Project 02
Model Predictive Control

Prem Sagar S - AE14B021

December 3, 2017

1 Formulation

With usual notations, the control input plan is determined by optmising the objective function:

$$J = \sum_{i=1}^{p} \left(\hat{Y}_{k+i/k} - Y^{ref}_{k+i}\right)^T Q^y \left(\hat{Y}_{k+i/k} - Y^{ref}_{k+i}\right) + \sum_{j=0}^{m-1} u^T_{m+j} Q^u u_{m+j}$$

The prediction at k+i time step can be written as:

$$\begin{aligned}
\hat{Y}_{k+i/k} &= CA^i \hat{X}_{k/k} + CA^{i-1}Bu_k + CA^{i-2}Bu_{k+1} + \ldots + CA^0 Bu_{k+i-1} \\
&= CA^i \hat{X}_{k/k} + \begin{bmatrix} CA^{i-1}B & CA^{i-2}B & \ldots & CA^0 B \end{bmatrix} \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+i-1} \end{bmatrix} \\
&= CA^i \hat{X}_{k/k} + CAB_i \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+m-1} \end{bmatrix} \\
&= CA^i \hat{X}_{k/k} + CAB_i U
\end{aligned}$$

where,

$$CAB_i = \begin{cases} \begin{bmatrix} CA^{i-1}B & CA^{i-2}B & \ldots & CA^0 B & 0 & 0 & \ldots & 0 \end{bmatrix}_{1\times m} & i \le m \\ \begin{bmatrix} CA^{i-1}B & CA^{i-2}B & \ldots & CA^{i-m+1}B & C\left(A^0 + A^1 + ... + A^{(i-m)}\right) B \end{bmatrix}_{1\times m} & i > m \end{cases}$$

Using this, the objective function can be written in the form below

$$J = \frac{1}{2} U^T H U + f^T U + K$$

where,

$$\begin{aligned}
H &= \sum_{i=1}^{p} CAB_i^T Q^y CAB_i + \begin{bmatrix} Q^u & 0 & \cdots & 0 \\ 0 & Q^u & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & Q^u \end{bmatrix}_{m\times m} \\
f &= -\sum_{i=1}^{p} \hat{X}^T_{k/k} \left(A^i\right)^T C^T Q^y CAB_i + \sum_{i=1}^{p} \left(Y^{ref}_{k+i}\right)^T Q^y CAB_i
\end{aligned}$$

U is then solved for using the matlab function quadprog.

To have a handle on the measurements being contorlled, the C matrix can be premultipied with an appropriate identity matrix. For example, to control first and last measurement,

