PROJECT 01
EXTENDED KALMAN FILTER FOR NON LINEAR DYNAMIC MODEL

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1. Algorithm

(1) An initial state $Y_{0/0}$ and $P_{0/0}$ is assumed. The process and measurement error covariances Q and R are used for tuning.

(2) **Model Identification/Representation:** The given system is in the following state space form

$$\dot{Y} = f(Y, u)$$

$$Z = h(Y, u)$$

where,

$$Y = \begin{bmatrix} C_{rc} \\ O_d \\ T_{rg} \end{bmatrix}$$

$$u = \begin{bmatrix} F_a & F_{sc} \end{bmatrix}$$

 (a) **Process Model:** This system can be linearized by using numerically calculated jacobians with respect to the states and inputs. Also this will be in continuous domain and has to be converted into discrete domain. It can then be treated like a linear time invariant state space system. Also it is assumed that the stochastic errors can be added linearly, so that their jacobians are identity matrices.

$$A_c(k) = \frac{\partial f[Y(k), u(k)]}{\partial Y(k)}$$

$$B_c(k) = \frac{\partial f[Y(k), u(k)]}{\partial u(k)}$$

 (b) **Measurement model:** The measurment model is constant with time and is set initially.

  (i) Case 1 : T_{rg} is measured.

$$C_c = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

  (ii) Case 2 : C_{rc} and T_{rg} are measured.

$$C_c = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(3) **Prediction:**

$$\hat{Y}_{k+1/k} = A_k \hat{Y}_{k+1/k} + B_k u_k$$

$$P_{k+1/k} = A_k P_{k/k} A_k^T + Q$$

(4) **Correction:**

$$K_{k+1} = \frac{P_{k+1/k} C^T}{C P_{k+1/k} C^T + R}$$

$$\hat{Y}_{k+1/k+1} = \hat{Y}_{k+1/k} + K_{k+1} \left(Z_{k+1} - C \hat{Y}_{k+1/k} \right)$$

$$P_{k+1/k+1} = (I - K_{k+1} C) P_{k+1/k}$$

(5) Repeat 2 - to determine linearinzed model approximate, 3 - predict, 4 - correct so on...

(6) The states estimated are compared directly with the measurement for tuning sinces the states itself are being measured.

2. Case 1

Only the third state T_{rg} is measured. The initial estimates taken for the shown plots are given below.

$Y_{0/0} = \begin{bmatrix} 10 \\ 10 \\ 100 \end{bmatrix}$	$P_{0/0} = 0.1 * \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$R = [0.0366]$

Since the measurements for all time steps were available, the variance was taken as R.

