Worksheet 02

Solve the homogenous D·E's

(1) $(y^2 - xy)\,dx + x^2\,dy = 0$ Ans: $x/y = \ln(x) + c$

step 1: Identification

Implicit function: Reducible to variable seperable, but degree of all terms is identical = homogeneous function

Step 2: Rewrite & Substitute

$(y^2 - xy)\,dx = -x^2\,dy$ → $y = u \cdot x$

$\frac{dy}{dx} = x\frac{du}{dx} + u$

$\frac{xy - y^2}{x^2} = \frac{dy}{dx}$

$\frac{x(u \cdot x) - (u \cdot x)^2}{x^2} = \frac{du}{dx}x + u$

$\frac{ux^2 - u^2x^2}{x^2} - \frac{u}{1} = x\frac{du}{dx} \Rightarrow \cancel{u} - u^2 - \cancel{u} = x\frac{du}{dx}$

$-u^2 = x\frac{du}{dx} \Rightarrow \boxed{\int \frac{dx}{x} = -\int \frac{du}{u^2}}$

$\int \frac{1}{x}\,dx = -\int \frac{1}{u^2}\,du \Rightarrow \ln(x) = -\frac{u^{-2+1}}{-2+1} + c$

$\ln(x) = +u^{-1} + c \Rightarrow \ln(x) = \frac{1}{u} + c$, $u = y/x$

$$\boxed{\ln(x) = \frac{x}{y} + c}$$

② $(x^2-y^2)\,dx + 2xy\,dy = 0$ Ans: $x^2+y^2 = ax$

step 1: Identification

Implicit function + degree of each term identical → Homogeneous

step 2: Rewrite & Substitute

$(x^2-y^2)\,dx = -2xy\,dy$; $y = u\cdot x$

$$\frac{y^2-x^2}{2xy} = \frac{dy}{dx} \qquad \frac{dy}{dx} = x\frac{du}{dx} + u$$

step 3: Replace & integrate

$$\frac{(u\cdot x)^2 - x^2}{2x(ux)} = x\frac{du}{dx} + u$$

$$\frac{u^2x^2-x^2}{2x^2u} = x\frac{du}{dx} + u$$

$$\frac{\cancel{x^2}(u^2-1)}{\cancel{x^2}(2u)} - \frac{u}{1} = x\frac{du}{dx}$$

$$\frac{u^2-1-2u^2}{2u} = x\frac{du}{dx}$$

$$\frac{-u^2-1}{2u} = x\frac{du}{dx}$$

$$\int\frac{dx}{x} = \int du \div \left(\frac{-u^2-1}{2u}\right)$$

$$\ln(x)+c = -\int\frac{2u}{u^2+1}\,du$$

$\int\frac{2u}{u^2+1}\,du$, let $a = u^2+1$, $da = 2u\,du$

$$\int\frac{da}{a} = \ln(a) = \ln(u^2+1)$$

$$\ln(u^2+1) = \ln\left(\left(\frac{y}{x}\right)^2+1\right)$$

$$\int\frac{dx}{x} = -\int\frac{2u}{u^2+1}\,du$$

$$\ln(x) = -\ln\left(\frac{y^2}{x^2}+1\right)+c$$

$$\ln|x| + \ln\left(\frac{y^2}{x^2}+1\right) = c$$

$$\ln(x) + \ln\left(\frac{y^2+x^2}{x^2}\right) = c \Rightarrow (\cancel{x})\left(\frac{y^2+x^2}{x^{\cancel{2}}}\right) = c$$

