



# GAME PLAYING MINIMAX ALGORITHM ALPHA-BETA PRUNING

Fundamentals of Artificial Intelligence

Session 12

Pramod Sharma  
pramod.sharma@prasami.com

2

## Agenda



4/13/2024

pra-sâmi

3

## Games

- ❑ So far single Agent
- ❑ There can be multi agent problems
- ❑ Good reasoning problems
  - ❖ Formal rules
  - ❖ Non-trivial
  - ❖ Time tested
- ❑ Direct comparison to human performance!

4/13/2024

pra-sâmi

4

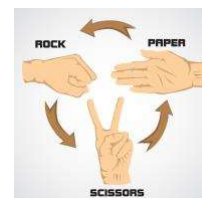
## Type of Games



Sequence of moves



Opponent also moves



Both play simultaneously

- ❑ Rewards for each move
  - ❖ May be, may be not
- ❑ Defined win conditions

4/13/2024

pra-sâmi

5

## Search versus Games

- ❑ Search – no adversary
  - ❖ Solution is (heuristic) method for finding goal
  - ❖ Heuristics and Constraints Satisfaction Problems techniques can find optimal solution
  - ❖ Evaluation function: estimate of cost from start to goal through given node
  - ❖ Examples: path planning, scheduling activities

4/13/2024

pra-sâmi

6

## Search versus Games

- ❑ Games – adversary
  - ❖ Unpredictable opponent
    - You make a move,
    - Opponent makes a move
  - ❖ Solution is strategy
    - Strategy specifies move for every possible opponent's reply
  - ❖ Optimality depends on opponent. Why?
  - ❖ Time limits
    - Large problems, huge state space, not enough compute power
    - Force an approximate solution
  - ❖ Evaluation function: evaluate "goodness" of game position
  - ❖ Examples: chess, checkers, Othello, backgammon

4/13/2024

pra-sâmi

7

## Adversarial Search

- ❑ Examine the problems that arise when we try to plan ahead in a world where other agents are planning against us
- ❑ A good example is board games
- ❑ Adversarial games, while much studied in AI, are a small part of game theory in economics

4/13/2024

pra-sâmi

8

## Games - Typical Assumptions

- ❑ Two agents whose actions alternate
  - ❖ May be more than 2
- ❑ Utility values for each agent are the opposite of the other
  - ❖ Creates the adversarial situation
  - ❖ You want to maximize your position so does your opponent
  - ❖ Both functions are interdependent
  - ❖ Simply put, you want to maximize your position and your opponent wants to minimize your position
- ❑ Fully observable environments
- ❑ In game theory terms: Zero-sum games of perfect information.
- ❑ Generalizes to stochastic games, multiple players, non-zero-sum, etc.
- ❑ Relaxation in these assumptions make it more complex

4/13/2024

pra-sâmi

9

## Types of Games

|   | Deterministic                | Chance Moves                       |
|---|------------------------------|------------------------------------|
| Perfect Information                             | Chess, Checkers, Go, Othello | Backgammon, Monopoly               |
| Imperfect Information<br>(Initial Chance Moves) | Skat, Battleship, Kriegspiel | Bridge, Poker, Scrabble, Blackjack |

Not Considered: Physical games like tennis, croquet, ice hockey, etc.  
(but see "robot soccer" <http://www.robocup.org/>)

4/13/2024

pra-sâmi

10

## Perfect and Imperfect Information Games

- A perfect information or Markov game is fully observed



Chess



Backgammon



Othello



Go



Checkers

- An imperfect information game is partially observed
  - ❖ Scrabble
  - ❖ Poker

4/13/2024

pra-sâmi

11

## Environment Types

- ❑ Turn-taking: Semi-dynamic
- ❑ Deterministic and non-deterministic

4/13/2024

pra-sâmi

12

## Size of Search Trees

- ❑  $b$  = branching factor
- ❑  $d$  = number of moves by both players
- ❑ Search tree is  $O(b^d)$
- ❑ Chess
  - ❖  $b \sim 35$
  - ❖  $D \sim 100$ 
    - Search tree is  $35^{100} = 2.5516E+154$  (!!)
    - Completely impractical to search this
- ❑ Game-playing emphasizes being able to make optimal decisions in a finite amount of time
  - ❖ Somewhat realistic as a model of a real-world agent
  - ❖ Even if games themselves are artificial

4/13/2024

pra-sâmi

13

## Two Fundamental Branches

### ❑ Reinforcement Learning:

- ❖ The environment is initially unknown
- ❖ The agent interacts with the environment
- ❖ The agent improves its policy

### ❑ Planning:

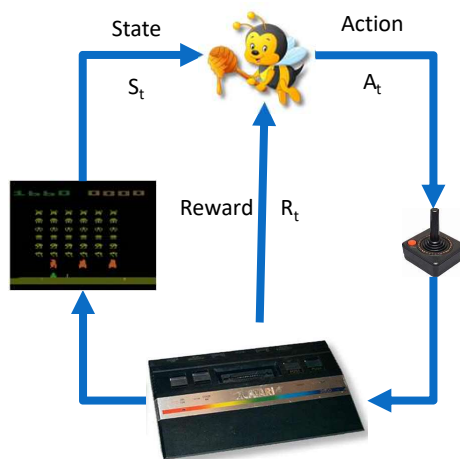
- ❖ A model of the environment is known
- ❖ The agent performs computations with its model (without any external interaction)
- ❖ The agent improves its policy a.k.a. deliberation, reasoning, introspection, pondering, thought, search

4/13/2024

pra-sâmi

14

## Atari Example: Reinforcement Learning



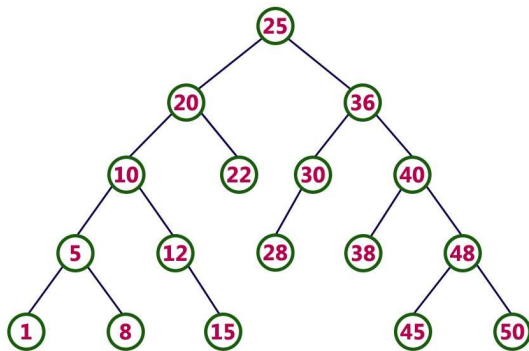
- ❑ Rules of the game are unknown
- ❑ Learn directly from interactive game-play
- ❑ Pick actions on joystick, see pixels and scores

4/13/2024

pra-sâmi

15

## Atari Example : Planning



- ❑ Rules of the game are known
- ❑ Can query emulator
  - ❖ Perfect model inside agent's brain?
- ❑ If I take action 'a' from state 's':
  - ❖ What would the next state be?
  - ❖ What would the score be?
- ❑ Plan ahead to find optimal policy
  - ❖ Tree search

4/13/2024

pra-sâmi

16

## Game Setup

- ❑ A two-player game has two (alternating) players: MAX and MIN
  - ❖ MAX moves first and they take turns until the game is over
- ❑ Games as search:
  - ❖ Initial state: e.g. board configuration of chess
  - ❖ Successor function: list of ( move, state) pairs specifying legal moves.
  - ❖ Terminal test: Is the game finished?
  - ❖ Utility function: Gives numerical value of terminal states
- ❑ A zero sum game has equal and opposite rewards for black and white
  - ❖  $R_w + R_b = 0$
  - ❖ Winner gets reward, loser gets penalty
  - ❖ Tic-Tac-Toe : win (+1), lose (-1) and draw (0)
  - ❖ Chess : win (+1), lose (-1) and draw ( $\frac{1}{2}$ )
- ❑ Both methods are applicable in such cases
  - ❖ Game tree search (i.e. planning)
  - ❖ Self-play reinforcement learning

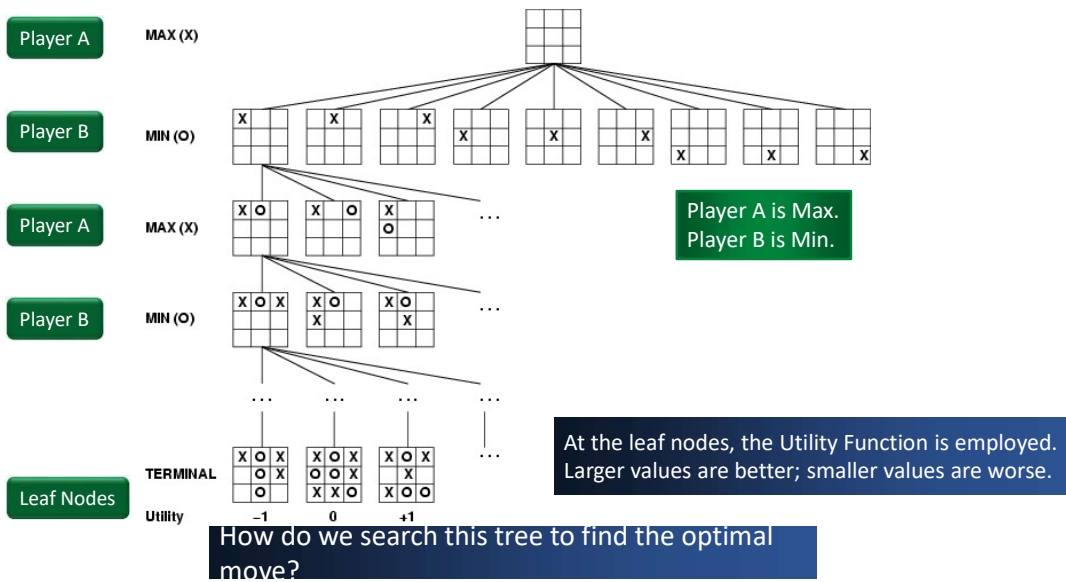
4/13/2024

pra-sâmi



17

## Game Tree (2-player, Deterministic, Turns)



4/13/2024

pra-sâmi

18

## Minimax Terminology

- ❑ Move: a move by both players
- ❑ Ply : one players move; a half-move
- ❑ Utility Function : function applied to leaf node
- ❑ Backed-up values
  - ❖ For Max : the values of largest successor
  - ❖ For Min : the values of smallest successor
- ❑ Minimax Procedure:
  - ❖ Search down several levels
  - ❖ At the bottom apply utility function
  - ❖ Back-up values all the way up to root node
  - ❖ Root selects the move

