Dynamics and Control Of Spacecraft Astrosat Controller Design

Rishi - AE
Kavita Sivaraman - AE16M006
Prem Sagar S - AE14B021
Daniel Satke - AE17F010

I. Introduction

We are planning to design a preliminary attitude control system for ASTROSAT satellite. Since this satellite is an inertial pointing satellite, we have chosen to stabilize the satellite using zero momentum biased stabilization method with 4 wheel tetrahedron configuration. Star sensors senses and provide the values of roll, pitch and yaw Euler angles along with gyroscopes. Our design also includes the momentum dumping, as and when any of the wheels reach their maximum angular rate capacity.

We have initially derived the kinematic equations of motion using 2-3-1 rotation sequence to get the Euler angles. Euler angles are then converted to quaternions before integration to avoid any singularities like gimbal lock phenomenon. Dynamic equations of motion for inertially pointing satellite, with zero momentum biased system configuration is derived using Euler equations. Gravity gradient torque is also considered as disturbance torque, apart from the disturbance torque given in the problem which is in the order of milli N-m. Since our's is an inertially pointing satellite, our roll,pitch and yaw equations of motion will be decoupled.

We have studied the motion of satellite, in roll, pitch and yaw, found that the solutions are unstable in all the 3 DOF in the presence of disturbances and hence we have concluded to go for active stabilization technique based on Proportional-Derivative (PD) controller using momentum wheels in tetrahedral configuration. The gains for our control system design are chosen in such a way that the maximum allowable steady state error does not exceed 0.005 deg about all the axes (zero momentum biased system).

II. Astrosat

It is an inertial pointing spacecraft in a 650km 6 deg inclined circular orbit; 4-wheel tetrahedron wheel system is used for momentum management; Actual MI properties of the s/c after deployment.

$$J_c = \begin{bmatrix} 1763 & -52 & -16 \\ -52 & 1591 & 25 \\ -16 & 25 & 1185 \end{bmatrix} kg - m^2$$

Mass of the s/c = 1542 Kg. [x,y,z] axes are yaw, roll and pitch respectively. Initially we have assumed that the cross product of inertias are negligible and designed the control system. Then once the control design is completed, we have used the actual inertia matrix to check the performance variations. The momentum dumping is achieved by using 60 A-m^2 torque rods about all the three axes. Disturbance torques are T_x= T_z=$2 * 10^{-3}$ Nm and T_y=10^{-4} Nm and satellite's orbital angular velocity, $\omega_o = 1.0741 * 10^{-3}$rad/sec.

III. Kinematics

We are choosing 2-3-1 (ψ,θ, ϕ) rotation for our kinematics. The angle rates and angular velocity can be related as below,

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & -cos\phi tan\theta & sin\phi tan\theta \\ 0 & sin\phi & cos\phi \\ 0 & cos\phi sec\theta & -sin\phi sec\theta \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

IV. Dynamics

Since we are using a zero momentum biased control system, the following holds true. Where $\vec{h_i}$ are the wheel angular momentum.

$$\vec{h}_1 + \vec{h}_2 + \vec{h}_3 + \vec{h}_4 = 0 \tag{IV.1}$$

Also each of XYZ components must be zero.

$$h_{wx} = \frac{h_1}{\sqrt{3}} - \frac{h_2}{\sqrt{3}} - \frac{h_3}{\sqrt{3}} + \frac{h_4}{\sqrt{3}} = 0 \tag{IV.2}$$

$$h_{wy} = \frac{h_1}{\sqrt{3}} - \frac{h_2}{\sqrt{3}} + \frac{h_3}{\sqrt{3}} - \frac{h_4}{\sqrt{3}} = 0 \quad \text{(IV.3)}$$

$$h_{wz} = \frac{h_1}{\sqrt{3}} + \frac{h_2}{\sqrt{3}} - \frac{h_3}{\sqrt{3}} - \frac{h_4}{\sqrt{3}} = 0 \quad \text{(IV.4)}$$

The dynamics of the wheels is governed by,

$$\frac{d}{dt}\left(\vec{H}_w\right) = -\vec{T}_c$$

where $\vec{T}_c$ is the control torque.

