AS 5540 Assignment 1

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January 31, 2017

Problem 1

Given the initial position as

$$r_0 = 6837432.552\hat{i} + 1868795.099\hat{j} + 1455480.629\hat{k}m$$

and inital velocity

$$v_0 = -2294.079\hat{i} + 6758.849\hat{j} + 2049.468\hat{k}m/sec.$$

Using these data in the orbit.m code, the position and velocity of the satellite is calculated for 200 minutes.
From the results (subsequent pages 3-9), the plot of the trajectory is elliptic.
minimum altitude = 852.503 km at **0.280533 h** and 875.06 km at **1.12901 h**.
The total time elapsed in moving from minimum altitude to maximum altitude is half the time period.
Therefore, $T = 2 * 60 * (1.12901 - 0.0280533) = 101.8174$ minutes.
The length of the major axis = sum of minimum and maximum altitude+earth diameter

$$2a = 875.503 + 875.06 + 2 * 6378.137 = 14483.837km$$

$$a = 7241.9185km$$

The ellipse centre to earth's centre measures **ae**

$$ae = 7241.9185 - 6378.137 - 852.503$$

which gives,

$$e = 0.0015574$$

Type of trajectory	elliptic
semi-major axis	7241.9185 km
Eccentricity	0.0015574
Time period	101.8174 min

Similar orbit in Moon

If we consider an orbit with similar altitude in moon,

$$a = \frac{852.503 + 875.06 + 2 * 1737}{2} = 2600.7815$$

The initial altitude is taken same as in the previous case,

$$r_0^{moon} = (\left|r_0^{earth}\right| - 6378.137 + 1737)\frac{r_0^{earth}}{\left|r_0^{earth}\right|} = 2446.8\hat{i} + 668.8\hat{j} + 520.4\hat{k}km$$

$$\epsilon = -\frac{\mu}{2a} = \frac{v_0^2}{2} - \frac{\mu}{r_0}$$

From this,

$$|v_0^{moon}| = 1.3768km/s$$

The velocity is taken along the same direction given before, thus

$$v_0^{moon} = |v_0^{moon}|\frac{v_0}{|v_0|} = -0.4260\hat{\imath} + 1.2551\hat{\jmath} + 0.6806\hat{k}$$

Using these initial conditions, the orbit.m code is run.

Altitude	Speed	time
851.957 km (min)	1.37927 km/s	3.48498h
873.97 km (max)	1.36764 km/s	1.83926h

Time period = 2*(3.48498-1.83926)*60=197.48 minutes.
(Code runs are in page 9 & 10)

Problem 2

From Kepler's third law,

$$\frac{R_{mars}}{R_{earth}} = (\frac{T_{mars}}{T_{earth}})^{2/3}$$

therefore the mean radius of mars is given by,

$$R_{mars} = (\frac{687}{365.26})^{2/3} * 149.59787 * 10^6 = 227.9441713 * 10^6 km$$

Orbital speed of Earth,

$$v_{earth} = R_{earth}\omega_{earth} = \frac{R_{earth}2\pi}{T_{earth}} = \frac{2\pi * 149.59787 * 10^6}{365.26 * 24 * 60 * 60} = 29.784 km/s$$

Orbital speed of Mars,

$$v_{mars} = R_{mars}\omega_{mars} = \frac{R_{mars}2\pi}{T_{mars}} = \frac{2\pi * 227.9441713 * 10^6}{687 * 24 * 60 * 60} = 24.129 km/s$$

Assuming the mean radius, gravitational force of Sun on Earth, ($\hat{r_{se}}$ is directed from Earth to sun and vice versa)

$$F_{es} = -\frac{Gm_{earth}m_{sun}}{R_{earth}^2}\hat{r_{es}} = -\frac{398600 km^3/s^2 * 1.989\times10^{30} kg}{(149.59787 * 10^6)^2 km^2}\hat{r_{es}} = -3.542593 * 10^{19} N\hat{r_{es}}$$

From Newton's third law, force on sun due to earth is

$$F_{se} = -F_{es} = +3.542593 * 10^{19} N\hat{r_{es}}$$

Assuming the mean radius, gravitational force of Sun on Mars, ($\hat{r_{ME}}$ is directed from Mars to sun and vice versa)

$$F_{ms} = -\frac{Gm_{mars}m_{sun}}{R_{mars}^2}\hat{r_{ms}} = -\frac{42828 km^3/s^2 * 1.989\times10^{30} kg}{(385.884508 * 10^6)^2 km^2}\hat{r_{ms}} = -5.720683 * 10^{17} N\hat{r_{ms}}$$

From Newton's third law, force on Sun due to Mars is

$$F_{sm} = -F_{ms} = +5.720683 * 10^{17} N\hat{r_{ms}}$$

```
clc; close all; clear all
%...Input data:
% Earth:
m1 = 5.974e24;% *4903/398600;                %moon %5.974e24 - earth;
R =6378.137 ;                           %6378.137%1737;
m2 = 1000;
v0 = [-2.294079 6.758849 2.049468] ;
r0 = [6837.432552   1868.795099  1455.480629 ];
t0 = 0;
tf = 200*60;                           %200*hours;
%...End input data

orbit(m1,m2,R,r0,v0,t0,tf)
```

```
t =

   1.0e+04 *

         0
    0.0095
    0.0189
    0.0284
    0.0379
    0.0472
    0.0561
    0.0651
    0.0740
    0.0829
    0.0919
    0.1010
    0.1101
    0.1192
    0.1283
    0.1375
    0.1467
    0.1560
    0.1653
    0.1746
    0.1840
    0.1934
    0.2027
    0.2119
    0.2211
    0.2303
    0.2396
    0.2488
    0.2581
    0.2675
    0.2768
    0.2862
    0.2956
    0.3050
    0.3145
    0.3239
```