4/13/2024

pra-sâmi

19

## Minimax Search

- ❑ Formal definition as a search problem:
  - ❖ Initial state: Set-up specified by the rules, e.g., initial board configuration of chess.
  - ❖ Player(s): Defines which player has the move in a state.
  - ❖ Actions(s): Returns the set of legal moves in a state.
  - ❖ Result( s, a): Transition model defines the result of a move.
  - ❖ Successor function: list of ( move, state) pairs specifying legal moves
  - ❖ Terminal-Test(s): Is the game finished? True if finished, false otherwise.
  - ❖ Utility function( s, p ): Gives numerical value of terminal state 's' for player p.
  
- ❑ MAX uses search tree to determine next move

4/13/2024

pra-sâmi

20

## An Optimal Procedure: The Minimax Method

- ❑ Designed to find the optimal strategy for Max and find best move
  - ❖ Assuming an infallible MIN opponent
  
- ❑ Assumption: Both players play optimally!
  
- ❑ Definition of optimal play for MAX assumes MIN plays optimally:
  - ❖ Maximize worst-case outcome for MAX
  
- ❑ But if MIN does not play optimally, MAX will do even better

4/13/2024

pra-sâmi

21

## An Optimal Procedure: The Minimax Method

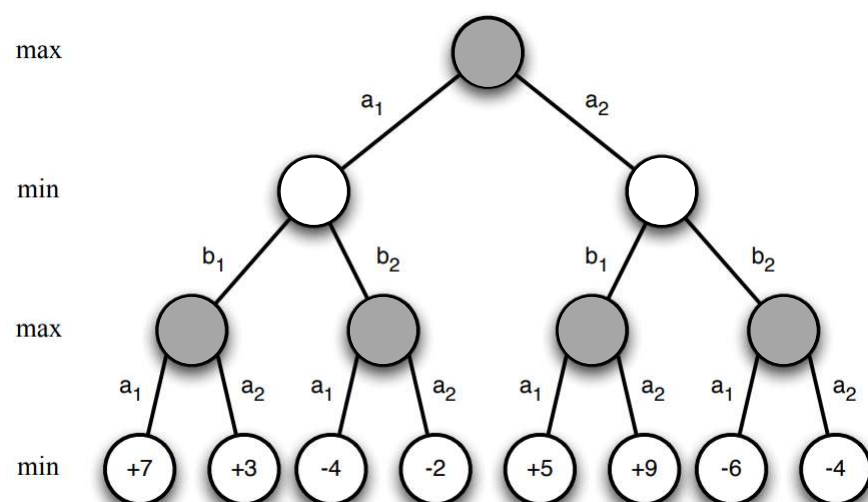
- ❑ Step 1. Generate the whole game tree, down to the leaves
- ❑ Step 2. Apply utility (payoff) function to each leaf
- ❑ Step 3. Back-up values from leaves through branch nodes:
  - ❖ a Max node computes the Max of its child values
  - ❖ a Min node computes the Min of its child values
- ❑ Step 4. At root: choose the move leading to the child of highest value
- ❑ Minimax values can be found by depth-first game-tree search
- ❑ Introduced by Claude Shannon: Programming a Computer for Playing Chess
  - ❖ Ran on paper!