$$\frac{y^2+x^2}{x} = c \Rightarrow \boxed{y^2+x^2 = cx}$$

(3) $x(y-x)\frac{dy}{dx} = y(y+x)$ Ans: $y/x - \ln xy = c$

Step 1: Identification : Implicit + homogeneous

Step 2: Rewrite & Substitute

$z = y+x$ $y = u \cdot x$

$$\frac{dy}{dx} = \frac{y(y+x)}{x(y-x)}$$

$$\frac{dy}{dx} = x\frac{du}{dx} + u$$

$$x\frac{du}{dx} + u = \frac{y^2 + xy}{xy - x^2}$$

Step 3: Replace & Integrate

$$x\frac{du}{dx} + u = \frac{(u\cdot x)^2 + x(u\cdot x)}{x(u\cdot x) - x^2}$$

$$x\frac{du}{dx} + u = \frac{u^2x^2 + ux^2}{ux^2 - x^2}$$

$$x\frac{du}{dx} + u = \frac{x^2(u^2+u)}{x^2(u-1)}$$

$$x\frac{du}{dx} + u = \frac{u^2+u}{u-1}$$

$$x\frac{du}{dx} = \frac{u^2+u}{u-1} - \frac{u}{1}$$

$$x\frac{du}{dx} = \frac{u^2+u-u(u-1)}{u-1}$$

$$x\frac{du}{dx} = \frac{u^2+u-u^2+u}{u-1}$$

$$\boxed{x\frac{du}{dx} = \frac{2u}{u-1}}$$

$$x\frac{du}{dx} = \frac{2u}{u-1}$$

$$du \div \left(\frac{2u}{u-1}\right) = \frac{1}{x}\,dx$$

$$\int \frac{u-1}{2u}\,du = \int \frac{1}{x}\,dx$$

$$\int \frac{u\,du}{2u} - \int \frac{du}{2u} = \int \frac{1}{x}\,dx$$

$$\frac{1}{2}\int du - \frac{1}{2}\int \frac{du}{u} = \int \frac{1}{x}\,dx$$

$$\frac{1}{2}(u) - \frac{1}{2}\ln(u) = \ln(x) + c$$

$$\frac{1}{2}(y/x) - \frac{1}{2}\ln(y/x) = \ln(x) + c$$

$$\frac{1}{2}(y/x) = \ln(x) + \frac{1}{2}\ln(y/x) + c$$

$$\frac{u}{2} = \ln(x) + \ln(u)^{1/2} + c$$

$$\frac{u}{2} = \ln(xu^{1/2}) + c$$

$$u = 2\ln(xu^{1/2}) + 2c \rightarrow c$$

$$u = \ln(xu^{1/2})^2 + c$$

$$u = \ln(x u^{1/2})^2 + c \Rightarrow u = \ln(x^2 u) + c$$

$$u - \ln(x^2 u) = c \Rightarrow \frac{y}{x} - \ln(x^2 (y/x)) = c$$

$$\boxed{\frac{y}{x} - \ln(xy) = c}$$

(4) $x(x-y)\,dy + y^2\,dx = 0$ .. Ans: $y = xy \ln c$

<u>Step 1: Identification : Homogeneous + Implicit</u>

<u>Step 2: Rewrite & Substitute</u>

$$x(x-y)\,dy = -y^2\,dx \qquad y = u \cdot x$$

$$\frac{dy}{dx} = \frac{-y^2}{x^2 - xy} \qquad \frac{dy}{dx} = x\frac{du}{dx} + u$$

