Home Work #1 MVA Sagar Kalauni page-1

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2.25 Let X have covariance matrix $\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$

a) Determine ρ and $V^{1/2}$

$\begin{vmatrix} 1 \\ 2 \\ 4 \end{vmatrix} \begin{vmatrix} 4 \\ 6 \\ 14 \\ \hline 24 \end{vmatrix} \begin{matrix} 4 \\ 6 \\ 14 \\ \hline 24 \end{matrix}$ $\frac{24}{24}$

b) Multiply your matrices to check the relation $V^{1/2} \rho V^{1/2} = \Sigma$

Soln

given a covariance matrix $\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$

(a) we know that, $V^{1/2}$ is a standard deviation matrix and is given by

$$V^{1/2} = \begin{bmatrix} \sqrt{\sigma_{11}} & 0 & 0 \\ 0 & \sqrt{\sigma_{22}} & 0 \\ 0 & 0 & \sqrt{\sigma_{33}} \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Now since we have Σ and $V^{1/2}$, we can find correlation matrix ρ using the relation

$$\rho = V^{-1/2} \Sigma V^{-1/2}$$

$$\Rightarrow \boxed{\rho = (V^{1/2})^{-1} \Sigma (V^{1/2})^{-1}}$$

The inverse of diagonal matrix $V^{1/2}$ is given as

$$(V^{1/2})^{-1} = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

So $\rho = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix} \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

$$= \begin{bmatrix} \frac{25}{5} & \frac{-2}{5} & \frac{4}{5} \\ \frac{-2}{2} & \frac{4}{2} & \frac{1}{2} \\ \frac{4}{3} & \frac{1}{3} & \frac{9}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{25}{5\cdot5} & \frac{-2}{5\cdot2} & \frac{4}{5\cdot3} \\ \frac{-2}{2\cdot5} & \frac{4}{2\cdot2} & \frac{1}{2\cdot3} \\ \frac{4}{3\cdot5} & \frac{1}{3\cdot2} & \frac{9}{3\cdot3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\frac{1}{5} & \frac{4}{15} \\ \frac{-1}{5} & 1 & \frac{1}{6} \\ \frac{4}{15} & \frac{1}{6} & 1 \end{bmatrix}$$

Another approach

Since we know that $\rho = [\rho_{ij}]$ $\quad \rho_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}}\sqrt{\sigma_{jj}}}$ for $i \neq j$

$= 1$ for $i = j$

$$\rho = \begin{bmatrix} 1 & \frac{-2}{5.2} & \frac{4}{5.3} \\ \frac{-2}{2.5} & 1 & \frac{1}{2.3} \\ \frac{4}{3.5} & \frac{1}{3.2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{-1}{5} & \frac{4}{15} \\ -\frac{1}{5} & 1 & \frac{1}{6} \\ \frac{4}{15} & \frac{1}{6} & 1 \end{bmatrix}$$

b) Checking for the relation $V^{1/2} \rho V^{1/2} = \Sigma$

$$LHS = V^{1/2} \rho V^{1/2}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{5} & \frac{4}{15} \\ -\frac{1}{5} & 1 & \frac{1}{6} \\ \frac{4}{15} & 1/6 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & \frac{4}{3} \\ \frac{-2}{5} & 2 & \frac{1}{3} \\ \frac{4}{5} & \frac{1}{2} & 3 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix} = \Sigma = RHS. \ \square$$

4/4

2.26] Use Σ as given in exercise 2.25

a) Find ρ_{13}

b) Find the correlation between X_1 and $\frac{1}{2}X_2+\frac{1}{2}X_3$.

Solu: We are given $\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$
$\quad X_1 \quad X_2 \quad X_3$

a) The standard deviation matrix is

$$V^{1/2} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

So, correlation matrix is $\rho = \begin{bmatrix} 1 & -1/5 & 4/15 \\ -1/5 & 1 & 1/6 \\ 4/15 & 1/6 & 1 \end{bmatrix}$

Hence $\rho_{13} = 4/15$. $\square$

b) To find correlation between two items, we need to first find co-variance between them and divide that by their individual variances.