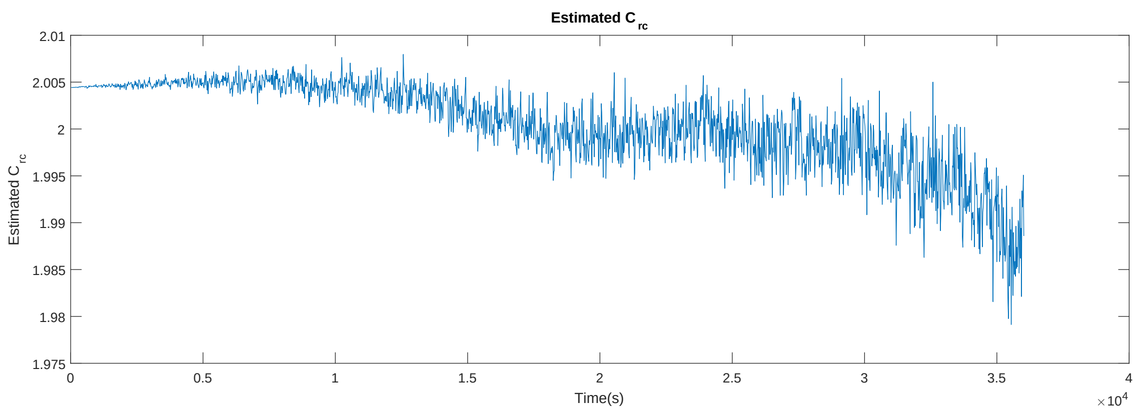

FIGURE 2.1. Estimated C_{rc}

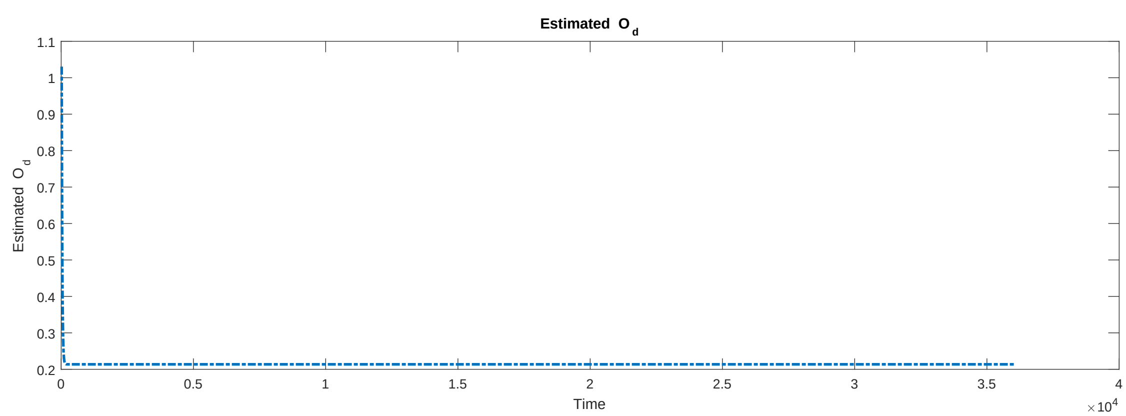

FIGURE 2.2. Estimated O_d

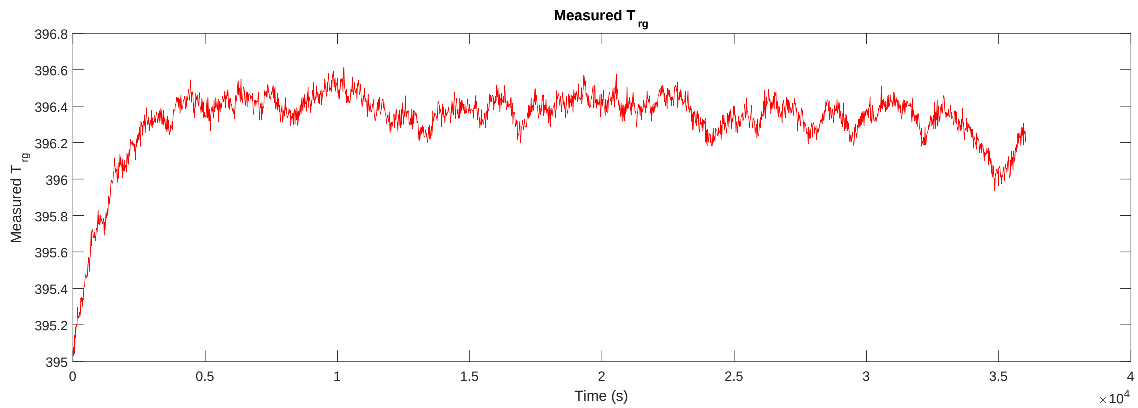

FIGURE 2.3. Measured T_{rg}

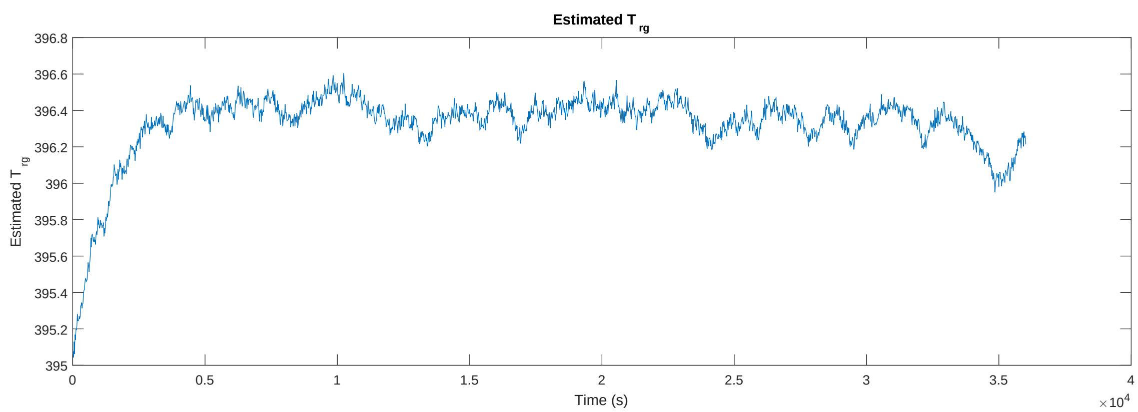

FIGURE 2.4. Estimated T_{rg}

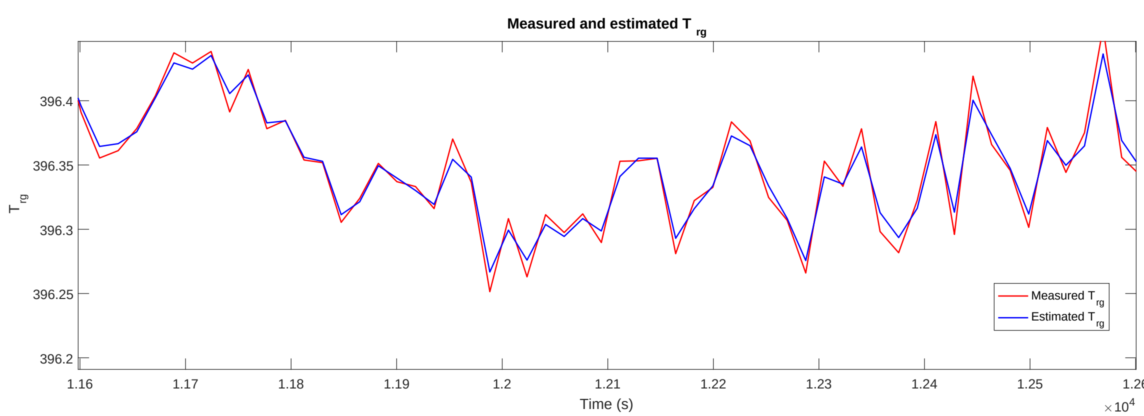

FIGURE 2.5. Zoomed plot of measured and estimated T_{rg}

Trial	$Y_{0/0}$	$P_{0/0}$	Q	R	Max error b/w measured and estimated T_{rg}
1	$Y_{0/0} = \begin{bmatrix} 10 \\ 10 \\ 100 \end{bmatrix}$	$P_{0/0} = 0.1 * \begin{bmatrix} 1 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$R = [0.0366]$	3.114130e-02
2	$Y_{0/0} = \begin{bmatrix} 2 \\ 2 \\ 150 \end{bmatrix}$	$P_{0/0} = 0.1 * \begin{bmatrix} 1 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$R = [0.0366]$	3.129204e-02
3	$Y_{0/0} = \begin{bmatrix} 2 \\ 2 \\ 150 \end{bmatrix}$	$P_{0/0} = 0.1 * \begin{bmatrix} 1 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$Q = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$	$R = [0.0366]$	7.007393e-02
4	$Y_{0/0} = \begin{bmatrix} 2 \\ 2 \\ 150 \end{bmatrix}$	$P_{0/0} = 0.01 * \begin{bmatrix} 1 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$R = [0.0366]$	3.129204e-02
5	$Y_{0/0} = \begin{bmatrix} 2 \\ 2 \\ 150 \end{bmatrix}$	$P_{0/0} = 0.001 * \begin{bmatrix} 1 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$R = [0.0366]$	4.539038e-02

Table 1. Error dynamics

The error doesn't seem to vary much for two different initial guesses chosen. Reducing Q by a factor or 10 seems to have increased the error by a bit. P was scaled down by 10 times, not much error varition. Further reduction of P by 10 more times increased the error a bit. All in all, all of the errors are small compared the values of the state itself. Hence, EKF has performed well.

3. Case 2

First and third states are measured. The plots are shown for this particular initial guesses:

$Y_{0/0} = \begin{bmatrix} 10 \\ 10 \\ 100 \end{bmatrix}$	$P_{0/0} = 0.1 \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$R = \begin{bmatrix} 0.0000 & 0 \\ 0 & 0.00354 \end{bmatrix}$

Since all the measurements were available, the variance is taken as R. Off diagonal terms are assumed zero, that is there is no correlation between the measured variables.