The Euler equations for the spacecraft system can be rewritten as below,

$$J_c\dot{\omega} + \omega \times (J_c\omega) = T_c + T_d$$

$$\begin{aligned} &J_x\ddot{\omega}_x + \frac{I_w\dot{\omega}_1}{\sqrt{3}} - \frac{I_w\dot{\omega}_2}{\sqrt{3}} + \frac{I_w\dot{\omega}_3}{\sqrt{3}} - \frac{I_w\dot{\omega}_4}{\sqrt{3}} \\ &-\omega_z\left(J_y\omega_y + \frac{I_w\omega_1}{\sqrt{3}} - \frac{I_w\omega_2}{\sqrt{3}} + \frac{I_w\omega_3}{\sqrt{3}} - \frac{I_w\omega_4}{\sqrt{3}}\right) \\ &+\omega_y\left(J_z\omega_z + \frac{I_w\omega_1}{\sqrt{3}} + \frac{I_w\omega_2}{\sqrt{3}} - \frac{I_w\omega_3}{\sqrt{3}} - \frac{I_w\omega_4}{\sqrt{3}}\right) = 0 \end{aligned} \quad \text{(IV.5)}$$

$$\begin{aligned} &J_y\ddot{\omega}_y + \frac{I_w\dot{\omega}_1}{\sqrt{3}} + \frac{I_w\dot{\omega}_2}{\sqrt{3}} - \frac{I_w\dot{\omega}_3}{\sqrt{3}} + \frac{I_w\dot{\omega}_4}{\sqrt{3}} \\ &+\omega_z\left(J_x\omega_x + \frac{I_w\omega_1}{\sqrt{3}} - \frac{I_w\omega_2}{\sqrt{3}} - \frac{I_w\omega_3}{\sqrt{3}} + \frac{I_w\omega_4}{\sqrt{3}}\right) \\ &-\omega_x\left(J_z\omega_z + \frac{I_w\omega_1}{\sqrt{3}} + \frac{I_w\omega_2}{\sqrt{3}} - \frac{I_w\omega_3}{\sqrt{3}} - \frac{I_w\omega_4}{\sqrt{3}}\right) = 0 \end{aligned} \quad \text{(IV.6)}$$

$$\begin{aligned} &J_z\ddot{\omega}_z + \frac{I_w\dot{\omega}_1}{\sqrt{3}} - \frac{I_w\dot{\omega}_2}{\sqrt{3}} - \frac{I_w\dot{\omega}_3}{\sqrt{3}} + \frac{I_w\dot{\omega}_4}{\sqrt{3}} \\ &-\omega_y\left(J_x\omega_x + \frac{I_w\omega_1}{\sqrt{3}} - \frac{I_w\omega_2}{\sqrt{3}} - \frac{I_w\omega_3}{\sqrt{3}} + \frac{I_w\omega_4}{\sqrt{3}}\right) \\ &+\omega_x\left(J_y\omega_y + \frac{I_w\omega_1}{\sqrt{3}} - \frac{I_w\omega_2}{\sqrt{3}} + \frac{I_w\omega_3}{\sqrt{3}} - \frac{I_w\omega_4}{\sqrt{3}}\right) = 0 \end{aligned} \quad \text{(IV.7)}$$

After doing small angle approximations the euler equations can be written as follows,

$$\begin{aligned} &J_x\ddot{\phi} + \frac{I_w\dot{\omega}_1}{\sqrt{3}} - \frac{I_w\dot{\omega}_2}{\sqrt{3}} + \frac{I_w\dot{\omega}_3}{\sqrt{3}} - \frac{I_w\dot{\omega}_4}{\sqrt{3}} \\ &-\dot{\theta}\left(\frac{I_w\omega_1}{\sqrt{3}} - \frac{I_w\omega_2}{\sqrt{3}} + \frac{I_w\omega_3}{\sqrt{3}} - \frac{I_w\omega_4}{\sqrt{3}}\right) \\ &+\dot{\psi}\left(\frac{I_w\omega_1}{\sqrt{3}} + \frac{I_w\omega_2}{\sqrt{3}} - \frac{I_w\omega_3}{\sqrt{3}} - \frac{I_w\omega_4}{\sqrt{3}}\right) = 0 \end{aligned} \quad \text{(IV.8)}$$