So $Var(X_1) = Var(X_1) = \sigma_{11} = 25$

$Var(\frac{1}{2}X_2 + \frac{1}{2}X_3) = (\frac{1}{2})^2 Var(X_2) + (\frac{1}{2})^2 Var(X_3) + 2\cdot\frac{1}{2\cdot 2} Cov(X_2, X_3)$

$= \frac{1}{4} Var(X_2) + \frac{1}{4} Var(X_3) + \frac{1}{2} Cov(X_2, X_3)$

$$= \frac{1}{4}\sigma_{22} + \frac{1}{4}\sigma_{33} + \frac{1}{2}\sigma_{23}$$

$$= \frac{1}{4}[\sigma_{22} + \sigma_{33}] + \frac{1}{2}\sigma_{23}$$

$$= \frac{1}{4}[4+9] + \frac{1}{2}\cdot 1$$

$$= \frac{13}{4} + \frac{1}{2}$$

$$= \frac{15}{4}$$

#Note

$$Var(aX_1+bX_2) = a^2 var(X_1) + b^2 var(X_2) + 2ab\,Cov(X_1, X_2)$$

Now $Cov\left(X_1, \frac{1}{2}X_2 + \frac{1}{2}X_3\right)$

$$= \frac{1}{2}Cov(X_1, X_2) + \frac{1}{2}Cov(X_1, X_3)$$

$$= \frac{1}{2}\sigma_{12} + \frac{1}{2}\sigma_{13}$$

$$= \frac{1}{2}(-2) + \frac{1}{2}\cdot(4)$$

$$= -1 + 2$$

$$= 1$$

finally, correlation between X_1 & $\frac{1}{2}X_2 + \frac{1}{2}X_3$ is given by

$$\rho_{X_1, \frac{1}{2}X_2+\frac{1}{2}X_3} = \frac{Cov\left(X_1, \frac{1}{2}X_2+\frac{1}{2}X_3\right)}{\sqrt{Var(X_1)}\sqrt{Var\left(\frac{1}{2}X_2+\frac{1}{2}X_3\right)}}$$

$$= \frac{1}{\sqrt{25}\sqrt{15/4}}$$

$$= \frac{2}{5\sqrt{15}}$$

$$= 0.1032$$

And from this result we can say there is <u>weak</u> correlation between X_1 and $\frac{1}{2}X_2 + \frac{1}{2}X_3$.

$\frac{6}{6}$

Q.27 Derive expression for the mean and varionce of the following linear Combinations in terms of the means and covariances of the random variable X_1, X_2, and X_3. [2]

a) $X_1 - 2X_2$

b) $-X_1 + 3X_2$

c) $X_1 + 2X_2 + X_3$

d) $X_1 + 2X_2 - X_3$

e) $X_1 + 2X_2 - X_3$

f) $3X_1 - 4X_2$ if X_1 and X_2 are independent Random variables

ASK Her: Is this X_1 and X_2 independent given only for part f or in general?

I have solved assuming only for last part but if it is ingeneral then $cov(X_1, X_2) = 0$ in all parts.

Soln

a) $X_1 - 2X_2$

For mean: $E(X_1 - 2X_2) = E(X_1) - 2E(X_2)$

For Variance: $Var(X_1 - 2X_2) = 1^2 Var(X_1) + (-2)^2 Var(X_2) + 2\cdot1(-2)Cov(X_1,X_2)$

$$\boxed{\because Var(aX_1 + bX_2) = a^2 Var(X_1) + b^2 Var(X_2) + 2ab\,Cov(X_1, X_2)}$$

$= Var(X_1) + 4Var(X_2) - 4\,Cov(X_1, X_2)$

(b) $-X_1 + 3X_2$

For mean: $E(-X_1 + 3X_2) = -E(X_1) + 3E(X_2)$

For Variance : $Var(-X_1 + 3X_2)$

$$= (-1)^2 Var(X_1) + (3)^2 Var(X_2) + 2\cdot(-1)(3)\, Cov(X_1, X_2)$$

$$= Var(X_1) + 9\, Var(X_2) - 6\, Cov(X_1, X_2)$$

ⓒ $X_1 + X_2 + X_3$

For mean : $E(X_1 + X_2 + X_3) = E(X_1) + E(X_2) + E(X_3)$

For Variance : $Var[X_1 + (X_2 + X_3)]$

$$= (1)^2 Var(X_1) + (1)^2 Var(X_2 + X_3) + 2\cdot(1)(1)\, Cov(X_1, X_2 + X_3)$$

$$= Var(X_1) + Var(X_2 + X_3) + 2\, Cov(X_1, X_2 + X_3)$$

$$= Var(X_1) + Var(X_2) + Var(X_3) + 2\, Cov(X_2, X_3) + 2\, Cov(X_1, X_2 + X_3)$$

$$= Var(X_1) + Var(X_2) + Var(X_3) + 2\, Cov(X_2, X_3) + 2\, Cov(X_1, X_2) + 2\, Cov(X_1, X_3$$

