Floating Rate Products, Interest Rate and Currency Swaps

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Equivalence between yields, spot rates, forward rates, discount factors

$$\sum_{t=1}^{T} \frac{c_t}{(1+y)^t} = \sum_{t=1}^{T} \frac{c_t}{(1+z_t)^t} = \sum_{t=1}^{T} \frac{c_t}{\prod_{j=1^t}(1+f_{t-1,t})} = \sum_{t=1}^{T} c_t d_t.$$

c_t: cashflows

y: yield to maturity

z_t: zero-coupon rates.

$f_{t-1,t}$: forward rate between time $t-1$ and t.

d_t: discount factor

Forward Rate Agreement (FRA)

The FRA is a contract made today to lend or borrow at a fixed rate for a forward period. For example, company A may contract with bank B to fix the lending rate at 6% for the forward period from the end of the 6th month to the end of the 12th month from today. This is denoted as the FRA(6,12), where the 6 and 12 denote the end of the starting and ending months for the loan contract underlying the agreement.

- *Settlement by Performance.* In this method, the actual borrowing or lending occurs.

- *Settlement at Maturity.* Settlement at the start of the FRA period. Suppose the contracted FRA(6,12) borrowing rate for company A (quoted by bank B) was 6%, and at maturity the 6-month zero-coupon rate turned out to be 5%,

$$\text{Settlement amount} = \frac{(0.06 - 0.05) \times 0.5}{1 + 0.05/2}.$$

- *Settlement in arrears.* In this approach, the settlement takes place at the end of the FRA period and is for the difference in rates at maturity.

Arbitrage if FRA not at fwd rate

Riskless arbitrage: A trading strategy that generates non-negative cashflows at all times with at least one positive cashflow.

Arbitrage Example: Let the (0,6) zero-coupon rate be 2% per year. The notation (0,6) stands for the period from time 0 to the end of the 6th month. Also, let the (0,12) zero-coupon rate be 3.9902% per year (this is just the one-year zero coupon rate). These two rates imply that the (6,12) forward rate is 6%. Lets check this:

$$(1 + 0.02/2) \times (1 + f(6, 12)/2) = (1 + 0.039902/2)^2.$$

You can easily check that the forward rate between 6 and 12 months. i.e. $f(6, 12) = 0.06$.

This calculation provides us with a hint as to how the arbitrage can be constructed. We can set up the arbitrage portfolio using the 6-month and 1-year zero-coupon bonds. If bank B quoted the FRA(6,12) at 6.1%, then the arbitrage is as follows.

- First, company A would immediately lock in an agreement to *lend* to bank B via the FRA(6,12) at 6.1%.
- Next, company A would buy the ZCB(0,6) at its traded price. The price of the 6-month zero-coupon bond is

$$ZCB(0, 6) = \frac{1}{1 + 0.02/2} = 0.990099.$$

Closing out the arbitrage

- Then, company A would short sell a certain number of units of the ZCB(0,12) at its traded price so as to fund the purchase of the ZCB(0,6). The price is

$$ZCB(0,12) = \frac{1}{(1 + 0.039902/2)^2} = 0.961261.$$

The number of units short sold will be

$$\frac{ZCB(0,6)}{ZCB(0,12)} = 1.03.$$

- Note that, after these two trades the cashflow at inception, i.e. at time zero, is zero, since the buy and sell of the two ZCBs cancels out.

- After 6 months, company A redeems the ZCB(0,6) for its face value of \$1, and lends it to bank B at the precontracted FRA(6,12) rate of 6.1%. Hence, the \$1 inflow is matched by the lending outflow, and cashflow at time 6 months is also zero. So far, the strategy has resulted in zero cashflow at both time 0 and time 6 months.

- Finally, at time 12 months, there are two cashflows. First, the lending transaction is closed out, and company A receives it \$1 with interest for a cash inflow of \$1.0305. Company A also has to redeem its short sale of ZCB(0,12) which closes out at its face value of \$1, for 1.03 units, i.e. a cash outflow of \$1.03. Therefore, the net cash flow is 0.0005 at time 12 months.

- This cashflow is received *for sure*, and does not depend in any way on what happens to interest rates over the entire year. Hence, this result is "locked in" and comprises a "riskless arbitrage". It is a free lunch, for company A has been able to construct a portfolio which at all three time periods, provides zero or positive cashflows, but no negative amounts.

FRA Notation

The convention in this chapter for quoting FRAs takes on the form *FRA*(t, $t+m$), where t is the time the FRA starts, and ($t+m$) is the time the FRA period ends.

Alternate convention: The FRA may be quoted as *FRA*(t,m), where the first variable is the start of the FRA period, but the second variable indicates the length of the FRA period. Therefore, instead of FRA(6,12), the alternate convention would be to state it as FRA(6,6).

Both methods are simply conventions, and do not change in any way, the valuation of the FRA. However, while trading, it is useful to clarify which convention is being used, else counterparties may encounter subsequent contracting confusion.

- A *forward-rate agreement* or FRA is an agreement (made at time t, say) to exchange fixed-rate payments k for LIBOR i on a principal amount A for the period T to $T+m$.
- The settlement of the FRA is made in discounted form at date T.
- The day-count notation is as follows:
  - d: No of days between T and $T+m$.
  - D: No of days in the reference period.

Mark-to-market (MTM) of FRAs

Prior to maturity, the value of the FRA contract is determined by comparing the FRA rate with the current forward rate for the period of the FRA, determining the difference, and discounting this cashflow back to the valuation date. For example, suppose that an FRA(6,12) was written at a contracted rate of 6%. To be more specific, company A has agreed to borrow in the (6,12) period from bank B at 6%. Three months later, the rate for the same period is now 6.3%, i.e. forward rates have risen. Hence, from the point of view of company A, its FRA is "in-the-money". What is the value at time 3 months now of this FRA to company A? This is computed as follows.