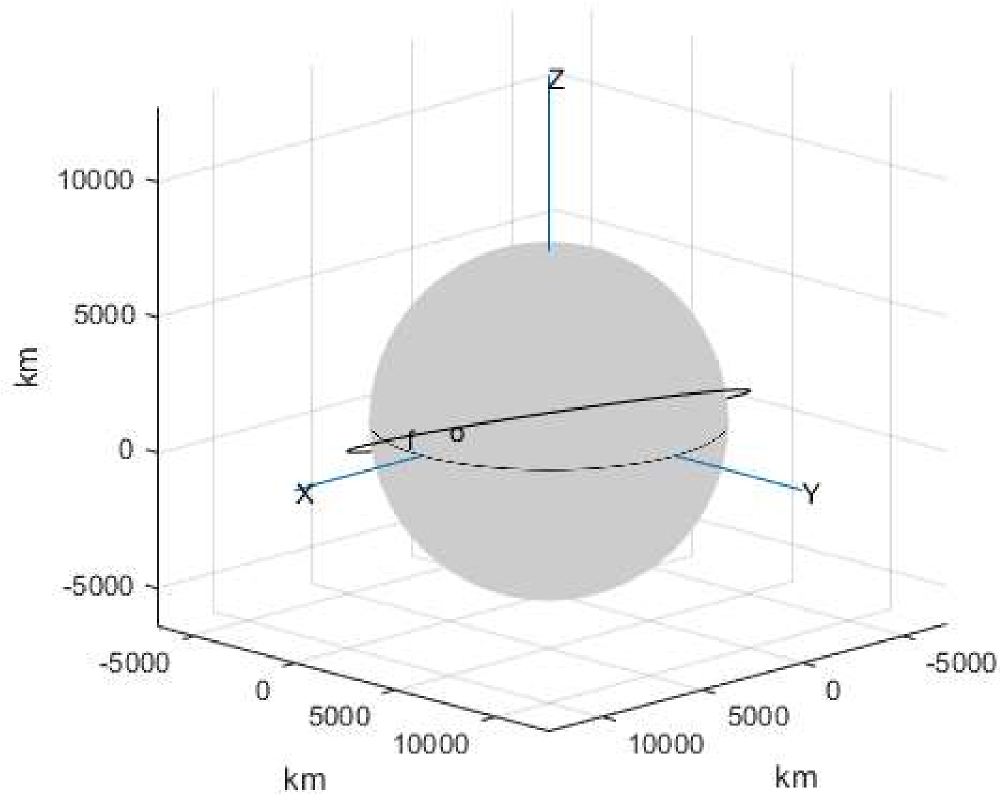
0.3334
0.3430
0.3523
0.3614
0.3703
0.3793
0.3883
0.3974
0.4064
0.4156
0.4247
0.4339
0.4431
0.4524
0.4617
0.4710
0.4804
0.4898
0.4993
0.5086
0.5178
0.5270
0.5363
0.5455
0.5548
0.5641
0.5734
0.5828
0.5921
0.6015
0.6109
0.6203
0.6298
0.6393
0.6488
0.6581
0.6672
0.6761
0.6850
0.6939
0.7029
0.7120
0.7210
0.7301
0.7393
0.7484
0.7576
0.7669
0.7762
0.7855
0.7949
0.8043
0.8137
0.8228
0.8320
0.8412
0.8505

```
    0.8597
    0.8690
    0.8784
    0.8877
    0.8971
    0.9065
    0.9159
    0.9253
    0.9348
    0.9443
    0.9538
    0.9632
    0.9724
    0.9813
    0.9903
    0.9992
    1.0083
    1.0174
    1.0265
    1.0356
    1.0448
    1.0540
    1.0633
    1.0726
    1.0819
    1.0912
    1.1007
    1.1101
    1.1196
    1.1287
    1.1379
    1.1471
    1.1564
    1.1657
    1.1750
    1.1843
    1.1936
    1.2000


y =

   1.0e+03 *

    6.8374    1.8688    1.4555   -0.0023    0.0068    0.0020
    6.5886    2.4984    1.6422   -0.0030    0.0065    0.0019
    6.2777    3.1044    1.8135   -0.0036    0.0063    0.0017
    5.9066    3.6829    1.9681   -0.0042    0.0059    0.0015
    5.4781    4.2281    2.1044   -0.0048    0.0055    0.0013
    5.0112    4.7196    2.2178   -0.0053    0.0051    0.0011
    4.5158    5.1553    2.3086   -0.0057    0.0046    0.0009
    3.9850    5.5453    2.3794   -0.0061    0.0041    0.0007
    3.4209    5.8889    2.4304   -0.0065    0.0036    0.0005
    2.8249    6.1845    2.4611   -0.0068    0.0030    0.0002
    2.2023    6.4287    2.4709   -0.0070    0.0024   -0.0000
    1.5580    6.6186    2.4595   -0.0072    0.0018   -0.0002
    0.8978    6.7519    2.4268   -0.0073    0.0011   -0.0005
```

0.2271	6.8267	2.3727	-0.0074	0.0005	-0.0007
-0.4480	6.8417	2.2975	-0.0074	-0.0002	-0.0009
-1.1217	6.7962	2.2018	-0.0073	-0.0008	-0.0012
-1.7880	6.6897	2.0859	-0.0072	-0.0015	-0.0014
-2.4407	6.5226	1.9509	-0.0069	-0.0021	-0.0016
-3.0741	6.2957	1.7975	-0.0067	-0.0028	-0.0017
-3.6821	6.0101	1.6271	-0.0063	-0.0034	-0.0019
-4.2591	5.6677	1.4408	-0.0060	-0.0039	-0.0021
-4.7996	5.2704	1.2400	-0.0055	-0.0045	-0.0022
-5.2891	4.8299	1.0304	-0.0050	-0.0050	-0.0023
-5.7247	4.3522	0.8144	-0.0045	-0.0054	-0.0024
-6.1097	3.8358	0.5912	-0.0039	-0.0058	-0.0025
-6.4417	3.2833	0.3619	-0.0033	-0.0062	-0.0025
-6.7170	2.6996	0.1285	-0.0027	-0.0065	-0.0025
-6.9325	2.0897	-0.1066	-0.0020	-0.0067	-0.0025
-7.0857	1.4592	-0.3415	-0.0013	-0.0069	-0.0025
-7.1747	0.8138	-0.5739	-0.0006	-0.0070	-0.0025
-7.1983	0.1593	-0.8016	0.0001	-0.0070	-0.0024
-7.1558	-0.4984	-1.0225	0.0008	-0.0070	-0.0023
-7.0470	-1.1531	-1.2345	0.0015	-0.0069	-0.0022
-6.8725	-1.7989	-1.4356	0.0022	-0.0068	-0.0021
-6.6333	-2.4296	-1.6239	0.0029	-0.0066	-0.0019
-6.3313	-3.0392	-1.7974	0.0035	-0.0063	-0.0017
-5.9686	-3.6221	-1.9546	0.0041	-0.0060	-0.0016
-5.5480	-4.1726	-2.0937	0.0047	-0.0056	-0.0014
-5.0849	-4.6734	-2.2107	0.0052	-0.0051	-0.0011
-4.5920	-5.1179	-2.3048	0.0057	-0.0047	-0.0009
-4.0653	-5.5148	-2.3785	0.0061	-0.0042	-0.0007
-3.5044	-5.8656	-2.4323	0.0064	-0.0036	-0.0005
-2.9111	-6.1686	-2.4658	0.0067	-0.0031	-0.0003
-2.2902	-6.4203	-2.4785	0.0070	-0.0025	-0.0000
-1.6470	-6.6179	-2.4698	0.0072	-0.0019	0.0002
-0.9870	-6.7588	-2.4398	0.0073	-0.0012	0.0004
-0.3158	-6.8413	-2.3882	0.0074	-0.0006	0.0007
0.3606	-6.8638	-2.3155	0.0074	0.0001	0.0009
1.0363	-6.8254	-2.2220	0.0073	0.0007	0.0011
1.7053	-6.7260	-2.1082	0.0072	0.0014	0.0013
2.3614	-6.5656	-1.9750	0.0070	0.0020	0.0015
2.9988	-6.3449	-1.8234	0.0067	0.0027	0.0017
3.6114	-6.0651	-1.6544	0.0064	0.0033	0.0019
4.1935	-5.7279	-1.4693	0.0060	0.0039	0.0020
4.7396	-5.3354	-1.2694	0.0055	0.0044	0.0022
5.2361	-4.8978	-1.0600	0.0051	0.0049	0.0023
5.6765	-4.4246	-0.8451	0.0045	0.0054	0.0024
6.0669	-3.9122	-0.6225	0.0040	0.0058	0.0025
6.4047	-3.3631	-0.3937	0.0034	0.0061	0.0025
6.6861	-2.7823	-0.1607	0.0027	0.0064	0.0025
6.9079	-2.1748	0.0745	0.0021	0.0067	0.0025
7.0677	-1.5462	0.3095	0.0014	0.0068	0.0025
7.1635	-0.9020	0.5422	0.0007	0.0070	0.0025
7.1940	-0.2481	0.7705	-0.0000	0.0070	0.0024
7.1583	0.4095	0.9922	-0.0007	0.0070	0.0023
7.0564	1.0647	1.2053	-0.0014	0.0069	0.0022
6.8887	1.7116	1.4076	-0.0021	0.0068	0.0021
6.6562	2.3441	1.5972	-0.0028	0.0066	0.0019
6.3607	2.9561	1.7724	-0.0034	0.0063	0.0018
6.0044	3.5419	1.9313	-0.0041	0.0060	0.0016