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} C_{new}$$

2 Control System

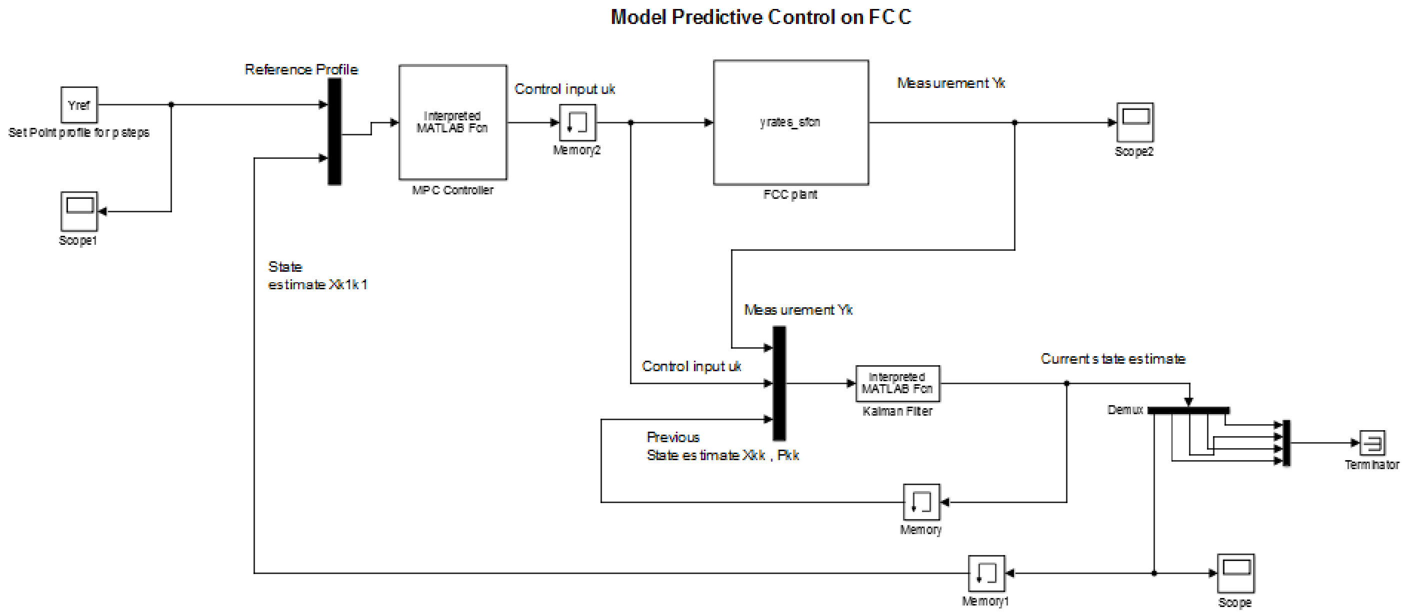


Figure 2.1: Schematic of Control on FCC

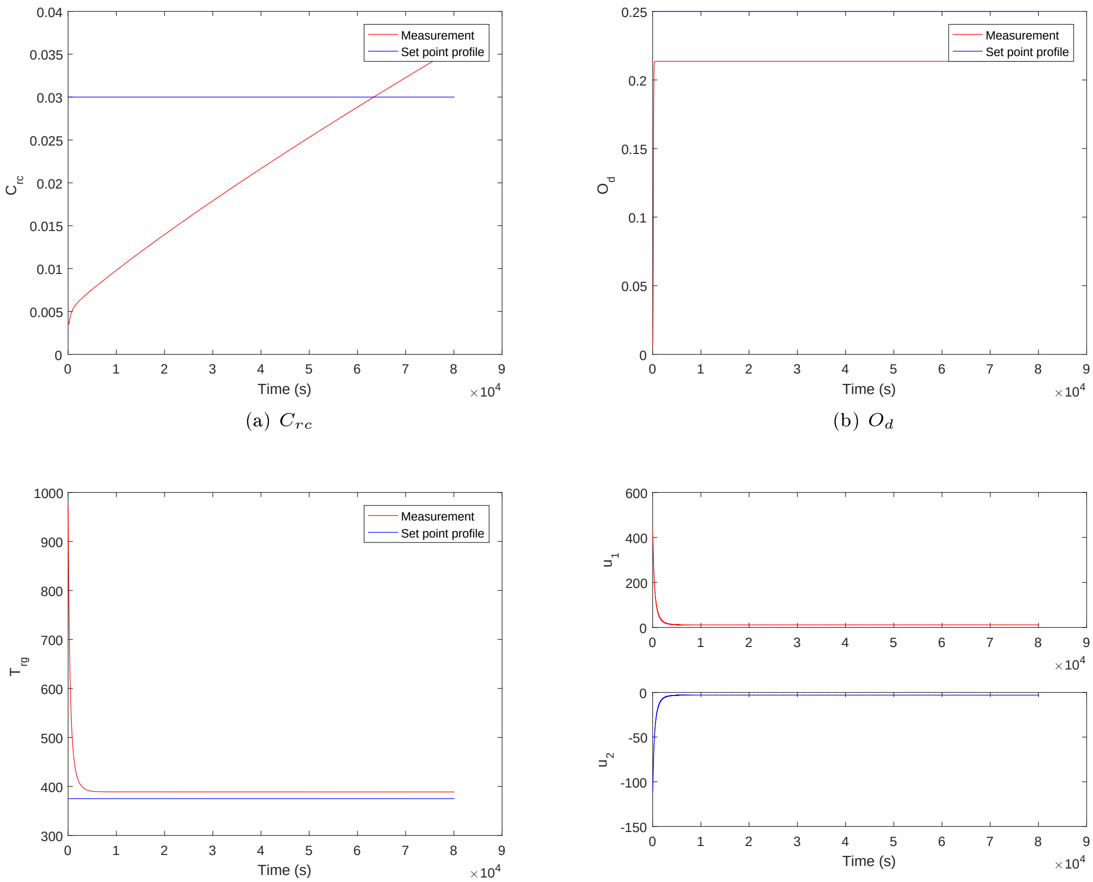
The above schematic shows the implementation of MPC in Simulink. (For representative purpose only) The plant measurements are simulated by solving the plant's differential equations. The measurement is fed to the Kalman filter that works on the linearized model. The filtered state estimate is fed to the controller which gives the optimal control input to the plant. This goes on.

3 Measurements with No Noise

3.1 Constant set-point profile

Kalman Filter	**Controller**
$X_0 = \begin{bmatrix} 0.0472 \\ -0.0576 \\ -0.1655 \\ -0.1065 \end{bmatrix}$	p=7
$P_{0/0} = 30000 \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$	m=3
$Q = 10e-3 \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$Q_y = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$
$R = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 100 \end{bmatrix}$	$Q_u = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$

Table 1: Tuned parameters



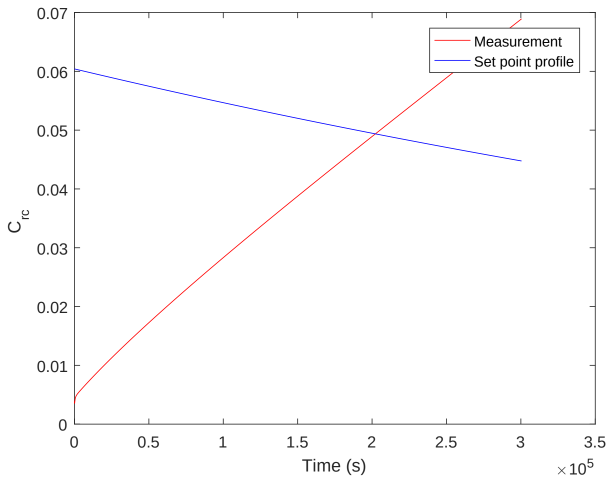
(a) C_{rc} (b) O_d (c) T_{rg} (d) Inputs