First, determine the settlement value of the FRA at maturity. This is

$$\text{Settlement at time 6 months} = \frac{(0.063 - 0.060) \times 0.5}{1 + 0.063/2} = 0.001454.$$

Value 3 months into the FRA (the 3 month T-bill is priced at 99c per dollar:

$$0.99 \times 0.001454 = 0.0014397$$

MTM Value is different from the settlement amount

Formal exposition of FRA payoffs

Consider a pay-fixed/receive-floating FRA. Suppose the LIBOR rate realized at date T for the period T to $T+m$ is i. Then, the fixed payment due at $T+m$ is

$$-A \cdot \left(k \frac{d}{D}\right)$$

k = fixed payment
D = days in ref period
d = actual days

while the floating receipt at $T+m$ is

$$+A \cdot \left(i \frac{d}{D}\right)$$

Thus, the difference between paying fixed and receiving floating at time $T+m$ is

$$A(i-k)\left(\frac{d}{D}\right).$$

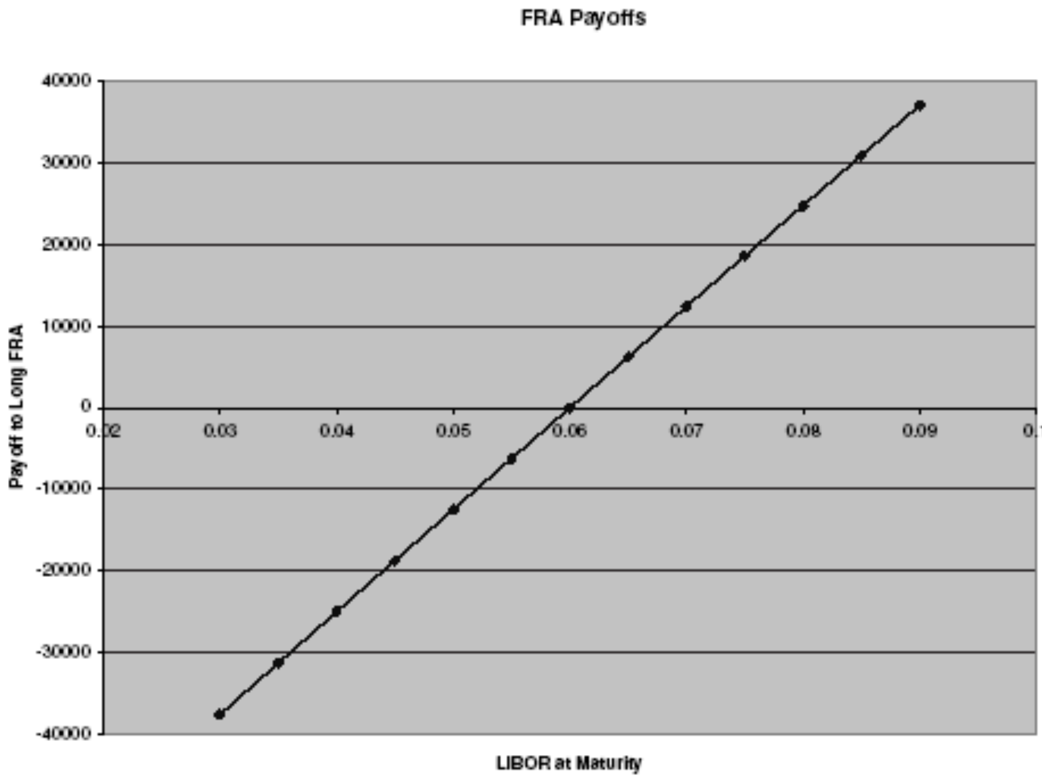
Since the FRA is settled in discounted form at T, the settlement amount is

$$A\left[\frac{(i-k)\left(\frac{d}{D}\right)}{1+i\left(\frac{d}{D}\right)}\right]$$

D is usually 360 in Libor mkts

FRA Payoffs

Principal Amount:	$5,000,000
Dealing date:	July 11, 2001
Spot Date:	July 13, 2001
Settlement Date:	September 13, 2001
Interest Rate Index:	3–month LIBOR
Contract rate (k):	6%



Value of an FRA

$$V = \frac{(f-k)\frac{d}{D}A}{1+f\frac{d}{D}}B(t,T) = AB(t,T)\left[\frac{\left(1+f\frac{d}{D}\right)-\left(1+k\frac{d}{D}\right)}{1+f\frac{d}{D}}\right]$$

$$= AB(t,T) - AB(t,T)\left[\frac{1+k\frac{d}{D}}{1+f\frac{d}{D}}\right]$$

$$= A\left[B(t,T) - \left(1+k\frac{d}{D}\right)B(t,T+m)\right]$$

FRA price: $$B(t,T+m) = \left(\frac{B(t,T)}{1+k\frac{d}{D}}\right) \Rightarrow k^* = \left[\frac{B(t,T)}{B(t,T+m)} - 1\right]\frac{D}{d}$$

Hedging FRAs (zero bond method)

Recall that the FRA value at time t is

$$V_t = A \cdot B(t,T) - A \cdot B(t,T+m)\left(1 + k\frac{d}{D}\right)$$

Thus, the interest rate exposure in the FRA comes from the possibility of changes in the zero bond prices $B(t,T)$ and $B(t,T+m)$. To offset this risk, we can

- take a short position in T–maturity zero-coupon bonds with face value A, and
- take a long position in $(T+m)$–maturity zeros with face value $A[1+k(d/D)]$.

For example, consider the 3×6 FRA of the example in Section 1.3.1. In this example,

- A \$1 increase in $B(t,T)$ will increase the value of the FRA by \$5,000,000.
- A \$1 increase in $B(t,T+m)$ will decrease the value of the FRA by $\$(5,000,000)(1+(0.06)(91/360)) = \$5,075,833$.