5.5898	4.0959	2.0723	-0.0046	0.0056	0.0014
5.1313	4.6016	2.1915	-0.0052	0.0052	0.0012
4.6423	5.0513	2.2879	-0.0056	0.0047	0.0010
4.1214	5.4518	2.3635	-0.0061	0.0042	0.0007
3.5659	5.8069	2.4194	-0.0064	0.0037	0.0005
2.9775	6.1149	2.4553	-0.0067	0.0032	0.0003
2.3611	6.3723	2.4704	-0.0070	0.0026	0.0001
1.7218	6.5761	2.4644	-0.0072	0.0019	-0.0002
1.0650	6.7238	2.4370	-0.0073	0.0013	-0.0004
0.3964	6.8135	2.3883	-0.0074	0.0007	-0.0007
-0.2781	6.8436	2.3184	-0.0074	0.0000	-0.0009
-0.9528	6.8134	2.2278	-0.0073	-0.0007	-0.0011
-1.6215	6.7222	2.1169	-0.0072	-0.0013	-0.0013
-2.2782	6.5703	1.9866	-0.0070	-0.0020	-0.0015
-2.9170	6.3583	1.8378	-0.0068	-0.0026	-0.0017
-3.5319	6.0874	1.6715	-0.0064	-0.0032	-0.0019
-4.1172	5.7590	1.4891	-0.0061	-0.0038	-0.0020
-4.6674	5.3754	1.2918	-0.0056	-0.0043	-0.0022
-5.1725	4.9430	1.0831	-0.0051	-0.0049	-0.0023
-5.6197	4.4762	0.8695	-0.0046	-0.0053	-0.0024
-6.0175	3.9696	0.6481	-0.0041	-0.0057	-0.0024
-6.3632	3.4257	0.4201	-0.0035	-0.0061	-0.0025
-6.6531	2.8494	0.1876	-0.0028	-0.0064	-0.0025
-6.8839	2.2456	-0.0473	-0.0022	-0.0066	-0.0025
-7.0530	1.6198	-0.2824	-0.0015	-0.0068	-0.0025
-7.1584	0.9776	-0.5156	-0.0008	-0.0069	-0.0025
-7.1986	0.3248	-0.7447	-0.0001	-0.0070	-0.0024
-7.1728	-0.3326	-0.9675	0.0006	-0.0070	-0.0023
-7.0806	-0.9887	-1.1820	0.0013	-0.0069	-0.0022
-6.9226	-1.6373	-1.3861	0.0020	-0.0068	-0.0021
-6.6996	-2.2723	-1.5777	0.0027	-0.0066	-0.0020
-6.4133	-2.8878	-1.7552	0.0033	-0.0064	-0.0018
-6.0656	-3.4780	-1.9166	0.0040	-0.0061	-0.0016
-5.6594	-4.0372	-2.0604	0.0046	-0.0057	-0.0014
-5.2048	-4.5524	-2.1834	0.0051	-0.0053	-0.0012
-4.7187	-5.0110	-2.2831	0.0056	-0.0048	-0.0010
-4.2022	-5.4185	-2.3617	0.0060	-0.0043	-0.0008
-3.6504	-5.7808	-2.4205	0.0063	-0.0038	-0.0005
-3.0648	-6.0962	-2.4593	0.0067	-0.0032	-0.0003
-2.4504	-6.3613	-2.4773	0.0069	-0.0026	-0.0001
-1.8124	-6.5729	-2.4741	0.0071	-0.0020	0.0002
-1.1561	-6.7285	-2.4494	0.0073	-0.0014	0.0004
-0.4872	-6.8259	-2.4034	0.0073	-0.0007	0.0006
0.1884	-6.8638	-2.3360	0.0074	-0.0001	0.0008
0.8649	-6.8410	-2.2477	0.0073	0.0006	0.0011
1.5361	-6.7571	-2.1390	0.0072	0.0012	0.0013
2.1961	-6.6121	-2.0107	0.0070	0.0019	0.0015
2.8388	-6.4067	-1.8637	0.0068	0.0025	0.0017
3.4583	-6.1418	-1.6990	0.0065	0.0031	0.0018
4.0487	-5.8190	-1.5178	0.0061	0.0037	0.0020
4.6045	-5.4403	-1.3216	0.0057	0.0043	0.0021
5.1167	-5.0111	-1.1132	0.0052	0.0048	0.0023
5.5688	-4.5492	-0.9007	0.0047	0.0053	0.0024
5.9720	-4.0468	-0.6801	0.0041	0.0057	0.0024
6.3236	-3.5066	-0.4526	0.0035	0.0060	0.0025
6.6197	-2.9334	-0.2205	0.0029	0.0064	0.0025
6.8570	-2.3323	0.0143	0.0022	0.0066	0.0025

```
   7.0329   -1.7085    0.2495    0.0016    0.0068    0.0025
   7.1452   -1.0678    0.4831    0.0009    0.0069    0.0025
   7.1925   -0.4158    0.7127    0.0002    0.0070    0.0024
   7.1797    0.0330    0.8654   -0.0003    0.0070    0.0024


--------------------------------------------------------

 Earth Orbit
 30-Jan-2017 18:47:35

 The initial position is [6837.43, 1868.8, 1455.48] (km).
 Magnitude = 7236.11 km

 The initial velocity is [-2.29408, 6.75885, 2.04947] (km/s).
 Magnitude = 7.42598 km/s

 Initial time = 0 h.
 Final time = 3.33333 h.

 The minimum altitude is 852.503 km at time = 0.280533 h.
 The speed at that point is 7.43159 km/s.

 The maximum altitude is 875.06 km at time = 1.12901 h.
 The speed at that point is 7.40848 km/s

--------------------------------------------------------
```



