)_2}$

$f(x)_2 = 7$

$\boxed{f(3)_2 = 7}$

$f(x)_3 = x^2-1$

$f(3)_3 = (3)^2-1$

$f(3)_3 = 9-1 = 8$

$\boxed{f(3)_3 = 8}$

$\lim \boxed{f(x)_2 \ne f(x)_3}$

f(x) is discontinuous at $\boxed{x=3}$

33) $f(x) = \begin{cases} 5x & \text{if } x < 4 \\ 21 & \text{if } x = 4 \\ x^2+4 & \text{if } x > 4 \end{cases}$

:- $f(x)_1 = 5x,\ x < 4$

$f(x)_2 = 21,\ x = 4$

$f(x)_3 = x^2+4,\ x > 4$

$\lim_{x \to 4} f(x)_1 = 5x$

$\boxed{f(4)_1 = 20}$

$f(x)_2 = 21$

$\boxed{f(4)_2 = 21}$

$\lim_{x \to 4} \boxed{f(x)_1 \ne f(x)_2}$

f(x) is discontinuous at $\boxed{x=4}$ and f(x) is exactly equal to 4.

34) $f(x) = \begin{cases} 7x & \text{if } x < 6 \\ 41 & \text{if } x = 6 \\ x^2+6 & \text{if } x > 6 \end{cases}$

