Assignment 1
Unconstrained Single Variable Optimization
Pipeline Problem

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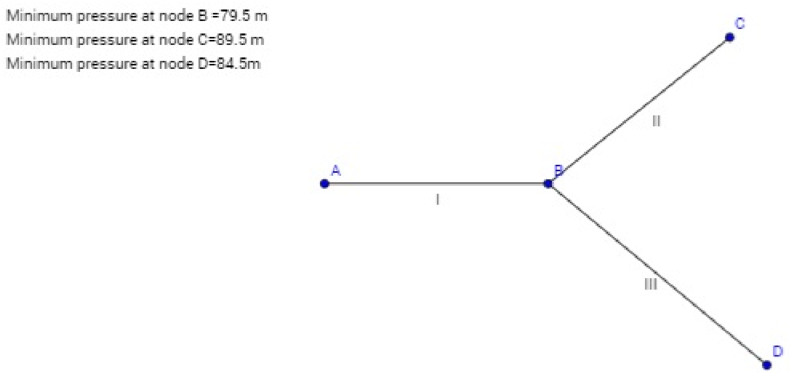
August 19, 2016

1 Problem Description

Roll number AE14B021

Enter your roll number in the box below and press the button titled Click



2 Heads at all nodes

1. Node 1 :

$$\begin{aligned} H_1 = H_0 - \Delta H_{01} = 100 - \frac{4.457 * 10^8 * 300 * 9^{1.85}}{D_1^{4.87}} \\ = 100 - \frac{7.7895 * 10^{12}}{D_1^{4.87}} \end{aligned} \tag{1}$$

2. Node 2 :

$$\begin{aligned} H_2 = H_1 - \Delta H_{12} = H_1 - \frac{4.457 * 10^8 * 500 * 3^{1.85}}{D_2^{4.87}} \\ = 100 - \frac{4.457 * 10^8 * 300 * 9^{1.85}}{D_1^{4.87}} - \frac{4.457 * 10^8 * 500 * 3^{1.85}}{D_2^{4.87}} \\ = 100 - \frac{7.7895 * 10^{12}}{D_1^{4.87}} - \frac{1.7 * 10^{12}}{D_2^{4.87}} \end{aligned} \tag{2}$$

3. Node 3 :

$$\begin{aligned} H_3 = H_1 - \Delta H_{13} = H_1 - \frac{4.457 * 10^8 * 400 * 2^{1.85}}{D_3^{4.87}} \\ = 100 - \frac{4.457 * 10^8 * 300 * 9^{1.85}}{D_1^{4.87}} - \frac{4.457 * 10^8 * 400 * 2^{1.85}}{D_3^{4.87}} \\ = 100 - \frac{7.7895 * 10^{12}}{D_1^{4.87}} - \frac{6.4269 * 10^{11}}{D_3^{4.87}} \end{aligned} \tag{3}$$

3 Cost function

Given the cost function per unit length, we get the follwing expression for the total cost for the entire network.

$$C = 1.2654[300D_1^{1.327} + 500D_2^{1.327} + 400D_3^{1.327}] \tag{4}$$

4 Formulation of the problem

At the optimum, the heads must be the minimum specified in the problem. But, for the flow to sustain, the constraint on H_1 is redundant because,
$H_1 > H_2 \geqslant 89.5m > 79.5m$ and
$H_1 > H_3 \geqslant 84.5m > 79.5m$
Therefore, $H_2 = 89.5m$ and $H_3 = 84.5m$. The problem boils down to finding H_1.

With these heads at nodes 2 and 3, the following relations for the diameters in terms of H_1 alone can be derived:

$$\begin{aligned} D_1 &= \frac{443.739}{(100 - H_1)^{\frac{1}{4.87}}} \\ D_2 &= \frac{324.664}{(H_1 - 89.5)^{\frac{1}{4.87}}} \\ D_3 &= \frac{265.853}{(H_1 - 84.5)^{\frac{1}{4.87}}} \end{aligned} \tag{5}$$

The cost function can be written in terms of H_1 alone, with all the constraints satisfied, the problem is now converted to an **unconstrained optimization problem in one variable.**

$$\begin{aligned} C = 1.2654[300(\frac{443.739}{(100 - H_1)^{\frac{1}{4.87}}})^{1.327} + 500(\frac{324.664}{(H_1 - 89.5)^{\frac{1}{4.87}}})^{1.327} \\ +400(\frac{265.853}{(H_1 - 84.5)^{\frac{1}{4.87}}})^{1.327}] \end{aligned} \tag{6}$$

Since $H_1 < 100m$ for the flow to sustain, our solution must have $100m > H_1 > 89.5m$. The optimal H_1 must be sought in this interval.