Figure 3.1: ***Constant set-point profile - Measurements with No Noise:*** *First state keeps increasing, while the other two saturate with an offset from the set point*

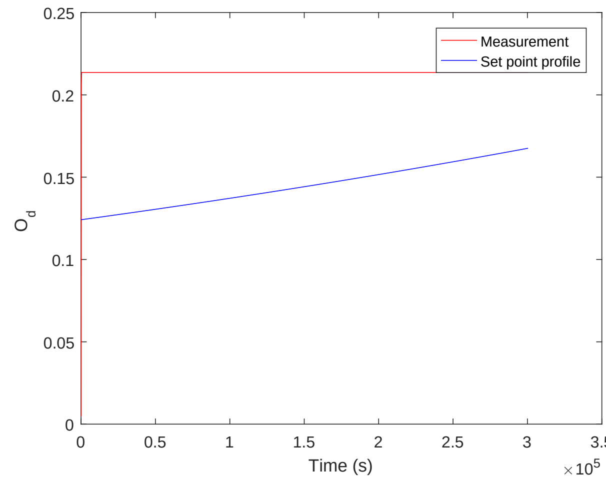
3.2 Non constant set-point profile

Kalman Filter	Controller
$X_0 = \begin{bmatrix} 0.0472 \\ -0.0576 \\ -0.1655 \\ -0.1065 \end{bmatrix}$	p=7
$P_{0/0} = 30000 \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$	m=3
$Q = 10e-3 \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$Q_y = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$
$R = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 100 \end{bmatrix}$	$Q_u = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$

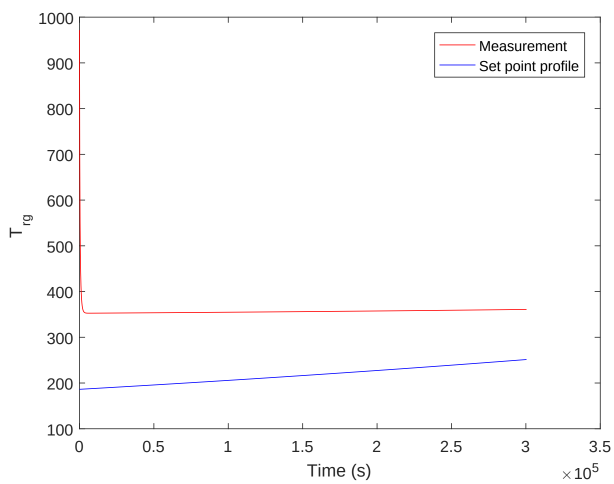
Table 2: Tuned parameters



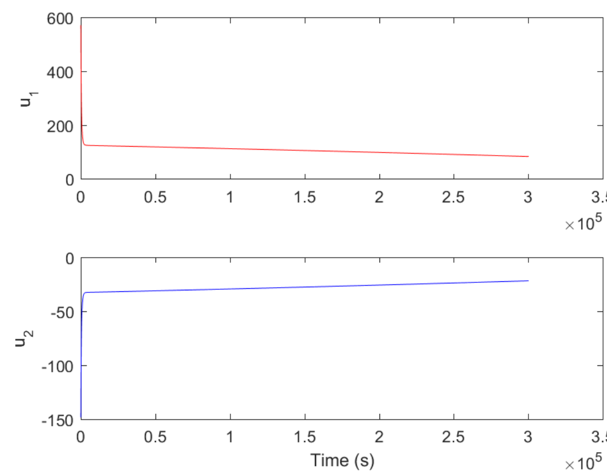
(a) C_{rc}



(b) O_d



(c) T_{rg}



(d) Inputs

Figure 3.2: ***Non Constant set-point profile - Measurements with no noise:*** *The behaviour is same as in previous case. The saturation value for the last two states have reduced. The saturated states stay that way and don't seem to make an effort to reach the set point.*

4 Measurements with Gaussian Noise

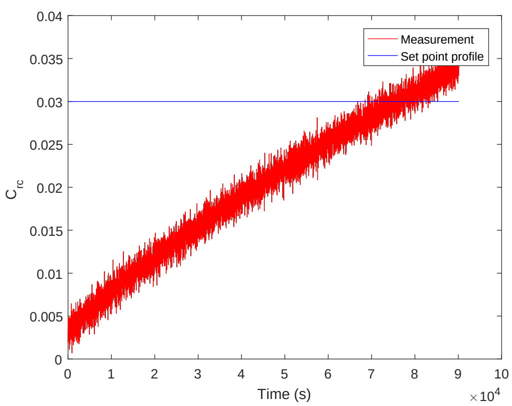
Gaussian noise $N(0, \sigma)$ is added to the plant generated measurements.