<u>Step 3: Replace & Integrate</u>

$$x\frac{du}{dx} + u = \frac{-(ux)^2}{x^2 - x(ux)}$$

$$x\frac{du}{dx} + u = \frac{-u^2 x^2}{x^2 - x^2 u}$$

$$x\frac{du}{dx} + u = \frac{-u^2}{1-u}$$

$$x\frac{du}{dx} = \frac{-u^2}{1-u} - \frac{u}{1}$$

$$x\frac{du}{dx} = \frac{-u^2 - u(1-u)}{1-u}$$

$$x\frac{du}{dx} = \frac{-u^2 - u + u^2}{1-u}$$

$$x\frac{du}{dx} = \frac{-u}{1-u}$$

$$\boxed{x\frac{du}{dx} = \frac{u}{u-1}}$$

$$\int x\frac{du}{dx} = \int \frac{u}{u-1}$$

$$\int \frac{u-1}{u}\,du = \int \frac{dx}{x}$$

$$\int \frac{u\,du}{u} \int \frac{du}{u} = \int \frac{dx}{x}$$

$$u - \ln(u) = \ln(x) + c$$

$$u = \ln(x) + \ln(u) + c$$

$$u = \ln(x) + \ln(y/x) + c$$

$$\frac{y}{x} = \ln\left(x \times \frac{y}{x}\right) + c$$

$$y = x\ln(y) + c$$

(5) $\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$

Step 1: Identification : Implicit + homogeneous

Step 2: Rewrite & Substitute

Step 3: Rewrite & integrate

$y = u \cdot x$

$\frac{dy}{dx} = x\frac{du}{dx} + u$

$\frac{dy}{dx} = \frac{2y-x}{2x-y}$

$x\frac{du}{dx} + u = \frac{2(u \cdot x) - x}{2x - ux}$

$x\frac{du}{dx} + u = \frac{2ux - x}{2x - ux}$

$x\frac{du}{dx} + u = \frac{x(2u-1)}{x(2-u)}$

$x\frac{du}{dx} + u = \frac{2u-1}{2-u}$

$x\frac{du}{dx} + u = \frac{2u-1}{2-u}$

$x\frac{du}{dx} = \frac{2u-1}{2-u} - u$

$x\frac{du}{dx} = \frac{2u-1-u(2-u)}{2-u}$

$x\frac{du}{dx} = \frac{\cancel{2u}-1-\cancel{2u}+u^2}{2-u}$

$x\frac{du}{dx} = \frac{u^2-1}{2-u}$

$\left[du \div \left(\frac{u^2-1}{2-u}\right)\right]\frac{x}{dx} = 1$

$\boxed{\frac{2-u}{u^2-1}du = \frac{dx}{x}}$

$\frac{2-u}{u^2-1}du = \frac{dx}{x}$

$\int \frac{2-u}{u^2-1}du = \int \frac{dx}{x}$

$\int \frac{2}{u^2-1}du - \int \frac{u\,du}{u^2-1} = \int \frac{dx}{x}$

$\boxed{\int \frac{2\,du}{(u-1)(u+1)} - \int \frac{u}{u^2-1}du = \int \frac{dx}{x}}$

$\Rightarrow \int \frac{2\,du}{(u-1)(u+1)}$

$\frac{2}{(u-1)(u+1)} = \frac{A}{u-1} + \frac{B}{u+1}$

$2 = A(u+1) + B(u-1)$

$2 = u(A+B) + A - B$

$A + B = 0$; $A - B = 2$

put $A = 2+B$ $\quad \boxed{A = 2+B}$

$2+B+B = 0$ $\quad$ put $B = -1$

$2 + 2B = 0$ $\quad A = 2 - 1$

$2B = -2$ $\quad \boxed{A = 1}$

$\boxed{B = -1}$

$\int \frac{2\,du}{(u-1)(u+1)} = \int \frac{du}{u-1} - \int \frac{du}{u+1}$

$$\int \frac{2\,dy}{(u-1)(u+1)} \xrightarrow{\text{partial}} - \int \frac{u\,dy}{u^2-1} \xrightarrow{\text{u-sub}} = \int \frac{dx}{x} \xrightarrow{\text{log}}$$

$a = u^2 - 1$

$da = 2u\,dy$

$\frac{da}{2} = u\,dy$

$$\int \frac{dy}{u-1} - \int \frac{dy}{u+1} - \int \frac{u\,dy}{u^2-1} = \int \frac{dx}{x}$$

$$\ln|u-1| - \ln|u+1| - \frac{1}{2}\int \frac{da}{a} = \ln|x| + \ln|c|$$

$$\ln|u-1| - \ln|u+1| - \frac{1}{2}\ln|a| = \ln|xc|$$

$$\ln\left|\frac{u-1}{u+1}\right| - \frac{1}{2}\ln|u^2-1| = \ln|xc|$$

$$\ln\left|\frac{u-1}{u+1}\right| - \ln|(u^2-1)^{1/2}| = \ln|xc|$$

$$\ln\left|\frac{u-1}{u+1}\right| - \ln|\sqrt{u^2-1}| = \ln|xc|$$

$$\ln\left|\left(\frac{u-1}{u+1}\right) \div \sqrt{u^2-1}\right| = \ln|xc|$$

$$\ln\left|\frac{u-1}{u+1} \times \frac{1}{\sqrt{u^2-1}}\right| = \ln|xc| \quad ; \; u = y/x$$

