AE5545 – DYNAMICS AND CONTROL OF SPACECRAFT

Assignment No. 2 – 100 points
(Attitude Kinematics)
Due: 07 September 2017

31 August 2017

1. The kinematic differential equation of the direction cosine matric is given by $\dot{\mathbf{C}} + \boldsymbol{\omega} \times \mathbf{C} = \mathbf{0}.$ Show that
$$\omega_1 = \dot{C}_{21}C_{31} + \dot{C}_{22}C_{32} + \dot{C}_{23}C_{33};$$
$$\omega_2 = \dot{C}_{31}C_{11} + \dot{C}_{32}C_{12} + \dot{C}_{33}C_{13};$$
$$\omega_3 = \dot{C}_{11}C_{21} + \dot{C}_{12}C_{22} + \dot{C}_{13}C_{23}$$

2. Parameterize the direction cosine matrix **C** in terms (2-3-2) Euler angles. Also find appropriate inverse transformations from **C** back to (2-3-2) angles. Find the kinematic differential equations of (2-3-2) Euler angles. What is the geometric condition for which these equations will encounter the kinematical singularity?
3. Roll, Pitch and Yaw are Euler angles and are sometimes defined as (1-2-3) sequence and sometimes as (3-2-1) sequences. Suppose the Roll, Pitch and Yaw angles are small, how do the form of the (1-2-3) and (3-2-1) matrices look? Are they different or same? Explain.
4. The initial (3-2-1) Euler angles Yaw, Pitch and Roll of a vehicle are $\{\psi \quad \theta \quad \varphi\}$ = $\{40 \quad 30 \quad 80\}$ degrees at time t=0. The angular velocity of the vehicle in body frame co-ordinates are given by $\boldsymbol{\omega}$ = $\begin{Bmatrix} \sin 0.1t \\ 0.01 \\ \cos 0.1t \end{Bmatrix} 20\frac{deg}{sec}$.
   (a) Translate this initial attitude description into the corresponding Euler parameters. (b) Write a matlab code to integrate the Euler angles and parameters for 2 minutes and plot the four Euler angle and parameter time histories. (c) Check the constraint ${q_1}^2 + {q_2}^2 + {q_3}^2 + {q_4}^2 = 1$ and normalize if they do not satisfy this constraint. Compare Euler parameters and Euler angles through conversions for the entire 2 minutes and specifically before and after normalization process. Comment on your results.

5. The state vector for a given instant of Indian Remote Sensing Satellite IRS-P5, which is an earth pointing satellite is $\bar{r}$ = [4172.39512860, -854.25163879,5538.75384795] Kms and $\bar{v}$ = [5.3839064673, -2.8358389294, -4.4805571591] Km/s. The orbit reference frame is defined

as follows: yaw axis = - unit position vector=$\frac{\bar{r}}{|\bar{r}|}$, pitch axis is = - $\frac{(\hat{r}\times\hat{v})}{|(\hat{r}\times\hat{v})|}$ and roll axis is defined to form an orthogonal triad. (a) Find out the DC matrix $\bar{C}_{oi}$ connecting orbit reference frame to inertial reference frame. The attitude errors between orbit frame to body frame was computed as a (1-2-3) rotation matrix $\bar{C}_{bo}$ as

$$\bar{C}_{bo} = \begin{bmatrix} 0.9958533 & 0.088210 & -0.022251 \\ -0.087126 & 0.995147 & 0.045733 \\ 0.026177 & -0.043604 & 0.998706 \end{bmatrix}$$

Also, the star sensor co-ordinates wrt the body co-ordinates is given by the rotation matrix

$$\bar{C}_{sb} = \begin{bmatrix} -0.731354 & 0.0 & -0.681998 \\ 0.222037 & 0.945519 & -0.238105 \\ 0.644842 & -0.325568 & -0.691509 \end{bmatrix}$$

(b) Now convert the above rotation matrices into quaternions and use the quaternion multiplication matrix to find out the attitude of the spacecraft in quaternion for this instant. (c) Convert the computed quaternion into rotation matrix and show that the matrix obtained is

$$\bar{C}_{si} = \begin{bmatrix} 0.666832 & 0.532724 & 0.521094 \\ 0.685653 & -0.164683 & -0.709056 \\ -0.291916 & 0.830111 & -0.475080 \end{bmatrix}$$

Provide all the steps and quaternions computed together with all matrices. You can do this exercise using Matlab as well.