4/13/2024

pra-sâmi

22

## Minimax Search Example

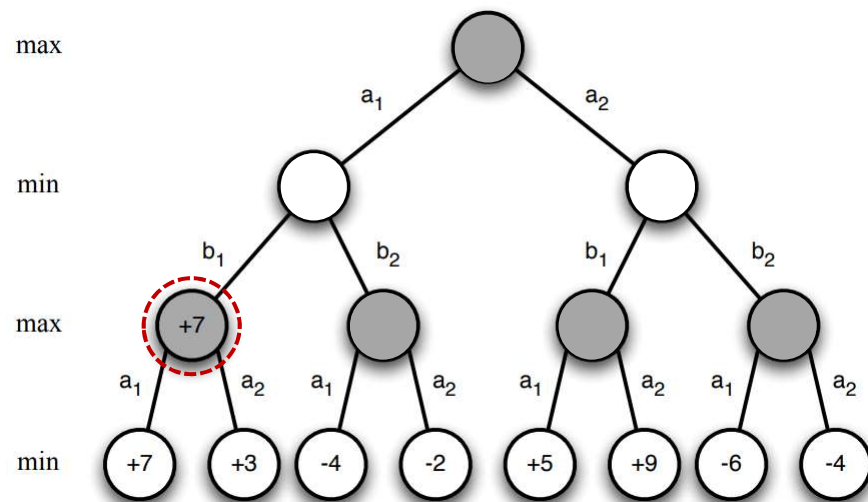


4/13/2024

pra-sâmi

23

## Minimax Search Example

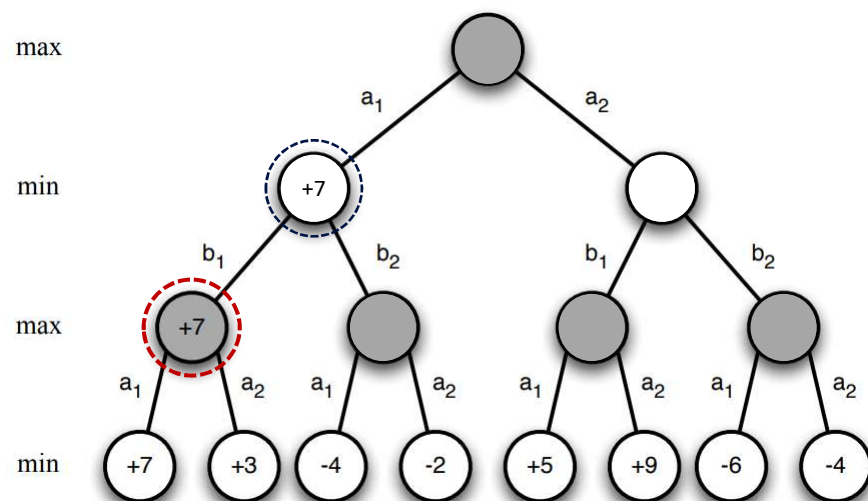


4/13/2024

pra-sâmi

24

## Minimax Search Example

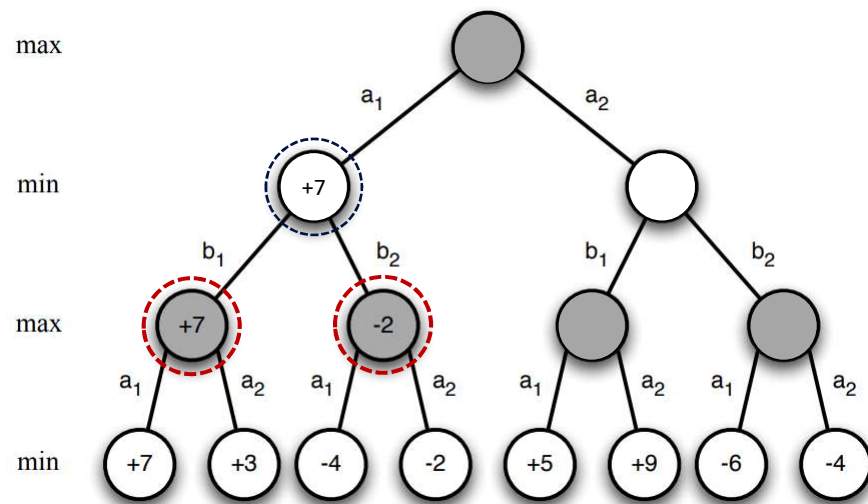


4/13/2024

pra-sâmi

25

## Minimax Search Example

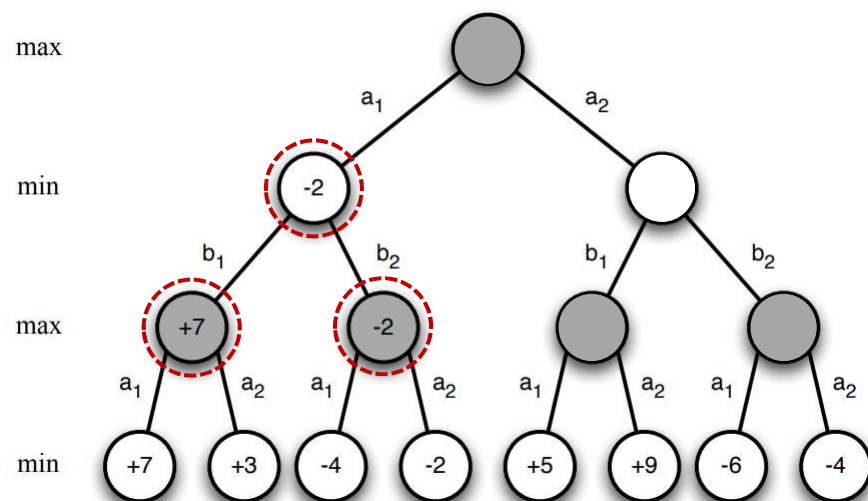


4/13/2024

pra-sâmi

26

## Minimax Search Example

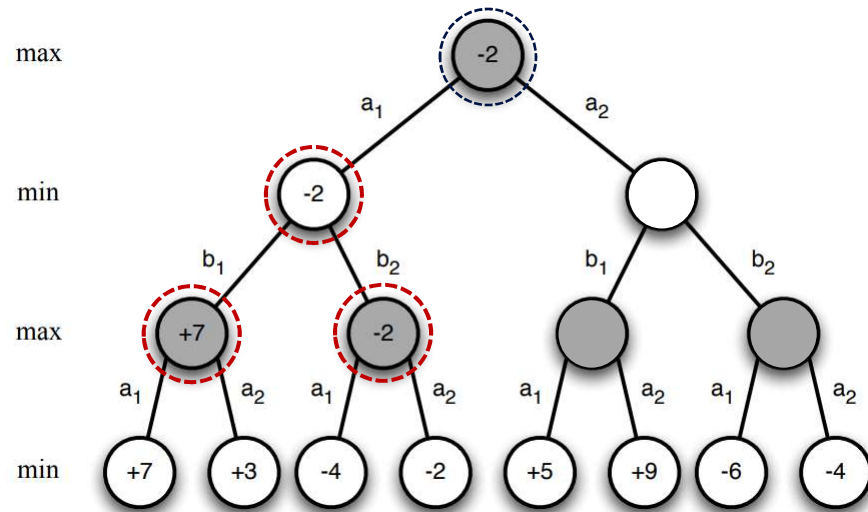


4/13/2024

pra-sâmi

27

## Minimax Search Example

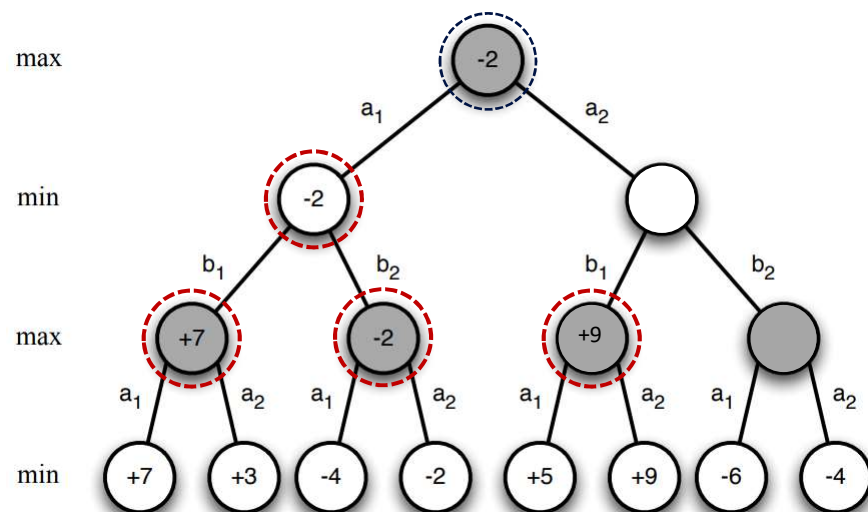


4/13/2024

pra-sâmi

28

## Minimax Search Example

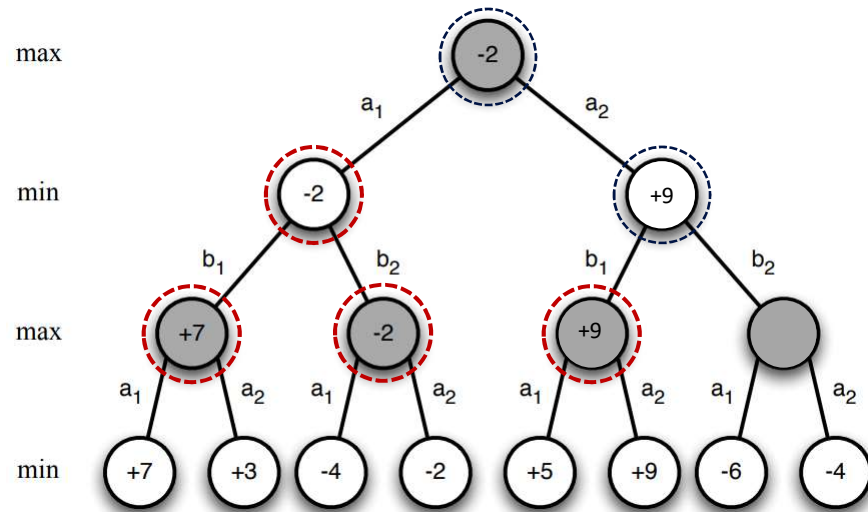


4/13/2024

pra-sâmi

29

## Minimax Search Example

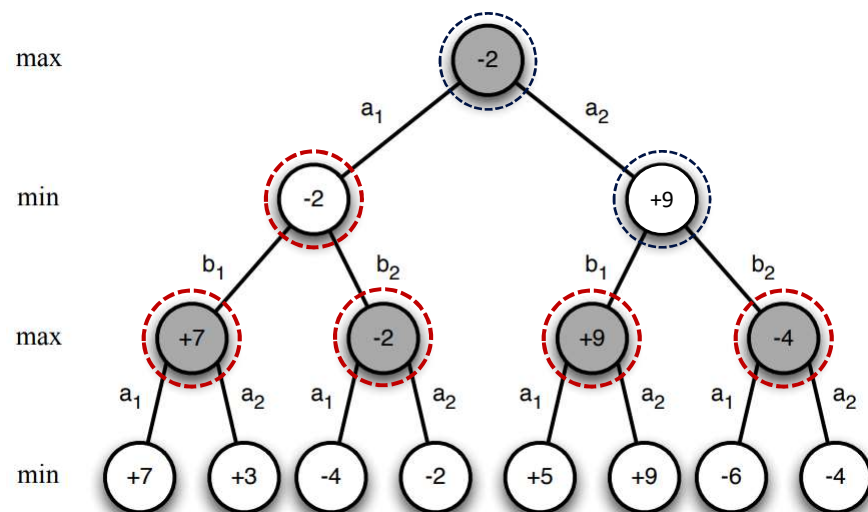


4/13/2024

pra-sâmi

30

## Minimax Search Example

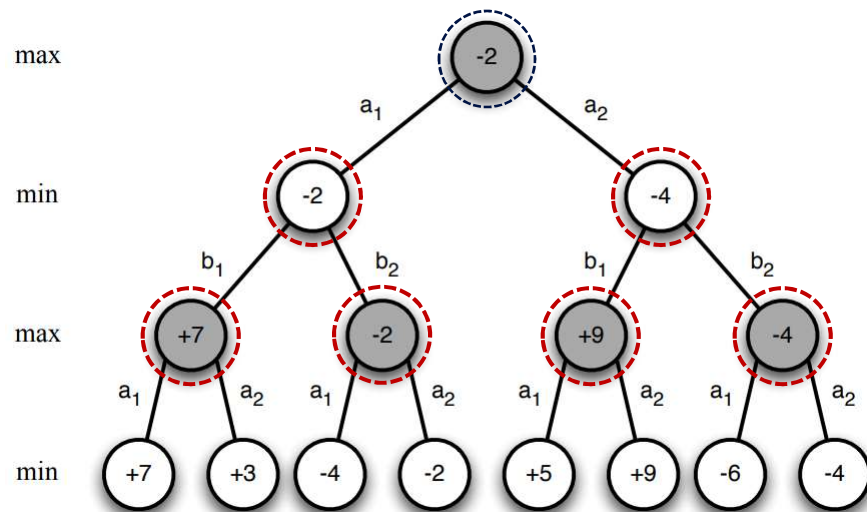


4/13/2024

pra-sâmi

31

## Minimax Search Example

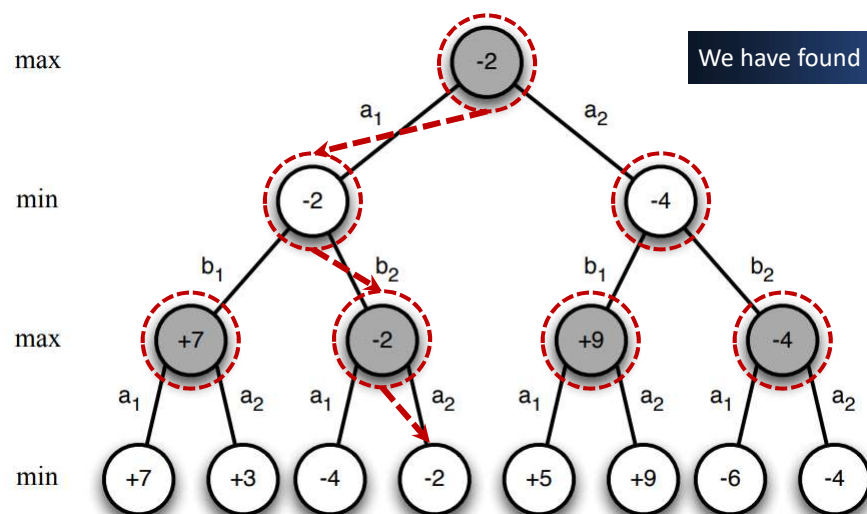


4/13/2024

pra-sâmi

32

## Minimax Search Example



4/13/2024

pra-sâmi



33

## Static (Heuristic) Evaluation Functions

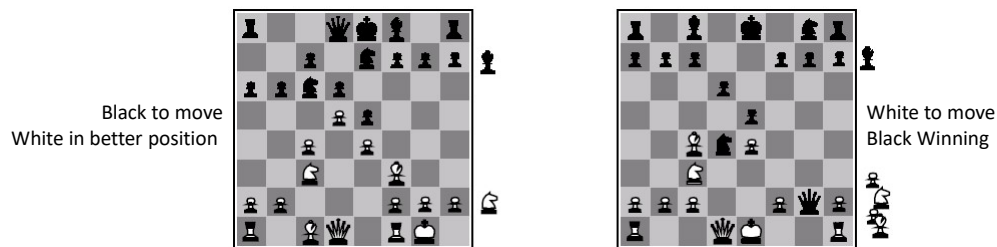
- ❑ An Evaluation Function:
  - ❖ Estimates how good the current board configuration is for a player.
  - ❖ Typically, one figures how good it is for the player, and how good it is for the opponent, and subtracts the opponents score from the players
  - ❖ Othello: Number of white pieces - Number of black pieces
  - ❖ Chess: Value of all white pieces - Value of all black pieces
- ❑ Typical values from -infinity (loss) to +infinity (win) or [-1, +1].
- ❑ If the board evaluation is X for a player, it's -X for the opponent.
- ❑ Many clever ideas about how to use the evaluation function.
  - ❖ e.g. null move heuristic: let opponent move twice.
- ❑ Example: Evaluating chess boards, Checkers, Tic-tac-toe

4/13/2024

pra-sâmi

34

## Evaluation Function



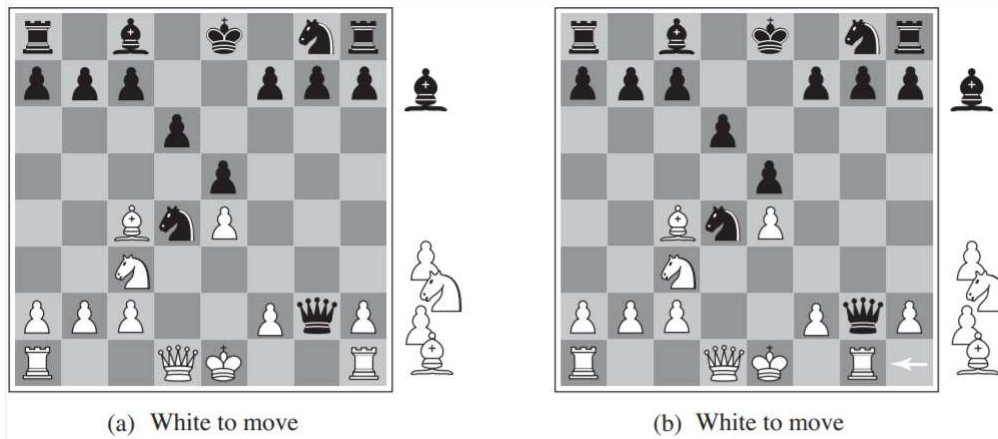
- ❑ For Chess, use linear weighted sum of features
  - ❖  $E(s) = w_1 \times f_1(s) + w_2 \times f_2(s) + \dots + w_n \times f_n(s)$
  - ❖ Where  $w_n$  is weight assigned to any coin (say  $w_n = 9$  for queen and  $w_n = 1$  for Pawn)
  - ❖  $f_n(s)$  = number of white pieces – number of black pieces
