ANALYSIS OF VARIANCE (ANOVA)

CHAPTER 9

OBJECTIVES:

1. Determine the use and limitation of analysis of variance
2. Test the hypothesis regarding ANOVA
3. Learn and appreciate the use of F-Table
4. Make meaningful interpretations
5. Compute ANOVA using MS- Excel

ANALYSIS OF VARIANCE (ANOVA or F-TEST)

The analysis of variance (ANOVA) is used to determine whether there are any significant differences between the means of three or more independent (unrelated) groups. This guide will provide a brief introduction to the one-way ANOVA including the assumptions of the test to interpret the output. The one-way ANOVA compares the means between the groups you are interested in and determines whether any of those means are significantly different from each other.

The basic ANOVA situation

One variable: 1 Categorical, 1 Quantitative

Main Question: Do the (means of) the quantitative variables depend on which group (given by categorical variable) the individual is in?

If categorical variable

has only 2 values: 2-sample t-test /z-test
ANOVA allows for 3 or more groups

Example of ANOVA situation

Subjects: 25 patients with blisters

Treatments: Treatment A, Treatment B, Placebo

Measurement: Number of days until blisters heal

Data [and means]:

Treatment A : 5,6,6,7,7,8,9,10 [7.25]

Treatment B : 7,7,8,9,9,10,10,11 [8.875]

Placebo : 7,9,9,10,10,10,11,12,13 [10.11]

One-Way ANOVA

There is only one factor being studied as the independent variable.

Independent variable may be different experimental conditions, teaching methods, guidance technique, values education approaches, educational attainment, socio economic status or other factors that may have two or more levels.

Steps to follow in dealing with One-Way ANOVA

1. State the Hypothesis:

   Null Hypothesis (H_o): The means of all the groups are equal.

   Alternative Hypothesis (H_a): Not all the means are equal (do not say how or which ones differ).

2. Set the level of significance (α).
3. Choose the statistical test appropriate to test the hypothesis.
4. Determine the tabular value for the test.
5. Compute the value of the statistical test.
6. Determine the significance of the computed value.
7. Interpret and discuss the result.

Formula in computing One- Way ANOVA

a. Sum of the squares

$$TSS = \sum x^2 - \frac{(\sum x)^2}{N}$$

$$SS_b = \frac{1}{r}\sum\left(\sum x_{ij}\right)^2 - \frac{\left(\sum x\right)^2}{N}$$

$$SS_b = \frac{\left(\sum x_{i1}\right)^2}{n_1} + \frac{\left(\sum x_{i2}\right)^2}{n_2} + \ldots + \frac{\left(\sum x_{ij}\right)^2}{n_j} - \frac{\left(\sum x\right)^2}{N}$$

$$SS_w = TSS - SS_b$$

where:

$\sum x_{ij}$ *= sum of each column*

$\sum x$ *= Sum of the values of all items*

TSS *= total sum of squares*

SS_b *= sum of squares between column*

SS_w = *sum of squares within column*

r or n_1 *… = number of items per column*

b. Degrees of freedom

$$df_T = N - 1$$
$$df_b = k - 1$$
$$df_w = df_T - df_b$$

where

df_T = *Total degrees of freedom*

df_b = *degrees of freedom between column*

df_w = *degrees of freedom within column*

k = number of columns

N = total number of entries

c. Mean sum of Squares

$$MSS_b = \frac{SS_b}{df_b}$$

$$MSS_w = \frac{SS_w}{df_w}$$

where

MSS_b = *mean sum of squares between column*

MSS_w = *mean sum of squares within column*

d. Locating the tabular value and calculating the computed value and comparing them

1. Locate the tabular value of F_T by following the format.

$$F_t = \frac{df_b}{df_w} \rightarrow \text{locate(Refer to Appendix C)}$$

2. Calculate F_c

$$F_C = \frac{MSS_b}{MSS_w}$$

3. Compare the Computed against the tabular value

   a. Reject Ho if the computed value is greater than or equal to the tabular value. $\left(F_C \geq F_T\right)$
   b. Do not reject Ho if the computed value is less than the tabular value. $\left(F_C < F_T\right)$.

Table for the One-Way ANOVA

Source of Variation	Sum Of Squares	Degree of Freedom	Mean Sum of Squares	FT	FC
Between Columns					
Within Columns					
Total					

Example

On the following four groups of teaching attitude, test the null hypothesis that academic performance does not vary due to teaching attitude.

Superior	Above Average	Average	Below Average
90	85	80	78
89	86	82	76
88	84	83	75
94	83	81	77
93	88	80	75

Solution

Step 1: Formulate your Hypothesis

H_o: The academic performance does not vary due to teaching attitude.

H_a: The academic performance varies due to teaching attitude.

