Chapter-8

* Chapter-8 નો Main goal [illegible] concept of statistics like sample mean and sample variance શીખવા નો and to derive the properties of such statistics.

Th.1
* $X_1, X_2, \ldots, X_n$ is random sample from $f(X)$, $E(X_i)=\mu$, $Var(X_i)=\sigma^2 < \infty$. Then

Imp. ① $E(\bar{X})=\mu$ (expected value of sample mean), $Var(\bar{X}) = \frac{\sigma^2}{n}$ $(T=\bar{X})$

(Variance of sample mean)

② $E(S^2)=\sigma^2$ (expected value of sample variance), &

$$Var(S^2) = \left(\mu_4 - \frac{n-3}{n-1}\sigma^4\right)\Big/ n, \quad n>1 \;\&\; \mu_4 = E(X-\mu)^4$$

$\llcorner$ 4th moment.

* Sample mean $(\bar{X}) = \frac{\sum X_i}{n}$, Sample variance $(S^2) = \frac{1}{n-1}\sum(X_i-\bar{X})^2$

* formulae $Var(X) = E(X^2) - \{E(X)\}^2$

$\therefore E(X^2) = Var(X) + \{E(X)\}^2$

* Theorem-2

$X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, Then

1) $\bar{X} \sim N(\mu, \sigma^2/n)$ (sample mean [illegible] distribute [illegible])
→ (mgf technique થી proof કરી શકાય)

2) $\bar{X}$ and $X_1-\bar{X}, X_2-\bar{X}, \ldots, X_n-\bar{X}$ are indep

3) $\bar{X}$ and S^2 are indep

4.) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}$

* Two quantity say Y_1 & Y_2 लाई independent देखाउनु छ भने, we can use either covariance technique or joint pdf technique. Covariance technique normal हुदा काम गर्छ but joint pdf technique more general तो always काम गर्छ।

* χ^2 distribution is special case of gamma distribution i.e

$$X \sim \chi^2(\nu) \iff X \sim \text{gamma}(\underset{\downarrow \text{scale}}{2}, \nu/2)$$

ν — degree of freedom

* properties ① $E(X) = 2 * \frac{\nu}{2} = \nu$

② $Var(X) = 2^2 * \frac{\nu}{2} = 2\nu$

③ $X \sim \text{gamma}(\theta, K)$ — θ scale, K shape → यो लाई mgf technique ले prove गर्न सकिन्छ

$$Y = \frac{2X}{\theta} \sim \text{gamma}(2, K) \equiv \chi^2(2K)$$

* मतलब यदी Y यसरी distribute भएको छ भने Y चाँही chi-squared distribute हुन्छ with 2K degree of freedom.

* If $X_i \overset{ind}{\sim} \text{gamma}(\theta, K_i)$. Then

$$Y = \sum X_i \sim \text{gamma}(\theta, \sum K_i)$$ (this can be shown using mgf technique)

4) * If $X_i \overset{ind}{\sim} \chi^2(\nu_i) \equiv \text{gamma}\left(2, \frac{\nu_i}{2}\right)$

$$Y = \sum X_i \sim \chi^2\left(\sum \nu_i\right) \equiv \text{gamma}\left(2, \frac{\sum \nu_i}{2}\right)$$

5) If $Z \sim N(0,1)$. Then

$$X = Z^2 \sim \chi^2(1)$$ → यसलाई पनि mgf technique ले देखाउन सकिन्छ

मतलब यदी normal का random variable लाई squre गरेर add गर्ने हो भने त्यो ले chi-square distribution दिन्छ and its degree of freedom is number of normal random variable added.