    - For 2 white knight and 1 black knight;  $f_{knight}(s) = 1$

4/13/2024

pra-sâmi

35

## Evaluation Function



Two chess positions that differ only in the position of the rook at lower right. In (a), Black has an advantage of a knight and two pawns, which should be enough to win the game. In (b), White will capture the queen, giving it an advantage that should be strong enough to win. Adding up the values of features apparently reasonable, but involves a strong assumption; The contribution of each feature is independent of the values of the other features. For example, assigning the value 3 to a bishop ignores the fact that bishops are more powerful in the endgame, when they have a lot of space to maneuver

4/13/2024

pra-sâmi

36

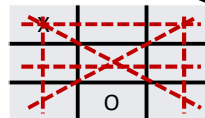
## Applying Minimax to Tic-tac-toe

- The static heuristic evaluation function  $E(n) = M(n) - O(n)$

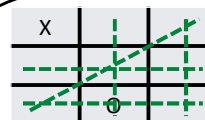
- ❖  $M(n)$  : total of Max's possible winning lines
- ❖  $O(n)$  : total of Min's possible winning lines
- ❖  $E(n)$  : total Evaluation of the State

|   |   |  |
|---|---|--|
| X |   |  |
|   |   |  |
|   | O |  |

- X has 6 possible win paths



- O has 5 possible win paths



- Therefore  $E(n) = 6 - 5 = 1$

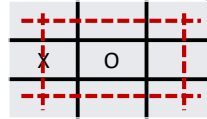
4/13/2024

pra-sâmi

37

## Applying Minimax to Tic-tac-toe

- X has 4 possible win paths



- O has 6 possible win paths



- Therefore  $E(n) = 4 - 6 = -2$

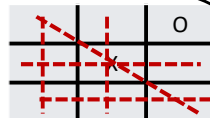
4/13/2024

pra-sâmi

38

## Applying Minimax to Tic-tac-toe

- X has 5 possible win paths



- O has 4 possible win paths



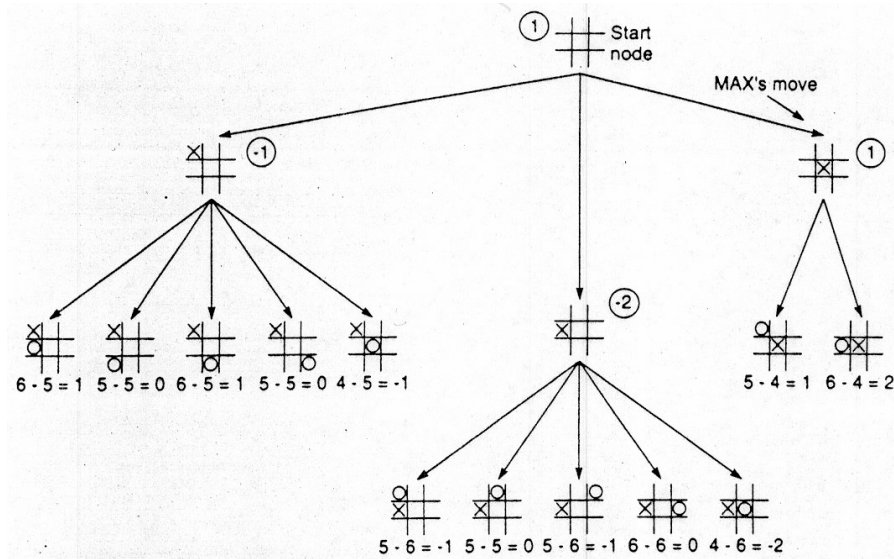
- Therefore  $E(n) = 5 - 4 = 1$

4/13/2024

pra-sâmi

39

## Two-ply Minimax Applied to the Opening Move of Tic-tac-toe

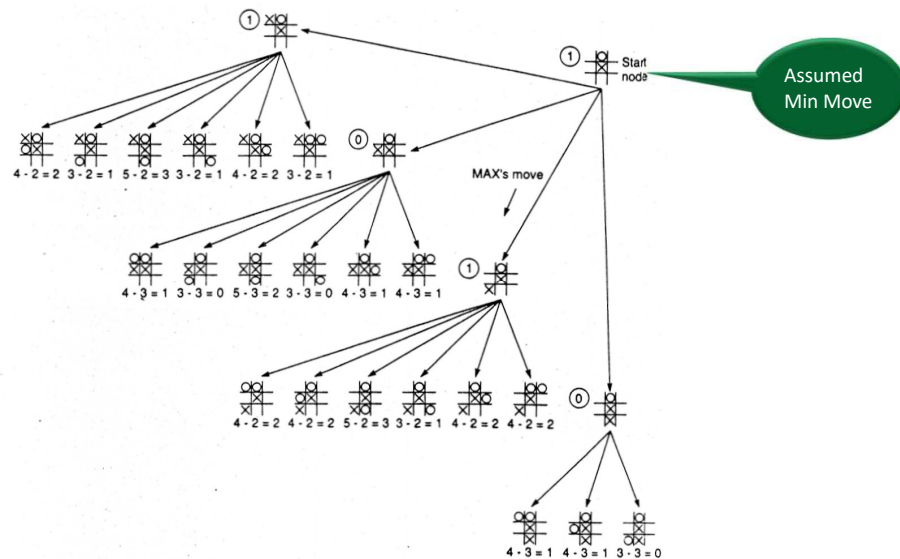


4/13/2024

pra-sâmi

40

## Two-ply Minimax Applied to X's Second Move of Tic-tac-toe

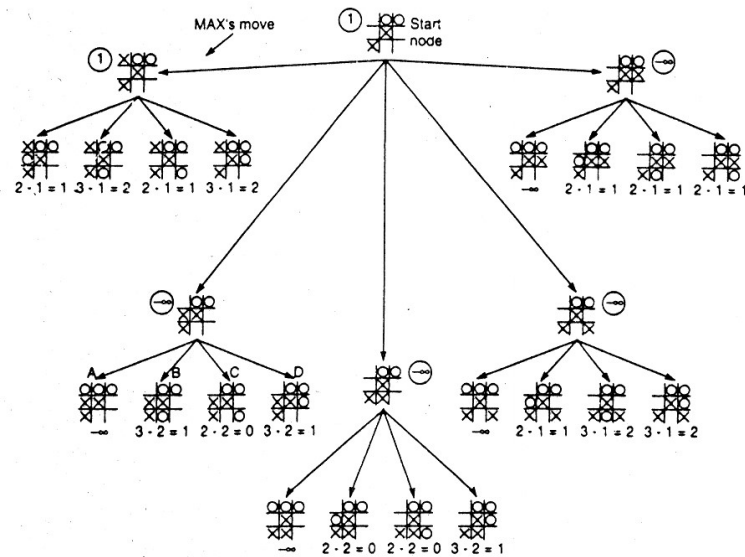


4/13/2024

pra-sâmi

41

## Two-ply Minimax Applied to X's Move Near End Game



4/13/2024

pra-sâmi

42

## Limitation

Minimax search is suitable for games like tic-tac-toe and not suitable for game of chess.