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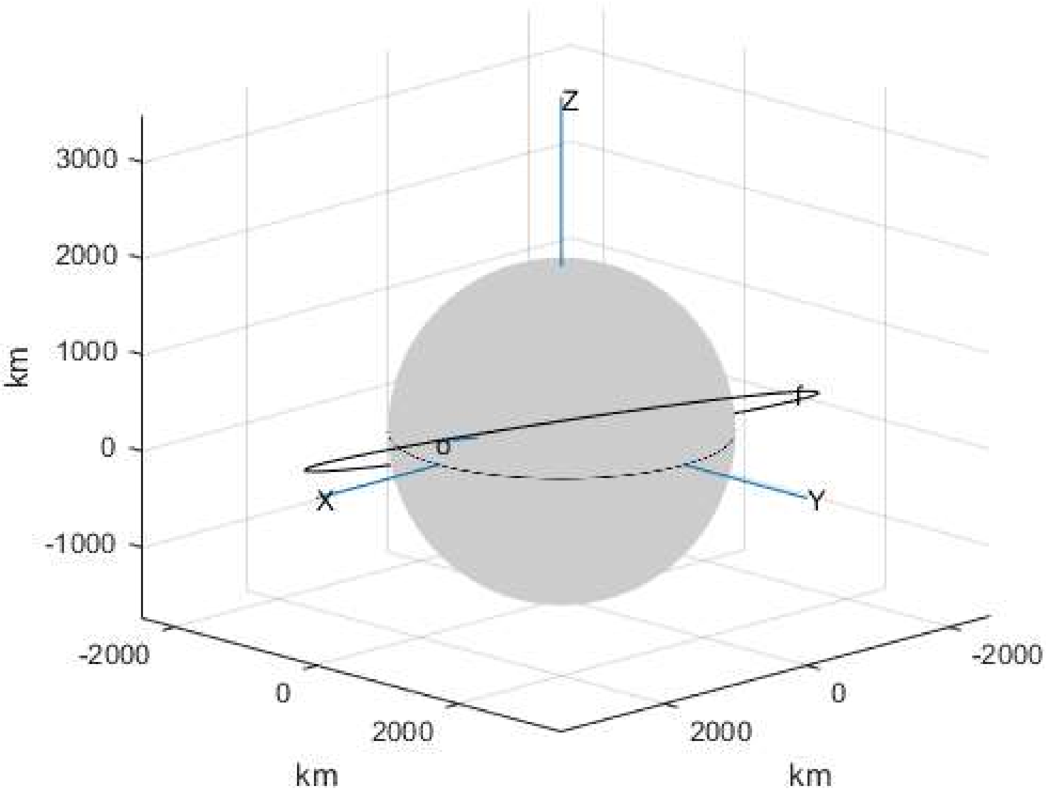
```
clc; close all; clear all
%...Input data:
% Moon:
m1 = 5.974e24*4903/398600;                %moon %5.974e24 - earth;
R =1737 ;                           %6378.137%1737;
m2 = 1000;
v0 = [-0.4260    1.2551    0.3806] ;
r0 = 1.0e+03 *[2.4468    0.6688    0.5209];
t0 = 0;
tf = 250*60;                        %200*hours;
%...End input data

orbit(m1,m2,R,r0,v0,t0,tf)
```

```
--------------------------------------------------------


 Earth Orbit
 30-Jan-2017 21:30:56

 The initial position is [2446.8, 668.8, 520.9] (km).
 Magnitude = 2589.49 km

 The initial velocity is [-0.426, 1.2551, 0.3806] (km/s).
 Magnitude = 1.37899 km/s

 Initial time = 0 h.
 Final time = 4.16667 h.

 The minimum altitude is 851.957 km at time = 3.48498 h.
 The speed at that point is 1.37927 km/s.

 The maximum altitude is 873.97 km at time = 1.83926 h.
 The speed at that point is 1.36764 km/s

--------------------------------------------------------
```



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Problem 3

It is assumed that the satellite is orbiting the earth. Given that the satellite is moving in an elliptical orbit, the specific energy is given by,

$$\epsilon = -\frac{\mu}{2a} = -16.61320 km^2/s^2$$

Perigee radius is given by r_p =**a(1-e)**

It can also be calculated as,

$$r_p = \frac{h^2/\mu}{1 + ecos(0)} = \frac{h^2/\mu}{1+e}$$

$$a(1-e) = \frac{h^2/\mu}{1+e}$$

Thus, the specific angular momentum can be obtained as,

$$h = \sqrt{\mu a(1-e^2)}$$

The code enclosed has assumed satellite mass is negligible compared to Earth's mass.

$$\epsilon = -16.61320 km^2/s^2$$

$$h = 6.726037 * 10^4 km^2/s$$

Radial velocity

$$v_r = \dot{r} = \frac{d}{dt}(\frac{h^2/\mu}{1+ecos\theta}) = \frac{h^2/\mu}{-(1+ecos\theta)^2}(-esin\theta)\dot{\theta}$$

$$= \frac{h^4/\mu^2}{(1+ecos\theta)^2}(esin\theta)\dot{\theta} * \frac{1}{h^2/\mu} = r^2\dot{\theta} * esin\theta * \frac{\mu}{h^2}$$

$$= h * esin\theta * \frac{\mu}{h^2} = \frac{\mu esin\theta}{h}$$

Normal velocity

The angular momentum is given by,

$$h = \vec{r} \times \vec{v} = \vec{r} \times (\vec{v_\perp} + \vec{v_r}) = \vec{r} \times \vec{v}_\perp + \vec{r} \times \vec{v}_r = \vec{r} \times \vec{v_\perp} + 0$$

where, $\vec{v_\perp}$and $\vec{v_r}$ are the normal and radial velocities respectively.

This simplifies to,

$$h = rv_\perp$$

$$v_\perp = \frac{h}{r} = \mu\frac{1+ecos\theta}{h}$$

Flight path angle

Flight path angle γ is the angle the velocity vector makes with the normal velocity.

$$tan\gamma = \frac{v_r}{v_\perp}$$

$$\gamma = tan^{-1}[\frac{\frac{\mu esin\theta}{h}}{\mu\frac{1+ecos\theta}{h}}] = tan^{-1}(\frac{esin\theta}{1+ecos\theta})$$

- The plots of radial and normal velocities against the true anomaly are given in the subsequent pages 14-19.

- **Positive** radial velocity means that the satellite is moving **away** from the earth in the radial direction and vice versa. Therefore the satellite is moving away when it goes from perigee to apogee and is moving towards the earth when it goes from apogee to perigee.
- Normal velocity is maximum at perigee and minimum at apogee. It decreases as the satellite moves from perigee to apogee and increases as the satellite moves from apogee to perigee.
- The plots are inherently sinusoidal but the normal and radial velocities have a phase difference of $\frac{\pi}{2}$.
- For the bigger orbit, the magnitude of radial and velocities increase.
- For the first case,i.e, e=0.2334 the flight path angle against θ is also sinusoidal.
- For the second case with high eccentricity e=0.95, the change in path angle is abrupt at $\theta = 180^0$. Thus, the plot looks like a skewed sine wave.
- When the eccentricity is changed from 0.2334 to 0.95, the ellipse's major axis shifts by $\frac{\pi}{2}$. Also by increasing **a,** the orbit becomes larger.

Codes for *(i)a=12000km, e=0.2334 - Page 14-16*
(ii) a=140000km, e=0.95 - Page 17-19

