Name: KEY

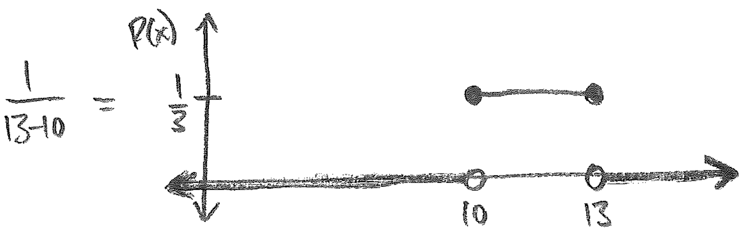
Worksheet 9: Uniform and Normal Distributions

1) I think a coffee dispenser machine fills cups at amounts that are uniformly distributed. Specifically, if I select a small 12 oz coffee, the machine dispenses between 10 and 13 oz. of coffee, following a uniform distribution.

a) Is the amount of coffee dispensed by the machine a discrete or continuous random variable? Explain.

Continuous: we are measuring, not counting: we can get any amount from 10 to 13 oz.

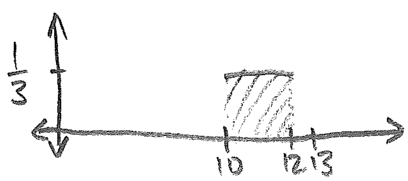
b) Graph the probability function for P(X), where X is the amount of coffee dispensed when I select a small coffee.



c) What is P(X = 12) ?

0.

d) Find the probability that my 12 oz. cup will not overflow, i.e. P(X ≤ 12).



$\text{Area} = \text{base} \times \text{height} = 2 \times \frac{1}{3} = \frac{2}{3}$

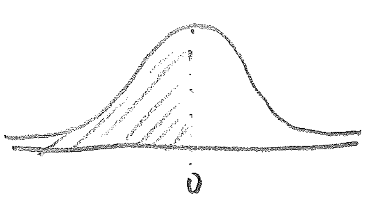
e) How large of a coffee cup should I have if I want to be 95% confident that the dispenser won't overfill the cup? (Hint: find P_{95}).

$\text{Area} = .95 = \text{base} \times \text{height} = b \times \frac{1}{3}$

$b = \frac{.95}{1/3} = 2.85 \Rightarrow$ Coffee cup size $= 10 + 2.85 = 12.85$ oz.

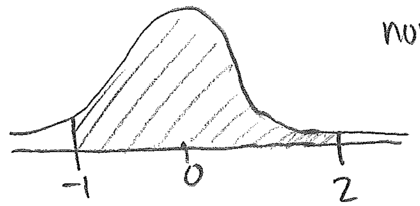
2. For this exercise, consider the standard normal distribution, where the mean $\mu = 0$ and the standard deviation $\sigma = 1$. Because we usually use this distribution after standardizing our data using z-scores, it is customary to use Z as our random variable instead of X when using the standard normal distribution.

a) Find P(Z < 0)



normalcdf$(-\infty, 0, \underset{\mu}{0}, \underset{\sigma}{1}) = 0.5$

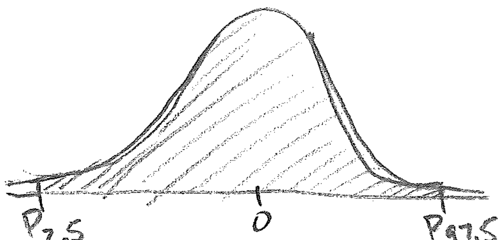
b) Find P(-1 < Z < 2).



normalcdf (-1, 2, 0, 1) = 0.8186

Now let's find the middle 95% of the distribution. We can do this by finding $P_{2.5}$ and $P_{97.5}$.

c) Shade in the area of interest on the graph of a standard normal curve.



d) Find $P_{2.5}$ for the standard normal distribution.

invNorm(0.025, 0, 1) = -1.96

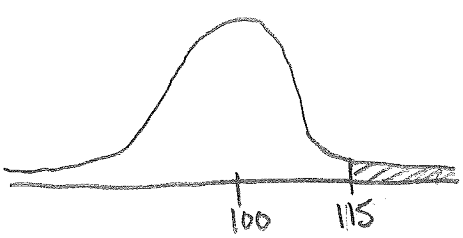
e) Find $P_{97.5}$ for the standard normal distribution.

invNorm(0.975, 0, 1) = 1.96

3. IQ test scores follow a normal distribution, and are standardized such that the mean is 100 and the standard deviation is 15.

a) What is the median IQ test score? The normal distribution is symmetric, so the median is equal to the mean. The median is 100.

b) What is the probability that a randomly selected person has an IQ above 115? Shade in the area of interest under the normal curve.



$P(X>115) = \text{normalcdf}(115, \infty, \underset{\mu}{100}, \underset{\sigma}{15})$

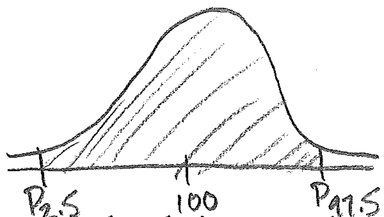
$= 0.1587$

c) MENSA, the high IQ society, only accepts membership into their organization if an applicant's IQ score is at or above the 98th percentile. What score would someone need to have in order to get into MENSA?

P_{98} = invNorm (0.98, 100, 15) = 130.81

Someone would need an IQ score above 130 to get into MENSA. (IQ scores are whole numbers)

d) Using the method from problem 2, what range of IQ scores should we expect the middle 95% of the population to have?



$P_{97.5}$ = invNorm(0.975, 100, 15) = 129.4

$P_{2.5}$ = invNorm(0.025, 100, 15) = 70.6

To include these, we need a range of IQ scores from 70 to 130.

4. A researcher designs a study where she goes to high schools throughout Oregon and measures the pulse rates of a group of female seniors, and then compares the averages of her sample at each school to the averages at other schools in Oregon.

a) What distribution would you expect the average pulse rates to follow?

Sample averages follow a normal distribution, by the Central Limit Theorem.

From a previous study, it is known that the mean pulse rates of female high school seniors is 77.5 beats per minute (bpm), with a standard deviation of 11.6 bpm.

b) Find the percentiles P_1 and P_{99} of the high school female pulse rates.

P_{99} = invNorm(0.99, 77.5, 11.6) = 104.49 bpm

P_1 = invNorm(0.01, 77.5, 11.6) = 50.51 bpm

c) Estimate the mean and standard deviation of a sample of n = 36 female high school seniors.

$$\mu_{\bar{x}} \approx \mu = 77.5$$

$$\sigma_{\bar{x}} \approx \frac{\sigma}{\sqrt{n}} = \frac{11.6}{\sqrt{36}} = 1.933$$

d) Find the probability that a school's average female pulse rate is between 70 and 85, i.e. find the probability $P(70 < \bar{x} < 85)$ when the sample size is n = 25 female high school seniors. Shade in the area of interest on a normal probability curve.

$$\mu_{\bar{x}} = 77.5$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{11.6}{\sqrt{25}} = 2.32$$

$P(70 < \bar{x} < 85)$ = normalcdf(70, 85, 77.5, 2.32)

= 0.9988