- ❑ Time complexity of the search is  $O(b^d)$ 
  - ❖  $b$  : number of branches
  - ❖  $d$  : depth of the tree
- ❑ Hence for the example with  $d = 3$ 
  - ❖ Time complexity will be  $2^3 = 8$
- ❑ In regular game of chess ,
  - ❖ Average choices are 35,
  - ❖ Average game lasts for about 50 moves by a player → Total 100 moves
- ❑ Hence complexity will be  $35^{100} = 2.5516 \times 10^{154}$

4/13/2024

pra-sâmi

43

## Solution to the Complexity Problem

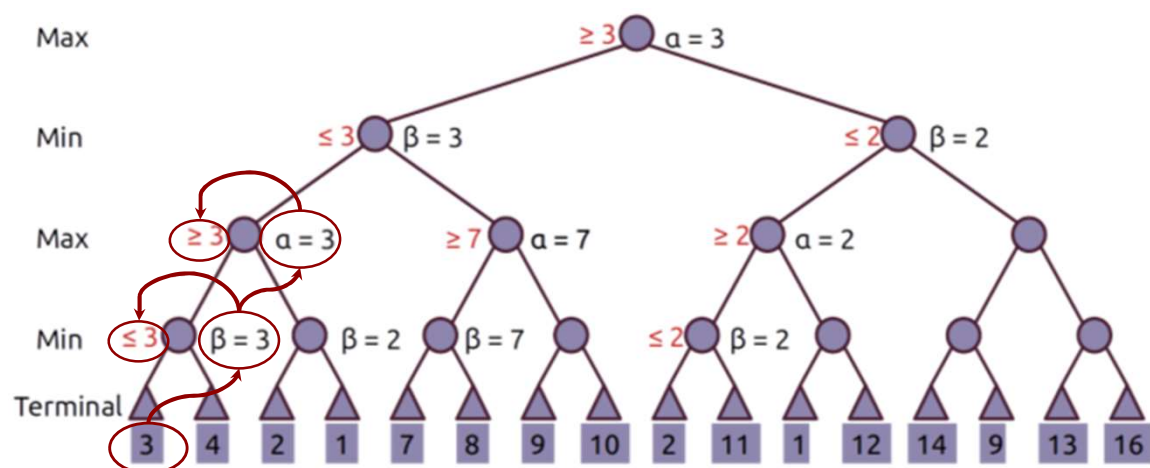
- ❑ Two solutions
  - ❖ Identify a provably suboptimal branch of the search tree before it is fully explored
  - ❖ Eliminate the suboptimal branch
  - ❖ It is called Alpha – Beta Pruning
  - ❖ Some branches will never be played by rational players since they include sub-optimal decisions for either player
- ❑ Early cutoff the search tree
- ❑ Use imperfect minimax value estimate for non-terminal states (positions)
  - ❖  $\alpha$  = highest-value choice that we can guarantee for MAX so far in the current subtree.
  - ❖  $\beta$  = lowest-value choice that we can guarantee for MIN so far in the current subtree.
  - ❖ Update values of  $\alpha$  and  $\beta$  during search and prunes remaining branches as soon as the value is known to be worse than the current  $\alpha$  or  $\beta$  value for MAX or MIN respectively.