$$\ln\left|\frac{y/x - 1}{y/x + 1} \times \frac{1}{\sqrt{(y/x)^2 - 1}}\right| = \ln|xc|$$

$$\ln\left|\frac{y-x/x}{y+x/x} \times \frac{1}{\sqrt{y^2/x^2 - 1}}\right| = \ln|xc|$$

$$\ln\left|\left(\frac{y-x}{x} \div \frac{y+x}{x}\right) \times \frac{1}{\sqrt{\frac{y^2}{x^2} - 1}}\right| = \ln|xc|$$

$$\ln\left|\left(\frac{y-x}{\cancel{x}} \times \frac{\cancel{x}}{y+x}\right) \times \frac{1}{\sqrt{\frac{y^2-x^2}{x^2}}}\right| = \ln|xc|$$

$$\ln\left|\left(\frac{y-x}{y+x}\right) \times \left(1 \div \sqrt{\frac{y^2-x^2}{x^2}}\right)\right| = \ln|xc|$$

$$\ln\left|\left(\frac{y-x}{y+x}\right)\times\left(1\div\frac{\sqrt{y^2-x^2}}{\sqrt{x^2}}\right)\right| = \ln|xc|$$

$$\ln\left|\frac{y-x}{y+x}\times\left(1\times\frac{x}{\sqrt{y^2-x^2}}\right)\right| = \ln|xc|$$

$$\cancel{\ln}\left|\left(\frac{y-x}{y+x}\right)\times\left(\frac{x}{\sqrt{y^2-x^2}}\right)\right| = \cancel{\ln}|xc|$$

$$\frac{y-x}{y+x}\times\frac{x}{\sqrt{y^2-x^2}} = xc$$

$$\frac{y-x}{y+x} = xc\div\frac{x}{\sqrt{y^2-x^2}}$$

$$\frac{y-x}{y+x} = xc\times\frac{\sqrt{y^2-x^2}}{x}$$

$$\frac{y-x}{y+x} = c\sqrt{y^2-x^2}$$

$$\frac{y-x}{y+x} = c\sqrt{(y-x)(y+x)}$$

$$\frac{y-x}{y+x} = c\sqrt{y-x}\sqrt{y+x}$$

$$\frac{y-x}{\sqrt{y-x}} = c\sqrt{y+x}\,(y+x)$$

$$(y-x)^{1/2} = c\,(y+x)^{3/2}$$

$$(y-x)^{1/2} = c(y+x)^{3/2}$$

Squaring on both sides

$$(y-x)^{\frac{1}{2}\times 2} = c\,(y+x)^{\frac{3}{2}\times 2}$$

$$(y-x) = c\,(y+x)^3$$

$$\boxed{y-x = c\,(y+x)^3}$$

⑥ $\frac{dy}{dx} = \tan\left(\frac{y}{x}\right) + \frac{y}{x}$ Ans: $\sin\left(\frac{y}{x}\right) = cx$