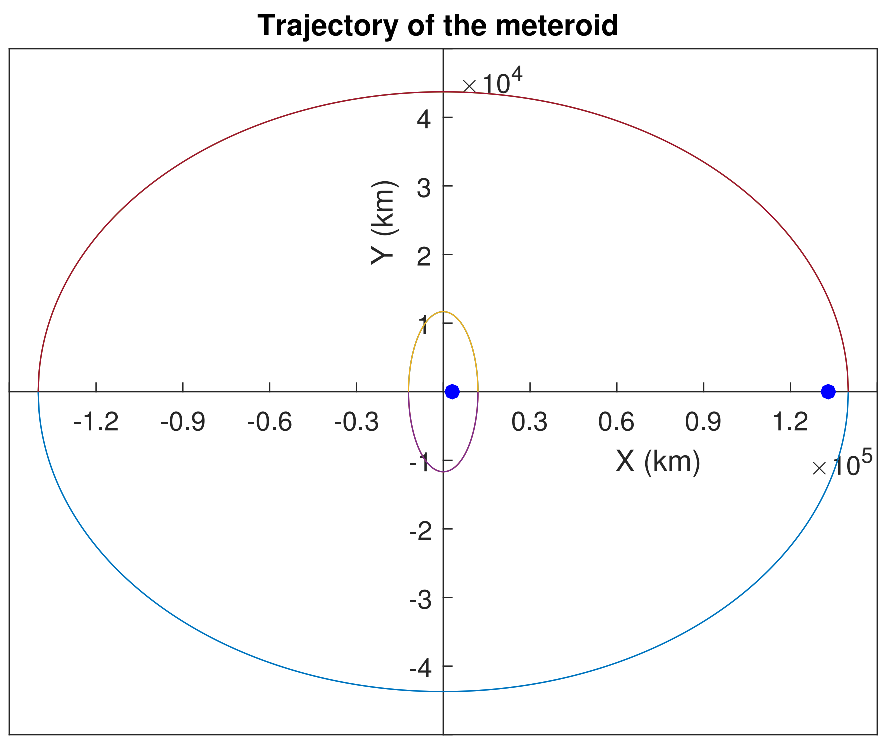
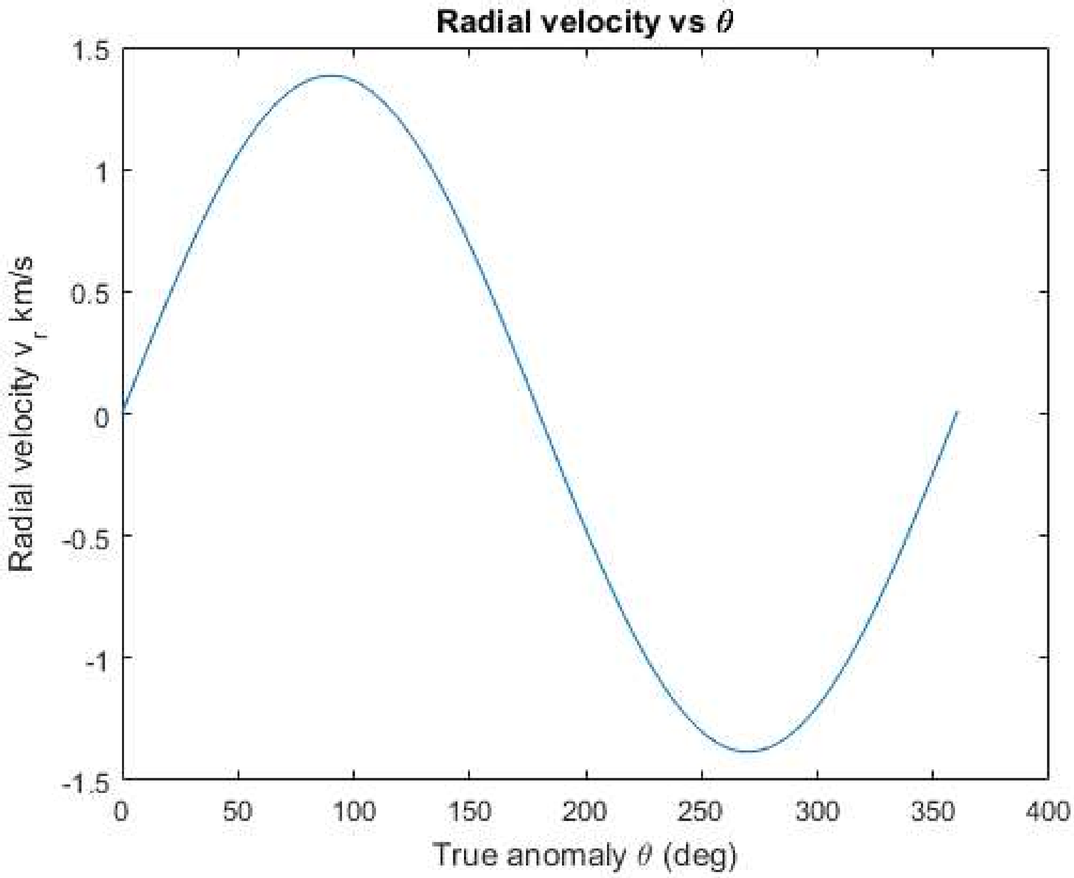
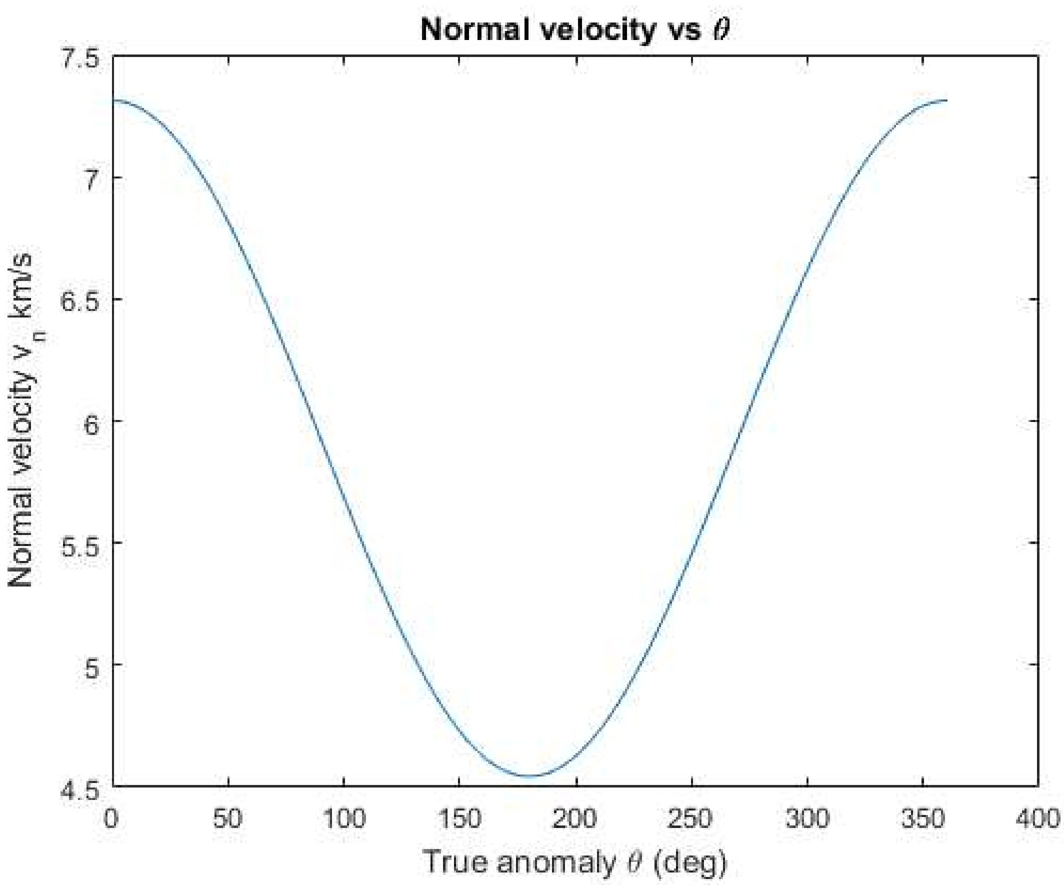