ⓓ $X_1 + 2X_2 - X_3$

For mean : $E(X_1 + 2X_2 - X_3) = E(X_1) + 2E(X_2) - E(X_3)$

For Variance : $Var[X_1 + (2X_2 - X_3)]$

$$= (1)^2 Var(X_1) + (1)^2 Var(2X_2 - X_3) + 2\cdot(1)(1)\, Cov(X_1, 2X_2 - X_3)$$

$$\boxed{\because Var(aX_1 + bX_2) = a^2 Var(X_1) + b^2 Var(X_2) + 2ab\, Cov(X_1, X_2)}$$

$= Var(X_1) + Var(2X_2 - X_3) + 2\,cov(X_1, 2X_2 - X_3)$

$= Var(X_1) + 4\,Var(X_2) + Var(X_3) - 4\,cov(X_2, X_3) + 2\,cov(X_1, 2X_2 - X_3)$

$= Var(X_1) + 4\,Var(X_2) + Var(X_3) - 4\,cov(X_2, X_3) + 4\,cov(X_1, X_2) - 2\,cov(X_1, X_3)$

(f) $3X_1 - 4X_2$. If X_1 and X_2 are independent Random variable.

For mean : $E(3X_1 - 4X_2) = 3E(X_1) - 4E(X_2)$

For variance : $Var(3X_1 - 4X_2)$

$= (3)^2 var(X_1) + (-4)^2 var(X_2) + 2\cdot(3)\cdot(-4)\,cov(X_1, X_2)$

$= 9\,var(X_1) + 16\,var(X_2) - 24\,\cancel{cov(X_1, X_2)}^{\,0}$

$cov(X_1, X_2) = 0$ because X_1 and X_2 are independent given

$= 9\,var(X_1) + 16\,var(X_2)$. $\square$

41. You are given the random vector $X' = [X_1, X_2, X_3, X_4]$ with mean vector $\mu'_X = [3, 2, -2, 0]$ and variance-covariance matrix

$$\Sigma_X = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}.$$

Let

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & 1 & 3 \end{bmatrix}$$

a) Find $E(AX)$, the mean of AX

b) Find $\mathrm{COV}(AX)$, the variance and covariance of AX

c) Which pairs of linear combinations have zero covariances?

Solu

a) Mean of AX : $E(AX)$

$$= A\,E(X)$$

$$= \underset{3\times 4}{\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & 1 & 3 \end{bmatrix}} \underset{4\times 1}{\begin{bmatrix} 3 \\ 2 \\ -2 \\ 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 1 \\ 9 \\ 3 \end{bmatrix}$$

b) $Cov(AX) = A\Sigma_x A'$

#Note:
$Cov(CX) = C\Sigma_x C'$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -3 & 0 & 0 \\ 3 & 3 & -6 & 0 \\ 3 & 3 & 3 & 9 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 36 \end{bmatrix}$$

Now

c) $$AX = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

$$= \begin{bmatrix} X_1 - X_2 \\ X_1 + X_2 - 2X_3 \\ X_1 + X_2 + X_3 + X_4 \end{bmatrix}$$

$$= \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}$$ say also let $Y = AX$

So clearly we have

$$\operatorname{cov}(AX) = \operatorname{cov}(Y) = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 36 \end{bmatrix} = \Sigma_Y$$

So

$$\operatorname{Var}(Y_1) = \operatorname{Var}(X_1 - X_2) = 6$$

$$\operatorname{Var}(Y_2) = \operatorname{Var}(X_1 + X_2 - 2X_3) = 18$$

$$\operatorname{Var}(Y_3) = \operatorname{Var}(X_1 + X_2 + X_3 + X_4) = 36$$

$$\operatorname{Cov}(Y_1, Y_2) = \operatorname{Cov}(X_1 - X_2, X_1 + X_2 - 2X_3) = 0 = \operatorname{Cov}(Y_2, Y_1)$$

$$\operatorname{Cov}(Y_1, Y_3) = \operatorname{Cov}(X_1 - X_2, X_1 + X_2 + X_3 + X_4) = 0 = \operatorname{Cov}(Y_3, Y_1)$$

$$\operatorname{Cov}(Y_2, Y_3) = \operatorname{Cov}(X_1 + X_2 - 2X_3, X_1 + X_2 + X_3 + X_4) = 0 = \operatorname{Cov}(Y_3, Y_2)$$

So, clearly All pairs of Linear combination have zero covariance. $\square$

* * The End * *

Q.30] You are given the random vector $X' = [X_1, X_2, X_3, X_4]$ with mean vector $\mu_X' = [4, 3, 2, 1]$ and Variance-covariance matrix

$$\Sigma_X = \left[\begin{array}{cc|cc} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ \hline 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{array}\right]$$

partition X as $X = \left[\begin{array}{c} X_1 \\ X_2 \\ \hline X_3 \\ X_4 \end{array}\right] = \left[\begin{array}{c} X^{(1)} \\ \hline X^{(2)} \end{array}\right]$