$$\sigma = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

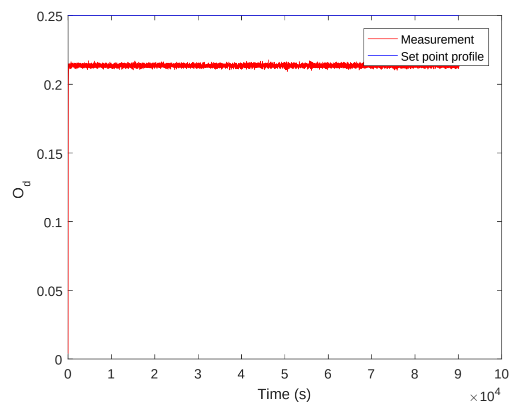
4.1 Constant set-point profile

Kalman Filter	Controller
$X_0 = \begin{bmatrix} 0.0472 \\ -0.0576 \\ -0.1655 \\ -0.1065 \end{bmatrix}$	p=7
$P_{0/0} = 30000 \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$	m=3
$Q = 10e-3 \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$Q_y = \begin{bmatrix} 80 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 50 \end{bmatrix}$
$R = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.002 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$	$Q_u = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$

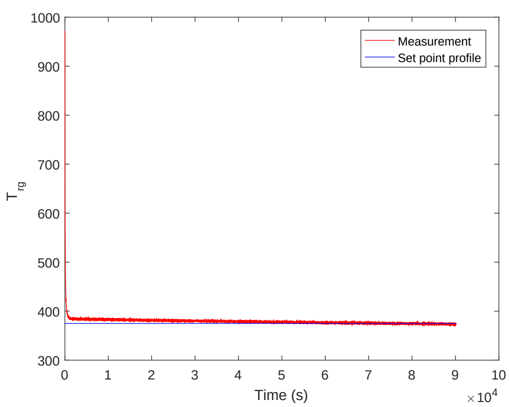
Table 3: Tuned parameters



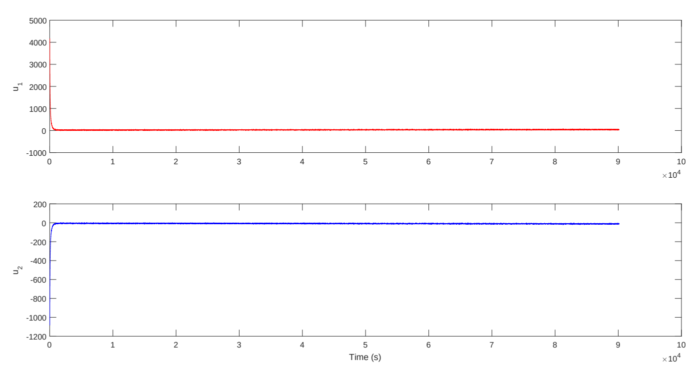
(a) C_{rc}



(b) O_d



(c) T_{rg}



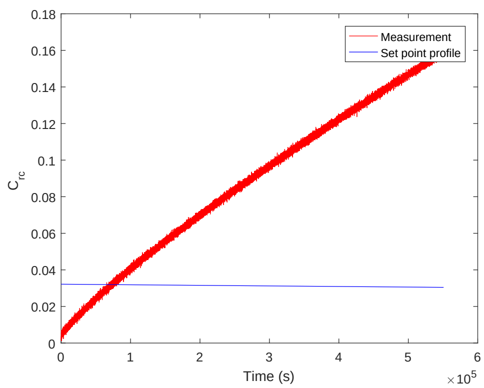
(d) Inputs

Figure 4.1: ***Constant set-point profile - Measurements with Gaussian Noise:*** *Here too the trend is the same, unaffected by addition of noise. The last state reaches the set point. Second state still has the same bias as seen previously.*

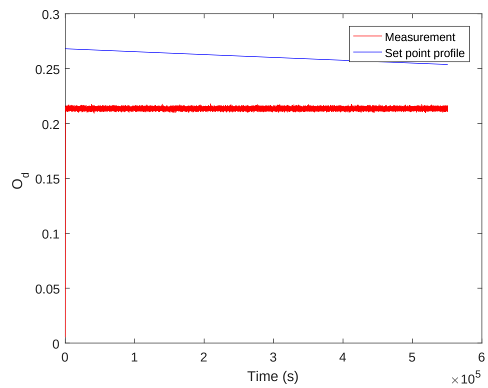
4.2 Non Constant set-point profile

Kalman Filter	**Controller**
$X_0 = \begin{bmatrix} 0.0472 \\ -0.0576 \\ -0.1655 \\ -0.1065 \end{bmatrix}$	p=7
$P_{0/0} = 30000 \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$	m=3
$Q = 10e-3 \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$Q_y = \begin{bmatrix} 80 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 50 \end{bmatrix}$
$R = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.002 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$	$Q_u = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$