Step 2: Set the level of significance

α = 0.01 or 0.05

Step 3: Choose the statistical test then apply it

ANOVA or F- test

Worksheet table for the One-Way ANOVA

Superior		Above Average		Average		Below Average	
X_1	X_1^2	X_2	X_2^2	X_3	X_3^2	X_4	X_4^2
90	8100	85	7225	80	6400	78	6084
89	7921	86	7396	82	6724	76	5776
88	7744	84	7056	83	6889	75	5625
94	8836	83	6889	81	6561	77	5929
93	8649	88	7744	80	6400	75	5625
454	41250	426	36310	406	32974	381	29039

Summation

$$\sum X_T = 1667$$

$$\sum X_T^2 = 139573$$

$$N_T = 20$$

Compute Sum of Squares

$$Tss = \Sigma x^2 - \frac{(\Sigma x)^2}{N}$$

$$Tss = 139573 - \frac{(1667)^2}{20}$$

$$Tss = 628.55$$

$$SS_b = \frac{1}{r}\sum\left(\sum x_{ij}\right)^2 - \frac{\left(\sum x\right)^2}{N}$$

$$SS_b = \frac{1}{5}\sum(454^2 + 426^2 + 406^2 + 381^2) - \frac{(1667)^2}{20}$$

$$SS_b = 573.35$$

$$SS_w = 628.55 - 573.35$$

$$SS_w = 55.2$$

Degrees of Freedom

$$df_T = \mathrm{N} - 1 \qquad df_b = \mathrm{k}\text{-}1 \qquad df_w = df_T - df_b$$

$$df_T = 20 - 1 \qquad df_b = 4\text{-}1 \qquad df_w = 19 - 3$$

$$df_T = 19 \qquad df_b = 3 \qquad df_w = 16$$

Mean Sum of Squares

$$MSS_b = \frac{SS_b}{df_b} \qquad MSS_w = \frac{SS_w}{df_w}$$

$$MSS_b = \frac{573.35}{3} \qquad MSS_w = \frac{55.2}{16}$$

$$MSS_b = 191.12 \qquad MSS_w = 3.45$$

Step 4: Determine the tabular value

$$F_T = \frac{df_b}{df_w} \qquad F_T = \frac{3}{16}$$

@ 0.05 → **3.239**
@ 0.01 → **5.29**

Step 5: Compute the value of the statistical test

$$F_C = \frac{MSS_b}{MSS_w} \qquad F_C = \frac{191.12}{3.45} \qquad F_C = 55.40$$

Source of Variation	Sum of Squares	Degree of Freedom	Mean sum of squares	F_T	F_c
Between Column	573.35	3	191.12	3.239 @0.05	55.40
Within Column	55.2	16	3.45	5.292 @0.01	
Total	628.55	19			

Step 6: Determine the significance of the computed value

Since the value of $F_c > F_T$, therefore reject the null hypothesis (H_o) accept alternative hypothesis (H_a)

Step 7: Interpret the result

Decision: Reject the null hypothesis.
Interpretation: The academic performance varies due to teaching attitude

EXERCISES

Name:_____ Score: _____

Course & Year: _____ Date: _____

DIRECTION: Solve each of the following problems completely and followed the steps in hypothesis testing.

1. The table below shows the achievement of 4 groups of students who were subjected to different teaching approaches (A, B, C & D). Determine if different teaching approaches affect student achievement.

A		B		C		D	
93	87	77	81	75	85	80	86
85	80	80	80	78	80	86	82
81	78	80	78	79	79	85	79
80	86	85	79	78	88	85	80
89	85	84	80	72	81	84	90

2. A researcher wishes to study the effectiveness of various methods of individuals' therapy: Reality therapy, Behavior therapy, Psychoanalysis and Gestalt Therapy. Ten Patients were randomly assigned to each group. At the conclusion of the study, changes in self concept are measured for each patient. The purpose of the study is to determine if one method was more effective than the other methods. The data on changes in self concept are as follow:

Reality Therapy	36	14	15	20	23	16	25	30	20	29
Behavior Therapy	20	25	40	15	17	18	34	23	24	10
Psycho-analysis	40	37	35	30	28	32	24	42	38	33
Gestalt Therapy	15	15	12	16	20	30	13	26	18	13

3. A 30-items research made test on reading comprehension was administered to five groups of fourth year students from different high school in Olongapo City. Their scores are shown below. Perform the analysis of variance and indicate whether or not their means score differ.

A	B	C	D	E
23	26	15	16	23
30	19	20	28	30
25	23	22	15	26
15	27	25	20	28
24	20	30	25	25
23		18	23	
25		19	20	
		25		

4. The following are the numbers of mistakes made on six occasions by four machine operators in making a technical report. Can we conclude that there is a real difference in the numbers of mistakes made in general by the four machine operators?

Machine Operator 1	11	14	10	12	13	15
Machine Operator 2	12	11	7	17	14	13
Machine Operator 3	9	14	12	10	15	11
Machine Operator 4	16	6	10	12	9	8

5. Suppose that a random sample of n = 7 was selected from the vineyard properties for sale in Cebu Philippines, in each of three years. The following data are consistent with summary information on price per acre for disease-resistant grape vineyards in Cebu. Determine whether there is evidence to support the claim that the mean price per acre for vineyard land in Cebu was not the same for each of the three years considered.

Year	1	2	3	4	5	6	7
2000	30000	34000	36000	38000	40000	42000	44000
2001	30000	35000	40000	45000	50000	55000	60000
2002	40000	42000	43000	45000	48000	50000	54000

6. A research study was conducted to examine the clinical efficacy of a new antidepressant. Depressed patients were randomly assigned to one of three groups: a placebo group, a group that received a low dose of the drug, and a group that received a moderate dose of the drug. After four weeks of treatment, the patients completed the Beck Depression Inventory. The higher the score, the more depressed the patient. The data are presented below. Compute the appropriate test.