Hence we may establish the following hedge position: a long position in 5,000,000 zero bonds maturing 13 September 2001, and a short position in 5,075,833 zero bonds maturing 13 December 2001.

Hedging FRAs (PVBP method)

$$\frac{\partial V_t}{\partial y(t,T)} = \frac{\partial V_t}{\partial B(t,T)} \frac{\partial B(t,T)}{\partial y(t,T)}$$

$$\frac{\partial B}{\partial y} = \frac{1}{(1+y)^{d/365}} - \frac{1}{(1+y+0.0001)^{d/365}}$$

As an example, consider the 3×6 FRA of Section 1.3.1. In t

$$B(t,T) = 0.9856 \quad \text{and} \quad B(t,T+m) = 0.9694.$$

(0.98558 and 0.96935 after 1 basis point shift)

Thus, we have

$$y(t,T) = 5.990\% \quad \text{and} \quad y(t,T+m) = 6.395\%$$

And we finally obtain:

$$\text{PVBP}(T) = 115.91 \quad \text{and} \quad \text{PVBP}(T+m) = -117.11$$

Floating Rate Note (FRN)

Coupon on these notes floats (based on an index rate). E.g. 6-month \$LIBOR. Reset occurs each coupon date.

How do we price a bond when its cashflows are not known?

Lets assume a \$100, 2-year FRN, semi-annual basis, indexed to 6-month LIBOR. The current value of 6-month LIBOR is 1%, and hence the first coupon is set to \$0.5. The following table describes the rates environment at time zero.

Period	Zero-coupon rate	Forward rate
(0,6)	0.010000	0.010000
(6,12)	0.013644	0.017296
(12,18)	0.017030	0.023817
(18,24)	0.020175	0.029642

At time 0, the only known cashflow is at time 6 months, and is equal to \$0.5. We can use the forward rates to determine the other three cashflows, at times 12, 18 and 24 months. These are as follows:

$$100 \times 0.017296/2 = 0.864800$$
$$100 \times 0.023817/2 = 1.190850$$
$$100 \times 0.029642/2 = 1.482100$$

Now, that these cashflows are generated, we can proceed to find the value of the FRN. The discounting equation is:

$$\begin{aligned} \text{FRN} &= \frac{0.500000}{1+\frac{0.01}{2}} + \frac{0.864800}{(1+\frac{0.013644}{2})^2} + \frac{1.190850}{(1+\frac{0.01703}{2})^3} + \frac{100+1.482100}{(1+\frac{0.020175}{2})^4} \\ &= 100. \end{aligned}$$

FRN short-form approach

Only the first reset date matters, the rest is at par.

The short-form approach directly recognizes this fact, and states that the only calculation required to value the FRN is to add the principal to the set coupon and dicount this to present time. Hence, the value of the FRN is obtained from the following short calculation:

$$\text{FRN} = \frac{100 + 0.5}{1 + 0.01/2} = 100.$$

Suppose 3 months elapse and we again want to determine the price of the FRN. Let the price of the (3,6) zero-coupon bond at time 3 months be equal to 0.99. In this case the FRN will be valued at

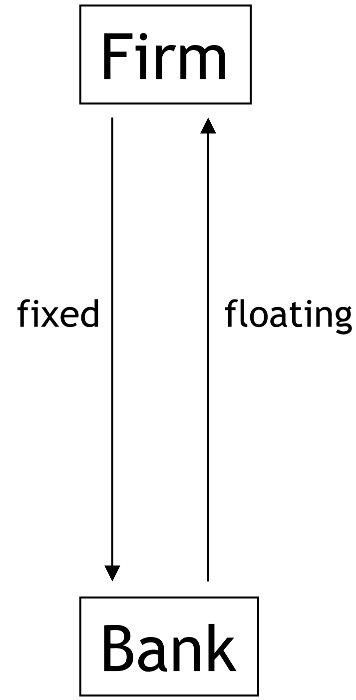
$$\text{FRN at time 3 months} = (100 + 0.5) \times 0.99 = 99.495,$$

which is trading below par because the zero-coupon rate for the period (3,6) is greater than 1% (the rate at which the time 6 month coupon was set). Likewise, if the zero-coupoikn rate were lower than 1%, then the FRN would trade over par.

How do we handle FRNs at a spread?

Interest Rate Swaps

A transaction in which one party pays periodic amounts of a given currency based on a specified fixed rate and the other party pays periodic amounts of the same currency based on a specified floating rate that is reset periodically, such as the London interbank offered rate; all calculations are based on a notional amount of the given currency.



Suppose the notional amount is $1000,000. The contract may be set at a fixed rate of 7% per year, in return for 6-month LIBOR. At the outset of the contract, i.e. at time 0 months, the 6-month LIBOR rate is, say 5%. On this date the first payment by the bank is "set" at 5%. Now, at the end of 6 months, bank B will pay $50,000 to company A, and in return will receive a payment at 7%, i.e. $70,000. In reality, two separate payments are never made, and only the net difference of $20,000 is paid by the company to bank B.

Further, at time 6 months, the new LIBOR rate is taken into account in determining the next floating payment. Suppose at this time, 6-month LIBOR is trading at 7%. Then the 12 month floating rate payment by bank B is now reset to $70,000. At time 12 months, the net payment is now zero, since both the fixed and floating legs of the swap are for equal amounts.

Types of Swaps

- Off-market swaps.
- Zero-coupon swaps.
- Changing fixed rates.
- Spreads to Libor.
- Forward start swaps.
- Amortizing/accreting/roller-coaster swaps.

