LA IDCT TRAMITE LA IDFT

Vogliamo calcolare $y = IDCT(c)$.

Cominciamo moltiplicando i c_k per $\alpha_k e^{i\frac{\pi k}{2N}}$:

$$\tilde{c}_k = \alpha_k c_k e^{i\frac{\pi k}{2N}}$$

calcoliamo la IDFT di $\tilde{c}$:

$$\tilde{y}_j = \sum_k \omega_N^{kj} \tilde{c}_k =$$

$$= \sum_k e^{i\frac{2\pi}{N}kj} \alpha_k c_k e^{i\frac{\pi k}{2N}} =$$

$$= \sum_k \alpha_k c_k e^{i\left[\frac{2\pi}{N}kj + \frac{\pi k}{2N}\right]} =$$

$$= \sum_k \alpha_k c_k \left\{ \cos\left[\frac{2\pi}{N}kj + \frac{\pi k}{2N}\right] + i \sin\left[\frac{2\pi}{N}kj + \frac{\pi k}{2N}\right]\right\}$$

Osserviamo che $\frac{2\pi}{N}kj + \frac{\pi k}{2N} = k\pi \cdot \left[2j + \frac{1}{2}\right] \cdot \frac{1}{N}$ quindi

$$Re\{\tilde{y}_j\} = \sum_k \alpha_k c_k \cos\left[k\pi\left(2j + \frac{1}{2}\right)\cdot\frac{1}{N}\right]$$

$2j$

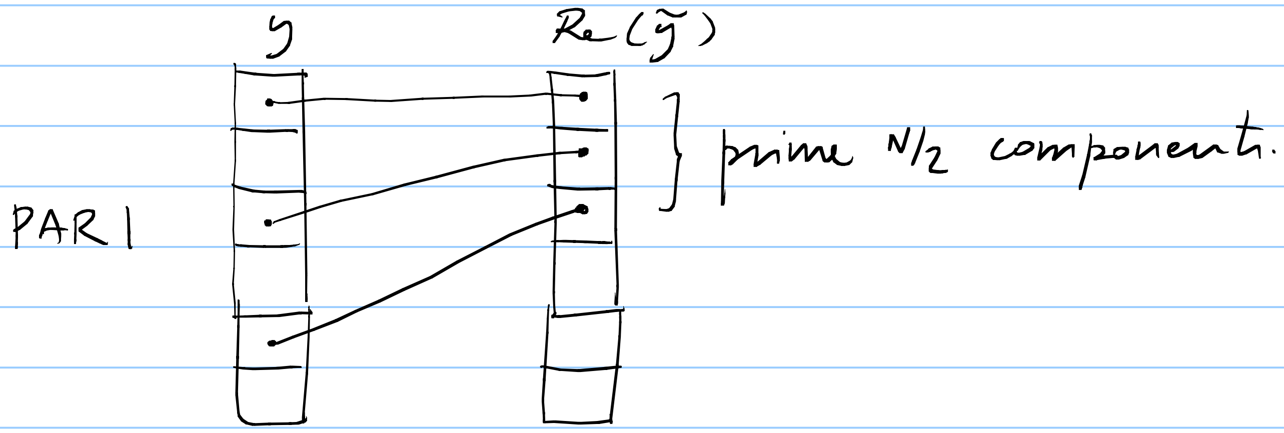
j

Ora $y = IDCT(c)$ significa

$$y_j = \sum_k \alpha_k c_k \cos\left[k\pi\left(j + \frac{1}{2}\right)\cdot\frac{1}{N}\right]$$

e quindi $y_{2j} = Re\{\tilde{y}_j\} \quad j=0,\dots,\frac{N}{2}-1$

Quindi le componenti PARI di y sono le prime $N/2$ componenti di $Re\{\tilde{y}_j\}$



Vediamo ora che le componenti dispari di y sono le ultime $N/2$ componenti di $Re\{\tilde{y}\}$ prese al contrario.

ultime $N/2$ di $Re\{\tilde{y}\}$: $\boxed{Re\{\tilde{y}_j\} \ ; \quad j = N/2, \dots, N-1}$

cambiamo l'indice : $j \leftrightarrow (N-1)-j$ in modo che le ultime $N/2$ componenti di $Re\{\tilde{y}\}$ in ordine inverso sono :

$$\boxed{Re\{\tilde{y}_{(N-1)-j}\} \ , \ j=0,\dots,\frac{N}{2}-1}$$

$$Re\{\tilde{y}_{(N-1)-j}\} = \sum_k \alpha_k c_k \cos\left[k\pi\left[2(N-1-j)+\frac{1}{2}\right]\cdot\frac{1}{N}\right]$$

$$= \sum_k \alpha_k c_k \cos\left[k\pi\left[2N-2-2j+\frac{1}{2}\right]\cdot\frac{1}{N}\right] =$$

$$= \sum_k \alpha_k c_k \cos\left[k\pi\left(2N-2j-\frac{3}{2}\right)\cdot\frac{1}{N}\right] =$$

(periodicità)

$$= \sum_k \alpha_k c_k \cos\left[k\pi\left(-2j-\frac{3}{2}\right)\cdot\frac{1}{N}\right] =$$

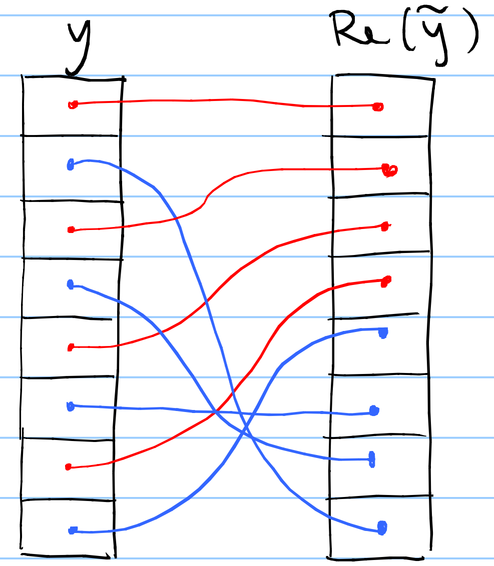
(parità)

$$= \sum_k \alpha_k c_k \cos\left[k\pi\left(2j+\frac{3}{2}\right)\cdot\frac{1}{N}\right] =$$

$$= \sum_k \alpha_k c_k \cos\left[k\pi\left((2j+1)+\frac{1}{2}\right)\cdot\frac{1}{N}\right]$$

$$= y_{2j+1}$$

ovvero $\quad y_{2j+1} = Re\{\tilde{y}_{(N-1)-j}\} \quad j = 0, \ldots, N/2 - 1$



~ PARI

~ DISPARI

ALGORITMO $\quad y = \mathrm{IDCT}(c)$

- moltiplicare c_k per $\alpha_k e^{i\frac{\pi k}{2N}}$:

$$\tilde{c}_k = \alpha_k c_k e^{i\frac{\pi k}{2N}}$$

- calcolare la IDFT di $\tilde{c}$

$$\tilde{y} = \mathrm{IDFT}(\tilde{c}) \qquad \text{(via IFFT)}$$

- risulta:

$$\boxed{\begin{array}{ll} y_{2i} = \mathrm{Re}\{\tilde{y}_i\} & i = 0, \dots, N/2 - 1 \\ y_{2i+1} = \mathrm{Re}\{\tilde{y}_{(N-1)-i}\} & i = 0, \dots, N/2 - 1 \end{array}}$$

~