* Note $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$ & $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1)$

* T-distribution :-

If $Z \sim N(0,1)$, $V \sim \chi^2(\nu)$, Z indep V. Then

$$T = \frac{Z}{\sqrt{V/\nu}} \sim t(\nu)$$

* V को जे degree of freedom छ त्यही t को नि हुने हो

properties

$E(T) = 0 \qquad \nu > 1$

$Var(T) = \frac{\nu}{\nu - 2} \qquad \nu > 2$

* Theorem :-

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1), \text{ where } X_1, \ldots, X_n \sim N(\mu, \sigma^2)$$

* जसरी $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$ normaly distribute भएको छ

त्यसरी नै $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$. (Actually a theorem)

* F distribution :-

Theorem :- $X_1 \sim \chi^2(\nu_1)$, $X_2 \sim \chi^2(\nu_2)$ X_1 indep of X_2

degree of freedom of denominator

$$Y = \frac{X_1/\nu_1}{X_2/\nu_2} \sim F(\nu_1, \nu_2)$$

degree of freedom of nemurator

properties *

① $X \sim F(P,Q)$, then $Y = \frac{1}{X} \sim F(Q,P)$

② If $X \sim t(Q)$, then $Y = X^2 \sim F(1,Q)$

i.e यदी X चाँही t distribute भएको छ भने, X^2 चाँही F distribute हुन्छ ।

③ $X \sim F(P,Q)$ then $Y = \frac{(P/Q)X}{1+(P/Q)X} \sim \text{beta}\left(\frac{P}{2}, \frac{Q}{2}\right)$

* Random Variable भनेको sample पिच्छे value change हुन सक्ने i.e values can change from sample to sample. eg sample mean($\bar{X}$) [capital X]

* exponential distribution special case of gamma distribution हो ।

$X \sim GAM(\theta, K)$ ← exponential when $K=1$

scale shape

* $\Gamma(K) = (K-1)\Gamma(K-1)$ & $\Gamma(K) = (K-1)!$

* Suppose तपाईंलाई कुनै question आएको छ, $P(X<10)$ where $X \sim GAM(\theta, k)$ 4.2
Gamma को CDF त गाह्रो हुन्छ, integration sucks and table नि छैन। In this case the best idea is to transform it to chi-squared distribution.

* जसरी यदी random variables हरु Normally distribute भएका छन भने then their sum ΣX_i पनि Normally नै distribute हुन्छ। सो त्यसरी नै यदी random variables हरु यदी χ^2 (chi-squared) distribute भएका छन भने, तिनिहरुको sum पनि χ^2 (chi-squared) नै distribute हुन्छ। But Normal distribution को case मा sample mean $(\bar{X})$ पनि normally नै distribute हुन्छ that not the same case in χ^2 (chi-squared) distribution. i.e $\bar{X} \not\sim \chi^2$. But we still can use our χ^2 table to get probability of the things other than just a sum of χ^2-dists.

* But But But the only time a linear combination of χ^2-distributed R.V.s is itself going to be χ^2-distributed is if the coefficients are all positive 1. $(X+Y+Z+\dots)$. The normal distribution is much more flexible in comparison; coefficients may be >0, <0 etc.

* So the difference of two χ^2-distribution is not one of our known special distribution. We can not say anything about difference of two χ^2-distribution.

* So यदी question मा चाँही probability Z^2 wala term मागेर सोधेको भने, we can answer it using Normal distribution or χ^2-distribution. Eg

$P(Z^2 < 4.6)$

① First way: using standard normal table [need to go from Z^2 to Z]

② second way: using $\chi^2(1)$

* हामीलाई S^2 को distribution कस्तो idea छैन but हामी त्यसलाई अलि modify गर्न सक्छौं because we know the distribution of $\frac{(n-1)S^2}{\sigma^2}$ which is distributed χ^2 with $(n-1)$ degree of freedom. i.e $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

* Note:- S^2 को distribution σ^2 मा मात्र depend गर्छ
but $\bar{X}$ को distribution μ and σ^2 both मा depend गर्छ।

* V. imp Note:-

$$Z_i \sim \frac{Y_i - \mu_2}{\sigma_2^2} \sim N(0,1)$$

$$\sum_{i=1}^{m} Z_i \sim N(0,m)$$

$$Z = \frac{1}{\sqrt{m}} \sum_{i=1}^{m} Z_i \sim N(0,1)$$

$$Z_i' = \frac{X_i - \mu_1}{\sigma_1} \sim N(0,1)$$

$$Z_i'^2 = \left(\frac{X_i - \mu_1}{\sigma_1}\right)^2 \sim \chi^2(1)$$

$$\sum_{i=1}^{m} (Z_i')^2 \sim \chi^2(m)$$