$$\begin{aligned} &J_y\ddot{\psi} + \frac{I_w\dot{\omega}_1}{\sqrt{3}} + \frac{I_w\dot{\omega}_2}{\sqrt{3}} - \frac{I_w\dot{\omega}_3}{\sqrt{3}} + \frac{I_w\dot{\omega}_4}{\sqrt{3}} \\ &+\dot{\theta}\left(\frac{I_w\omega_1}{\sqrt{3}} - \frac{I_w\omega_2}{\sqrt{3}} - \frac{I_w\omega_3}{\sqrt{3}} + \frac{I_w\omega_4}{\sqrt{3}}\right) \\ &-\dot{\phi}\left(\frac{I_w\omega_1}{\sqrt{3}} + \frac{I_w\omega_2}{\sqrt{3}} - \frac{I_w\omega_3}{\sqrt{3}} - \frac{I_w\omega_4}{\sqrt{3}}\right) = 0 \end{aligned} \quad \text{(IV.9)}$$

$$\begin{aligned} &J_z\ddot{\theta} + \frac{I_w\dot{\omega}_1}{\sqrt{3}} - \frac{I_w\dot{\omega}_2}{\sqrt{3}} - \frac{I_w\dot{\omega}_3}{\sqrt{3}} + \frac{I_w\dot{\omega}_4}{\sqrt{3}} \\ &-\dot{\psi}\left(\frac{I_w\omega_1}{\sqrt{3}} - \frac{I_w\omega_2}{\sqrt{3}} - \frac{I_w\omega_3}{\sqrt{3}} + \frac{I_w\omega_4}{\sqrt{3}}\right) \\ &+\dot{\phi}\left(\frac{I_w\omega_1}{\sqrt{3}} - \frac{I_w\omega_2}{\sqrt{3}} + \frac{I_w\omega_3}{\sqrt{3}} - \frac{I_w\omega_4}{\sqrt{3}}\right) = 0 \end{aligned} \quad \text{(IV.10)}$$

V. Controller Design

Without consideration of disturbance torque

$$lim_{s\to 0}\frac{sR(s)}{1+G_0(s)}$$

A. *Modified PD controller*

$$\begin{aligned} \lim_{s\to 0}\frac{sR(s)}{1+\left(\frac{1}{Is^2+K_ds}\right)G_c(s)} &= \lim_{s\to 0}\frac{s(Is^2+K_ds)}{Is^2+K_ds+G_c(s)}R(s) \\ &= \lim_{s\to 0}\frac{Is+K_d}{I+\frac{K_d}{s}+\frac{G_c(s)}{s^2}}R(s) \end{aligned}$$

Test signals $R(s)=\frac{1}{s}$, $R(s)=\frac{1}{s^2}$,$R(s)=\frac{1}{s^3}$,

1) Step Input:

$$\begin{aligned} \lim_{s\to 0}\frac{Is+K_d}{I+\frac{K_d}{s}+\frac{G_c(s)}{s^2}}\frac{1}{s} &= \lim_{s\to 0}\frac{Is+K_d}{Is+K_d+\frac{G_c(s)}{s}} \\ &= \lim_{s\to 0}\frac{Is+K_d}{Is+K_d+\frac{K_p}{s}} \\ &= 0 \end{aligned}$$

2) Ramp Input:

$$\begin{aligned} \lim_{s\to 0}\frac{Is+K_d}{I+\frac{K_d}{s}+\frac{G_c(s)}{s^2}}\frac{1}{s^2} &= \lim_{s\to 0}\frac{Is+K_d}{Is^2+K_ds+K_p} \\ &= \frac{K_d}{K_p} \end{aligned}$$

3) Accelerating Input:

$$\begin{aligned} \lim_{s\to 0}\frac{Is+K_d}{I+\frac{K_d}{s}+\frac{G_c(s)}{s^2}}\frac{1}{s^3} &= \lim_{s\to 0}\frac{Is+K_d}{Is^3+K_ds^2+sK_p} \\ &= K_d \end{aligned}$$