4/13/2024

pra-sâmi

44

## Alpha Beta Pruning

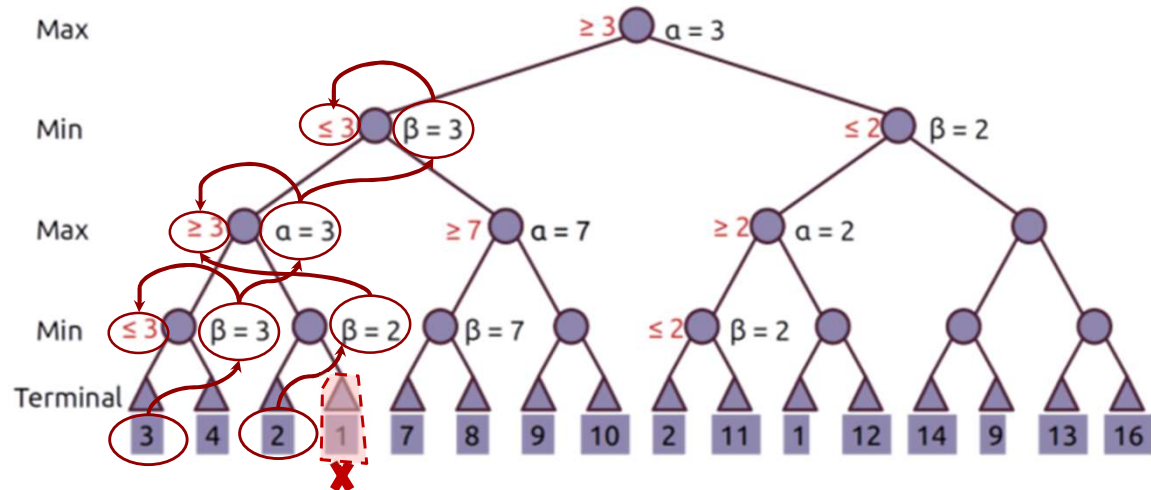


4/13/2024

pra-sâmi

45

## Alpha Beta Pruning

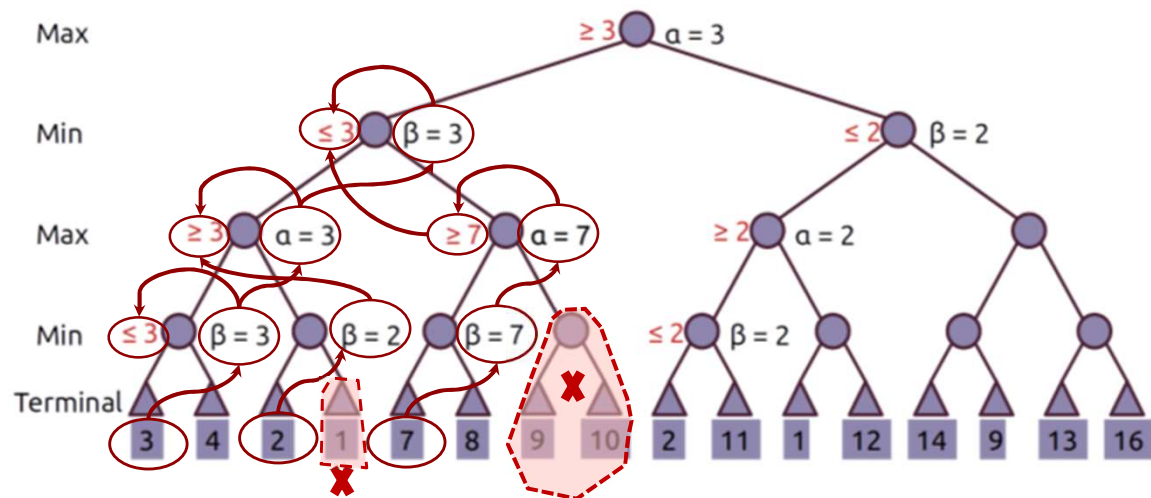


4/13/2024

pra-sâmi

46

## Alpha Beta Pruning

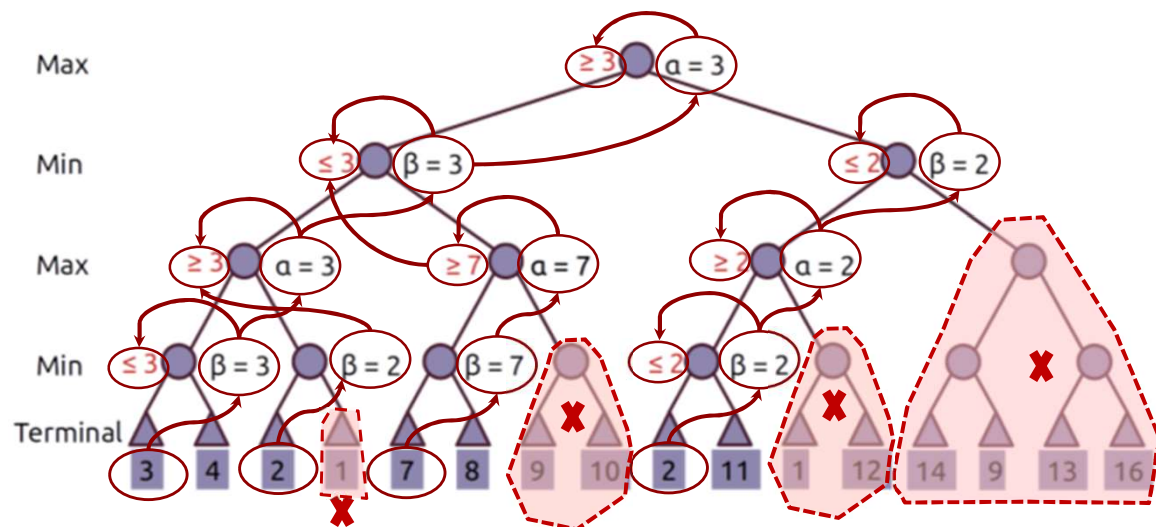


4/13/2024

pra-sâmi

47

## Alpha Beta Pruning



4/13/2024

pra-sâmi

48

## Effectiveness of Alpha-Beta Search

- ❑ Worst-Case
  - ❖ Branches are ordered so that no pruning takes place
  - ❖ Gives no improvement over exhaustive search
- ❑ Best-Case
  - ❖ Each player's best move is the left-most child (i.e., evaluated first)
  - ❖ In practice, performance is closer to best rather than worst-case
    - E.g., sort moves by the remembered move values found last time
    - E.g., expand captures first, then threats, then forward moves, etc.
    - E.g., run Iterative Deepening search, sort by value last iteration
- ❑ In practice often get  $O(b^{\frac{d}{2}})$  rather than  $O(b^d)$ 
  - ❖ This is the same as having a branching factor of  $\sqrt{b}$ ,
    - $\sqrt{b}^d = b^{\frac{d}{2}}$ , i.e., we effectively go from  $b$  to square root of  $b$
  - ❖ e.g., in chess go from  $b \sim 35$  to  $b \sim 6$ 
    - This permits much deeper search in the same amount of time

4/13/2024

pra-sâmi



49

## Final Comments about Alpha-Beta Pruning

- ❑ Pruning does not affect final results
- ❑ Entire subtrees can be pruned
- ❑ Good move ordering improves effectiveness of pruning
- ❑ Repeated states are again possible
  - ❖ Store them in memory = transposition table

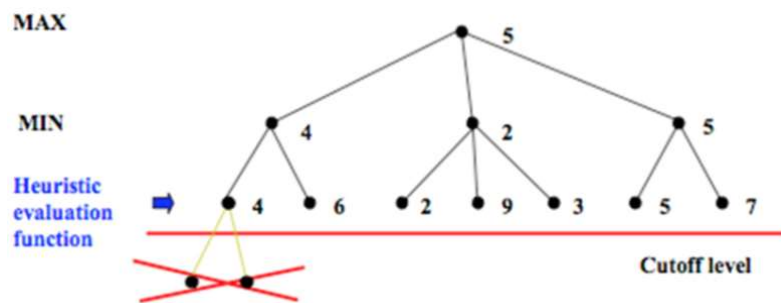
4/13/2024

pra-sâmi

50

## Early Cutoff the Search Tree

- ❑ Using Minimax value estimates
  - ❖ Cutoff the search tree before the terminal state is reached
  - ❖ Use imperfect estimate of the minimax value at the leaves
    - Evaluation function



4/13/2024

pra-sâmi

51

## Practical Implementation

- ❑ How do we make these ideas practical in real game trees?
- ❑ Standard approach:
  - ❖ Cutoff test: (where do we stop descending the tree)
    - Depth limit
    - Better: iterative deepening
    - Cutoff only when no big changes are expected to occur next
  - ❖ Evaluation function
    - When the search is cut off, we evaluate the current state by estimating its utility using an evaluation function.

4/13/2024

pra-sâmi

52

## Iterative (Progressive) Deepening

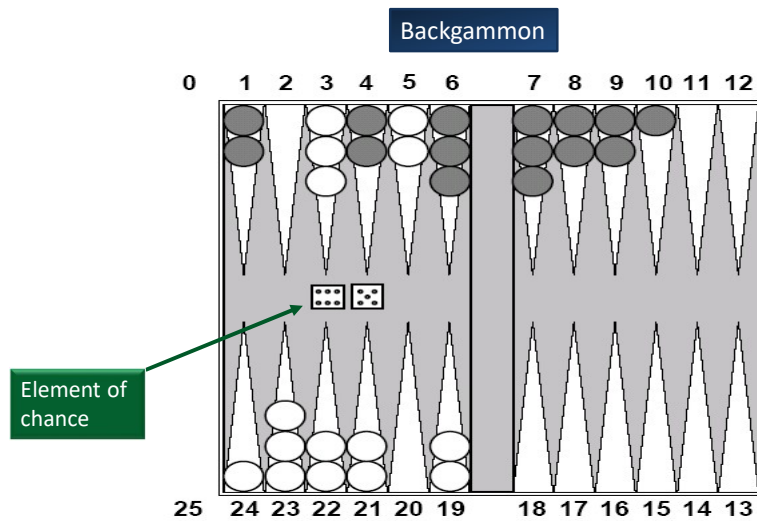
- ❑ In real games, there is usually a time limit  $T$  on making a move
- ❑ How do we take this into account?
  - ❖ Using alpha-beta we cannot use “partial” results with any confidence unless the full breadth of the tree has been searched
  - ❖ So, we could be conservative and set a conservative depth-limit which guarantees that we will find a move in time  $< T$ 
    - Disadvantage is that we may finish early, could have done more search
- ❑ In practice, iterative deepening search (IDS) is used
  - ❖ IDS runs depth-first search with an increasing depth-limit
  - ❖ When the clock runs out we use the solution found at the previous depth limit