Uses of Swaps

1. Taking gapping positions.

2. Closing maturity mismatches on balance sheets – e.g. Sallie Mae.

3. Credit Spread Arbitrage.

4. Synthetic Financing.

Credit Spread Arbitrage

Type of Financing	Company A	Bank B	Difference
Fixed	$11\frac{1}{2}\%$	13%	$1\frac{1}{2}\%$
Floating	$L+\frac{3}{8}\%$	$L+1\frac{1}{8}\%$	$\frac{3}{4}\%$

- First, company A borrows funds from the fixed rate markets at 11.5%.
- Bank B raises money in the floating rate markets at $L+1\frac{1}{8}\%$.
- Next, both counterparties enter into the following swap: A agrees to pay B at a floating rate of $L+1\frac{1}{8}$ and receive fixed at a rate of $12\frac{5}{8}$.

Company A - net funding cost.

$$\text{Net cost to A} = 11\frac{1}{2} + (L + 1\frac{1}{8}) - 12\frac{5}{8} = L, (\text{saves}\frac{3}{8}\%).$$

Bank B - net funding cost.

$$\text{Net cost to B} = L + 1\frac{1}{8} - (L + 1\frac{1}{8}) + 12\frac{5}{8} = 12\frac{5}{8}, (\text{saves}\frac{3}{8}\%).$$

Synthetic Financing

Suppose company A can raise fixed rate financing at 10%. It can raise money in the floating rate markets at $L + \frac{3}{4}\%$. It also finds that the current fixed rate versus 1-year LIBOR being offered to it in the swap market is $9\frac{1}{8}\%$. It is easy to show that instead of locking into the fixed rate of 10% quoted above, the firm can actually do better. Here is how:

- First, company A raises money in the *floating rate* markets at $L + \frac{3}{4}\%$.
- Next, company A enters into a swap with bank B, where it pays fixed $9\frac{1}{8}\%$ and receives floating LIBOR.

The net costs from these transactions is as follows:

$$\text{Net cost per period to company A} = L + \frac{3}{4} + 9\frac{1}{8} - L = 9\frac{7}{8}.$$

Hence, this is $\frac{1}{8}\%$ cheaper than it is in the fixed rate markets. Note that at the end of this transaction, company A has effectively obtained fixed rate financing, but cheaper than by going directly to the fixed rate markets.

Valuing Swaps

- As an exchange of a bonds: where company A pays fixed and receives floating. From the point of view of A, the value of this swap at any time before its expiration will be

  Swap value to A = Value of floating rate bond − Value of fixed rate bond.

- As a collection of forwards. In particular, swaps are simply portfolios of FRAs, where in the simplest cases, the contracted rate is the same across all FRAs. For example, if we consider a 5-year, semi-annual swap with floating payments based on LIBOR, and a fixed rate of 10%, then it is equivalent to the portfolio comprising ten LIBOR FRAs, each with a contracted rate of 10%, and maturing at six month intervals from each other.

Swaps: An example

Spot Date:	June 11, 2001
Term:	3 years
Interest Rate Index:	6–month LIBOR
Reset interval:	6 months
Swap rate (k):	6%

Fixed leg: 30/360 basis
Floating leg: Actual/360

Notional: $100,000,000

We shall compute the cash flows to the holder of the long swap. Suppose the initial LIBOR rate is 6.50%. The computations are as follows:

- Floating payment on December 11:

$$(100,000,000) \times \left(0.065 \times \frac{183}{360}\right) = 3,304,167$$

- Fixed payment on December 11:

$$(100,000,000) \times \left(0.060 \times \frac{180}{360}\right) = 3,000,000$$

- Therefore, net receipts of long position on Dec 11:

$$3,304,167 - 3,000,000 = 304,167$$

First reset date

6-month Libor = 7%

- Floating payment on June 11, 2002:

$$(100,000,000) \times \left(0.07 \times \frac{182}{360}\right) = 3,538,889$$

- Fixed payment on June 11, 2002:

$$(100,000,000) \times \left(0.060 \times \frac{180}{360}\right) = 3,000,000$$

- Therefore, net receipts of long position on June 11, 2002:

$$3,538,889 - 3,000,000 = 538,889$$

Table 1.2: Three-year pay fixed, receive floating swap entered into on June 11, 2001. Floating rate: 6-month LIBOR. Swap rate: 6.00% The table presents the net payoffs on the reset days.

Time	Days from last reset	LIBOR at last reset	Swap rate	Fixed Payment	Floating Receipt	Net
11-Dec-01	183	6.50%	6.00%	(3,000,000)	3,304,167	304,167
11-Jun-02	182	7.00%	6.00%	(3,000,000)	3,538,889	538,889
11-Dec-02	183	6.50%	6.00%	(3,000,000)	3,304,167	304,167
11-Jun-03	182	6.25%	6.00%	(3,000,000)	3,159,722	159,722
11-Dec-03	183	5.75%	6.00%	(3,000,000)	2,922,917	(77,083)
11-Jun-04	183	5.25%	6.00%	(3,000,000)	2,668,750	(331,250)

Swaps: Example 2

Valuation date: 13 August, 2001.

Payment dates:	11 Dec 01, 11 June 02, 11 Dec 02, 11 June 03, 11 Dec 03
Swap rate:	6%
LIBOR at last reset:	6.50%
Notional Principal:	$100,000,000
Long/Short:	Long

Input Information		
Dates	No. of Days	Annualized Yields
13-Aug-01		
13-Nov-01	92	4.9880
11-Dec-01	120	
13-Feb-02	184	5.1109
11-Jun-02	302	
13-Aug-02	365	5.4898
11-Dec-02	485	
11-Jun-03	667	
13-Aug-03	730	5.9748
11-Dec-03	850	
13-Aug-04	1096	6.2369

Completing the Yield Curve			
Dates	No. of Days	Annualized Yields	Obtained by
13-Aug-01			
13-Nov-01	92	4.9880	Input information
11-Dec-01	120	*5.0254*	Linear interpolation
13-Feb-02	184	5.1109	Input information
11-Jun-02	302	*5.3579*	Linear interpolation
13-Aug-02	365	5.4898	Input information
11-Dec-02	485	*5.6493*	Linear interpolation
11-Jun-03	667	*5.8911*	Linear interpolation
13-Aug-03	730	5.9748	Input information
11-Dec-03	850	*6.0607*	Linear interpolation
13-Aug-04	1096	6.2369	Input information

$$y_{120} = \left(\frac{184-120}{184-92}\right) y_{92} + \left(\frac{120-92}{184-92}\right) y_{194}$$