Let $A = [1\ 2]$ and $B = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$

and consider the linear combinations $AX^{(1)}$ and $BX^{(2)}$. Find

a.) $E(X^{(1)})$

b.) $E(AX^{(1)})$

c.) $Cov(X^{(1)})$

d.) $COV(AX^{(1)})$

e.) $E(X^{(2)})$

f.) $E(BX^{(2)})$

g.) $Cov(X^{(2)})$

h.) $Cov(BX^{(2)})$

i.) $Cov(X^{(1)}, X^{(2)})$

j.) $Cov(AX^{(1)}, BX^{(2)})$.

Solu.

We are given that, a random vector $X' = [X_1\ X_2\ X_3\ X_4]$, with mean vector $\mu_X' = [4\ 3\ 2\ 1]$

and variance-covariance matrix

$$\Sigma_X = \left[\begin{array}{cc|cc} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ \hline 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{array}\right]$$

consequently their partitions

$$\Sigma_{11} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Sigma_{12} = \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix} = \Sigma_{21}^{T}$$

and $\Sigma_{22} = \begin{bmatrix} 9 & -2 \\ -2 & 4 \end{bmatrix}$, for $X = \begin{bmatrix} X_1 \\ X_2 \\ \hdashline X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} X^{(1)} \\ \hdashline X^{(2)} \end{bmatrix}$

Also given $A = [1 \ 2]$ and $B = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ and their linear combination $AX^{(1)}$ and $BX^{(2)}$.

a) $E(X^{(1)}) = E\left(\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}\right) = \left(\begin{bmatrix} E(X_1) \\ E(X_2) \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

b) $E(AX^{(1)}) = A\,E(X^{(1)}) = [1 \ 2]\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$$= 4+6$$
$$= 10$$

c) $cov(X^{(1)}) = \Sigma_{11} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

d) $cov(AX^{(1)}) = A\,cov(X^{(1)})\,A'$

#Note:
$cov(CX) = C\,\Sigma_X\,C'$

$$= \underset{1\times 2}{\begin{bmatrix} 1 & 2 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}$$

$$\left\{ \because A = \begin{bmatrix} 1 & 2 \end{bmatrix},\quad A' = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \;\&\; cov(X^{(1)}) = \Sigma_{11} \right\}$$

$$= \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= 7$$

e.) $E(X^{(2)}) = E\left(\begin{bmatrix} X_3 \\ X_4 \end{bmatrix}\right) = \left(\begin{bmatrix} E(X_3) \\ E(X_4) \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

f.) $E(BX^{(2)}) = B\,E(X^{(2)}) = \underset{2\times 2}{\begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}$

$$= \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

g.) $COV(X^{(2)}) = \Sigma_{22} = \begin{bmatrix} 9 & -2 \\ -2 & 4 \end{bmatrix}.$

h.) $Cov(BX^{(2)}) = B\,cov(X^{(2)})\,B^T$

$$\boxed{\#\text{Note} \quad cov(CX) = C\,\Sigma_X C'}$$

$$= \underset{2\times 2}{\begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 9 & -2 \\ -2 & 4 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}}$$

$$= \begin{bmatrix} 13 & -10 \\ 20 & -8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}.$$

$$= \begin{bmatrix} 33 & 36 \\ 36 & 48 \end{bmatrix}$$

i.) $cov(X^{(1)}, X^{(2)})$

$$= cov\left(\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \begin{bmatrix} X_3 \\ X_4 \end{bmatrix}\right)$$

$$= \begin{bmatrix} cov(X_1, X_3) & cov(X_1, X_4) \\ cov(X_2, X_3) & cov(X_2, X_4) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix} = \Sigma_{12}$$

j.) $cov(AX^{(1)}, BX^{(2)})$

$$= E(AX^{(1)} - \mu(AX^{(1)}))(BX^{(2)} - \mu(BX^{(2)}))^T$$

$$= A \cdot E(X^{(1)} - \mu(X^{(1)}))((X^{(2)})^T B^T - \mu(X^{(2)})^T B^T)$$

$$= A \cdot cov(X^{(1)}, X^{(2)}) \cdot B^T \longrightarrow$$

Ask her - The source for this relation is chatgpt, so I am not 100% sure and look for explination on this.

$$= \underset{1\times 2}{\begin{bmatrix} 1 & 2 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}}$$

$$= \underset{1\times 2}{\begin{bmatrix} 4 & 2 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}}$$

$$= \begin{bmatrix} 0 & 6 \end{bmatrix}$$

$\frac{14}{14}$