Placebo	Low Dose	Moderate Dose
38	22	14
47	19	26
39	8	11
25	23	18
42	31	5

7. A researcher is concerned about the level of knowledge possessed by university students regarding United States history. Students completed a high school senior level standardized U.S. history exam. Major for students was also recorded. Data in terms of percent correct is recorded below for 32 students. Compute the appropriate test for the data provided below.

Education	Business/Management	Behavioral/Social Science	Fine Arts
62	72	42	80
81	49	52	57
75	63	31	87
58	68	80	64
67	39	22	28
48	79	71	29
26	40	68	62
36	15	76	45

8. Below are the bowling scores of four groups of 5 players each. Find out if there is unusual variation among the four groups.

Player	Group A	Group B	Group C	Group D
A	97	102	89	93
B	79	97	92	90
C	94	92	108	99
D	114	87	88	96
E	98	95	78	84

9. Sixteen midsize cars were chosen at random to test the mileage claims of four different brands of gasoline. The table lists the mileage (in miles/gallon) achieved with each brand. Is there any evidence here that all gasolines are NOT the same?

Gas Brand A	Gas Brand B	Gas Brand C	Gas Brand D
22.3	24.9	26.0	19.7
20.9	19.7	20.0	22.2
21.5	22.0	24.5	19.9
22.0	21.9	23.4	22.3

CHI-SQUARE TEST FOR ASSOCIATION

CHAPTER 10

OBJECTIVES:

1. Test two variables for independence
2. Learn and appreciate the use of x^2 table
3. Make meaningful interpretations
4. Compute Chi-square using MS- Excel

CHI-SQUARE TEST FOR ASSOCIATION

The Chi-Square test for independence, also called Pearson's Chi-square test or the Chi-square test of association is used to discover if there is a relationship between two categorical or nominal variables.

Steps to follow in dealing with Chi-Square test

1. State the Hypothesis:
2. Set the level of significance (α).
3. Compute the value of the statistical test.
4. Compare the value of the statistical test to the tabular value.
5. Interpret and discuss the result.

Example

Does college academic grade depend on the high school NCAE results for the following 200 students? Use 0.01 level of significance

Academic Grade	**NCAE Results**		
	Low	Average	High
Above 85	13	25	21
75-85	18	31	38
Below 75	14	20	20

Solution

Step 1: Formulate your Hypothesis

Ho: College academic grade does not depend on the high school NCAE results.
Ha: The College academic grade depends on the high school NCAE results.

Step 2: Set the level of significance

$\propto: 0.01$

Step 3: Compute the value of the statistical test

Calculate x^2 using the formula below

$$\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e}$$

where

f_0 = *observed frequency*

f_e = *expected frequency*

$$f_e = \frac{(column\_total)(row\_total)}{Grand\_Total}$$

a. The first step to do is to obtain the marginal totals of the rows and of the columns as indicated below.

Academic Grade	NCAE Results			**Total**
	Low	Average	High	
Above 85	13	25	21	**59**
75-85	18	31	38	**87**
Below 75	14	20	20	**54**
Total	**45**	**76**	**79**	**200**

b. Compute the expected frequency by multiplying the column total and row total divided by the grand total

$$fe_1 = \frac{45*59}{200} = 13.28 \qquad fe_4 = \frac{76*59}{200} = 22.42$$

$$fe_2 = \frac{45*87}{200} = 19.58 \qquad fe_5 = \frac{76*87}{200} = 33.06$$

$$fe_3 = \frac{45*54}{200} = 12.15 \qquad fe_6 = \frac{76*54}{200} = 20.52$$

$$fe_7 = \frac{79*59}{200} = 23.31$$

$$fe_8 = \frac{79*87}{200} = 34.37$$

$$fe_9 = \frac{79*54}{200} = 21.33$$

c. Calculation of the Chi-square

Observed Frequency (fo)	Expected Frequency (fe)	$(fo - fe)$	$(fo - fe)^2$	$\frac{(fo - fe)^2}{fe}$
13	13.28	-0.28	0.08	0.01
18	19.58	-1.58	2.50	0.13
14	12.15	1.85	3.42	0.28
25	22.42	2.58	6.66	0.30
31	33.06	-2.06	4.24	0.13
20	20.52	-0.52	0.27	0.01
21	23.31	-2.31	5.34	0.23
38	34.37	3.63	13.18	0.38
20	21.33	-1.33	1.77	0.08
				$x^2 = 1.55$

Step 4: Compare the value of the statistical test to the tabular value.
Statistical value = 1.55

Tabular value

$$df = (c-1)(r-1)$$

where

c = no. of columns
r = no. of rows

$$df = (3-1)(3-1)$$

$df = 4 \rightarrow$ locate (refer to Appendix D)

$x^2_T = 13.28 \rightarrow$ tabular value

Step 5: Interpret and discuss the result

Since the statistical value for chi-square is 1.55 is less than 13.28 which is the tabular value at 0.01 level of probability using 4 degrees of freedom. Therefore, we do not reject the null hypothesis. Hence, college academic grade does not depend on the high school NCAE results.