:- $f(x)_1 = 7x,\ x < 6$

$f(x)_2 = 41,\ x = 6$

$f(x)_3 = x^2+6,\ x > 6$

$\lim_{x \to 6} f(x)_1 = 7x$

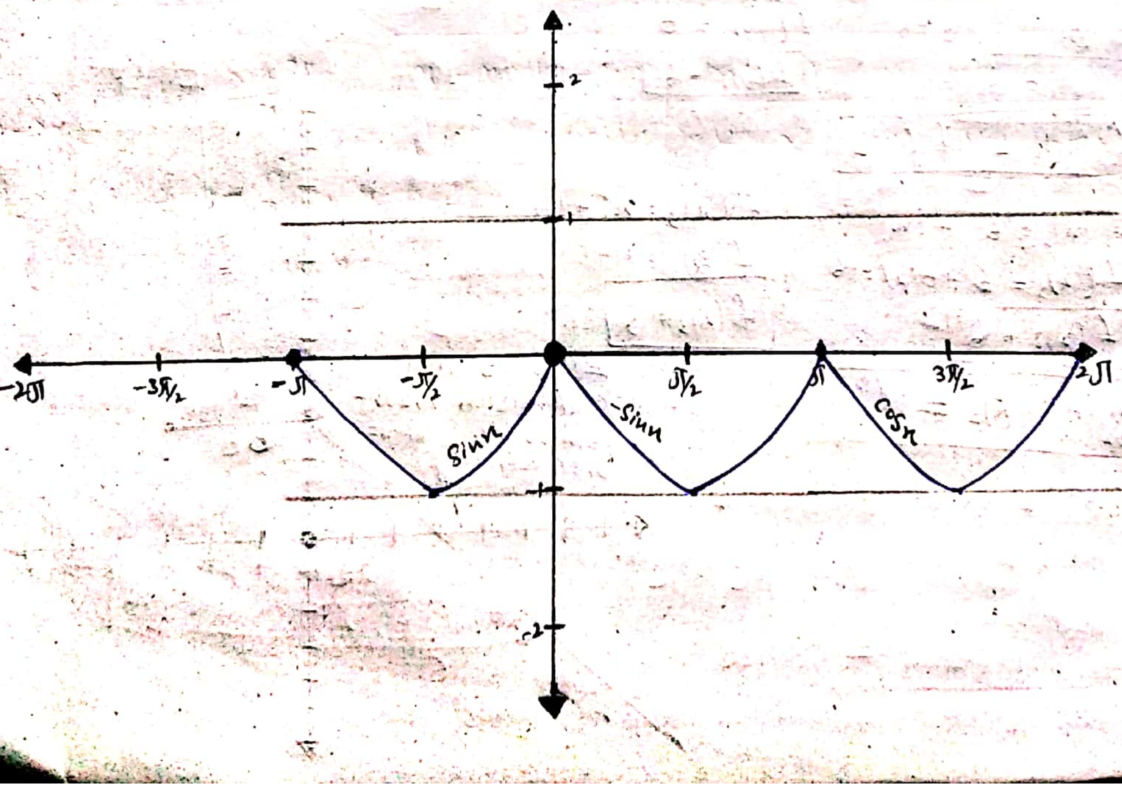
$\boxed{f(6)_1 = 42}$

$\lim_{x \to 6} f(x)_2 = 41$

$\boxed{f(6)_2 = 41}$

$\lim \boxed{f(x)_1 \ne f(x)_2}$ f(x) is discontinuous at $\boxed{x=6}$

35) $f(x) = \begin{cases} \sin x, & -\pi \le x < 0 \\ -\sin x, & 0 \le x < \pi \\ \cos x, & \pi \le x \le 2\pi \end{cases}$



$$\begin{cases} -\sin x & , 0 \le x < \pi \\ \sin x & , \pi \le x \le 2\pi \end{cases}$$

:- $A\cos(Bx+C)+D$

$-\cos x$

Amplitude $= A = -1$ $[-1,1]$

Period $= B = 1$

Horizontal shift $= C = 0$

vertical shift $= D = 0$

Period $= \frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

$\boxed{\text{Period} = 2\pi}$

$A\sin(Bx+C)+B$

$-\sin x$

$\boxed{A = -1}$ Amplitude $[-1,1]$

Period $= \frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

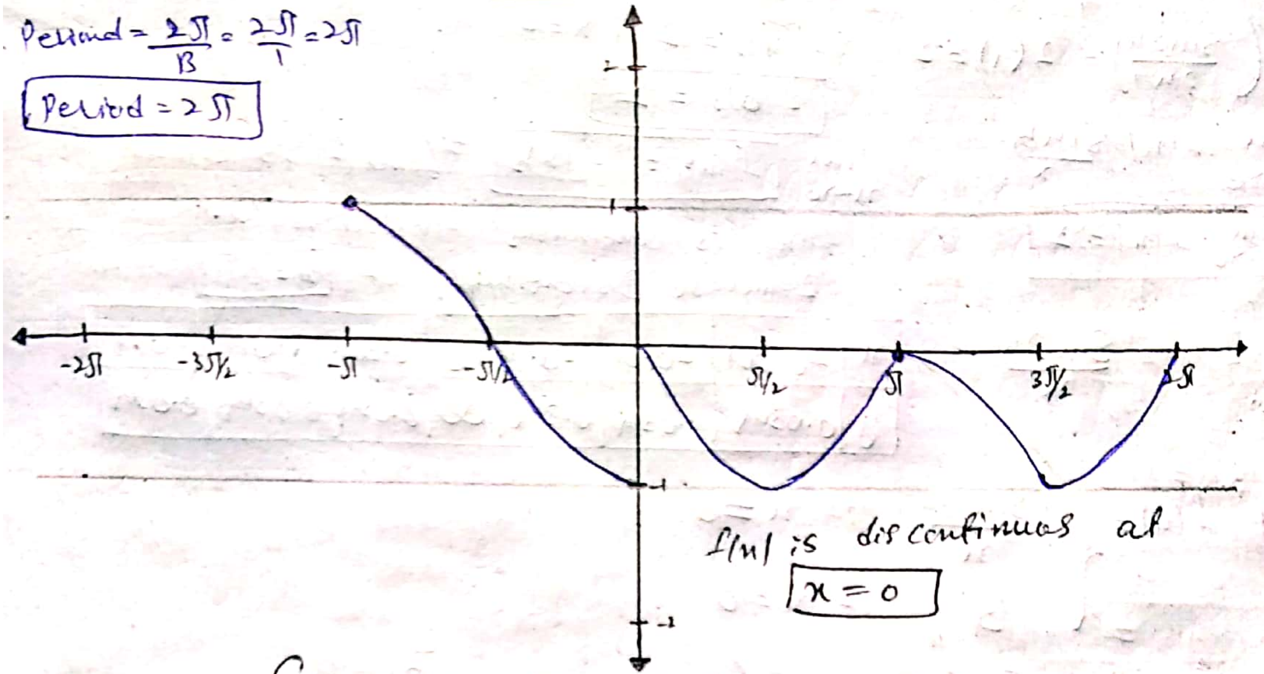
$\boxed{\text{Period} = 2\pi}$

$A\sin(Bx+C)+D$

$\boxed{A = 1}$ $[-1,1]$

Period $= \frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$ ✓

$\boxed{\text{Period} = 2\pi}$



$f(x)$ is discontinuos at $\boxed{x = 0}$

37) $f(x) = \begin{cases} -1, & \text{if } x \text{ is an integer} \\ 1, & \text{if } x \text{ is not an integer} \end{cases}$