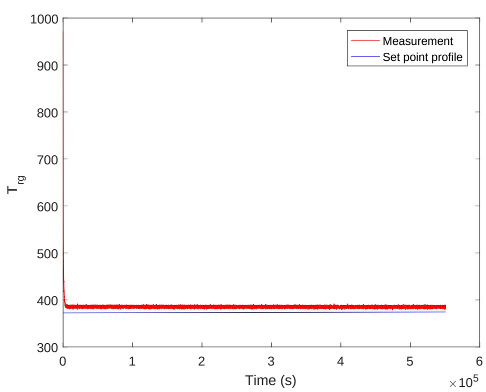
Table 4: Tuned parameters



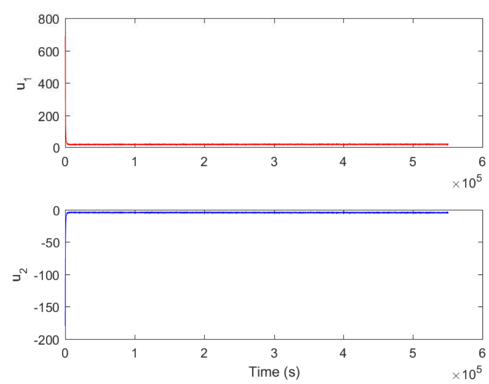
(a) C_{rc}



(b) O_d



(c) T_{rg}



(d) Inputs

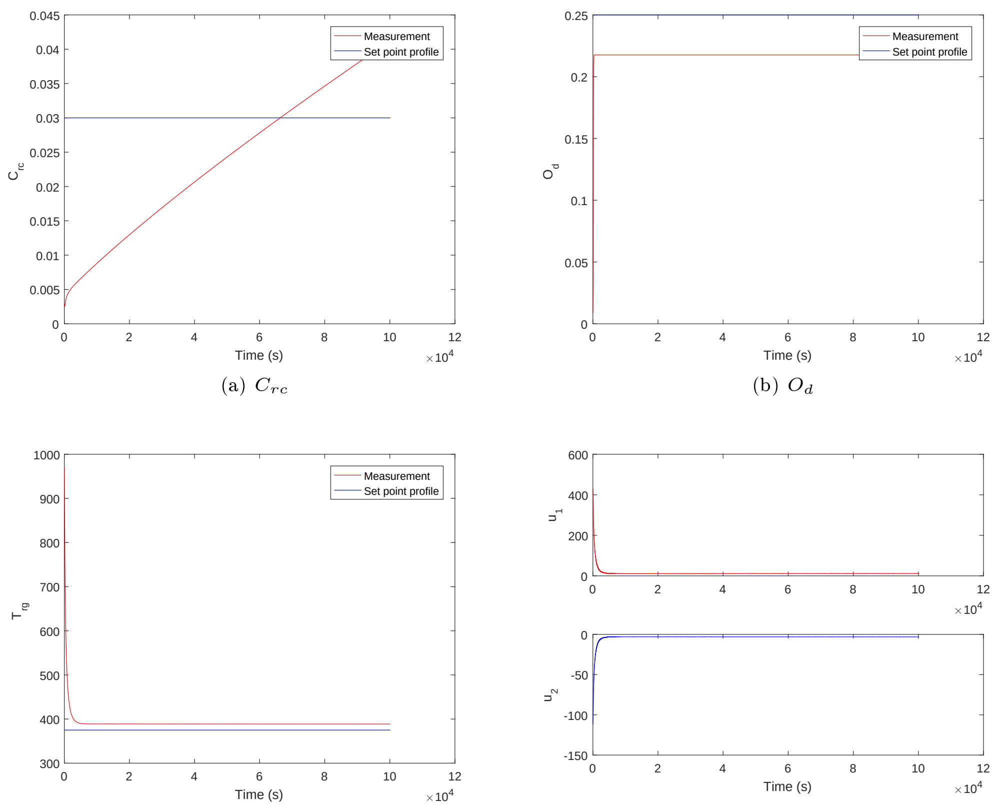
Figure 4.2: ***Non Constant set-point profile - Measurements with Gaussian noise:*** *This is similar to the case seen without noise. The first state just crosses the set point and the other saturate without approaching the set point themselves. It takes a lot of time for them to cross to see what happens after that. It was not easy to speed this up because large control inputs would lead to unstably high increments crashing some optimisers.*

5 Measurements with Fixed Bias

5.1 Constant set-point profile

Kalman Filter	Controller
$X_0 = \begin{bmatrix} 0.0472 \\ -0.0576 \\ -0.1655 \\ -0.1065 \end{bmatrix}$	p=7
$P_{0/0} = 30000 \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$	m=3
$Q = 10e-3 \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$Q_y = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$
$R = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.002 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$	$Q_u = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$

Table 5: Tuned parameters



(a) C_{rc}

(b) O_d

(c) T_{rg}

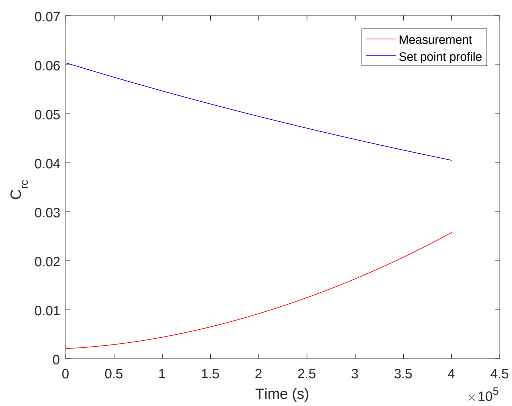
(d) Inputs

Figure 5.1: ***Non Constant set-point profile - Measurements with fixed bias:*** *As such the bias in measurents has not changed much when the other cases are considered. The states still behave the same way. The saturation values have changed a bit not directly related to the bias.*

