HOMEWORK 3

Prem Sagar S - AE14B021

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1 Regression problem

Pipe diameters	Cost per length
80	424
100	570
125	767
150	977
200	1431
250	1924
300	2451
350	3008
400	3591
450	4198
500	4828
600	6149
700	7545
750	8269

Table 1: Pipe cost

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The costs per length of the pipe seems to be increasing almost linearly with the pipe diameter. Hence a least squared linear regression solution would give a near approximate correlation between them.

1. Let the error be **e** for between the actual costs y_i and the predicted cost y_p for corresponding diameters x_i. The following objective function is to be minimized. (n - number of diameters considered)

$$e^2 = \sum(Y - y_i)^2 \quad (1)$$

$$y_p = mx_i + b \quad (2)$$

2. Combining (1.1) and (1.2),
$e^2 = \sum_i (y_i - mx_i - b)^2 = \sum_i (m^2 x_i^2 + 2mbx_i - 2mx_i y_i + b^2 - 2by_i + y_i^2)$

3. The error is a quadratic funtion of **m** and **b.** This convex function and has a least optimal solution when,

$$\frac{de^2}{dm} = 0 = 2m \sum x_i^2 + 2b \sum x_i - 2 \sum (x_i y_i) \tag{3}$$

and

$$\frac{de^2}{db} = 0 = 2m \sum x_i + 2 \sum b - 2 \sum y_i \tag{4}$$

4. Solving (1.3) & (1.4) for **m** and **b**,

$$m = \frac{n(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{n(\sum x_i^2) - (\sum x_i)(\sum x_i)} \tag{5}$$

$$b = \frac{\sum y_i - m \sum x_i}{n} \tag{6}$$

Sl.no	x_i	y_i	$x_i y_i$	x_i^2	x_i^3	x_i^4	$x_i^2 y_i$
1	80	424	33920	6400	512000	40960000	2713600
2	100	570	57000	10000	1000000	100000000	5700000
3	125	767	95875	15625	1953125	244140625	11984375
4	150	977	146550	22500	3375000	506250000	21982500
5	200	1431	286200	40000	8000000	1600000000	57240000
6	250	1924	481000	62500	15625000	3906250000	120250000
7	300	2451	735300	90000	27000000	8100000000	220590000
8	350	3008	1052800	122500	42875000	15006250000	368480000
9	400	3591	1436400	160000	64000000	25600000000	574560000
10	450	4198	1889100	202500	91125000	41006250000	850095000
11	500	4828	2414000	250000	125000000	62500000000	1207000000
12	600	6149	3689400	360000	216000000	129600000000	2213640000
13	700	7545	5281500	490000	343000000	240100000000	3697050000
14	750	8269	6201750	562500	421875000	316406250000	4651312500
$\sum$	**4955**	**46132**	**23800795**	**2394525**	**1361340125**	**844716350625**	**14835407975**

Table 2: Calculations required for **m** and **b,** n =14

- $m = \frac{14*23800795-46132*4955}{14*2394525-4955*4955} = 11.6624$
- $b = \frac{46132-11.6624*4955}{14} = -832.5093$
- The approximate correlation between the pipe diamter **x** and the cost per unit length **y** is given by (2),