(interpolation example)

Example 2 (continued)

Obtaining the Discount Factors			
Dates	No. of Days	Annualized Yields	Discount Factors
13-Aug-01			
13-Nov-01	92	4.9880	0.987806
11-Dec-01	120	5.0254	0.984009
13-Feb-02	184	5.1109	0.975185
11-Jun-02	302	5.3579	0.957735
13-Aug-02	365	5.4898	0.947959
11-Dec-02	485	5.6493	0.929581
11-Jun-03	667	5.8911	0.900683
13-Aug-03	730	5.9748	0.890420
11-Dec-03	850	6.0607	0.871945
13-Aug-04	1096	6.2369	0.833877

Pricing of Fixed vs Floating Swap					
Date	Discount Factor	Floating CF	PV(Floating CF)	Fixed CF	PV(Fixed CF)
11-Dec-01	0.984009	103,304,167	101,652,230	3,000,000	2,952,027
11-Jun-02	0.957735			3,000,000	2,873,205
11-Dec-02	0.929581			3,000,000	2,788,743
11-Jun-03	0.900683			3,000,000	2,702,049
11-Dec-03	0.871945			103,000,000	89,810,335
Totals			101,652,230		101,126,359

The initial swap rate

Find rate k that makes the swap value zero at inception.

In notational terms, given k, the value of the fixed rate bond is

$$A\left[k\left(\frac{s_0}{S}B(t,t_1)+\cdots+\frac{s_{N-1}}{S}B(t,t_N)\right)+B(t,t_N)\right]$$

Thus, the swap rate is the value k^* that satisfies

$$A \; - \; A\left[k\left(\frac{s_0}{S}B(t,t_1)+\cdots+\frac{s_{N-1}}{S}B(t,t_N)\right)+B(t,t_N)\right] \; = \; 0.$$

Solving, we obtain:

$$k^* \; = \; \left[\frac{1-B(t,t_N)}{B(t,t_1)\,\frac{s_1}{S}+\cdots+B(t,t_N)\,\frac{s_{N-1}}{S}}\right]$$

In the case of US swaps, the 30/360 convention therefore implies that

$$k^* \; = \; \frac{1-B(t,t_N)}{\frac{1}{2}\left[B(t,t_1)+\cdots+B(t,t_N)\right]}$$

Swap initial pricing example

(3yr swap as of 18 June 2001)

Maturity	Yield
6–month	5.1000%
1–year	5.4590%
2–year	5.9595%
3–year	6.2879%

30/360 basis

1. First, we identify the payment dates.
2. Then, we use interpolation to find the missing yields.
3. Finally, we convert the yields into discount factors.

Pricing a new swap

Date	No. of Days	Yield	Discount Factor	Source of Yield Information
18-Jun-01			1.000000	Input information
18-Dec-01	183	5.1000	0.975369	Input information
18-Jun-02	365	5.4590	0.948236	Input information
18-Dec-02	548	5.7099	0.920011	Linear interpolation
18-Jun-03	730	5.9595	0.890677	Input information
18-Dec-03	913	6.1237	0.861854	Linear interpolation
18-Jun-04	1096	6.2879	0.832676	Input information
Three-year swap rate: 0.06164288				

$$\frac{1}{2} k \sum_t B(t) + B(T) = 1 \quad \Rightarrow \quad k = \frac{1 - B(T)}{\frac{1}{2} \sum_t B(t)}$$

Off-market Swap

Off Market Swap Pricing					
Date	Discount Factor	Floating CFs	Fixed CFs	PV of Floating CFs	PV of Fixed CFs
18-Jun-01	1.000000	100,000,000		100,000,000	
18-Dec-01	0.975369		3,000,000		2,926,107
18-Jun-02	0.948236		3,000,000		2,844,708
18-Dec-02	0.920011		3,000,000		2,760,033
18-Jun-03	0.890677		3,000,000		2,672,031
18-Dec-03	0.861854		3,000,000		2,585,562
18-Jun-04	0.832676		103,000,000		85,765,628
Totals				100,000,000	99,554,069
Value				Long swap:	445,931

From the previous example, the break-even swap rate was 6.16%.

Zero-coupon Swap

Zero-Coupon Swap Pricing					
Date	Discount Factor	Floating CFs	Fixed CFs	PV of Floating CFs	PV of Fixed CFs
18-Jun-01	1.000000	100,000,000		100,000,000	
18-Dec-01	0.975369				
18-Jun-02	0.948236				
18-Dec-02	0.920011				
18-Jun-03	0.890677				
18-Dec-03	0.861854				
18-Jun-04	0.832676		120,094,731		100,000,000
Totals				100,000,000	100,000,000

$$k = \left(\frac{120{,}094{,}731}{100{,}000{,}000}\right)^{\frac{365}{1096}} - 1 = \left(\frac{1}{0.832676}\right)^{\frac{365}{1096}} - 1 = 0.062879$$

Changing fixed rates

In these swaps the fixed rate changes during the life of the swap in accordance with a pre-specified schedule. The approach to pricing such swaps is standard - once again, present value all payments, including the principal amounts. The floating leg is unaffected in these computations. Finally, identify initial value of swap (which need not typically be zero).