Step1: identification : Implicit + homogeneous

Step2: Rewrite & Substitute

$y = u \cdot x$

$\frac{dy}{dx} = x\frac{du}{dx} + u$

$\frac{dy}{dx} = \tan\left(\frac{y}{x}\right) + \frac{y}{x}$

Step3: Replace & Integrate

$x\frac{du}{dx} + u = \tan(u) + u$

$x\frac{du}{dx} = \tan(u)$

$\tan^{-1}(u)\,du = \frac{dx}{x}$

$\cot(u)\,du = \frac{dx}{x}$

$\boxed{\frac{\cos(u)}{\sin(u)}\,du = \frac{dx}{x}}$

u-sub   log

$\int \frac{\cos(u)}{\sin(u)}\,du = \int \frac{dx}{x}$

$a = \sin(u)$

$da = \cos(u)\,du$

$\int \frac{da}{a} = \int \frac{dx}{x}$

$\ln|a| = \ln|x| + \ln|c|$

$\cancel{\ln}|\sin(u)| = \cancel{\ln}|xc|$

$\sin(u) = xc$, $\boxed{u = y/x}$

$\boxed{\sin\left(\frac{y}{x}\right) = xc}$

(7) $\frac{dy}{dx} = \frac{3xy+y^2}{3x^2}$ Ans: $3x + y\ln x + cy = 0$

Step 1: Identification: Implicit + homogeneous

Step 2: Rewrite & Substitute

$\frac{dy}{dx} = \frac{3xy+y^2}{3x^2}$

$y = u \cdot x$

$\frac{dy}{dx} = x\frac{du}{dx} + u$

Step 3: Replace & Integrate

$x\frac{du}{dx} + u = \frac{3x(ux)+(u \cdot x)^2}{3x^2}$

$x\frac{du}{dx} + u = \frac{3ux^2 + u^2x^2}{3x^2}$

$x\frac{du}{dx} + u = \frac{x^2(3u+u^2)}{x^2(3)}$

$x\frac{du}{dx} + u = \frac{3u+u^2}{3}$

$x\frac{du}{dx} = \frac{3u+u^2}{3} - \frac{u}{1}$

$x\frac{du}{dx} = \frac{3u+u^2-3u}{3}$

$x\frac{du}{dx} = \frac{u^2}{3}$

$3x\frac{du}{dx} = u^2$

$\frac{3x\,du}{u^2} = dx$

$$\boxed{\frac{du}{u^2} = \frac{dx}{3x}}$$

$\int \frac{du}{u^2} = \int \frac{dx}{3x}$

$\frac{u^{-2+1}}{-2+1} = \frac{1}{3}\ln|x| + c$

$-\frac{1}{u} = \frac{1}{3}\ln|x| + c$

$-\frac{1}{y/x} = \frac{1}{3}\ln|x| + c$

$-\frac{x}{y} = \frac{1}{3}\ln|x| + c$

$-x = \frac{y\ln|x|}{3} + cy$

multiply 3 on b/s

Constant

$3c = c$

$-3x = \frac{3y\ln|x|}{3} + 3cy$

$-3x = y\ln|x| + cy$

$$\boxed{3x + y\ln|x| + cy = 0}$$

(8) $\frac{dy}{dx} = \frac{x^2 - 2y^2}{2xy}$ Ans: $4y^2 - x^2 = c/x^2$

Step 1: Identification : Implicit + homogeneous

Step 2: Rewrite & Substitute

$$\frac{dy}{dx} = \frac{x^2-2y^2}{2xy}$$

$y = u \cdot x$

$\frac{dy}{dx} = x\frac{du}{dx} + u$

Step 3: Replace & Integrate

$$x\frac{du}{dx} + u = \frac{x^2 - 2(u \cdot x)^2}{2x(u \cdot x)}$$

$$x\frac{du}{dx} + u = \frac{x^2 - 2u^2x^2}{2x^2u}$$

$$x\frac{du}{dx} + u = \frac{x^2(1-2u^2)}{x^2(2u)}$$

$$x\frac{du}{dx} + u = \frac{1-2u^2}{2u}$$

$$x\frac{du}{dx} = \frac{1-2u^2-u}{2u}$$

$$x\frac{du}{dx} = \frac{1-2u^2-2u^2}{2u}$$

$$x\frac{du}{dx} = \frac{1-2u^2-2u^2}{2u}$$

$$\boxed{\frac{2u}{1-2u^2-2u^2}du = \frac{dx}{x}}$$

$$\frac{2u}{1-4u^2}du = \frac{dx}{x}$$

Note: Any power or sum, subtraction, multiplication division to C is always C because it results in Constant.