EXERCISES

Name:_____ Score: _____

Course & Year: _____ Date: _____

I. Below are chi-square (x^2) computed values and tabular values. Write if is significant or insignificant at the right column blank.

	Computed (x^2)	Tabular (x^2)	Decision
1.	3.73	9.21	_____
2.	6.29	5.41	_____
3.	18.57	23.21	_____
4.	15.08	11.07	_____
5.	1.78	4.64	_____
6.	0.83	2.71	_____
7.	14.32	10.83	_____
8.	11.24	15.99	_____
9.	23.34	23.34	_____
10.	28.75	30.96	_____

II. Solve each of the following problems completely and follow the steps in hypothesis testing.

1. A public opinion poll surveyed a simple random sample of 1000 voters. Respondents were classified by gender (male or female) and by voting preference (Republican, Democrat, or Independent). Results are shown in the contingency table below.

Gender	**Voting Preferences**		
	Republican	Democrat	Independent
Male	200	150	50
Female	250	300	50

2. Each respondent in the Current Survey of Status of Employment was classified as employed, unemployed, or outside the labor force. The employment status of the men in a certain city with the age bracket of 35-44 was cross-tabulated with their marital status, as follows:

Status of Employment	**Marital Status**		
	Married	Separated	Single
Employed	379	52	98
Unemployed	32	8	20
Not in labor force	21	13	27

Determine if men of different marital status are related to different distributions of employment status.

3. Administrators in a large urban district take a random sample of 50 seventh graders and compare the pre-algebra achievement levels of those who attended pre-school and those who did not. If achievement is independent of attending pre-school then the proportions at each level should be equal. Use the table below to determine if there is an association between attending pre-school and pre-algebra achievement.

Attending Pre-school	Pre –algebra achievement		
	Below grade level	At grade level	Advanced
Pre –school	8	6	6
No Pre -school	6	15	9

4. A group of students were classified in terms of personality and in terms of color preference with the purpose of seeing whether there is an association between personality and color preference. Four hundred (400) students was collected and presented the data below:

Personality	Color Preference			
	Red	Yellow	Green	Blue
Introvert personality	20	6	30	44
Extrovert personality	180	34	50	36

5. Students in grades 4-6 were asked to choose whether good grades, athletic ability, or popularity was their most important factor. A two-way table separating the students by grade and by choice of most important factor is shown below:

Most Important Factor	Grade level		
	Grade 4	Grade 5	Grade 6
Good grade	49	50	69
Athletic ability	19	22	28
Popularity	24	36	38

APPENDIX C: Critical F- Value (ANOVA)

APPENDIX C: Tables of critical F values for α=0.05

ndf / ddf	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	161.469	199.493	215.737	224.500	230.066	234.001	236.772	238.949	240.496	241.838	242.968	243.880	244.798	245.260	245.956	246.422	246.890	247.360	247.596	248.068
2	18.5128	18.9995	19.1642	19.2467	19.2969	19.3299	19.3536	19.3710	19.3852	19.3963	19.4043	19.4122	19.4186	19.4250	19.4297	19.4329	19.4377	19.4409	19.4425	19.4457
3	10.1278	9.5522	9.2767	9.1173	9.0133	8.9408	8.8868	8.8452	8.8124	8.7857	8.7635	8.7446	8.7287	8.7150	8.7028	8.6923	8.6830	8.6745	8.6670	8.6602
4	7.7087	6.9444	6.5915	6.3882	6.2561	6.1631	6.0943	6.0411	5.9988	5.9644	5.9359	5.9117	5.8912	5.8733	5.8578	5.8440	5.8319	5.8211	5.8113	5.8025
5	6.6080	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8184	4.7725	4.7350	4.7039	4.6777	4.6552	4.6358	4.6187	4.6038	4.5904	4.5785	4.5679	4.5581
6	5.9874	5.1433	4.7570	4.5337	4.3874	4.2838	4.2067	4.1468	4.0990	4.0600	4.0275	3.9999	3.9764	3.9560	3.9381	3.9223	3.9083	3.8957	3.8844	3.8742
7	5.5914	4.7374	4.3469	4.1204	3.9715	3.8660	3.7870	3.7257	3.6767	3.6366	3.6030	3.5747	3.5504	3.5292	3.5107	3.4944	3.4799	3.4669	3.4552	3.4445
8	5.3177	4.4590	4.0662	3.8378	3.6875	3.5806	3.5004	3.4381	3.3881	3.3472	3.3130	3.2839	3.2590	3.2374	3.2184	3.2017	3.1867	3.1733	3.1613	3.1503
9	5.1174	4.2565	3.8626	3.6331	3.4817	3.3738	3.2928	3.2296	3.1789	3.1373	3.1025	3.0729	3.0475	3.0255	3.0061	2.9890	2.9737	2.9600	2.9476	2.9365
10	4.9647	4.1028	3.7083	3.4781	3.3258	3.2171	3.1355	3.0717	3.0204	2.9782	2.9429	2.9130	2.8872	2.8648	2.8450	2.8276	2.8120	2.7981	2.7855	2.7740
11	4.8443	3.9823	3.5875	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	2.8536	2.8179	2.7876	2.7614	2.7386	2.7186	2.7009	2.6851	2.6709	2.6581	2.6464
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.7173	2.6866	2.6602	2.6371	2.6169	2.5989	2.5828	2.5684	2.5554	2.5436
13	4.6672	3.8055	3.4106	3.1791	3.0255	2.9153	2.8321	2.7669	2.7144	2.6711	2.6347	2.6037	2.5769	2.5536	2.5331	2.5149	2.4987	2.4841	2.4709	2.4589
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6021	2.5655	2.5343	2.5073	2.4837	2.4630	2.4446	2.4282	2.4134	2.4000	2.3879
15	4.5430	3.6824	3.2874	3.0555	2.9013	2.7905	2.7066	2.6408	2.5877	2.5437	2.5068	2.4753	2.4481	2.4244	2.4034	2.3849	2.3683	2.3533	2.3398	2.3275
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4564	2.4247	2.3973	2.3733	2.3522	2.3335	2.3167	2.3016	2.2880	2.2756
17	4.4513	3.5916	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943	2.4499	2.4126	2.3807	2.3531	2.3290	2.3077	2.2888	2.2719	2.2567	2.2429	2.2303
18	4.4139	3.5546	3.1599	2.9278	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3742	2.3421	2.3143	2.2900	2.2686	2.2496	2.2325	2.2172	2.2033	2.1906
19	4.3808	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3780	2.3402	2.3080	2.2800	2.2556	2.2341	2.2149	2.1977	2.1823	2.1683	2.1555
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928	2.3479	2.3100	2.2776	2.2495	2.2249	2.2033	2.1840	2.1667	2.1511	2.1370	2.1242