Variable Fixed Rate Swap Pricing						
Date	Discount Factor	Fixed Rate	Floating CFs	Fixed CFs	PV of Floating CFs	PV of Fixed CFs
18-Jun-01	1.000000		100,000,000		100,000,000	
18-Dec-01	0.975369	5.00%		2,500,000		2,438,423
18-Jun-02	0.948236	5.50%		2,750,000		2,607,649
18-Dec-02	0.920011	6.00%		3,000,000		2,760,033
18-Jun-03	0.890677	6.50%		3,250,000		2,894,700
18-Dec-03	0.861854	6.50%		3,250,000		2,801,026
18-Jun-04	0.832676	6.50%		103,250,000		85,973,797
Totals					100,000,000	99,475,627

Spread over Libor Swap

Swap Pricing with Spread over LIBOR						
Date	Discount Factor	Fixed CFs	Floating CFs	Spread CFs	PV of Fixed CFs	PV of Floating CFs
18-Jun-01	1.000000		100,000,000			100,000,000
18-Dec-01	0.975369	3,233,655		152,500	3,154,007	148,744
18-Jun-02	0.948236	3,233,655		149,589	3,066,268	141,846
18-Dec-02	0.920011	3,233,655		152,500	2,974,998	140,302
18-Jun-03	0.890677	3,233,655		149,589	2,880,142	133,236
18-Dec-03	0.861854	3,233,655		152,500	2,786,939	131,433
18-Jun-04	0.832676	103,233,655		152,500	85,960,187	126,983
Totals					100,822,543	100,822,543

The initial value of this swap is zero, and the spread over LIBOR is 30 bps. In order to make this a zero net present value swap at inception, the fixed rate is set to 6.4673%.

Forward Start Swap

Forward Start Swap Pricing: Fixed Rate = 6.3878%					
Date	Discount Factor	Floating CFs	Fixed CFs	PV of Floating CFs	PV of Fixed CFs
18-Dec-01	0.975369	100,000,000		97,536,900	0
18-Jun-02	0.948236		3,193,907		3,028,578
18-Dec-02	0.920011		3,193,907		2,938,430
18-Jun-03	0.890677		3,193,907		2,844,740
18-Dec-03	0.861854		3,193,907		2,752,682
18-Jun-04	0.832676		3,193,907		2,659,490
18-Dec-04	0.807344		103,193,907		83,312,982
Totals				97,536,900	97,536,900

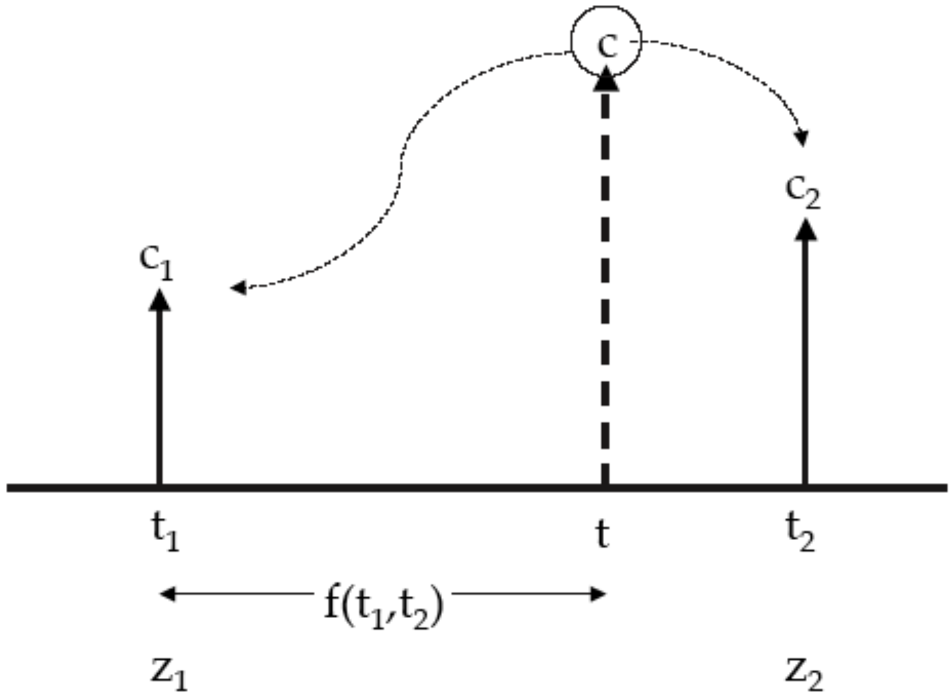
It is easy to see that the dates are moved forward 6 months. Hence the payments on the swap will begin in one year from inception.

Risks in Swaps

- Interest rate risk.
- Credit risk.
  - Counterparty risk.
  - Maturity.
  - Market risk effects.
  - Netting.
  - Multi-currency swaps (principal exposure).
  - MTM impact.
- Mismatch risk.
- Basis risk.

Hedging: swaps, bonds, futures.

Cashflow Bucketing



Preserve value: $$PV(c) = PV(c_1) + PV(c_2).$$

$$c\ \exp[-z_1 t_1 - f(t_1,t_2)(t-t_1)] = c_1\ \exp[-z_1 t_1] + c_2\ \exp[-z_2 t_2].$$

Preserve risk: $$PVBP(c) = PVBP(c_1) + PVBP(c_2).$$

$$t\ c\ \exp[-z_1 t_1 - f(t_1,t_2)(t-t_1)] = t_1\ c_1\ \exp[-z_1 t_1] + t_2\ c_2\ \exp[-z_2 t_2].$$

Caps and Floors

A cap (floor) is an option contract that pays off if the interest rate is above (below) a strike rate at maturity, like a call on interest rates.

A cap (floor) consists of a series of caplets (floorlets), single date options.

A cap (floor) is to an equity call (put), as an FRA is to a equity forward contract. Hence, the payoff to caps and floors are the maximum of zero and the payoff to an FRA.

Financing with Caps

2 yr financing:

- Semi-annual loan at 9%.
- FRN: LIBOR+10bps.

Current Libor = 8%

2 yr 8% cap premium = \$1.55/\$100.