Critically damped system

$$\xi = 1$$

$$K_d^2 = 4IK_p$$

Closed loop system pole

$$1 + G_c(s) = 0$$

$$1 + \frac{K_p}{Is^2 + K_d s} = 0$$

$$Is^2 + K_d s + K_p = 0$$

Type 1 system

$$s^2 + \frac{K_d}{I}s + \frac{K_p}{I} = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = \frac{K_p}{I}$$

$$2\xi\omega_n = \frac{K_d}{I}$$

$$\xi = \frac{K_d}{2\omega_n I}$$

$$\xi^2 = \frac{K_d^2}{4\frac{K_p}{I}I^2} = \frac{K_d^2}{4K_p I}$$

Control Torque

Euler dynamics

$$J_{cx}\dot{\omega}_x + \omega_y\omega_z(J_{cz} - J_{cy}) = T_{cx} + T_{dx}$$

$$J_{cy}\dot{\omega}_y + \omega_z\omega_x(J_{cx} - J_{cz}) = T_{cy} + T_{dy}$$

$$J_{cz}\dot{\omega}_z + \omega_x\omega_y(J_{cy} - J_{cx}) = T_{cz} + T_{dz}$$

$$T_c = \begin{bmatrix} K_{px}(R_\phi - \phi) + K_{dx}\dot{\phi} \\ K_{py}(R_\psi - \psi) + K_{dy}\dot{\psi} \\ K_{pz}(R_\theta - \theta) + K_{dz}\dot{\theta} \end{bmatrix}$$

The individual wheel torques are:

$$T_1 = \frac{1}{4}\left(\sqrt{3}T_{cx} + \sqrt{3}T_{cy} + \sqrt{3}T_{cz}\right)$$

$$T_2 = \frac{1}{4}\left(-\sqrt{3}T_{cx} - \sqrt{3}T_{cy} + \sqrt{3}T_{cz}\right)$$

$$T_3 = \frac{1}{4}\left(-\sqrt{3}T_{cx} + \sqrt{3}T_{cy} - \sqrt{3}T_{cz}\right)$$

$$T_4 = \frac{1}{4}\left(\sqrt{3}T_{cx} - \sqrt{3}T_{cx} - \sqrt{3}T_{cx}\right)$$

The wheel dynamics of wheel **i** are described by the equations below:

$$I\dot{\omega}_i = T_i$$

$$I\dot{\omega}_i + IC_{wb}\dot{\omega} + (\omega_i + C_{wb}\omega) \times (I\omega_i + C_{wb}\omega) = T_i$$

$$I\dot{\omega}_i = T_i - IC_{wb}\dot{\omega} - (\omega_i + C_{wb}\omega) \times (I\omega_i + C_{wb}\omega)$$

VI. RESULTS

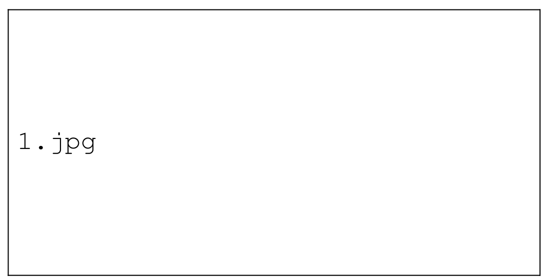

Figure VI.1. Click on image to change scale/image. Figure float can be copy pasted and you can change image for copied float

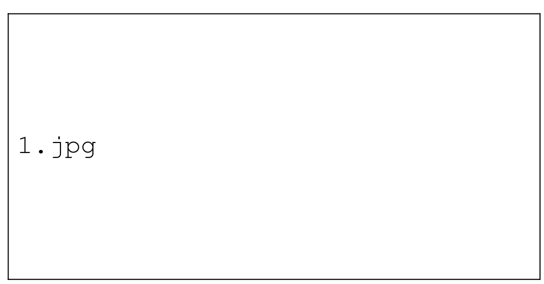

Figure VI.2. Click on image to change scale/image.Figure float can be copy pasted and you can change image for copied floa