ddf \ ndf	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3661	2.3209	2.2829	2.2504	2.2222	2.1975	2.1757	2.1563	2.1389	2.1232	2.1090	2.0960
22	4.3009	3.4434	3.0492	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2585	2.2258	2.1975	2.1727	2.1508	2.1313	2.1138	2.0980	2.0837	2.0707
23	4.2794	3.4221	3.0280	2.7955	2.6400	2.5276	2.4422	2.3748	2.3201	2.2747	2.2364	2.2036	2.1752	2.1503	2.1282	2.1086	2.0910	2.0751	2.0608	2.0476
24	4.2597	3.4029	3.0088	2.7763	2.6206	2.5082	2.4226	2.3551	2.3003	2.2547	2.2163	2.1834	2.1548	2.1298	2.1077	2.0880	2.0703	2.0543	2.0399	2.0267
25	4.2417	3.3852	2.9913	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821	2.2365	2.1979	2.1649	2.1362	2.1111	2.0889	2.0691	2.0513	2.0353	2.0207	2.0075
30	4.1709	3.3158	2.9223	2.6896	2.5335	2.4205	2.3343	2.2662	2.2107	2.1646	2.1255	2.0921	2.0629	2.0374	2.0148	1.9946	1.9765	1.9601	1.9452	1.9317
35	4.1214	3.2674	2.8742	2.6415	2.4851	2.3718	2.2852	2.2167	2.1608	2.1143	2.0749	2.0411	2.0117	1.9858	1.9629	1.9424	1.9240	1.9073	1.8922	1.8784
40	4.0848	3.2317	2.8388	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240	2.0773	2.0376	2.0035	1.9738	1.9476	1.9245	1.9038	1.8851	1.8682	1.8529	1.8389
45	4.0566	3.2043	2.8115	2.5787	2.4221	2.3083	2.2212	2.1521	2.0958	2.0487	2.0088	1.9745	1.9446	1.9182	1.8949	1.8740	1.8551	1.8381	1.8226	1.8084
50	4.0343	3.1826	2.7900	2.5572	2.4004	2.2864	2.1992	2.1299	2.0734	2.0261	1.9861	1.9515	1.9214	1.8949	1.8714	1.8503	1.8313	1.8141	1.7985	1.7841
55	4.0162	3.1650	2.7725	2.5397	2.3828	2.2687	2.1813	2.1119	2.0552	2.0078	1.9676	1.9329	1.9026	1.8760	1.8523	1.8311	1.8120	1.7946	1.7788	1.7644
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2540	2.1665	2.0970	2.0401	1.9926	1.9522	1.9174	1.8870	1.8603	1.8364	1.8151	1.7959	1.7784	1.7625	1.7480
65	3.9885	3.1381	2.7459	2.5130	2.3560	2.2417	2.1541	2.0844	2.0274	1.9798	1.9393	1.9044	1.8739	1.8470	1.8231	1.8017	1.7823	1.7648	1.7488	1.7342
70	3.9778	3.1277	2.7355	2.5027	2.3456	2.2312	2.1435	2.0737	2.0166	1.9689	1.9283	1.8932	1.8627	1.8357	1.8117	1.7902	1.7707	1.7531	1.7371	1.7223
75	3.9685	3.1186	2.7266	2.4937	2.3366	2.2221	2.1343	2.0645	2.0073	1.9595	1.9188	1.8836	1.8530	1.8259	1.8018	1.7802	1.7607	1.7431	1.7269	1.7121
80	3.9604	3.1107	2.7188	2.4859	2.3287	2.2142	2.1263	2.0564	1.9991	1.9512	1.9105	1.8753	1.8445	1.8174	1.7932	1.7716	1.7520	1.7342	1.7180	1.7032
85	3.9532	3.1039	2.7119	2.4790	2.3218	2.2072	2.1193	2.0493	1.9919	1.9440	1.9031	1.8679	1.8371	1.8099	1.7856	1.7639	1.7443	1.7265	1.7102	1.6953
90	3.9469	3.0977	2.7058	2.4729	2.3157	2.2011	2.1131	2.0430	1.9856	1.9376	1.8967	1.8613	1.8305	1.8032	1.7789	1.7571	1.7375	1.7196	1.7033	1.6883
95	3.9412	3.0922	2.7004	2.4675	2.3102	2.1955	2.1075	2.0374	1.9799	1.9318	1.8909	1.8555	1.8246	1.7973	1.7729	1.7511	1.7314	1.7134	1.6971	1.6820
100	3.9361	3.0873	2.6955	2.4626	2.3053	2.1906	2.1025	2.0323	1.9748	1.9267	1.8857	1.8502	1.8193	1.7919	1.7675	1.7456	1.7259	1.7079	1.6915	1.6764