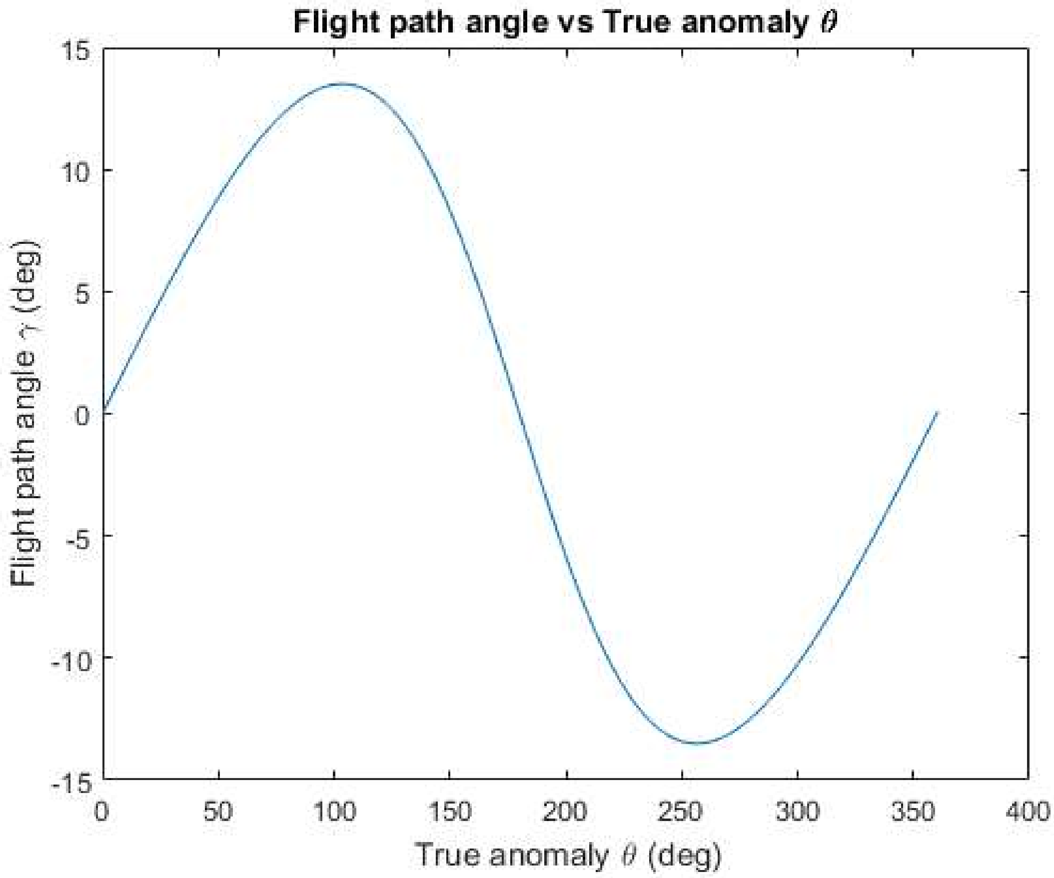
Figure 1: *Trajectories for (i)a=12000km, e=0.2334 - smaller ellipse (ii) a=140000km, e=0.95 (The orbits are not planet-shifted)*

```
clc; close all; clear all
a=12000  ;
e=0.2334;
m1=5.974e24;
m2=00;
mu=6.6742e-20*(m1+m2)
spec_e=-mu/2/a;
r_p=a*(1-e);
h=sqrt(r_p*mu*(1+e));

radVel_theta(e,h); % plots radial velocity against theta
normVel_theta(e,h); % plots normal velocity against theta
pathAngle(e); % plots flight path angle against theta

fprintf('\n\n--------------------------------------------------------\n')
fprintf('\n Specific energy\n')
fprintf(' %s\n',spec_e )
fprintf('\n Specific angular momentum \n')
fprintf(' %s\n', h )
fprintf('\n--------------------------------------------------------\n')
```

```
mu =

   3.9872e+05



--------------------------------------------------------

 Specific energy
 -1.661320e+01

 Specific angular momentum
 6.726037e+04

--------------------------------------------------------
```


Radial velocity vs θ
Radial velocity v_r km/s
True anomaly θ (deg)
1.5
1
0.5
0
-0.5
-1
-1.5
0
50
100
150
200
250
300
350
400


Normal velocity vs θ
Normal velocity v_n km/s
True anomaly θ (deg)
7.5
7
6.5
6
5.5
5
4.5
0
50
100
150
200
250
300
350
400



