1. (20pt) Assume X_1 and X_2 are i.i.d. normal random variables with mean μ variance σ^2. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$. Show that Y_1 and Y_2 are independent and find their distributions.

2. (20pt) $X_1, X_2, \cdots, X_n$ is an i.i.d sample from $N(\mu, \sigma^2)$. Let S^2 denote the sample variance. Find $E(S^2)$ and $Var(S^2)$.

3. (40pt) Let $Y_1, Y_2, \cdots, Y_5$ be a random sample of size 5 from a normal population with mean 0 and variance 1 and let $\bar{Y} = (1/5)\sum_{i=1}^{5} Y_i$. Let Y_6 be another independent observation from the same population. Find the distribution of the following and explain.

(a) $W = \sum_{i=1}^{5} Y_i^2$

(b) $U = \sum_{i=1}^{5} (Y_i - \bar{Y})^2$

(c) $U + Y_6^2$

(d)$2(5\bar{Y}^2 + Y_6^2)/U$

4. (20pt) Suppose that $X \sim \chi^2(m)$, $S = X + Y \sim \chi^2(m+n)$, and X and Y are independent. Use MGF to show that $S - X \sim \chi^2(n)$.

5. (Mandatory for Graduate Student. Extra credit for undergrad. 10pt)
Suppose that independent samples (of sizes n_i) are taken from each of k populations and that population i is normally distributed with mean μ_i and variance σ^2, $i = 1, 2, \cdots, k$. That is, all populations are normally distributed with the same variance but with (possibly) different means. Let $\bar{X}_i$ and S_i^2 , $i = 1, 2, \cdots, k$ be the respective sample means and variances. Let $\theta = c_1\mu_1 + c_2\mu_2 + \cdots + c_k\mu_k$, where $c_1, c_2, \cdots, c_k$ are given constants.

(a) Give the distribution of $\hat{\theta} = c_1\bar{X}_1 + c_2\bar{X}_2 + \cdots + c_k\bar{X}_k$.
(b) Give the distribution of

$$\frac{SSE}{\sigma^2}, \text{ where } SSE = \sum_{i=1}^{k} (n_i - 1)S_i^2.$$