Tables of critical F values for α=0.01

ddf \ ndf	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	4063.25	4992.22	5404.03	5636.51	5760.41	5889.88	5889.88	6025.30	6025.30	6025.30	6025.30	6167.09	6167.09	6167.09	6167.09	6167.09	6167.09	6167.09	6167.09	6167.09
2	98.5043	98.9977	99.1505	99.2654	99.3038	99.3422	99.3422	99.3806	99.3806	99.3806	99.4191	99.4191	99.4191	99.4191	99.4191	99.4191	99.4576	99.4576	99.4576	99.4576
3	34.1141	30.8155	29.4553	28.7098	28.2359	27.9103	27.6731	27.4893	27.3445	27.2285	27.1316	27.0533	26.9844	26.9248	26.8714	26.8270	26.7857	26.7504	26.7181	26.6888
4	21.1977	17.9994	16.6939	15.9766	15.5219	15.2072	14.9761	14.7990	14.6593	14.5460	14.4525	14.3737	14.3063	14.2485	14.1981	14.1542	14.1148	14.0792	14.0480	14.0196
5	16.2582	13.2737	12.0599	11.3919	10.9670	10.6724	10.4556	10.2893	10.1577	10.0509	9.9626	9.8884	9.8250	9.7699	9.7223	9.6804	9.6431	9.6095	9.5795	9.5527
6	13.7450	10.9246	9.7796	9.1482	8.7460	8.4663	8.2599	8.1018	7.9762	7.8741	7.7896	7.7182	7.6575	7.6049	7.5591	7.5185	7.4827	7.4507	7.4219	7.3957
7	12.2466	9.5467	8.4512	7.8465	7.4605	7.1914	6.9928	6.8401	6.7189	6.6201	6.5382	6.4690	6.4101	6.3589	6.3144	6.2750	6.2401	6.2088	6.1808	6.1555
8	11.2586	8.6492	7.5911	7.0060	6.6319	6.3707	6.1776	6.0288	5.9106	5.8143	5.7343	5.6667	5.6089	5.5589	5.5151	5.4766	5.4422	5.4117	5.3840	5.3591
9	10.5612	8.0216	6.9918	6.4221	6.0570	5.8018	5.6129	5.4672	5.3511	5.2565	5.1779	5.1114	5.0545	5.0053	4.9621	4.9240	4.8902	4.8599	4.8326	4.8079
10	10.0444	7.5594	6.5523	5.9943	5.6363	5.3857	5.2001	5.0567	4.9424	4.8491	4.7715	4.7059	4.6496	4.6008	4.5582	4.5205	4.4869	4.4570	4.4298	4.4054
11	9.6461	7.2057	6.2167	5.6684	5.3160	5.0692	4.8861	4.7444	4.6315	4.5393	4.4624	4.3974	4.3417	4.2932	4.2509	4.2134	4.1802	4.1503	4.1234	4.0990
12	9.3302	6.9266	5.9526	5.4120	5.0643	4.8206	4.6395	4.4993	4.3875	4.2961	4.2199	4.1553	4.0998	4.0518	4.0096	3.9723	3.9392	3.9095	3.8827	3.8584
13	9.0737	6.7009	5.7394	5.2054	4.8616	4.6204	4.4410	4.3021	4.1911	4.1002	4.0246	3.9603	3.9052	3.8574	3.8153	3.7783	3.7452	3.7155	3.6888	3.6646
14	8.8615	6.5148	5.5639	5.0354	4.6950	4.4558	4.2779	4.1399	4.0297	3.9394	3.8640	3.8002	3.7453	3.6975	3.6557	3.6187	3.5857	3.5561	3.5295	3.5052
15	8.6830	6.3589	5.4170	4.8932	4.5556	4.3182	4.1415	4.0044	3.8948	3.8049	3.7299	3.6663	3.6115	3.5639	3.5222	3.4853	3.4523	3.4228	3.3961	3.3719
16	8.5310	6.2263	5.2922	4.7726	4.4374	4.2016	4.0259	3.8896	3.7804	3.6909	3.6162	3.5527	3.4981	3.4506	3.4089	3.3720	3.3392	3.3096	3.2829	3.2587
17	8.3997	6.1122	5.1851	4.6690	4.3360	4.1015	3.9267	3.7910	3.6823	3.5931	3.5185	3.4552	3.4007	3.3534	3.3117	3.2748	3.2419	3.2124	3.1857	3.1615
18	8.2855	6.0129	5.0919	4.5790	4.2479	4.0147	3.8407	3.7054	3.5970	3.5082	3.4338	3.3706	3.3162	3.2689	3.2273	3.1905	3.1576	3.1280	3.1014	3.0771
19	8.1850	5.9260	5.0103	4.5002	4.1708	3.9386	3.7653	3.6305	3.5225	3.4338	3.3596	3.2966	3.2422	3.1949	3.1533	3.1165	3.0836	3.0540	3.0274	3.0031
20	8.0961	5.8489	4.9382	4.4307	4.1027	3.8714	3.6987	3.5644	3.4567	3.3682	3.2941	3.2311	3.1768	3.1296	3.0881	3.0512	3.0183	2.9887	2.9620	2.9377

