

With usual notations, the control input plan is determined by optimising the objective function:

$$J = \sum_{i=1}^p \left( Y_{k+i}^{ref} - \hat{Y}_{k+i/k} \right)^T Q^y \left( Y_{k+i}^{ref} - \hat{Y}_{k+i/k} \right) + \sum_{j=0}^{m-1} u_{m+j}^T Q^u u_{m+j}$$

The prediction at  $k+i$  time step can be written as:

$$\begin{aligned} \hat{Y}_{k+i/k} &= CA^i \hat{X}_{k/k} + CA^{i-1} Bu_k + CA^{i-2} Bu_{k+1} + \dots + CA^0 Bu_{k+i-1} \\ &= CA^i \hat{X}_{k/k} + [CA^{i-1}B \quad CA^{i-2}B \quad \dots \quad CA^0B] \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+i-1} \end{bmatrix} \\ &= CA^i \hat{X}_{k/k} + CAB_i \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+m-1} \end{bmatrix} \\ &= CA^i \hat{X}_{k/k} + CAB_i U \end{aligned}$$

where,

$$CAB_i = \begin{cases} \begin{bmatrix} CA^{i-1}B & CA^{i-2}B & \dots & CA^0B & 0 & 0 & \dots & 0 \end{bmatrix}_{1 \times m} & i \leq m \\ \begin{bmatrix} CA^{i-1}B & CA^{i-2}B & \dots & CA^{i-m+1}B & C(A^0 + A^1 + \dots + A^{(i-m)}) \end{bmatrix}_{1 \times m} B & i > m \end{cases}$$

Using this, the objective function can be written in the form below

$$J = \frac{1}{2} U^T H U + f^T U + K$$

where,

$$\begin{aligned} H &= CAB_i^T Q^y CAB_i + \begin{bmatrix} Q^u & 0 & \dots & 0 \\ 0 & Q^u & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & Q^u \end{bmatrix}_{m \times m} \\ f^T &= \hat{X}_{k/k}^T (A^i)^T C^T Q^y CAB_i - (Y_{k+i}^{ref})^T Q^y CAB_i \end{aligned}$$

$\mathbf{U}$  is then solved for using the matlab function quadprog.