"All-in-cost"

Time (months)	Cashflow	Present Value
0	98.45	98.4500
6	-4.05	-3.8763
12	-4.05	-3.7100
18	-4.05	-3.5508
24	-104.05	-87.3129
TOTAL	(IRR = 0.089637)	0.0000

Worst case still better than Fixed rate loan.

Financing with Floors

8% Floor premium = $0.40/$100
Sell floor to get subsidy.

Time (months)	Worst Case Cashflow	Best Case Cashflow
0	98.85	98.85
6	-4.05	-3.05
12	-4.05	-3.05
18	-4.05	-3.05
24	-104.05	-103.05
IRR	0.087392	0.067241

Dominates the pure use of caps.
But introduces a lower bound.

Swaptions

Option on a (forward start) swap.

No strike rate as exercise depends on the moneyness.
Strike is implicitly zero MTM value.

Swaption = option on a portfolio of FRAs.
Caps/Floors = portfolio of options on FRAs.

Which is worth more?

Put-Call Parity for Interest Rates

Swap(Pay X, Receive L) = Cap(X) - Floor(X)
= FRN(L) - Fixed Rate Bond(X)

PROOF:

We form 2 portfolios as follows:

- Portfolio A = Swap(Receive L, Pay X).
- Portfolio B = Cap(X) - Floor(X). This means a long cap position and a short floor position.

All securities have the same maturity. At each settlement date there are broadly speaking, two possible outcomes:

1. The LIBOR rate is greater than the strike, i.e. $L > X$. In this case portfolio A's cashflow is $[L - X] > 0$. The cap in portfolio B pays off an amount $[L - X]$, whereas the floor expires worthless. Therefore, the two portfolios return exactly the same cashflows.
2. The LIBOR rate is less than the strike, i.e. $L \leq X$. In this case portfolio A's cashflow is $[L - X] < 0$. The cap in portfolio B expires worthless, whereas the floor ends up in the money, and since it is in a short position the portfolio pays out $[-X + L] < 0$. Again, the two portfolios return exactly the same cashflows.

Foreign Exchange

Consider a domestic investor who wishes to trade T-periods forward on the \$-¥FX rate.

$$\boxed{F(T) = S \times \frac{(1 + r_{¥})^T}{(1 + r_{\$})^T}}$$

How do we arrive at this formula? It's simple, and is just an application of the law of one price. Suppose you own \$1 at the outset. There are two possible investment routes you can take:

1. You have the choice to invest in dollar bonds for T periods. The amount you will receive at maturity will be $(1 + r_{\$})^T$. We call this the *dollar route*.
2. You may also convert your dollar into Yen, invest the Yen for T periods, and finally convert the Yen back into dollars after T periods. To lock in the rate of conversion at the end, you book a forward FX contract today at forward price $F(T)$. The amount you will receive at maturity will then be $S \times (1 + r_{¥})^T \times \frac{1}{F(T)}$. Lets call this the *Yen route*.

To preclude arbitrage, i.e. abide by the law of one price, both amounts must be equal. Hence:

$$(1 + r_{\$})^T = S \times (1 + r_{¥})^T \times \frac{1}{F(T)}.$$

Example

Example: Suppose $S = 135$, $r_{¥} = 0.01$, and $r_{\$} = 0.03$. Let interest rates be compounded in annual periods. Then, the 1-year forward rate is $135 \times \frac{1.01}{1.03} = 132.3786$.

Why is the forward rate lower than the spot rate?

Rule: the currency at the higher (lower) interest rate trades at a discount (premium) in the forward markets.

WHY?

Forward Points

FX Forward Rates & Points (¥/$)				
	FX Rates		Points	
Maturity	Bid	Offer	Bid	Offer
0	134.9672	135.4672		
1	132.7981	133.3981	2.1691	2.0691
2	129.7906	130.4906	5.1767	4.9767
3	126.9866	127.7866	7.9806	7.6806
4	124.8685	125.7685	10.0987	9.6987
5	125.1499	126.1499	9.8173	9.3173

How do you skew your quoted rates when you want to be on one side of a trade?

Hedging Cross-Currency Borrowing

Fixed 5 yr US$ borrowing at 4.40%.
Fixed 5 yr Yen borrowing at 2.5%.
Which is cheaper?

Borrowing using FX Forward Markets					
	FX Rates		Yen cashflow	Conv Rate	$ cashflow
Maturity	Bid	Offer	at 2.50%		at **4.16%**
0	134.9672	135.4672	13500.0000	135.4672	99.6551
1	132.7981	133.3981	-337.5000	132.7981	-2.5415
2	129.7906	130.4906	-337.5000	129.7906	-2.6003
3	126.9866	127.7866	-337.5000	126.9866	-2.6578
4	124.8685	125.7685	-337.5000	124.8685	-2.7028
5	125.1499	126.1499	-13837.5000	125.1499	-110.5674

(Synthetic USD$ financing)