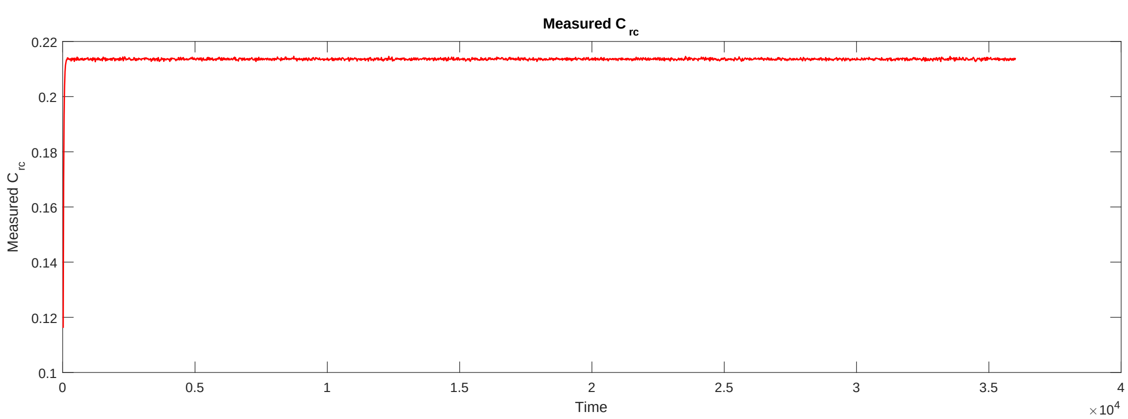

Figure 3.1. Measured C_{rc}

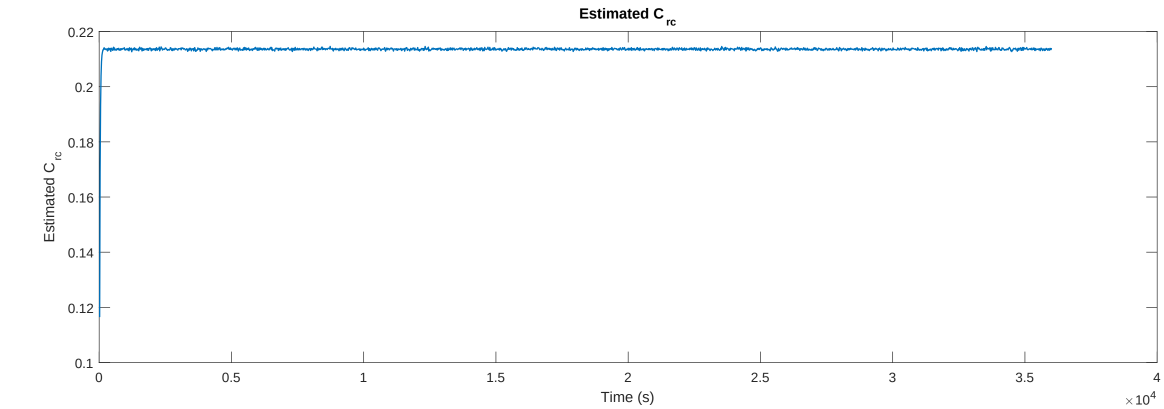

FIGURE 3.2. Estimated C_{rc}

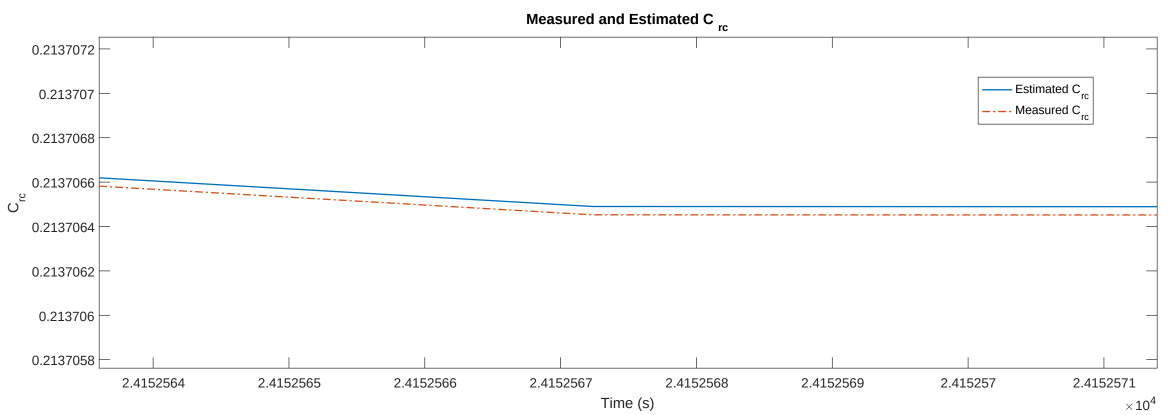

FIGURE 3.3. Zoomed plot of measured and estimated C_{rc}

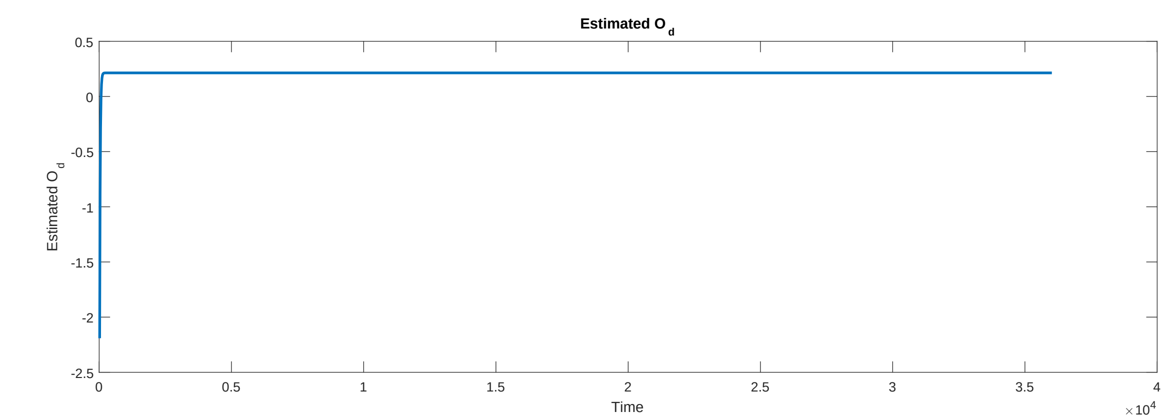

FIGURE 3.4. Estimated O_d

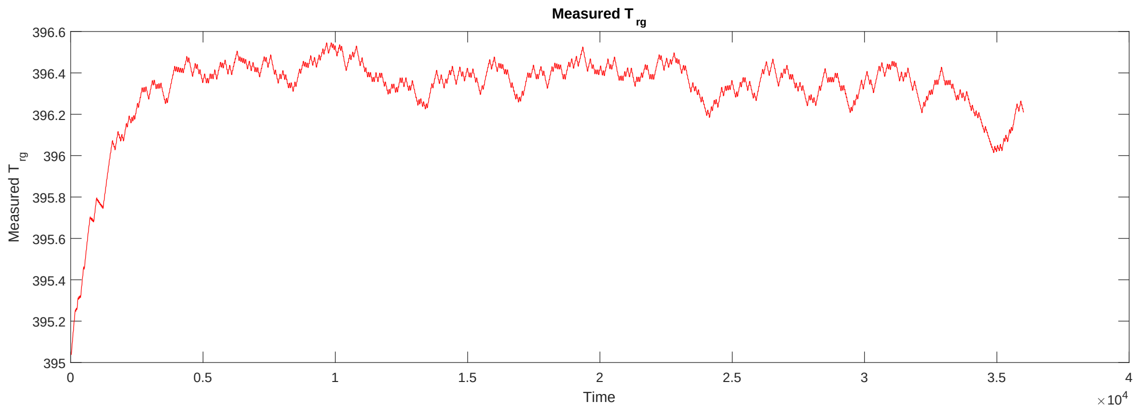

FIGURE 3.5. Measured T_{rg}

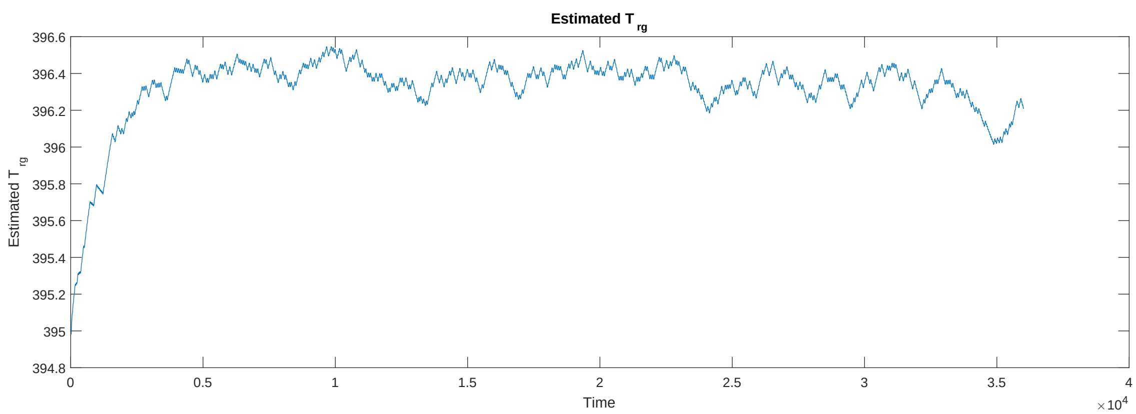

FIGURE 3.6. Estimated T_{rg}

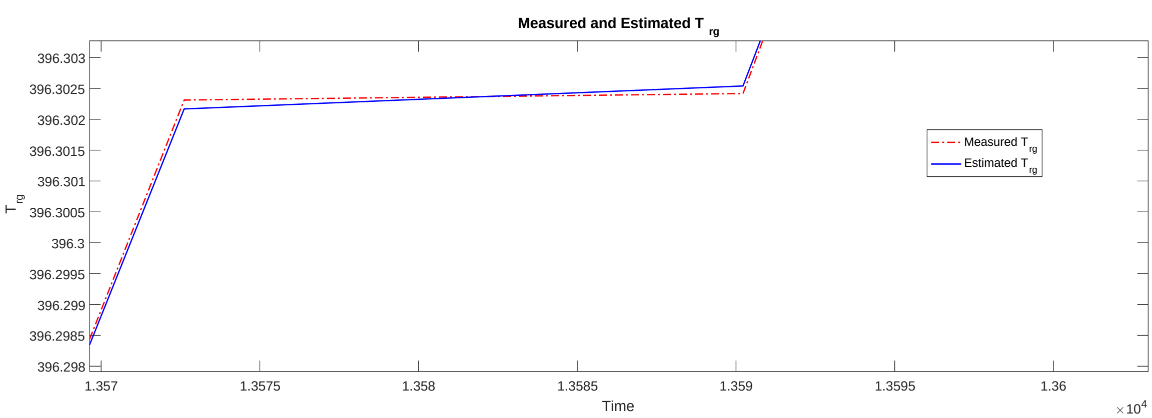

FIGURE 3.7. Zoomed plot of measured and estimated T_{rg}

Trial	$Y_{0/0}$	$P_{0/0}$	Q	R	Max error b/w measured and estimated C_{rc}	Max error b/w measured and estimated T_{rg}
1	$\mathrm{Y}_{0/0} = \begin{bmatrix} 10 \\ 10 \\ 100 \end{bmatrix}$	$\mathrm{P}_{0/0} = 0.1 * \begin{bmatrix} 1 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$\mathrm{Q}= \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$\mathrm{R}= \begin{bmatrix} 0.0366 \end{bmatrix}$	3.144432e-06	5.565311e-02
