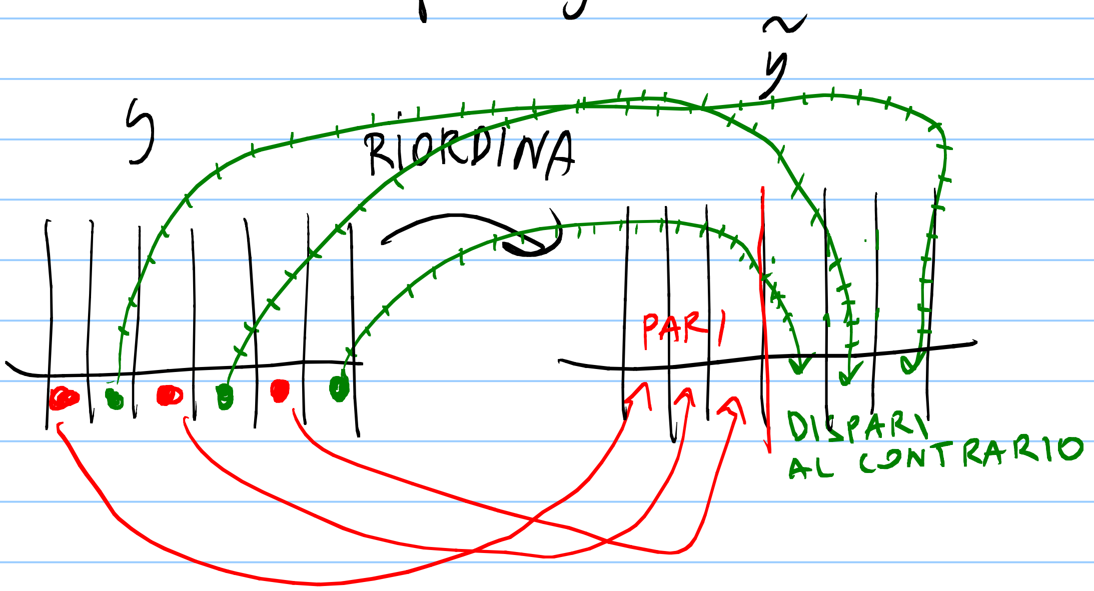
CALCOLO DELLA DCT CON LA DFT

4/12/2009

Riordino gli y:



$$\tilde{c} = \overset{(FFT)}{DFT} \text{ di } \tilde{y}$$

con $\tilde{c}$ costruisco $c := DCT$ di y.

$$\begin{tikzcd} y \arrow[r, squiggly, "\text{RIORDINO}"] \arrow[rrr, bend right, "DCT"'] & \tilde{y} \arrow[r, "\overset{(FFT)}{DFT}"] & \tilde{c} \arrow[r, "\overset{Re + Im}{\text{combino}}"] & c \end{tikzcd}$$

RIORDINO:

$$\begin{cases} \tilde{y}_i = y_{2i} & i = 0, \ldots, N/2 \\ \tilde{y}_{(N-1)-i} = y_{2i+1} & i = 0, \ldots N/2 - 1 \end{cases}$$

pari dritti

dispari al contrario

DCT:

$$c_k = \alpha_k \sum_{i=0}^{N-1} y_i \cos\left(k\pi \left(i + \frac{1}{2}\right) \cdot \frac{1}{N}\right) =$$

$$= \alpha_k \left(\sum_{\text{pari}} + \sum_{\text{dispari}} \right) =$$

è come questo!

$$= \boxed{\alpha_k} \left[\sum_{i=0}^{N/2-1} y_{2i} \boxed{\cos\left(k\pi \left(2i + \frac{1}{2}\right) \cdot \frac{1}{N}\right)} + \right.$$

pari

$$\left. + \sum_{i=0}^{N/2-1} y_{2i+1} \cos\left(k\pi \left((2i+1) + \frac{1}{2}\right) \frac{1}{N}\right) \right] =$$

dispari

inserisco gli $\tilde{y}_i$

$$= \boxed{\alpha_k} \left[\sum_{i=0}^{N/2-1} \tilde{y}_i \cos\left(k\pi \left(2i + \frac{1}{2}\right) \cdot \frac{1}{N}\right) + \right.$$

$$\left. + \sum_{i=0}^{N/2-1} \tilde{y}_{(N-1)-i} \cos\left(k\pi \left(2i + \frac{3}{2}\right) \cdot \frac{1}{N}\right) \right]$$

ora:

$$\sum_{i=0}^{N/2-1} \tilde{y}_{(N-1)-i} \cos(\ldots) =$$

(rigiro l'indice) $i \leftrightarrow (N-1) - i$

$$\begin{bmatrix} N-1 & 0 \\ N/2 & N/2-1 \end{bmatrix}$$

$$= \sum_{i=N/2}^{N-1} \tilde{y}_i \cos\left(k\pi \left(2[N-1-i] + \frac{3}{2}\right) \cdot \frac{1}{N}\right)$$

$$\parallel$$

$$\cos\left(k\pi \left(2N - \frac{1}{2} - 2i\right) \cdot \frac{1}{N}\right)$$

$$\parallel$$

$$\cos\left(k\pi \left(-\frac{1}{2} - 2i\right) \frac{1}{N}\right) = \boxed{\cos\left(k\pi \left(\frac{1}{2} + 2i\right) \frac{1}{N}\right)}$$

ok

(2) invece di (i)