4/13/2024

pra-sâmi

53

## Chance Games



4/13/2024

pra-sâmi

54

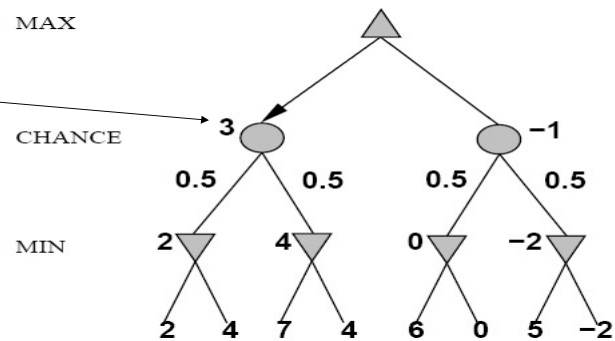
## Expected Minimax

$$v = \sum_{\text{chance nodes}} P(n) \times \text{Minimax}(n)$$

$$3 = 0.5 \times 4 + 0.5 \times 2$$

Interleave chance nodes  
with min/max nodes

Again, the tree is constructed  
bottom-up

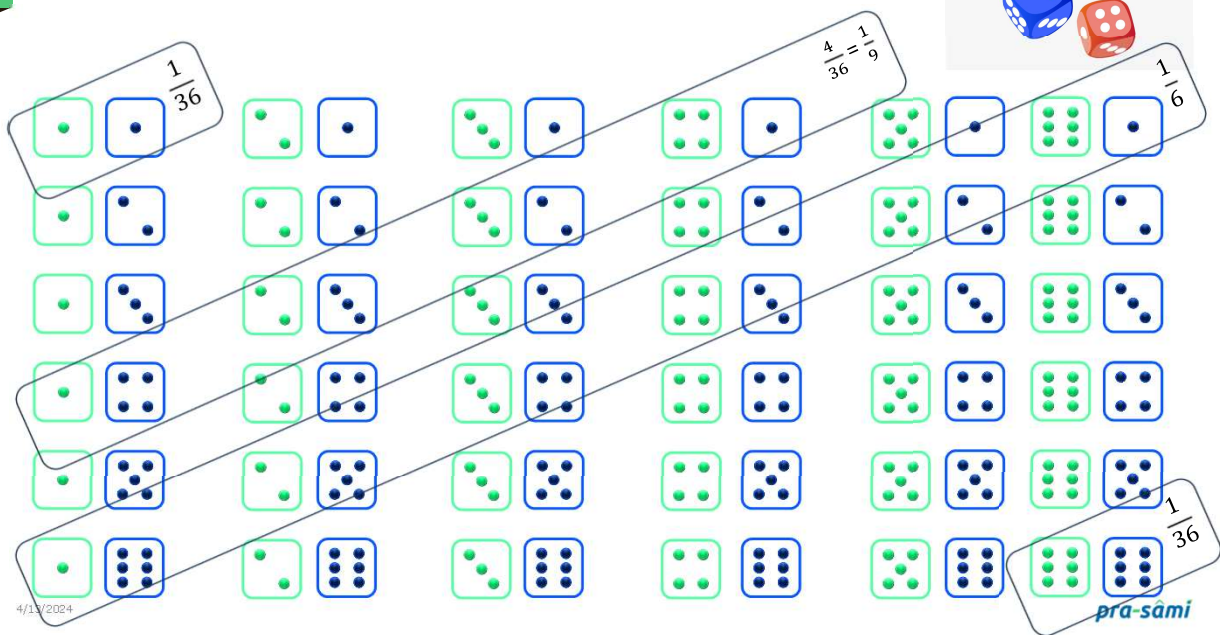


4/13/2024

pra-sâmi

55

## Dice World



56

## Game Tree for Backgammon

MAX

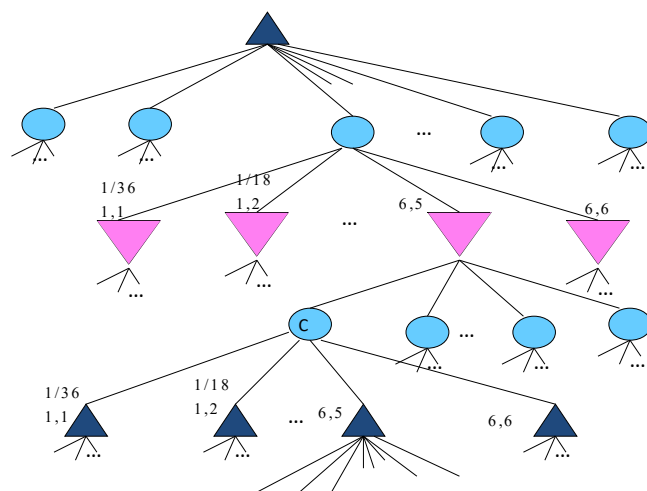
DICE

MIN

DICE

MAX

TERMINAL

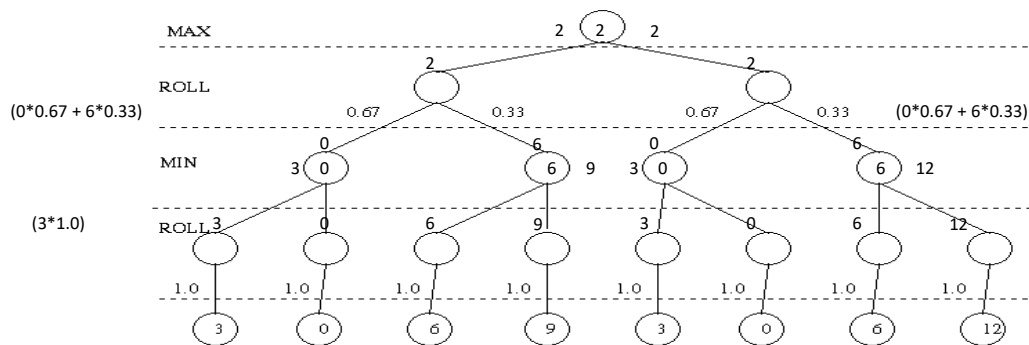


4/13/2024

pra-sâmi

58

## Expectiminimax Example

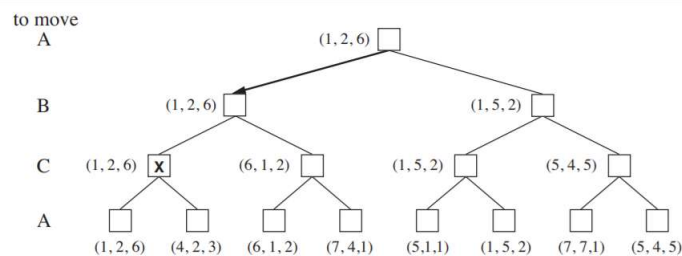


4/13/2024

pra-sâmi

59

## Optimal Decisions in Multiplayer Games



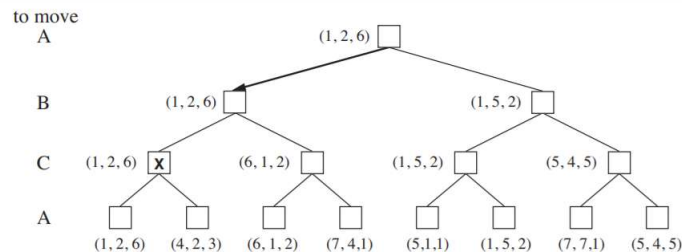
- ❑ A vector of values instead of a single value for each node
- ❑ Multiplayer games usually involve alliances, among the players
  - ❖ May be formal or informal
- ❑ Alliances are made and broken as the game proceeds
- ❑ Suppose A and B are in weak positions and C is in a stronger position
- ❑ Then it is often optimal for both A and B to attack C rather than each other
  - ❖ Collaboration emerges from purely selfish behavior

4/13/2024

pra-sâmi

60

## Optimal Decisions in Multiplayer Games



- ❑ Of course, as soon as C weakens under the joint onslaught, the alliance loses its value
- ❑ Either A or B could violate the agreement
- ❑ Explicit alliances, if any, merely make concrete what would have happened anyway
- ❑ In some cases, a social stigma attaches to breaking an alliance
  - ❖ Immediate advantage of breaking an alliance vs the long-term disadvantage of being perceived as untrustworthy

4/13/2024

pra-sâmi

61

## Nonzero Sum Game Trees

- ❑ The idea of “look ahead, reason backward” works for any game tree with perfect information.
  - ❖ I.e., also in cooperative games
- ❑ In AI, this is called retrograde analysis.
- ❑ In game theory, it is called backward induction or subgame perfect equilibrium.
- ❑ Can be extended to many games with imperfect information (sequential equilibrium).

4/13/2024

pra-sâmi

62

## A Bullfight or the Opera

### □ Background

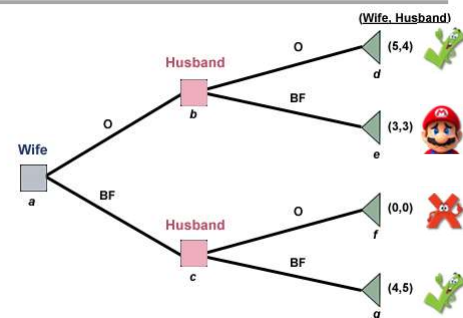
- ❖ A husband and wife are deciding whether to spend the evening at a Bullfight or the Opera.
- ❖ The husband prefers the bullfight to the opera, but the wife prefers the opera to the bullfight.
- ❖ Both would prefer to be together at either event rather than spending the evening apart.
- ❖ The worst outcome would be for the husband to spend the evening alone at the opera and the wife to spend the evening alone at the bullfight

### □ This game draws attention to the problem of co-ordination

- ❖ Players have divergent preferences
- ❖ A common interest in coordinating their strategies.

|      |    | Husband |      |
|------|----|---------|------|
|      |    | O       | BF   |
| Wife | O  | 5, 4    | 3, 3 |
|      | BF | 0, 0    | 4, 5 |

4/13/2024



63

## Compare to “Prisoner’s Dilemma”

### □ “Deterministic, NON-turn-taking, NON-zero-sum game of Imperfect information”



A prisoner's dilemma is a situation where individual decision-makers always have an incentive to choose in a way that creates a less than optimal outcome for the individuals as a group.

There are methods of overcoming prisoner's dilemmas to choose better collective results despite apparently unfavorable individual incentives.

- Dave and Henry - Two members of a gang of bank robbers; arrested ; being interrogated in separate rooms; No other witnesses; need confession
- Be loyal and remain silent; the authorities will only be able to convict them on a lesser charge
  - ❖ One year in jail for each (1 year for Dave + 1 year for Henry = 2 years total jail time)
- If one testifies and the other does not
  - ❖ The one who testifies will go free and the other will get five years (0 years for the one who defects + 5 for the one convicted = 5 years total)
- If both testify against the other
  - ❖ Each will get two years in jail for being partly responsible for the robbery (2 years for Dave + 2 years for Henry = 4 years total jail time)

4/13/2024

pra-sâmi

64

## Win as much as you can!

- ❑ Four Teams, choose either X or Y
- ❑ Pay off Schedule
  - ❖ 4 X's:
    - Lose \$ 1.00 each
  - ❖ 3 X's, 1 Y :
    - Xs: Win \$1.00 each; Ys : Lose \$3.00
  - ❖ 2 X's, 2 Y's :
    - Xs: Win \$ 2.00 each; Ys : Lose \$ 2.00 each
  - ❖ 1 X , 3 Y's:
    - Xs : Win \$ 3.00; Ys: Lose \$ 1.00 each
  - ❖ 4 Y's:
    - Win \$ 1.00 each

|                           | Round | Your Choice | Group's pattern of Choice | Payoff | Balance |
|---------------------------|-------|-------------|---------------------------|--------|---------|
|                           | 1     | X Y         | _X _Y                     |        |         |
|                           | 2     | X Y         | _X _Y                     |        |         |
|                           | 3     | X Y         | _X _Y                     |        |         |
|                           | 4     | X Y         | _X _Y                     |        |         |
| Bonus Round (Payoff X 3)  | 5     | X Y         | _X _Y                     |        |         |
|                           | 6     | X Y         | _X _Y                     |        |         |
| Leader's Conference       | 7     | X Y         | _X _Y                     |        |         |
| Bonus Round (Payoff X 5)  | 8     | X Y         | _X _Y                     |        |         |
|                           | 9     | X Y         | _X _Y                     |        |         |
| Bonus Round (Payoff X 10) | 10    | X Y         | _X _Y                     |        |         |

4/13/2024

pra-sâmi

65

## Summary

- ❑ Game playing can be effectively modeled as a search problem
- ❑ Game trees represent alternate computer/opponent moves
- ❑ Evaluation functions estimate the quality of a given board configuration for the Max player.
- ❑ Minimax is a procedure which chooses moves by assuming that the opponent will always choose the move which is best for them
- ❑ Alpha-Beta is a procedure which can prune large parts of the search tree and allow search to go deeper
- ❑ For many well-known games, computer algorithms based on heuristic search match or out-perform human world experts.