2	$\mathrm{Y}_{0/0} = \begin{bmatrix} 2 \\ 2 \\ 150 \end{bmatrix}$	$\mathrm{P}_{0/0} = 0.1 * \begin{bmatrix} 1 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$\mathrm{Q}= \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$\mathrm{R}= \begin{bmatrix} 0.0366 \end{bmatrix}$	3.144432e-06	3.129204e-02
3	$\mathrm{Y}_{0/0} = \begin{bmatrix} 2 \\ 2 \\ 150 \end{bmatrix}$	$\mathrm{P}_{0/0} = 0.1 * \begin{bmatrix} 1 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$\mathrm{Q}= \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$	$\mathrm{R}= \begin{bmatrix} 0.0366 \end{bmatrix}$	3.152068e-05	4.503534e-02
4	$\mathrm{Y}_{0/0} = \begin{bmatrix} 2 \\ 2 \\ 150 \end{bmatrix}$	$\mathrm{P}_{0/0} = 0.01 * \begin{bmatrix} 1 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$\mathrm{Q}= \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$\mathrm{R}= \begin{bmatrix} 0.0366 \end{bmatrix}$	3.155473e-06	4.374650e-01
5	$\mathrm{Y}_{0/0} = \begin{bmatrix} 2 \\ 2 \\ 150 \end{bmatrix}$	$\mathrm{P}_{0/0} = 0.001 * \begin{bmatrix} 1 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 20000 \end{bmatrix}$	$\mathrm{Q}= \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$\mathrm{R}= \begin{bmatrix} 0.0366 \end{bmatrix}$	3.154913e-06	4.109853e+00

TABLE 2. Error dynamics

For all the trials, first state doesn't seem to have much change in error. As compared to previous case, the error in third state is generally slightly higher but still in acceptable range. Error has increased a bit by scaling down Q 10 times. Scaling down $P_{0/0}$ by 10 times successively has shown that error increases significantly.