$$\int \frac{2u}{1-4u^2}du = \int \frac{dx}{x}$$

$a = 1-4u^2$; $da = -8u\,du$; $\frac{da}{-4} = 2u\,du$

$$-\frac{da}{4} = 2u\,du$$

$$-\frac{1}{4}\int \frac{da}{a} = \int \frac{dx}{x}$$

$$-\frac{1}{4}\ln|a| = \ln|x| + \ln|c|$$

$$-\frac{1}{4}\ln|1-4u^2| = \ln|x \cdot c|$$

$$\ln|1-4u^2| = -4\ln|x \cdot c|$$

$$\ln\left|1-\frac{4y^2}{x^2}\right| = \ln|(x \cdot c)^{-4}|$$

$$\cancel{\ln}\left|\frac{x^2-4y^2}{x^2}\right| = \cancel{\ln}\left|\frac{1}{x^4c^4}\right|$$

$\frac{x^2-4y^2}{x^2} = \frac{c}{x^4}$; $\boxed{\begin{array}{l} c^a = c' \\ 1/c = c \\ -c = c \end{array}}$

$$x^2 - 4y^2 = \frac{c}{x^2}$$

$$-(x^2-4y^2) = -\frac{c}{x^2}$$

$$\boxed{4y^2 - x^2 = \frac{c}{x^2}}$$

(8) $(x^2+y^2)\,dy = xy\,dx$ Ans: $\frac{-x^2}{2y^2} + \ln y = c$

Step 1: Identification: homogeneous + Implicit

Step 2: Rewrite & Substitute

$y = u \cdot x$

$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ $\frac{dy}{dx} = x\frac{du}{dx} + u$

Step 3: Replace & Integrate

$x\frac{du}{dx} + u = \frac{x(u \cdot x)}{x^2+(u \cdot x)^2}$

$x\frac{du}{dx} + u = \frac{ux^2}{x^2+u^2x^2}$

$x\frac{du}{dx} + u = \frac{x^2(u)}{x^2(1+u^2)}$

$x\frac{du}{dx} + u = \frac{u}{1+u^2}$

$x\frac{du}{dx} = \frac{u}{1+u^2} - u$

$x\frac{du}{dx} = \frac{u-u(1+u^2)}{1+u^2}$

$x\frac{du}{dx} = \frac{u-u-u^3}{1+u^2}$

$x\frac{du}{dx} = \frac{-u^3}{1+u^2}$

$\boxed{\frac{1+u^2}{-u^3}du = \frac{dx}{x}}$

$-\int \frac{1+u^2}{u^3}du = \int \frac{dx}{x}$

$-\int (1+u^2)u^{-3}du = \ln|x| + c$

$-\left[\int u^{-3}du + \int u^{-1}du\right] = \ln|x| + c$

$-\left[\frac{u^{-3+1}}{-3+1} + \ln|u|\right] = \ln|x| + c$

$-\left[-\frac{1}{2u^2} + \ln|u|\right] = \ln|x| + c$

$\frac{1}{2u^2} - \ln|u| = \ln|x| + c$

$\frac{1}{2\left(\frac{y}{x}\right)^2} = \ln|x| + \ln|u| + c$

$\frac{x^2}{2y^2} = \ln|x \cdot u| + c$

$\frac{x^2}{2y^2} = \ln\left|x \cdot \frac{y}{x}\right| + c$

$\frac{x^2}{2y^2} = \ln(y) + c$

$\frac{-x^2}{2y^2} + \ln(y) = -c$

$\boxed{\frac{-x^2}{2y^2} + \ln(y) = c}$

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(10) $\left[x\cos(y/x) + y\sin(y/x)\right]y - \left[y\sin(y/x) - x\cos(y/x)\right]x\dfrac{dy}{dx} = 0$