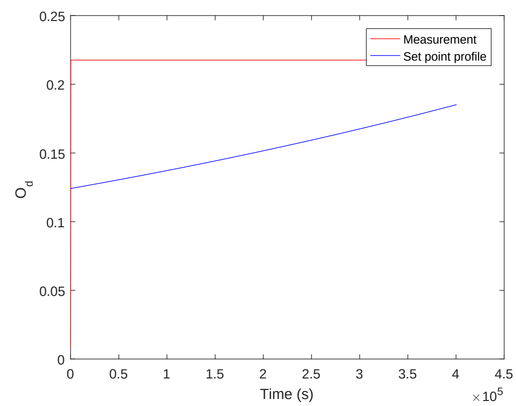
5.2 Non Constant set-point profile

Kalman Filter	Controller
$X_0 = \begin{bmatrix} 0.0472 \\ -0.0576 \\ -0.1655 \\ -0.1065 \end{bmatrix}$	p=7
$P_{0/0} = 30000 \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$	m=3
$Q = 10e-3 \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$Q_y = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$
$R = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.002 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$	$Q_u = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$

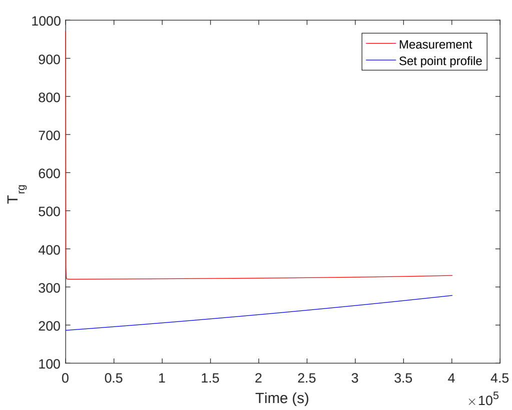
Table 6: Tuned parameters



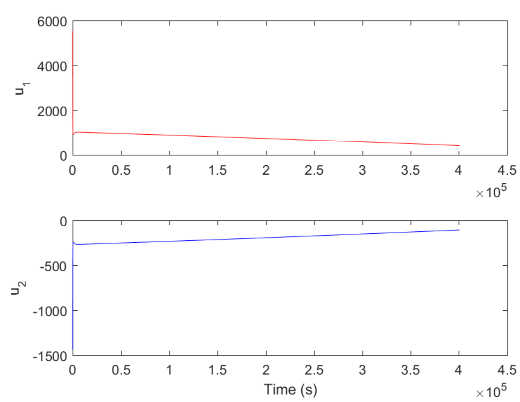
(a) C_{rc}



(b) O_d



(c) T_{rg}



(d) Inputs

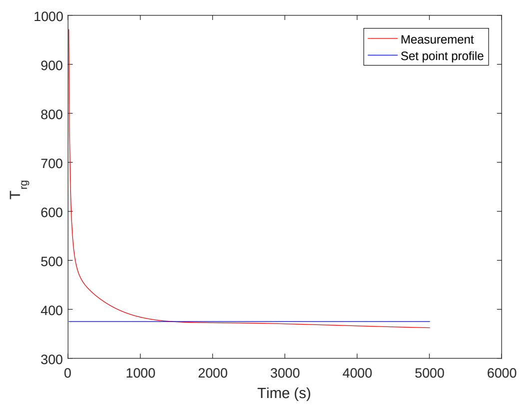
Figure 5.2: ***Non Constant set-point profile - Measurements with fixed bias:*** *Here too, the bias doesn't make much differemce. The states behave the same way as before and don't reach the set point on their own.*

6 Variation in X_0 Control Horizon and Prediction Horizon

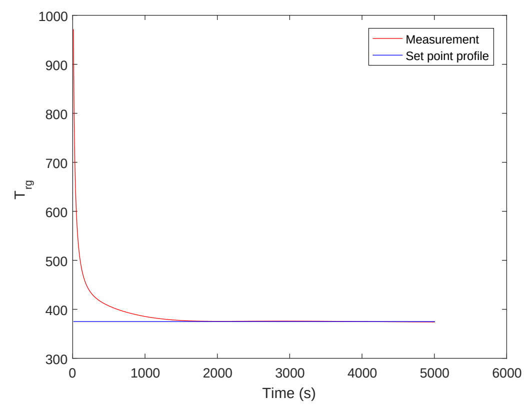
- To understand the behaviour of the control system, a rigorous and intensive parametric study is required, which is very time consuming.

- One trend is that for a very high p=45, m = 40, the bias in T_{rg} is removed. But again beyond some point, it departs from the set point. However, the other two states doesn't seem to converge to setpoint within this time. If for the same case, m is reduced to 10, T_{rg} stays at the set point reasonably. Even here, the other states don't converge.

- A significant change is not seen by varying m, p in small steps.