$*$ Ricompongo:

$$\boxed{c_k = \boxed{\alpha_k} \sum_{i=0}^{N-1} \tilde{y}_i \cos\left(k\pi\left(2i+\frac{1}{2}\right)\cdot\frac{1}{N}\right)}$$

ho scritto la DCT in termini degli $\tilde{y}_i$

ora faccio la DFT degli $\tilde{y}_i \rightarrow \tilde{c}_k$:

$$\tilde{c} = \frac{1}{N} \bar{F}_N \tilde{y}$$

versione con $\frac{1}{N}$ (cf Strang)

$$\tilde{c}_k = \frac{1}{N} \sum_{j=0}^{N-1} \bar{w}^{kj} \tilde{y}_j =$$

$$= \frac{1}{N} \sum_{j=0}^{N-1} \left[\cos\left(-k\,2\pi\frac{j}{N}\right) + i\sin\left(-k\,2\pi\frac{j}{N}\right)\right]\tilde{y}_j$$

$$= \frac{1}{N} \sum_{j=0}^{N-1} \left[\cos\left(k\cdot 2\pi\frac{j}{N}\right) - i\sin\left(k\cdot 2\pi\frac{j}{N}\right)\right]\tilde{y}_j$$

sviluppo il coseno nei c_k in $(*)$

$$c_k = \boxed{\alpha_k} \sum_{i=0}^{N-1} \tilde{y}_i \left[\cos\left(k(2\pi)\frac{i}{N}\right)\cos\left(\frac{k\pi}{2N}\right) - \sin\left(k(2\pi)\frac{i}{N}\right)\sin\left(\frac{k\pi}{2N}\right)\right]$$

quindi:

$$\boxed{c_k = \boxed{N\alpha_k}\left\{\left(\cos\left[\frac{k\pi}{2N}\right]\right)\mathrm{Re}\{\tilde{c}_k\} + \left(\sin\left[\frac{k\pi}{2N}\right]\right)\mathrm{Im}\{\tilde{c}_k\}\right\}}$$

$$y \xrightarrow{\text{Riordino}} \tilde{y} \xrightarrow{\text{DFT}} \tilde{c} \xrightarrow[\text{COMBINO}]{\mathrm{Re}+\mathrm{Im}} c$$

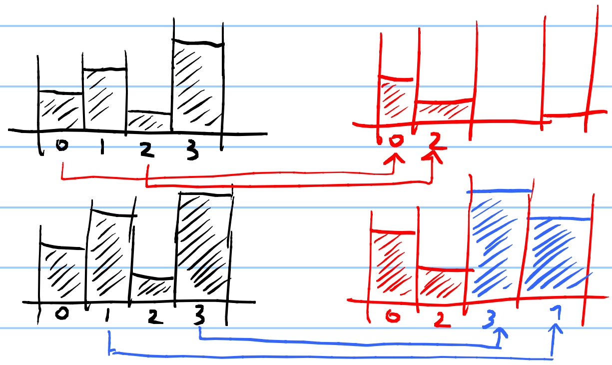
$$y \xrightarrow{\text{DCT}} c$$

CONFRONTO IN MATLAB

Con +1 sommato a tutti gli indici dei vettori usiamo le notazioni solite

```
%
% questo codice mostra come si ottiene la DCT dalla DFT (FFT)
%
clear
%
N=4;
y=rand(1,N);
%
% riordino gli y in yt
%
for i=0:N/2-1
    yt(i +1)=y(2*i +1);
end
%
for i=0:N/2-1
    yt(N-1-i +1)=y(2*i+1 +1);
end
%
% calcolo la dft degli yt
%
ct=fft(yt);
%
for k=0:N-1
    if k==0
        alfa(k +1)=1/sqrt(N);
    else
        alfa(k +1)=sqrt(2/N);
    end
end
%
% riassemblo i ct e ottengo i c
for k=0:N-1
    c(k +1)=alfa(k +1)*...
     (cos(k*pi/(2*N))*real(ct(k +1))+sin(k*pi/(2*N))*imag(ct(k +1)));
end
%
% confronto c con dct(y)
%
[c' dct(y)']
%
```

N deve essere pari.



Attenzione: in Matlab $\vec{c} = F_N \vec{y}$ quindi perdiamo un fattore N in $*$ (cf. Strang)

(*) [riga `c(k +1)=alfa(k +1)*...`]

```
>> dftdct

ans =

   1.013494961523395   1.013494961523395
  -0.064668718775003  -0.064668718775003
   0.315845629050704   0.315845629050704
  -0.166406306695575  -0.166406306695575

>>
```

Ok!