AE5545 – DYNAMICS AND CONTROL OF SPACECRAFT

Assignment No. 1 – 100 points
(Attitude Representations)
Due: 23 August 2017

16 August 2017

1. Consider 3 reference frames 'N', 'F' and 'B'. The co-ordinate axes of 'F' and 'B' frames are provided in terms of axes of 'N' frame as follows:
   $\hat{b}_1 = \frac{1}{3}\{1\ 2\ -2\ \};\ \hat{b}_2 = \frac{1}{\sqrt{2}}\{0\ 1\ 1\ \};\ \hat{b}_3 = \frac{1}{3\sqrt{2}}\{4\ -1\ 1\ \}$;

   $\hat{f}_1 = \frac{1}{4}\{3\ -2\ \sqrt{3}\ \};\ \hat{f}_2 = \frac{1}{2}\{-1\ 0\ \sqrt{3}\ \};\ \hat{f}_3 = \frac{-1}{4}\{\sqrt{3}\ 2\sqrt{3}\ 1\ \}$.
   Detect the direction cosine matrices of frames 'B' and 'F' in terms of 'N' and then compute the direction cosine matrices of frame 'B' relative to frame 'F'.
2. The orientation of a spacecraft B wrt an inertial frame N is given through the asymmetric (3-2-1) Euler angles $(30,\ -45,\ 60\ )^T$. Find out the orientation (attitude) matrix of the sequence. Identify that Euler angle value that could create singularity for the (3-2-1) rotation. Find the corresponding principal rotation axis and angle for the given (3-2-1) rotation sequence.
3. Consider the direction cosine matrix C between the two right hand orthogonal reference frames B and N obtained in problem 2.
   (a) Show that the direction cosine matrix is an orthonormal matrix (that is. $C\ C^T = I\ =\ C^T C$).
   (b) Show that ijth element of C is equal to the ijth cofactor of C (that is $C_{ij} = (-1)^{i+j} M_{ij}$).
   (c) Show the determinant of C = $|C|$ = 1.
   (d) Find the 6 row ortho-normality conditions of C and 6 column ortho-normality conditions of C. Are they same?
   (e) Finally show only three of the nine direction cosines are independent and they do not uniquely determine the attitude matrix C.
4. A spacecraft performs a 45^0 single axis rotation about an axis $\hat{e}$ = $\frac{1}{\sqrt{3}}\{1\ 1\ 1\ \}^T$. Find the corresponding 3-2-1 yaw, pitch and roll angles that relate the final attitude to the original attitude.
5. The attitude matrix between two reference frames F and N is given by $[C] = [\ \ 0.892539\ 0.157379\ -0.422618\ -0.275451\ 0.932257\ -0.234570\ \ \ 0.357073\ 0.$
   ; Calculate the four Euler parameters or quaternion for this matrix.