ddf \ ndf	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	8.0166	5.7805	4.8740	4.3688	4.0422	3.8117	3.6396	3.5056	3.3981	3.3098	3.2359	3.1730	3.1187	3.0715	3.0300	2.9931	2.9602	2.9306	2.9039	2.8796
22	7.9455	5.7190	4.8166	4.3134	3.9879	3.7583	3.5867	3.4530	3.3458	3.2576	3.1838	3.1209	3.0667	3.0195	2.9779	2.9411	2.9081	2.8785	2.8518	2.8275
23	7.8813	5.6636	4.7649	4.2636	3.9392	3.7102	3.5390	3.4057	3.2986	3.2106	3.1368	3.0740	3.0199	2.9727	2.9311	2.8943	2.8613	2.8316	2.8049	2.7805
24	7.8230	5.6135	4.7181	4.2184	3.8951	3.6667	3.4960	3.3628	3.2560	3.1681	3.0944	3.0316	2.9775	2.9303	2.8887	2.8519	2.8189	2.7892	2.7624	2.7380
25	7.7699	5.5680	4.6755	4.1774	3.8549	3.6271	3.4568	3.3239	3.2172	3.1294	3.0558	2.9931	2.9390	2.8918	2.8502	2.8133	2.7803	2.7506	2.7238	2.6993
30	7.5625	5.3904	4.5097	4.0178	3.6991	3.4735	3.3045	3.1726	3.0665	2.9791	2.9057	2.8431	2.7890	2.7418	2.7002	2.6632	2.6301	2.6003	2.5733	2.5486
35	7.4192	5.2680	4.3957	3.9083	3.5919	3.3679	3.2000	3.0687	2.9630	2.8758	2.8026	2.7400	2.6860	2.6387	2.5970	2.5599	2.5266	2.4967	2.4696	2.4448
40	7.3140	5.1785	4.3125	3.8283	3.5138	3.2910	3.1237	2.9930	2.8876	2.8005	2.7274	2.6648	2.6107	2.5634	2.5216	2.4844	2.4511	2.4210	2.3937	2.3689
45	7.2338	5.1104	4.2492	3.7674	3.4544	3.2325	3.0658	2.9354	2.8301	2.7432	2.6701	2.6075	2.5534	2.5060	2.4642	2.4269	2.3935	2.3633	2.3359	2.3109
50	7.1707	5.0566	4.1994	3.7195	3.4077	3.1864	3.0202	2.8900	2.7850	2.6981	2.6250	2.5625	2.5083	2.4609	2.4190	2.3816	2.3481	2.3178	2.2903	2.2652
55	7.1193	5.0132	4.1590	3.6809	3.3700	3.1493	2.9834	2.8534	2.7485	2.6618	2.5886	2.5261	2.4719	2.4244	2.3824	2.3450	2.3114	2.2810	2.2535	2.2283
60	7.0770	4.9774	4.1259	3.6491	3.3389	3.1187	2.9530	2.8233	2.7184	2.6317	2.5587	2.4961	2.4419	2.3944	2.3523	2.3148	2.2811	2.2507	2.2230	2.1978
65	7.0416	4.9475	4.0980	3.6223	3.3128	3.0930	2.9276	2.7980	2.6933	2.6066	2.5335	2.4710	2.4167	2.3691	2.3270	2.2895	2.2557	2.2252	2.1975	2.1722
70	7.0114	4.9218	4.0744	3.5996	3.2907	3.0712	2.9060	2.7766	2.6719	2.5852	2.5121	2.4496	2.3953	2.3477	2.3055	2.2679	2.2341	2.2036	2.1758	2.1504
75	6.9853	4.8998	4.0540	3.5801	3.2716	3.0525	2.8874	2.7580	2.6534	2.5668	2.4937	2.4311	2.3768	2.3292	2.2870	2.2493	2.2155	2.1849	2.1570	2.1316
80	6.9627	4.8808	4.0363	3.5631	3.2551	3.0361	2.8713	2.7419	2.6374	2.5508	2.4777	2.4151	2.3608	2.3131	2.2709	2.2332	2.1993	2.1687	2.1407	2.1153
65	7.0416	4.9475	4.0980	3.6223	3.3128	3.0930	2.9276	2.7980	2.6933	2.6066	2.5335	2.4710	2.4167	2.3691	2.3270	2.2895	2.2557	2.2252	2.1975	2.1722
85	6.9427	4.8639	4.0207	3.5482	3.2405	3.0218	2.8571	2.7278	2.6234	2.5368	2.4637	2.4011	2.3467	2.2990	2.2567	2.2190	2.1851	2.1544	2.1264	2.1009
90	6.9251	4.8490	4.0070	3.5350	3.2276	3.0091	2.8445	2.7154	2.6109	2.5243	2.4512	2.3886	2.3343	2.2865	2.2442	2.2064	2.1724	2.1417	2.1137	2.0882
95	6.9094	4.8359	3.9947	3.5232	3.2161	2.9978	2.8333	2.7042	2.5998	2.5133	2.4402	2.3775	2.3231	2.2753	2.2330	2.1952	2.1612	2.1304	2.1024	2.0768
100	6.8953	4.8239	3.9837	3.5127	3.2058	2.9877	2.8233	2.6943	2.5899	2.5033	2.4302	2.3676	2.3131	2.2654	2.2230	2.1852	2.1511	2.1203	2.0923	2.0666

