Chapter - 10

{Sufficient statistics}

- We are trying to find out the estimator of unknown parameter i.e population parameter using sample parameter.

- Now In this chapter we will look for what will be the sufficient things from sample distribution to ~~ap~~ estimate population parameter.

Def^n:- A statistics $T = T(\vec{X})$ is a sufficient statistics for $\vec{\theta}$ if the conditional joint distribution of $\vec{X}$ given T is free of $\vec{\theta}$.

i.e $$f_{X|T}(X_1, \ldots, X_n | T) = \frac{f_{\vec{X}}(\vec{X}/\theta)}{f_T(T/\theta)} \text{ free of } \theta.$$

यसलाई working def^n पनि भनिन्छ र यसले sufficient statistic दिंदैन but यदी तपाइले कुनै statistics दिनु भयो भने त्यो sufficient छ कि छैन त्यो चाही भन्दछ।

- Theorem:- [factorization]

$\vec{T} = T(\vec{X})$ is sufficient for θ if and only if there exist a function $g(t/\theta)$ and a function $h(\vec{x})$ such that

$$\underbrace{f_{\vec{X}}(\vec{X}/\theta)}_{\text{product of individual pdf}} = g(\vec{t}/\vec{\theta}) \cdot h(\vec{X})$$

* यस्ता question solve गर्दा जब सम्म सकिन्छ population parameter (like $\mu, \theta, \sigma^2, k, \ldots$ etc) लाई sample parameter ($X_i, \Sigma X_i, \Sigma X_i^2, \ldots$) बाट seperate गरेर two function बनाउन सकिन्छ

① having only population parameter

② other having only sample parameter.

कुनै पनि case मा population parameter बाट अलग्ग नहुने sample parameter लाई T ले denote गर्ने & त्यो लाई पनि population parameter wala group $g(T|\theta)$ मा हाली दिने other लाई $h(\bar{x})$ मा हाली दिने।

And that value of T is called sufficient statistics.