$x^{2n}-y^{2n}$ is divisible by $x+y$

P(1) :→ $x^2-y^2 = \frac{(x-y)\cancel{(x+y)}}{\cancel{(x+y)}}$

P(k) :→ $x^{2k}-y^{2k} = (x+y)m \Rightarrow x^{2k} = (x+y)m+y^{2k}$

P(k+1) ⇒ $x^{2(k+1)}-y^{2(k+1)} = (x+y)r$

$x^2\cdot x^{2k} - y^2\cdot y^{2k} =$

$x^2[(x+y)m+y^{2k}] - y^2\cdot y^{2k} =$

$x^2(x+y)m + x^2\cdot y^{2k} - y^2\cdot y^{2k} =$

$x^2(x+y)m + y^{2k}(x^2-y^2) =$

$x^2(x+y)m + y^{2k}(x+y)(x-y) =$

$(x+y)[mx^2 + y^{2k}(x-y)] =$

$3^n \geq 1+2n$

$\sum_{m=0}^{n} 3^m = \frac{3^{n+1}-1}{2}$, $n\geq 1$

$\underbrace{3^0+3^1}_{1+3=4}+3^2+\dots+3^n = \frac{3^{n+1}-1}{2}$

P(1) ⇒ $\frac{3^2-1}{2}=4$

P(k) ⇒ $3^0+3^1+3^2+\dots+3^k = \frac{3^{k+1}-1}{2}$ → i

P(k+1) ⇒ $\underbrace{3^0+3^1+3^2+\dots+3^k}+3^{k+1} = \frac{3^{k+2}-1}{2}$ → ii

$\frac{3^{k+1}-1}{2}+3^{k+1} =$

$\frac{3^{k+1}-1+2\cdot 3^{k+1}}{2} =$

$\frac{3^{k+1}(1+2)-1}{2} =$

$\frac{3\cdot 3^{k+1}-1}{2} =$

$\frac{3^{k+2}-1}{2} =$

6^n+4 is divisible by 5, $n\geq 0$

$0=5(0)$

P(1) ⇒ $6^1+4=5(2)$

P(k) ⇒ $6^k+4=5m \Rightarrow 6^k=5m-4$

P(k+1) ⇒ $6^{k+1}+4=5r$

$6^k\cdot 6+4=5r$

$6(5m-4)+4=5r$

$5(6m)-24+4=5r$

$5(6m)-20=5r$

$5(6m-4)=5r$

Proved