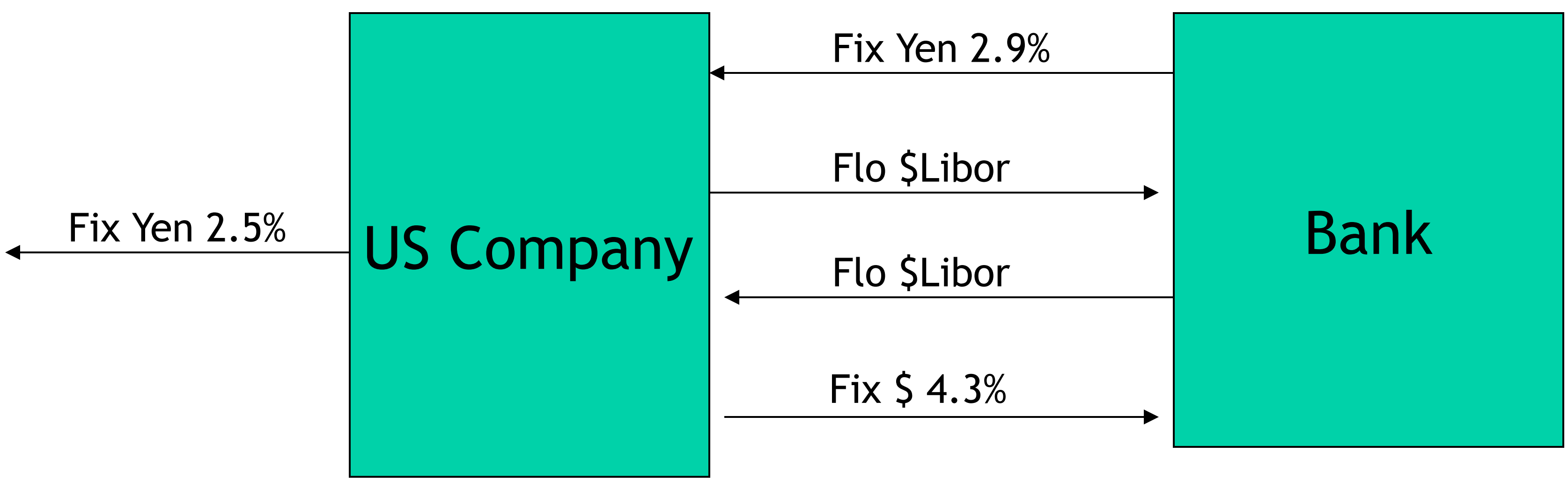
Currency Swaps

Usually fixed FCY vs floating $Libor.

Example:

1. Fixed Yen vs 1-yr $ LIBOR: 2.90-3.00%

2. Fixed $ vs 1-yr $ LIBOR: 4.20-4.30%.



Fixed $ with Yen bps drag

Borrowing using Currency Swaps

Maturity	Yen cf at 2.50%	Receive Yen at 2.90%	Pay $ at 4.30%	Net Yen
0	13500.00	-13500.00	100.00	0.00
1	-337.50	391.50	-4.30	54.00
2	-337.50	391.50	-4.30	54.00
3	-337.50	391.50	-4.30	54.00
4	-337.50	391.50	-4.30	54.00
5	-13837.50	13891.50	-104.30	54.00

Basis Points Conversion

Maturity	FX Rates Bid	FX Rates Offer	Yen cashflow	Conv Rt	$ cashflow **0.4203%**
0	134.9672	135.4672	-100.0000	135.2172	-0.7396
1	132.7981	133.3981	0.4000	133.3981	0.0030
2	129.7906	130.4906	0.4000	130.4906	0.0031
3	126.9866	127.7866	0.4000	127.7866	0.0031
4	124.8685	125.7685	0.4000	125.7685	0.0032
5	125.1499	126.1499	0.4000	126.1499	0.0032
5			100.0000		0.7396

Net all-in-cost = $4.30-$0.4203 = $3.88%

Commodity Swaps

Exchange a fixed amount for a floating payment that is based on the price of a commodity.

May be analyzed a collection of commodity forward contracts.

k = no of settlements per year.

Settlement: compare S_t with k.

Pricing a Commodity Swap

Goal: find the fixed payment Y that makes the swap fair (i.e. zero value) at inception.

$$\text{PV of the fixed payments on the swap} = Y \sum_{i=1}^{Tk} e^{-r(i/k)}$$

$$\text{PV of the floating payments on the swap} = \sum_{i=1}^{Tk} e^{-r(i/k)} F(i/k)$$

$$\boxed{Y = \frac{\sum_{i=1}^{Tk} e^{-r(i/k)} F(i/k)}{\sum_{i=1}^{Tk} e^{-r(i/k)}}}$$

Maturity (t)	Spot Rate (r_t)	Forward Price (F_t)	PV(Fixed=43.68)	PV(Floating)
1	0.01	44.66	44.22	43.25
2	0.0125	44.78	43.67	42.61
3	0.02	43.67	41.13	41.14
4	0.03	43.24	38.35	38.74
5	0.035	41.75	35.04	36.67
Total			202.41	202.41

ACME problem (from Mason, Merton, Perold & Tufano) "Cases in Financial Engineering"

1. Acme Manufacturing wants to raise \$100 million of 3-year debt in the Euromarket (where interest is quoted and paid on an annual basis). Its alternatives are either a 3-year fixed-rate note at a spread of 250 basis points over the 3-year U.S. Treasury (currently yielding 4.50%), or a 3-year floating-rate note on which it must pay yearly interest of 1-year LIBOR + 2% (1-year LIBOR currently yields 3.70%; therefore, the floating-rate note's first interest payment of \$5.7 million would be paid at the end of year 1). However, Acme wants to have a fixed-rate liability for the entire 3 years. Assuming the following instruments also are available to Acme (with National Trust, an AAA-rated bank, as the counterparty), which strategy will enable Acme to pay the lowest all-in fixed rate of interest (assume all rates are quoted on an annual basis)?

Swap:	3-year swap has all-in pay-fixed rate of 3-year Treasury + 30 basis points (= 4.50% + 0.30%) versus receiving 1-year LIBOR.
FRAs:[14]	12/24 FRA has all-in pay-fixed rate of 2-year Treasury + 90 basis points (= 4.10% + 0.90%) versus receiving 1-year LIBOR.
	24/36 FRA has all-in pay-fixed rate of 3-year Treasury + 150 basis points (= 4.50% + 1.50%) versus receiving 1-year LIBOR.

Continued overlead ….

Caps: Premiums for 3-year caps (with annual pay) on 1-year LIBOR:

Strike Rate	Premium
4.00%	2.23%
4.80%	0.70%
5.00%	0.49%
6.00%	0.08%

Floors: Premiums for 3-year floors (with annual pay) on 1-year LIBOR:

Strike Rate	Premium
4.00%	0.18%
4.80%	0.70%
5.00%	0.90%
6.00%	3.22%

What is the cheapest financing structure?