$$y = 11.6624x - 832.5093 \tag{7}$$

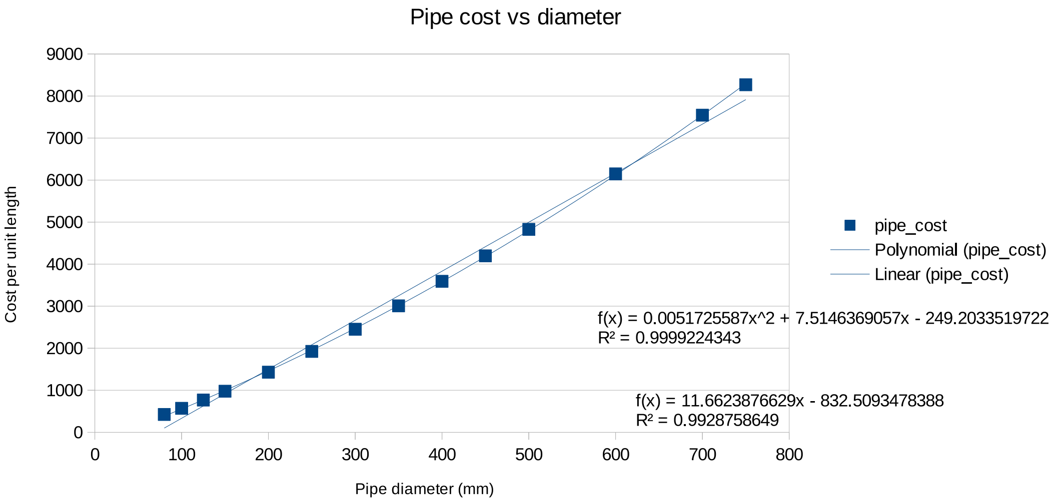

Figure 1: The regression equations verified on Libre office

2 Pipe optimization problem

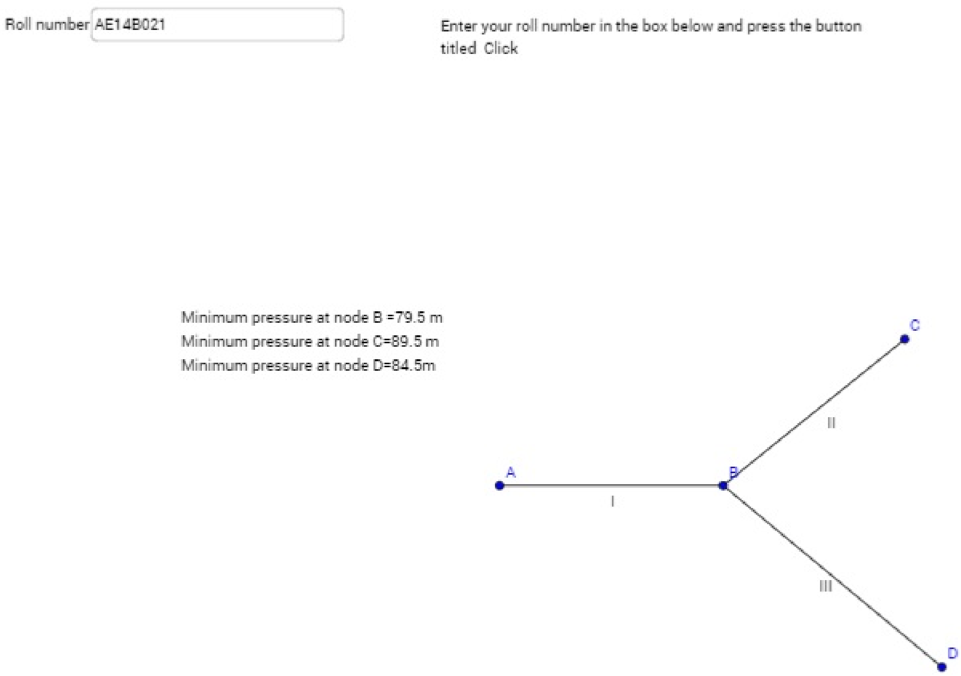

Figure 2: Network from HW1

The optimum diameters obtained were:

D_1	324.80mm	$9m^3/min$
D_2	225.27mm	$3m^3/min$
D_3	162.69mm	$2\ m^3/min$

From HW(1) the original cost function is,

$$C = 1.2654\left[300D_1^{1.327} + 500D_2^{1.327} + 400D_3^{1.327}\right]$$

Now in order to use the available diameters, each section of pipe is split such that the nearest available diameters are used. The length is kept the same

For AB, $L^I = 300 = L_1^I + L_2^I$ & nearest diameters are 300mm and 350mm.

For BC, $L^{II} = 500 = L_1^{II} + L_2^{II}$ & nearest diameters are 200mm and 250mm.

For BD, $L^{III} = 400 = L_1^{III} + L_2^{III}$ & nearest diameters are 150mm and 200mm.