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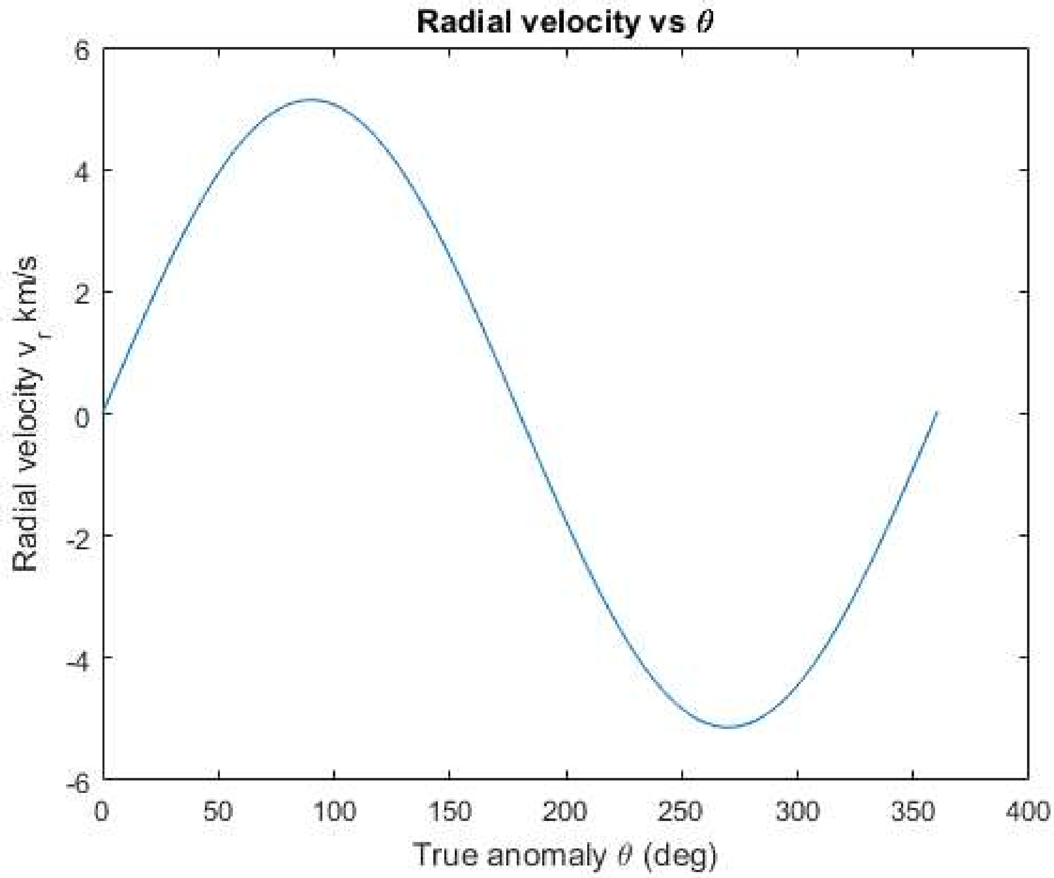
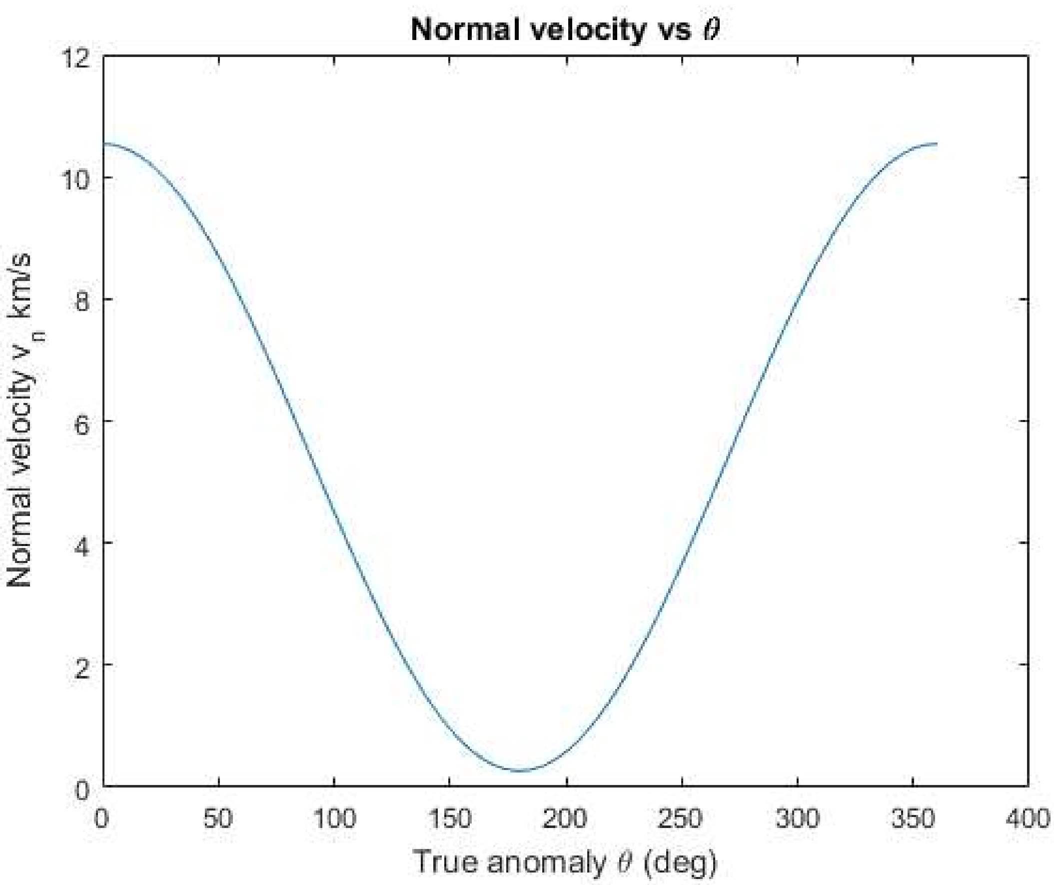
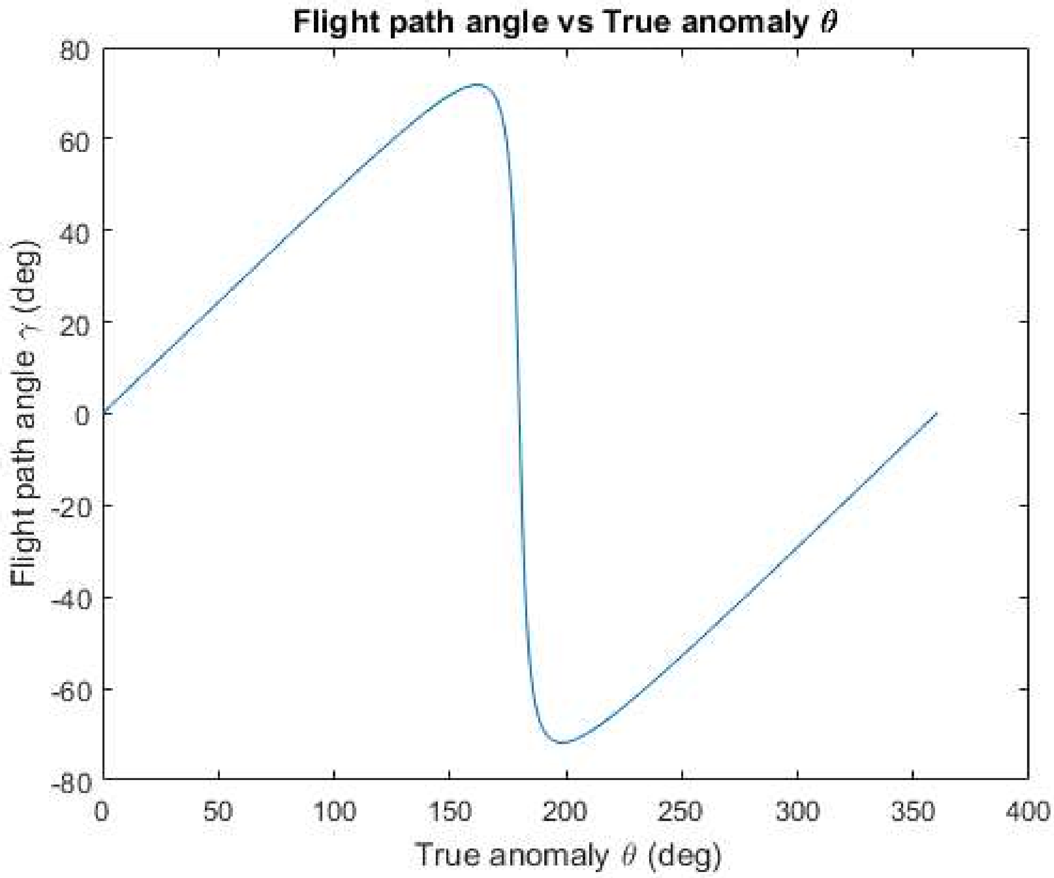
```matlab
clc; close all; clear all
a=140000  ;
e=0.95;
m1=5.974e24;
m2=00;
mu=6.6742e-20*(m1+m2)
spec_e=-mu/2/a;
r_p=a*(1-e);
h=sqrt(r_p*mu*(1+e));

radVel_theta(e,h); % plots radial velocity against theta
normVel_theta(e,h); % plots normal velocity against theta
pathAngle(e); % plots flight path angle against theta

fprintf('\n\n--------------------------------------------------------\n')
fprintf('\n Specific energy\n')
fprintf(' %s\n',spec_e )
fprintf('\n Specific angular momentum \n')
fprintf(' %s\n', h )
fprintf('\n--------------------------------------------------------\n')
```

```
mu =

   3.9872e+05



--------------------------------------------------------

 Specific energy
 -1.423988e+00

 Specific angular momentum
 7.377319e+04

--------------------------------------------------------
```


Radial velocity vs θ
Radial velocity v_r km/s
6
4
2
0
-2
-4
-6
0
50
100
150
200
250
300
350
400
True anomaly θ (deg)


Normal velocity vs θ
Normal velocity v_n km/s
12
10
8
6
4
2
0
0
50
100
150
200
250
300
350
400
True anomaly θ (deg)



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Problem 4

Given the meteoroid is at 402000km with $\theta = 150^0$and velocity v=2.23km/s, semi major axis can be computed from the specific energy relation.

$$\epsilon = \frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$$

$$a = \sqrt{\frac{\mu}{2(\frac{v^2}{2} - \frac{\mu}{r})}} =$$

From the previous problem, the total speed is obtained as the square of normal and radial velocities,

$$v^2 = v_r^2 + v_\perp^2 = [\frac{\mu esin\theta}{h}]^2 + [\frac{\mu(1+ecos\theta)}{h}]^2$$

$$v^2 = \frac{\mu^2}{h^2}[e^2sin^2\theta + 1 + e^2cos^2\theta + 2ecos\theta]$$

$$v^2 = \frac{\mu}{r(1+ecos\theta)}[e^2 + 1 + 2ecos\theta]$$

This can be simplified to a quadratic in **e,**

$$e^2 + (2 - \frac{v^2r}{\mu})cos\theta e + 1 - \frac{v^2r}{\mu} = 0$$

Since the meteoroid is known to be in a hyperbolic path, the positive root greater than 1 is taken as the eccentricity.