:-

$f(x)_1 = -1$, $x \in \mathbb{Z}$

for any integer a $f(a)_1$ is equal to -1

$a \in \mathbb{Z}$

$\boxed{f(a)_1 = -1}$

$f(x)_2 = 1$, $x \notin \mathbb{Z}$, let $a \in \mathbb{Q}$

$\boxed{f(a)_2 = 1}$, for any rational number a.

$\lim \boxed{f(x)_1 \neq f(x)_2}$ limit is not equal.

38) $f(x) = \begin{cases} 2 & \text{if } x \text{ is an odd integer} \\ -2 & \text{if } x \text{ is not an odd integer} \end{cases}$

:- $f(x)_1 = 2$, $x \in$ odd integer

let a be an odd integer

$f(x)_1 = 2 + 0x = 2 + 0(a) = 2$

$\boxed{f(x)_1 = 2}$ for any odd integer a.

$f(x)_2 = -2$, $x \notin$ odd integers

↳ $x \in$ even integer

$f(x)_2 = -2 + 0x$

let a be an even integer

$f(x)_2 = -2 + 0(a) = -2$

$\boxed{f(x)_2 = -2}$

$\lim \boxed{f(x)_1 \neq f(x)_2}$

$f(x)$ is discontinuous, limit is not equal.

39–42 determine for what numbers, if any, the function is discontinuous. Construct a table to find any required limit.

39) $f(x) = \begin{cases} \dfrac{\sin 2x}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$

:– $\boxed{\dfrac{\sin x}{x} = 1}$ $\qquad \dfrac{\sin 2x}{x} = \dfrac{2(\sin 2x)}{2(x)} = 2\left(\dfrac{\sin 2x}{2x}\right) = 2\left(\dfrac{\sin 2x}{2x}\right)$

$2\left(\dfrac{\sin 2x}{2x}\right) = 2(1) = 2$ $\qquad f(x)_2 = 2, \; x = 0$

$\boxed{f(0)_2 = 2}$

$\lim_{x \to 0} f(x) = \dfrac{\sin 2x}{x}$

$\boxed{\lim_{x \to 0} f(x) = 2}$

$\lim_{x \to 0} \boxed{f(x)_1 = f(x)_2}$ limit is equal

$f(x)$ is continuous for all real x.

Removable Discontinuity at $\boxed{x = 0}$

$\lim_{x \to 0} f(x) = \dfrac{\sin 2x}{x}$

x	−0.001	−0.01	−0.1	0	0.001	0.01	0.1
y	0.0349	0.0348	0.348	∞	0.0349	0.0348	0.0348

40) $f(x) = \begin{cases} \dfrac{\sin 3x}{x} & \text{if } x \neq 0 \\ 3 & \text{if } x = 0 \end{cases}$

:– $\dfrac{\sin 3x}{x} = \dfrac{3(\sin 3x)}{3(x)} = 3\left(\dfrac{\sin 3x}{3x}\right) = 3(1) = \boxed{3}$

$\lim_{x \to 0} f(x)_1 = \dfrac{\sin 3x}{x}$

$\lim_{x \to 0} \boxed{f(x)_1 = 3}$ $\qquad \boxed{f(x)_2 = 3}$

$\lim_{x \to 0} \boxed{f(x)_1 = f(x)_2}$

limit is equal.

Removable discontinuity at $\boxed{x = 0}$

$\lim_{x \to 0} f(x)_1 = \dfrac{\sin 3x}{x}$

x	−0.001	−0.01	−0.1	0	0.001	0.01	0.1
y	0.0524	0.0524	0.0524	∞	0.0524	0.0524	0.0524

41) $\begin{cases} \cos x \div x - \frac{\pi}{2}, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$