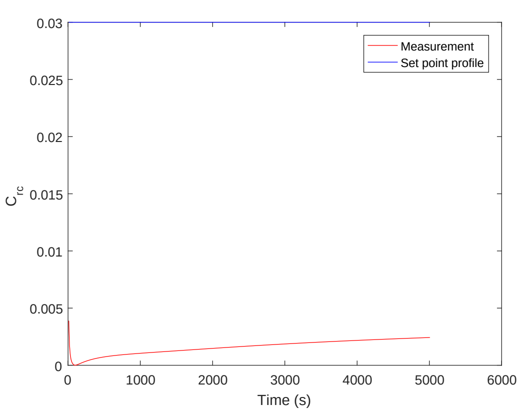


(a) p=45,m=40

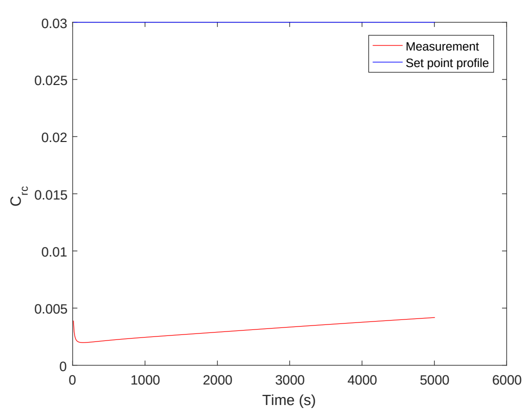


(b) p=45, m=10

Figure 6.1: ***Behaviour of m,p***

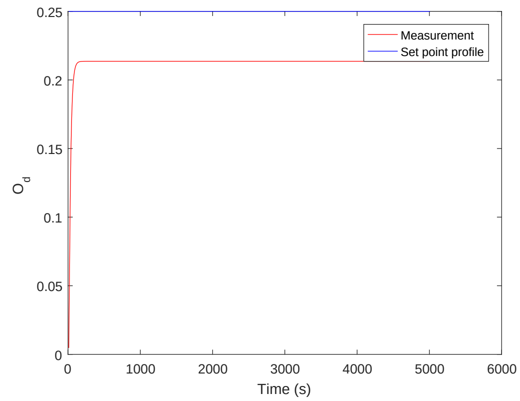


(a) p=45,m=40

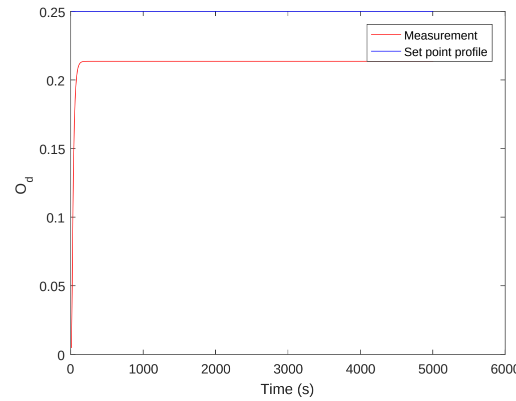


(b) p=45, m=10

Figure 6.2: ***Behaviour of m,p***



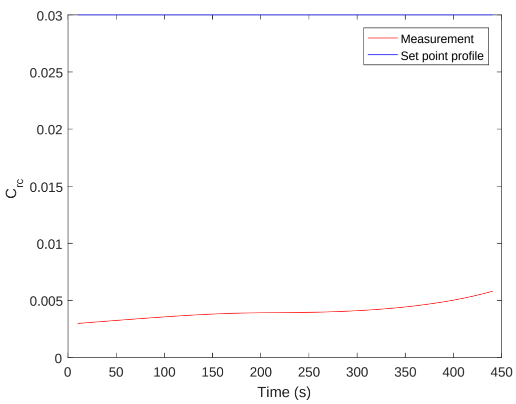
(a) p=45,m=40



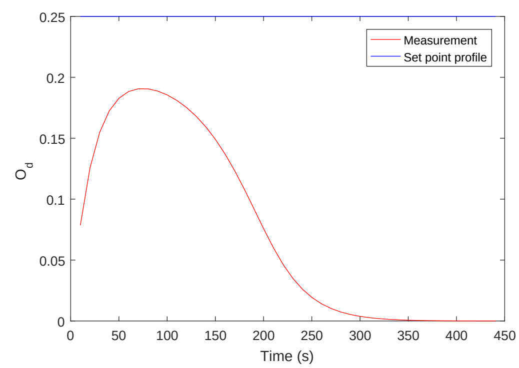
(b) p=45, m=10

Figure 6.3: ***Behaviour of m,p***

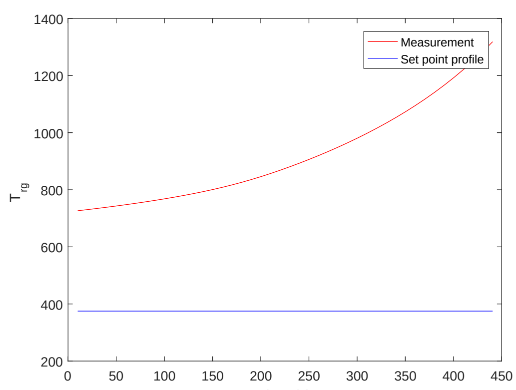
- The trends with X_0 is shown in the following figures.