$$e = 1.085983$$

The radius at closest approach= $r_p = a(e-1) = 1.146873 * 10^4$ km.
Thus, the alititude at closest approach = 11468.73-6378.14 km = 5090.59 km.
The velocity at closest approach can be computed from the energy relation,

$$\epsilon = \frac{\mu}{2a} = \frac{v_p^2}{2} - \frac{\mu}{r_p}$$

$$v_p = \sqrt{2(\frac{\mu}{2a} + \frac{\mu}{r_p})} = 8.514 km/s$$

Hyperbolic excess velocity is defined as the velocity when the body arrives at infinity. It can be computed from the specific energy.

$$\epsilon = \frac{v^2}{2} - \frac{\mu}{r} = \frac{v_\infty^2}{2} = \frac{\mu}{2a}$$

$$v_\infty = \sqrt{\frac{\mu}{a}} = 1.72894 km/s$$

Having known the semi major axis and eccentricity, the trajectory can be determined from the equation of hyperbola given by,

$$\frac{x^2}{a^2} - \frac{y^2}{a^2(1-e^2)} = 1$$

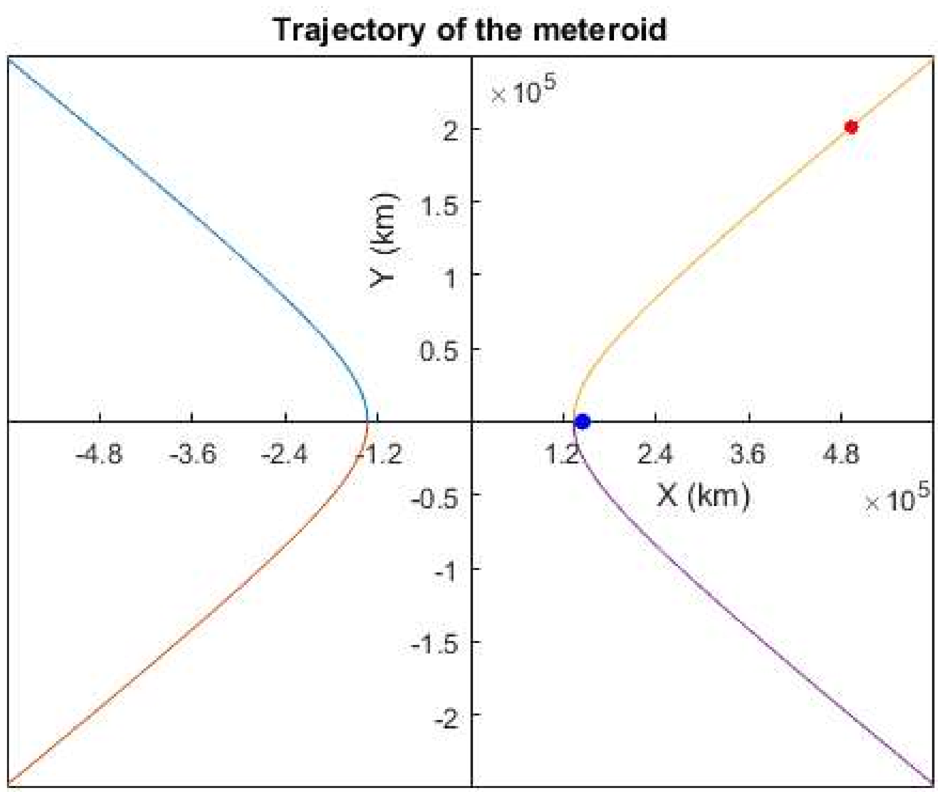
As r$\rightarrow \infty$, $(1 + ecos\theta_\infty) \rightarrow 0$, therfore,

$$\theta_\infty = cos^{-1}(\frac{-1}{e}) = cos^{-1}(\frac{-1}{1.085983}) = 157.0469^0$$

(Code runs are in pages 20-21)

```
clc; clear all;
r=402000;
R=6378;
mu=6.6742e-20*5.974e24;
v=2.23;
theta=150*pi/180;
a=mu*0.5/(0.5*v^2-mu/r); % semi major axis
c=1-v^2*r/mu;
e= max(roots([1 (1+c)*cos(theta) c])); %quadratic equation solvng eccentricity
r_p=a*(e-1); % altitude at closest approach
v_p=sqrt(mu*(1/2/a+1/r_p)); %speed at closest approach
v_inf=sqrt(mu/a); % hyperbolic excess velocity
b=a*sqrt(e^2-1);

%plots the trajectory

x1=linspace(-600000,-a,1000);
y1= b*sqrt(x1.^2/a^2-1);
y1a= -b*sqrt(x1.^2/a^2-1);
plot(x1,y1);
hold on
plot(x1,y1a);
x2=linspace( a, 600000,1000);
y2=b*sqrt(x2.^2/a^2-1);
y2a=-b*sqrt(x2.^2/a^2-1);
plot(x2,y2)
plot(x2,y2a)

newLim = get(gca,'XLim');
newx = linspace(newLim(1), newLim(2),11);
set(gca,'XTick', newx);
ax = gca;
ax.XAxisLocation = 'origin';
ax.YAxisLocation = 'origin';

 plot(a*e, 0,'.b', 'MarkerSize',20)
 plot(a*e-402000*cos(theta), 402000*sin(theta), '.r', 'MarkerSize',15)

 xlabel('X (km)')
ylabel('Y (km)')
title('Trajectory of the meteroid')

%displays results

fprintf('\n\n--------------------------------------------------------\n')
fprintf('\n Eccentricty \n')
fprintf(' %s\n',e )
fprintf('\n Altitude at closest approach\n')
fprintf(' %s km/s\n', r_p )
fprintf('\n Speed at closest approach\n')
fprintf(' %s km/s\n', v_p )
fprintf('\n Hyperbolic excess velocity\n')
fprintf(' %s km/s\n', v_inf )
fprintf('\n--------------------------------------------------------\n')
```

```
-------------------------------------------------------

 Eccentricty
 1.085983e+00

 Altitude at closest approach
 1.146873e+04 km/s

 Speed at closest approach
 6.021641e+00 km/s

 Hyperbolic excess velocity
 1.728940e+00 km/s

-------------------------------------------------------
```



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Problem 5

Given mean anomaly $M_e =$and eccentricity $e =$, the eccentric anomaly can be obtained from the Kepler's equation

$$M_e = E - esinE$$

The difficulty in obtaining an analytical solution for E gives way to using Newton Raphson method to obtain **E.**

$$E = 2.772571 radians$$

Pages 24 & 25 have the code runs.

```
% ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
function  kepler_E(e,M)
% ~~~~~~~~~~~~~~~~~~~~~~~~~~~
 e=0.7079772;
 M=144.225*pi/180;
%...Set an error tolerance:
error = 1.e-8;
%...Select a starting value for E:
if M < pi
E = M + e/2;
else
E = M - e/2;
end
%...Iterate on Equation 3.17 until E is determined to within
%...the error tolerance:
ratio = 1;
counter=1;
while abs(ratio) > error
    fprintf(' iteration = %d\n',counter)
    ratio = (E - e*sin(E) - M)/(1 - e*cos(E))
E = E - ratio;
counter=counter+1;
end

%kepler_E
fprintf('\n\n---------------------------------------------------------\n\n')
fprintf('\n Converged eccentric anomaly = %s  \n', E)
fprintf('\n---------------------------------------------------------\n\n')
```

```
 iteration = 1

ratio =

    0.0980

 iteration = 2

ratio =

   6.1100e-04

 iteration = 3

ratio =

   2.8679e-08

 iteration = 4

ratio =

  -2.6747e-16
```

```
-------------------------------------------------------

 Converged eccentric anomaly = 2.772571e+00

-------------------------------------------------------
```

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