Checked by Abhishek

IBA University

Student's Name: Irfan Ali	Class: BS(CS)-II	Instructor: Mr. Iftikhar Ahmed
Student's ID: 023-21- 0327	Subject: Discrete Structures	Total Marks:
Date: 17-February -2020	Department: Computer Science	Time Allowed:

Note: Attempt all questions.

Question No. 1.

a) (i) Translate into logical expression "you can access the Wi-Fi password from IBA campus only if you are a BSCS student or you are not a freshman".

(ii) P: "I bought a lottery ticket"; q: "I won the million-dollar jackpot"

Then express "$p \leftrightarrow q$", "$\neg p \wedge \neg q$" and "$p \vee q$" in English.

b) Determine $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a contradiction by using truth table.

c) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology by using truth table.

Question No. 2.

a) Show that by developing a series of logical equivalences.

i. $\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$

ii. $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q$

b) State the converse and contrapositive of the following statements:

(i) *If it snows today, I will stay at home.*

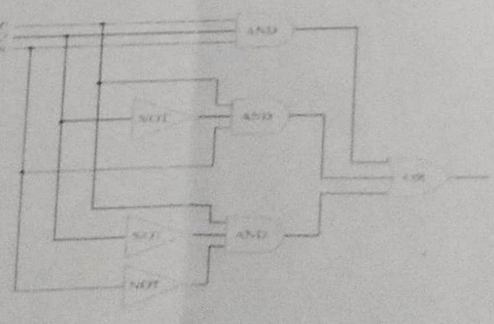
(ii) *We play the game if it is sunny.*

(iii) *If a positive integer is a prime then it has no divisor other than one and itself.*

a) Determine whether the following argument form is valid or invalid by drawing a truth table, indicating which columns represent the premises and which represent the conclusion, and annotating the table with a sentence of explanation. When you fill in the table, you only need to indicate the truth values for the conclusion in the rows where all the premises are true (the critical rows) because the truth values of the conclusion in the other rows are irrelevant to the validity or invalidity of the argument.

$$p \rightarrow q \vee \sim r$$
$$q \rightarrow p \wedge r$$
$$\therefore p \rightarrow r$$

b) Consider the combinational circuit shown below, find its input/output in table form.



Name: Abhishek Kumar

Date: ________

QNo. 1.

a) i

P: You can access the wifi password from IBA Campus

Q: You are a BSCS student

R: You are not a freshman.

$$P \rightarrow (q \vee r)$$

Ans

(ii) P: "I bought a lottery ticket"

q: "I won the million-dollar jackpot".

$p \leftrightarrow q$: I bought a lottery ticket if and only if I won the million-dollar jackpot.

$\sim p \wedge \sim q$: I did not buy a lottery ticket and I did not win the million-dollar jackpot.

$p \vee q$: I bought a lottery ticket or I won the million-dollar jackpot.

(b) $(\sim q \wedge (p \rightarrow q)) \longrightarrow \sim p$

Date:

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \wedge (p \to q)$	$(\sim q \wedge (p \to q)) \to \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Hence it is not contradiction, it is tautology.

(C) $(p \wedge q) \to (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Hence it is tautology.

Q No. 2

a) i. $\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$

$\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge q)$

$= (\sim(\sim p \wedge q) \wedge \sim(\sim p \wedge \sim q)) \vee (p \wedge q)$ ∴ Demorgan's law

$= (p \vee \sim q) \wedge (p \vee q) \vee (p \wedge q)$ ∴ Demorgan's

$= p \vee (\sim q \wedge q) \vee (p \wedge q)$ ∴ distributive law

$= p \vee (c) \vee (p \wedge q)$ ∴ Negative law

$= p \vee (p \wedge q)$ ∴ Identity law

$= p$ ∴ Absorption law

Proved

Date:

(ii) $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q$

Sol:

$(p \vee \sim q) \wedge (\sim p \vee \sim q)$

$= (\sim q \vee p) \wedge (\sim q \vee \sim p)$ ∴ Commutative pro

$= \sim q \vee (p \wedge \sim p)$ ∴ Distributive law

$= \sim q \vee (C)$ ∴ Negative law

$= \sim q \vee C$

$= \sim q$ ∴ Identify law

Proved

b) State the converse & contrapositive

(i) If it snows today, I will stay at ho

Converse : $q \rightarrow p$

If I stay at home, it will snow today

Contrapositive : $\sim q \rightarrow \sim p$

If I do not stay at home then it will not snow today.

(ii) We play the game if it is sunny

Converse : $q \rightarrow p$

If we play the game, then it is sunny

Contrapositive : $\sim q \rightarrow \sim p$

If we do not play the game, then it is not sunny.

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(iii) If a positive integer is a prime then it has no divisor other than one and ~~both~~ itself.

Converse: ~~If~~ $q \rightarrow p$

If a positive integer has no divisor other than one and ~~both~~ itself, then it is a prime.

Contra-positive: $\sim q \rightarrow \sim p$

If a positive integer has divisor other than one and itself, then it is not a prime.

Q 3 -

(a) $p \rightarrow q \vee \sim r$

$q \rightarrow p \wedge r$

$\therefore p \rightarrow r$

Hypothesis/premises — Conclusion

p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \rightarrow q \vee \sim r$	$q \rightarrow p \wedge r$	$p \rightarrow r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Date:

The 1^{st}, 4^{th}, 7^{th}. and 8^{th} rows are the critical rows, and 4^{th} row indicates that true premises have the false conclusion. So this is an invalide argument.

(b) $(P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R)$

Inputs

Q	R	~P	~Q	~R	P∧Q	P∧Q∧R	P∧~Q	P∧~Q∧R
1	1	0	0	0	1	1	0	0
1	0	0	0	1	1	0	0	0
0	1	0	1	0	0	0	1	1
0	0	0	1	1	0	0	1	0
1	1	1	0	0	0	0	0	0
1	0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0	0
0	0	1	1	1	0	0	0	0

P∧~Q∧~R	(P∧Q∧R)∨(P∧~Q∧R)∨(P∧~Q∧
0	1
0	0
0	1
1	1
0	0
0	0
0	0
0	0

outputs