- The new cost function is given by

$$
\begin{aligned}
C' &= 1.2654\left[L_1^I(300)^{1.327} + L_2^I(350)^{1.327} + L_1^{II}(200)^{1.327} + L_2^{II}(250)^{1.327} + L_1^{III}(150)^{1.327} + L_2^{III}(\right.\\
&= 2451.1378L_1^I + 3007.5034L_2^I + 1431.1812L_1^{II} + 1924.3950L_2^{II} + 977.0143L_1^{III} + 1431.1812L_2^{II}\\
&= 2451.1378L_1^I + 3007.5034(300 - L_1^I)\\
&\quad +1431.1812L_1^{II} + 1924.3950(500 - L_1^{II})\\
&\quad +977.0143L_1^{III} + 1431.1812(400 - L_1^{III})\\
&= 2436921 - 556.3656L_1^I - 493.2138L_1^{II} - 454.1669L_1^{III}
\end{aligned}
$$

This is equivalent to maximizing the objective function:

$$C = 556.3656L_1^I + 493.2138L_1^{II} + 454.1669L_1^{III} \tag{8}$$

  - Let X, Y & Z be the joints in AB, BC & BD connecting pipes of different diameters. For the minimum conditions, $H_C = 89.5m$ & $H_D = 84.5m$. The condition on H_B is redundant and thus $89.5 \leq H_B < 100$.

$$H_A - H_B = \Delta H_{AB} = 4.457 \times 10^8 L \frac{Q^{185}}{D^{4.87}}$$

$$
\begin{aligned}
\triangle H_{AB} &= \triangle H_{XB} + \triangle H_{AX} = 4.457{\times}10^8 L_1^I \frac{9^{1.85}}{300^{4.87}} + 4.457{\times}10^8 L_2^I \frac{9^{185}}{350^{4.87}} = 100 - H_B\\
H_B &= 100 - 2.2429 * 10^{-2} L_1^I - 1.0587 * 10^{-2}(300 - L_1^I)\\
&= 96.8239 - 1.1842 * 10^{-2} L_1^I \geq 89.5
\end{aligned}
$$

This condition is redundant for $0 < L_1^I < 300$.

$$
\begin{aligned}
\triangle H_{BC} &= \triangle H_{YC} + \triangle H_{BY} = 4.457 * 10^8 L_1^{II} \frac{3^{1.85}}{200^{4.87}} + 4.457 * 10^8 L_2^{II} \frac{3^{1.85}}{250^{4.87}} = H_B - H_C\\
H_B - H_c &= 2.1169 * 10^{-2} L_1^{II} + 0.7141 * 10^{-2}(500 - L_1^{II})\\
H_c &= 96.8239 - 1.1842 * 10^{-2} L_1^I - 1.4028 * 10^{-2} L_1^{II} - 3.5705\\
&= 93.2534 - 1.1842 * 10^{-2} L_2^I - 1.4028 * 10^{-2} L_2^{II} \geq 89.5\\
&\quad 1.1842L_1^I + 1.4028L_1^{II} \leq 375.34
\end{aligned}
$$

$$
\begin{aligned}
\triangle H_{BD} &= \triangle H_{ZD} + \triangle H_{BZ} = 4.457 * 10^8 L_1^{III} \frac{2^{1.85}}{150^{4.87}} + 4.457 * 10^8 L_2^{III} \frac{2^{1.85}}{200^{4.87}} = H_B - H\\
H_B - H_D &= 4.0587 * 10^{-2} L_1^{III} + 0.9998 * 10^{-2}(400 - L_1^{III}) = -3.0589 * 10^{-2} L_1^{III} + 3.9992\\
H_D &= 93.2713 - 1.1842 * 10^{-2} L_1^I - 3.0589 * 10^{-2} L_1^{III} + 3.9992\\
&= 77.0365 - 1.1842 * 10^{-2} L_1^I - 3.0589 * 10^{-2} L_2^{III} \geq 84.5\\
&\quad 1.1842L_1^I + 3.0589L_1^{III} \leq 832.47
\end{aligned}
$$