APPENDIX D: Critical Values of Chi-Square

df	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.002	0.001
1	1.642	2.706	3.841	5.024	5.412	6.635	7.879	9.550	10.828
2	3.219	4.605	5.991	7.378	7.824	9.210	10.597	12.429	13.816
3	4.642	6.251	7.815	9.348	9.837	11.345	12.838	14.796	16.266
4	5.989	7.779	9.488	11.143	11.668	13.277	14.860	16.924	18.467
5	7.289	9.236	11.070	12.833	13.388	15.086	16.750	18.907	20.515
6	8.558	10.645	12.592	14.449	15.033	16.812	18.548	20.791	22.458
7	9.803	12.017	14.067	16.013	16.622	18.475	20.278	22.601	24.322
8	11.030	13.362	15.507	17.535	18.168	20.090	21.955	24.352	26.124
9	12.242	14.684	16.919	19.023	19.679	21.666	23.589	26.056	27.877
10	13.442	15.987	18.307	20.483	21.161	23.209	25.188	27.722	29.588
11	14.631	17.275	19.675	21.920	22.618	24.725	26.757	29.354	31.264
12	15.812	18.549	21.026	23.337	24.054	26.217	28.300	30.957	32.909
13	16.985	19.812	22.362	24.736	25.472	27.688	29.819	32.535	34.528
14	18.151	21.064	23.685	26.119	26.873	29.141	31.319	34.091	36.123
15	19.311	22.307	24.996	27.488	28.259	30.578	32.801	35.628	37.697
16	20.465	23.542	26.296	28.845	29.633	32.000	34.267	37.146	39.252
17	21.615	24.769	27.587	30.191	30.995	33.409	35.718	38.648	40.790
18	22.760	25.989	28.869	31.526	32.346	34.805	37.156	40.136	42.312
19	23.900	27.204	30.144	32.852	33.687	36.191	38.582	41.610	43.820
20	25.038	28.412	31.410	34.170	35.020	37.566	39.997	43.072	45.315
21	26.171	29.615	32.671	35.479	36.343	38.932	41.401	44.522	46.797
22	27.301	30.813	33.924	36.781	37.659	40.289	42.796	45.962	48.268
23	28.429	32.007	35.172	38.076	38.968	41.638	44.181	47.391	49.728
24	29.553	33.196	36.415	39.364	40.270	42.980	45.559	48.812	51.179
25	30.675	34.382	37.652	40.646	41.566	44.314	46.928	50.223	52.620
26	31.795	35.563	38.885	41.923	42.856	45.642	48.290	51.627	54.052
27	32.912	36.741	40.113	43.195	44.140	46.963	49.645	53.023	55.476
28	34.027	37.916	41.337	44.461	45.419	48.278	50.993	54.411	56.892
29	35.139	39.087	42.557	45.722	46.693	49.588	52.336	55.792	58.301
35	41.778	46.059	49.802	53.203	54.244	57.342	60.275	63.955	66.619
40	47.269	51.805	55.758	59.342	60.436	63.691	66.766	70.618	73.402
45	52.729	57.505	61.656	65.410	66.555	69.957	73.166	77.179	80.077
50	58.164	63.167	67.505	71.420	72.613	76.154	79.490	83.657	86.661
55	63.577	68.796	73.311	77.380	78.619	82.292	85.749	90.061	93.168
60	68.972	74.397	79.082	83.298	84.580	88.379	91.952	96.404	99.607
65	74.351	79.973	84.821	89.177	90.501	94.422	98.105	102.691	105.988
70	79.715	85.527	90.531	95.023	96.388	100.425	104.215	108.929	112.317
75	85.066	91.061	96.217	100.839	102.243	106.393	110.286	115.125	118.599
80	90.405	96.578	101.879	106.629	108.069	112.329	116.321	121.280	124.839
85	95.734	102.079	107.522	112.393	113.871	118.236	122.325	127.401	131.041
90	101.054	107.565	113.145	118.136	119.648	124.116	128.299	133.489	137.208
95	106.364	113.038	118.752	123.858	125.405	129.973	134.247	139.546	143.344
100	111.667	118.498	124.342	129.561	131.142	135.807	140.169	145.577	149.449

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