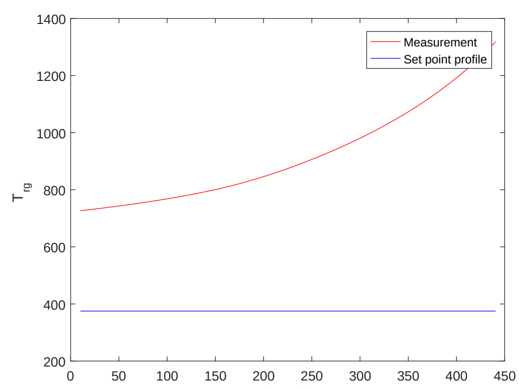
(a) C_{rc}



(b) O_d

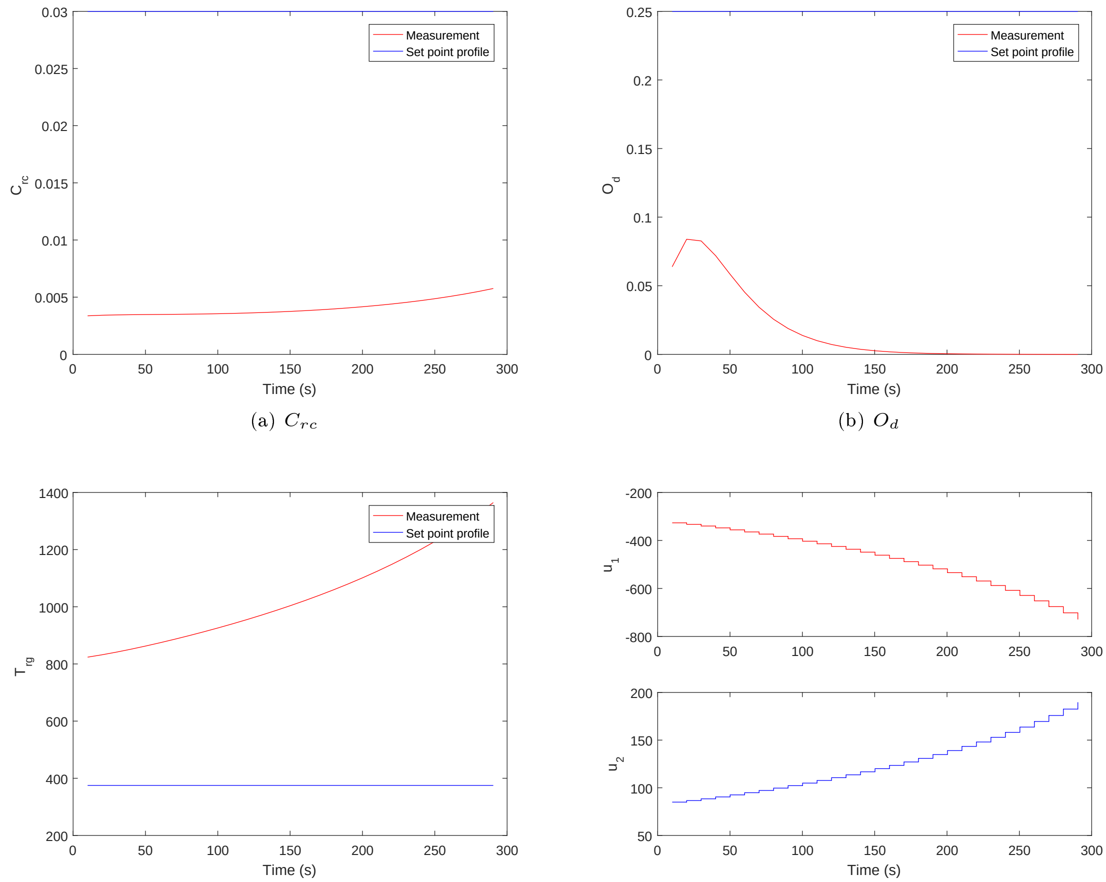


(c) T_{rg}



(d) Inputs

Figure 6.4: ***X0=[0.0354 -0.0432 -0.1241 -0.0798]:*** *The first state approches set point very slowly. It was not possible to increase it's approach rate even with very high weights. Second state comes closer and then moves away from the set point. It takes a saturated value it was always reaching. The third state just departs from the set point which is unusual. May further tuning could have alleviated this issue.*



(a) C_{rc} (b) O_d (c) T_{rg} (d) Inputs

Figure 6.5: ***X0=[0.0402 -0.0490 -0.1407 -0.0905]:*** *This is even closer to the set point. Strangely, the system is not able to achieve the set point. The states behave the same way.*

7 Conclusions

- Only the third state seems to be reasonably controllable and stable by suitably choosing large p and m. It can be made to stay at the desired set point.
- Second state always had an offset which could not be removed. By studying this bias variation and it's consequences, we can may be define a new state with this bias subtracted.
- First state was always increasing and it was never possible to saturate it. May be a threshold can be set. But essentially, it is possible to take the state to desired set point, but it cannot be made to stay there. So. technocally, it is "Controllable".
- Even if the system started from somewhere near the set point, the second state digresses to reach it's steady value. The third state completely diverges from the reference. First state just increases almost linearly as it always did.
- It has to be noted that the bias may be because the controller and the Kalman filter work on the linearized model. The non linear model can be implemented if it's really important to fix these issues. But then again, it's a tradeoff between calulation time and system performance.