Ans: $xy \cos y/x = a$

Step 1: Identification : Homogenous + Implicit

Step 2: Rewrite & Substitute $\quad y = ux$

$$\left[x\cos(y/x) + y\sin(y/x)\right]y - \left[y\sin(y/x) - x\cos(y/x)\right]x\frac{dy}{dx} = 0$$

$$-\left[y\sin(y/x) - x\cos(y/x)\right]x\frac{dy}{dx} = -\left[x\cos(y/x) + y\sin(y/x)\right]y$$

$$\frac{dy}{dx} = \frac{\left[x\cos(y/x) + y\sin(y/x)\right]y}{\left[y\sin(y/x) - x\cos(y/x)\right]x}$$

Step 3: Replace & Integrate

$$x\frac{du}{dx} + u = \frac{\left[x\cos(u) + (ux)\sin(u)\right](u \cdot x)}{\left[(u \cdot x)\sin(u) - x\cos(u)\right]x}$$

$$x\frac{du}{dx} + u = \frac{ux^2\cos(u) + u^2x^2\sin(u)}{ux^2\sin(u) - x^2\cos(u)}$$

$$x\frac{du}{dx} + u = \frac{x^2\left(u\cos(u) + u^2\sin(u)\right)}{x^2\left(u\sin(u) - \cos(u)\right)}$$

$$x\frac{du}{dx} + u = \frac{u\cos(u) + u^2\sin(u)}{u\sin(u) - \cos(u)}$$

$$x\frac{du}{dx} = \frac{u\cos(u) + u^2\sin(u) - u\left(u\sin(u) - \cos(u)\right)}{u\sin(u) - \cos(u)}$$

$$x\frac{du}{dx} = \frac{u\cos(u) + \cancel{u^2\sin(u)} - \cancel{u^2\sin(u)} + u\cos(u)}{u\sin(u) - \cos(u)}$$

$$x\frac{du}{dx} = \frac{2u\cos(u)}{u\sin(u) - \cos(u)}$$

$$x\frac{du}{dx} = \frac{2u\cos(u)}{u\sin(u)-\cos(u)}$$

$$\int \frac{u\sin(u)-\cos(u)}{2u\cos(u)}du = \int\frac{dx}{x}$$

$$\int \frac{\cancel{u}\sin(u)\,du}{2\cancel{u}\cos(u)} - \int\frac{\cancel{\cos(u)}\,du}{2u\cancel{\cos(u)}} = \int\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{\sin(u)}{\cos(u)}du - \frac{1}{2}\int\frac{du}{u} = \int\frac{dx}{x}$$

$a = \cos(u)$, $-da = \sin(u)\,du$

$$\frac{1}{2}\int\frac{-da}{a} - \frac{1}{2}\int\frac{du}{u} = \int\frac{dx}{x}$$

$$-\frac{1}{2}\ln|a| - \frac{1}{2}\ln|u| = \ln|x| + \ln|c|$$

$$-\frac{1}{2}\ln|\cos(u)| - \frac{1}{2}\ln|u| = \ln|x\cdot c|$$

$$-\frac{1}{2}\left[\ln|\cos(u)| + \ln|u|\right] = \ln|x\cdot c|$$

$$-\frac{1}{2}\left[\ln|u\cos(u)|\right] = \ln|x\cdot c|$$

$$\ln|u\cos(u)| = \ln|x\cdot c| \times -2$$

$$\ln|u\cos(u)| = -2\ln|x\cdot c|$$

$$\ln|u\cos(u)| = \ln|(x\cdot c)^{-2}|$$

$$\cancel{\ln}|u\cos(u)| = \cancel{\ln}\left|\frac{1}{x^2c^2}\right|$$

$$u\cos(u) = \frac{1}{x^2c^2}$$

$$u\cos(u) = \frac{1}{x^2c^2}\ ;\ \frac{h}{c^n} = C$$

$$u\cos(u) = \frac{C}{x^2}\ ,\ u = y/x$$

$$\frac{y}{x}\cos(y/x) = \frac{C}{x^2}$$

$$\frac{x^2y}{x}\cos(y/x) = C$$

$$\boxed{xy\cos(y/x) = C}$$