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Determination of a preliminary orbit by the Laplace method

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The spherical coordinates of a celestial object obtained from observations are represented by polynomials. The approximate values of the first and second derivatives of the spherical coordinates, needed to determine the orbit by the Laplace method, are determined by differentiating these polynomials. An iterative process enabling one to improve simultaneously these derivatives and the orbital elements is described. Examples of the determination of the orbits of geostationary earth satellites by the proposed method are given.

1. INTRODUCTION

Interest in the Laplace method of determining an initial orbit[1] has recently increased considerably, and various modifications of it have been developed.[2-5] This is due to improvement in the technology of earth satellite position measurement and the more frequent use of television methods that enable one to obtain dense runs of satellite observations over relatively short time intervals. The Laplace method is especially convenient for processing such runs, since it permits one to use extra observations, in contrast to the Gauss method, in which the number of observations is strictly limited.

The classical version of the method, suggested by P. S. Laplace in 1780, is based on the assumption that the spherical coordinates α, δ of a celestial object and their first and second derivatives $\dot\alpha$, $\ddot\alpha$, $\dot\delta$, and $\ddot\delta$ are known from observations at a certain time t_0. To determine the orbit, one uses the geometrical relation

$$\mathbf{r}=\rho\mathbf{D}+\mathbf{R}, \tag{1}$$

where $\mathbf{r}$ is the position vector of the celestial object, $\mathbf{R}$ is the position vector of the observer, ρ is the distance from the observer to the celestial object, and $\mathbf{D}$ is the unit vector in the direction of the object at time t_0, as well as the equation of motion

$$\ddot{\mathbf{r}}=-G\mathbf{r}r^{-3}, \tag{2}$$

where G is the gravitational constant of the central object (the earth or the sun).

From (1) we obtain

$$\dot{\mathbf{r}}=\dot\rho\mathbf{D}+\rho\dot{\mathbf{D}}+\dot{\mathbf{R}}, \tag{3}$$

$$\ddot{\mathbf{r}}=\ddot\rho\mathbf{D}+\rho\ddot{\mathbf{D}}+2\dot\rho\dot{\mathbf{D}}+\ddot{\mathbf{R}}, \tag{4}$$

where the dots denote time differentiation. The unit vector $\mathbf{D}=(D_1, D_2, D_3)^T$ and its derivatives may be expressed in terms of the quantities α, $\dot\alpha$, $\ddot\alpha$, δ, $\dot\delta$, and $\ddot\delta$, known at the time t_0 from observations:

$$\mathbf{D}=\begin{pmatrix}\cos\alpha\cos\delta\\ \sin\alpha\cos\delta\\ \sin\delta\end{pmatrix}\quad \dot{\mathbf{D}}=\begin{pmatrix}-\dot\alpha D_2-\dot\delta\cos\alpha D_3\\ \dot\alpha D_1-\dot\delta\sin\alpha D_3\\ \dot\delta\cos\delta\end{pmatrix},$$
$$\ddot{\mathbf{D}}=\begin{pmatrix}-(\dot\alpha^2+\dot\delta^2)D_1+2\dot\alpha\dot\delta\sin\alpha D_3-\ddot\alpha D_2-\ddot\delta\cos\alpha D_3\\ -(\dot\alpha^2+\dot\delta^2)D_2-2\dot\alpha\dot\delta\cos\alpha D_3+\ddot\alpha D_1-\ddot\delta\sin\alpha D_3\\ -\dot\delta^2\sin\delta+\ddot\delta\cos\delta\end{pmatrix}. \tag{5}$$

From (1), (2), and (4) we obtain the (Lagrange) equations

$$\begin{cases}\rho=P-Qr^{-3}\\ r^2=\rho^2+2\rho(\mathbf{DR})+R^2\end{cases} \tag{6}$$

and the equation for determining $\dot\rho$,

$$\dot\rho=P'-Q'\rho, \tag{7}$$

where P, Q, P', and Q' are the known coefficients:

$$P=-\frac{\ddot{\mathbf{R}}(\dot{\mathbf{D}}\times\mathbf{D})}{\ddot{\mathbf{D}}(\mathbf{D}\times\mathbf{D})},\quad Q=G\frac{\mathbf{R}(\dot{\mathbf{D}}\times\mathbf{D})}{\ddot{\mathbf{D}}(\dot{\mathbf{D}}\times\mathbf{D})}, \tag{8}$$

$$P'=-\frac{\mathbf{R}(\ddot{\mathbf{R}}\times\mathbf{D})}{2\mathbf{R}(\dot{\mathbf{D}}\times\mathbf{D})},\quad Q'=\frac{\mathbf{R}(\ddot{\mathbf{D}}\times\mathbf{D})}{2\mathbf{R}(\dot{\mathbf{D}}\times\mathbf{D})}. \tag{9}$$

After a simultaneous solution of Eqs. (6) for ρ and r and a calculation of $\dot\rho$ using (7), Eqs. (1) and (3) yield the position and velocity vectors $\mathbf{r}$ and $\dot{\mathbf{r}}$ of the body at time t_0.

In the classical version of the Laplace method, it has been assumed that the orbit can be determined from three observations. Three observations are obviously incapable of providing high accuracy in the derivatives of the spherical coordinates or of yielding a satisfactory orbit.

Attempts to improve the method have involved two independent approaches.

In the first of these, the approximate derivatives of the spherical coordinates are found by differentiation of interpolation polynomials constructed from three observations. If the orbit obtained from these derivatives represents the observations with large deviations, then the orbit is improved from observations in successive approximations. This direction includes the work of Hartzer and Leuschner[1] and of Danjon.[1,2]

In Hartzer's method, the corrections to $\mathbf{r}$ and $\dot{\mathbf{r}}$ at time t_0 are determined from the deviations from the observed spherical coordinates. Five observations are needed to set up the equations in Hartzer's method. In Leuschner's method, instead of corrections to $\mathbf{r}$ one determines corrections to ρ, so that four parameters are corrected instead of six. Leuschner uses three observations to construct the iterative process.

Danjon suggested a different course. Suppose that, by applying the Laplace method to first order, we obtain calculated positions that are displaced relative to the observations by the amount of the deviations, which considerably exceed the observation error. We have a right to expect that an orbit determined by the same method from fictitious positions, displaced with respect to the initial ones by the amount of the deviations taken with the opposite sign, will better represent the available observations. Having obtained new deviations,

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