5 Optimal solution

The solution can be sought in multiple ways. Analytical method involves finding the point where the deriavative is zero. Numeraical methods include Uniform search method, Golden search method, Fibbinocci search method etc.

5.1 Solution by Uniform Search method

The minimum of the cost function can be determined by choosing an interval of reasonable size, evaluating the cost at four equidistant points including the bounds. The equal part containing the highest cost can be eliminated. This is applicable only when the function is monotonous about the minimum. Once that part eliminated, the new band is divided again into three equal parts and the region containing the highest cost function is eliminated. This is done iteratively until we reach a convergent value for the cost function. The iterations for the problem with the initial band from 90m-99.5m is shown.

i	a	x_1	x_2	b	f(a)	f(x_1)	f(x_2)	f(b)
1	93.5	95.5	97.5	99.5	2116894	2074921	2136377	2606198
2	93.5	94.83333	96.16667	97.5	2116894	2078804	2081469	2136377
3	93.5	94.38889	95.27778	96.16667	2116894	2086873	2075109	2081469
4	94.38889	94.98148	95.57407	96.16667	2086873	2077087	2075112	2081469
5	94.98148	95.37654	95.7716	96.16667	2077087	2074886	2076256	2081469
6	94.98148	95.24486	95.50823	95.7716	2077087	2075232	2074936	2076256
7	95.24486	95.42044	95.59602	95.7716	2075232	2074859	2075193	2076256
8	95.24486	95.36191	95.47897	95.59602	2075232	2074905	2074891	2075193
9	95.36191	95.43995	95.51799	95.59602	2074905	2074861	2074956	2075193
10	95.36191	95.41394	95.46596	95.51799	2074905	2074860	2074877	2074956
11	95.36191	95.39659	95.43128	95.46596	2074905	2074868	2074859	2074877
12	95.39659	95.41972	95.44284	95.46596	2074868	2074859	2074862	2074877
13	95.39659	95.41201	95.42742	95.44284	2074868	2074861	2074858	2074862
14	95.41201	95.42229	95.43256	95.44284	2074861	2074859	2074859	2074862
15	95.41201	95.41886	95.42571	95.43256	2074861	2074859	2074858	2074859
16	95.41886	95.42343	95.42799	95.43256	2074859	2074858	2074858	2074859
17	95.42343	95.42647	95.42952	95.43256	2074858	2074858	2074859	2074859
18	95.42343	95.42546	95.42749	95.42952	2074858	2074858	2074858	2074859
19	95.42343	95.42478	95.42613	95.42749	2074858	2074858	2074858	2074858
20	95.42478	95.42568	95.42658	95.42749	2074858	2074858	2074858	2074858

H_1 tolerance=0.001m	
	Regions eliminated
	Solution interval
	Minimum Cost

The cost function reaches convergence in the region 95.42568m to 94.42658m. We may as well take **95.43m** to be the optimum, since the residue was set to be 0.001m. So the optimum cost of the pipeline network is **INR 20,74,858.37** The optimum diameters for the three links can be evaluated from equations (5) as below:

$$D_1 = 324.80mm$$
$$D_2 = 225.27mm$$
$$D_3 = 162.69mm$$

5.2 Solution by Golden Search method

Divide interval into 3 sections by adding two internal points between ends. Evaluate the function at the two internal points x1 and x2. If f(x1)<f(x2) the maximum is between x1 and xmx. Redefine range xmn = x1, xmx = xmx. Set the following conditions: (1) L = L1 + L2 (2) R = L/L2 = L2/L1. Thus, R = (sqrt(5)-1)/2 = .61803. (Golden ratio). **This method reduces function evaluation because every successive iteration re-uses one of the previous internal values.**

5.3 Solution by Fibbinocci Search method

The Fibonacci search is based on the use of the Fibonacci number series. Each n^{th} Fibonacci number is the sum of the preceding two numbers. The Fibonacci search begins by placing the experiments at a distance $d_1 = \frac{F_{n-1}}{F_n}L$ from each end of the range of values, where L is the range of lengths. The interval with highest functional value is eliminated as done previously. The new interval is again split with $d_2 = \frac{F_{n-3}}{F_{n-1}}L$. This is done iteratively to reach convergence. 'n' can be chosen to obtain the solution interval in 'n-1' iterations. This method in the limiting case becomes the Golden search method. This reaches faster convergence because it removes bigger chunks of intervals compared to the previous methods and is thus, the optimal method..