4/13/2024

pra-sâmi



66

## Summary

- ❑ Game systems rely heavily on
  - ❖ Search techniques
  - ❖ Heuristic functions
  - ❖ Bounding and pruning techniques
  - ❖ Knowledge database on game
- ❑ For AI, the abstract nature of games makes them a good subject for study:
  - ❖ State of the game is easy to represent;
  - ❖ Agents are usually restricted to a small number of actions whose outcomes are defined by precise rules
- ❑ Game playing was one of the first tasks undertaken in AI as soon as computers became programmable (e.g., Turing, Shannon, Wiener tackled chess).
- ❑ Game playing research has spawned a number of interesting research ideas on search, data structures, databases, heuristics, evaluations functions and many areas of computer science.

Games are fun:  
Teach your computer how to play a game!

4/13/2024

pra-sâmi

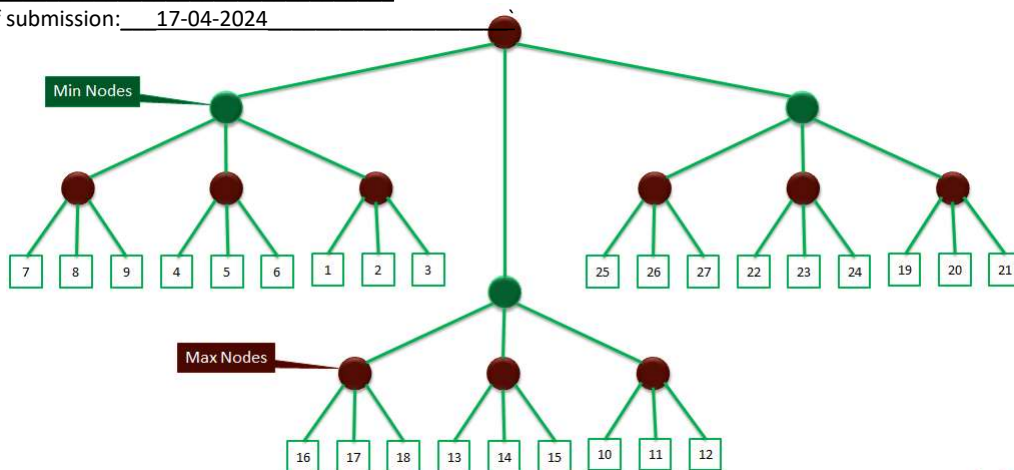
67

## PS\_03: Alpha Beta Pruning

Course & Batch : \_\_\_\_\_  
 Roll No: \_\_\_\_\_  
 Name: \_\_\_\_\_  
 Date of submission: 17-04-2024

Task:

- ❖ Perform alpha-beta pruning of following tree.
- ❖ Explain benefit you achieved over exhaustive search.



4/13/2024

pra-sâmi

68

## Reflect...

- ❑ Which search methodology may not necessarily guarantee the optimal solution but is generally faster in practice?
  - A. Uninformed search
  - B. Informed search
  - C. Heuristic search
  - D. Depth-first search

❑ Ans: C. Heuristic search

- ❑ Which classical search algorithm can get trapped in infinite loops if cycles are present in the search space?
  - A. Depth-first search
  - B. Breadth-first search
  - C. A\* search algorithm
  - D. Uniform-cost search

❑ Ans: A. Depth-first search

- ❑ In parallel search, what is the primary challenge associated with coordinating multiple processors or threads?
  - A. Load balancing
  - B. Increased memory usage
  - C. Decreased search space
  - D. Slower execution time

❑ Ans: A. Load balancing

- ❑ Which component of a search engine is responsible for determining the relevance of web pages to a given query?
  - A. Indexing
  - B. Query processing
  - C. Ranking algorithms
  - D. User authentication

❑ Ans: C. Ranking algorithms

4/13/2024

pra-sâmi

69

## Reflect...

- ❑ Which adversarial search algorithm uses a minimax strategy to make decisions?
  - A. Alpha-beta pruning
  - B. Monte Carlo Tree Search (MCTS)
  - C. Minimax
  - D. Expectimax

❑ Ans: C. Minimax

- ❑ Which type of search can be visualized as exploring a tree structure representing all possible states?
  - A. Informed search
  - B. Uninformed search
  - C. Adversarial search
  - D. Heuristic search

❑ Ans: B. Uninformed search

- ❑ Which classical search algorithm expands the node with the lowest path cost  $g(n)$  plus an estimated cost  $h(n)$  to the goal?
  - A. Depth-first search
  - B. Breadth-first search
  - C. A\* search algorithm
  - D. Uniform-cost search

❑ Ans: C. A\* search algorithm

- ❑ In parallel search, what is a potential drawback of using a large number of processors or threads?
  - A. Increased memory usage
  - B. Higher energy consumption
  - C. Slower execution time
  - D. Difficulty in implementation

❑ Ans: B. Higher energy consumption

4/13/2024

pra-sâmi

70

## Reflect...

- ❑ Which algorithm is commonly used to solve CSPs by systematically searching through the problem space?
  - A. A\* algorithm
  - B. Depth-First Search (DFS)
  - C. Breadth-First Search (BFS)
  - D. Hill Climbing

❑ Ans: B) Depth-First Search (DFS)

- ❑ In combinatorial optimization problems, what is the objective?
  - A. To find a feasible solution that satisfies a set of constraints
  - B. To find the solution that maximizes a certain criterion
  - C. To minimize the number of constraints
  - D. To randomly explore the search space

❑ Ans: B) To find the solution that maximizes a certain criterion

- ❑ Which of the following is an example of a combinatorial optimization problem?
  - A. Sorting
  - B. Linear Regression
  - C. Traveling Salesman Problem
  - D. Binary Search

❑ Ans: C) Traveling Salesman Problem

- ❑ Which algorithm is commonly used to solve combinatorial optimization problems by iteratively improving a candidate solution?
  - A. Simulated Annealing
  - B. Genetic Algorithm
  - C. Particle Swarm Optimization
  - D. Hill Climbing

❑ Ans: D) Hill Climbing

4/13/2024

pra-sâmi

71

## Reflect...

- ❑ Which of the following techniques is NOT typically used in Constraint Satisfaction Search?
  - A. Forward checking
  - B. Genetic Algorithm
  - C. Arc consistency
  - D. Backtracking

❑ Ans: B) Genetic Algorithm

- ❑ What does the term "arc consistency" refer to in Constraint Satisfaction Problems?
  - A. It ensures that all variables in a CSP are assigned a value
  - B. It prunes inconsistent values from the domains of variables
  - C. It involves traversing the graph representing the CSP
  - D. It is a heuristic function used in search algorithms

❑ Ans: B) It prunes inconsistent values from the domains of variables

- ❑ Which of the following is a limitation of the hill climbing algorithm in combinatorial optimization?
  - A. It always finds the global optimum
  - B. It is susceptible to getting stuck in local optima
  - C. It is computationally expensive
  - D. It requires complete knowledge of the search space

❑ Ans: B) It is susceptible to getting stuck in local optima

- ❑ Which of the following is NOT a step in the typical problem-solving process for Constraint Satisfaction Search?
  - A. Formulating the problem as a CSP
  - B. Propagating constraints to reduce the search space
  - C. Randomly assigning values to variables
  - D. Searching for a solution using a suitable algorithm

❑ Ans: C) Randomly assigning values to variables

4/13/2024

pra-sâmi

72

## Reflect...

- ❑ Which algorithm is commonly used to solve CSPs by systematically searching through the problem space?
  - A. A\* algorithm
  - B. Depth-First Search (DFS)
  - C. Breadth-First Search (BFS)
  - D. Hill Climbing

❑ Ans: B) Depth-First Search (DFS)

- ❑ In combinatorial optimization problems, what is the objective?
  - A. To find a feasible solution that satisfies a set of constraints
  - B. To find the solution that maximizes a certain criterion
  - C. To minimize the number of constraints
  - D. To randomly explore the search space

❑ Ans: B) To find the solution that maximizes a certain criterion

- ❑ Which of the following is an example of a combinatorial optimization problem?

- A. A) Sorting
- B. B) Linear Regression
- C. C) Traveling Salesman Problem
- D. D) Binary Search

❑ Ans: C) Traveling Salesman Problem

- ❑ Which algorithm is commonly used to solve combinatorial optimization problems by iteratively improving a candidate solution?

- A. A) Simulated Annealing
- B. B) Genetic Algorithm
- C. C) Particle Swarm Optimization
- D. D) Hill Climbing

❑ Ans: D) Hill Climbing

4/13/2024

pra-sâmi

73

## Reflect...

- ❑ Which of the following techniques is NOT typically used in Constraint Satisfaction Search?
  - A. Forward checking
  - B. Genetic Algorithm
  - C. Arc consistency
  - D. Backtracking

❑ Ans: B) Genetic Algorithm

- ❑ What does the term "arc consistency" refer to in Constraint Satisfaction Problems?
  - A. It ensures that all variables in a CSP are assigned a value
  - B. It prunes inconsistent values from the domains of variables
  - C. It involves traversing the graph representing the CSP
  - D. It is a heuristic function used in search algorithms

❑ Ans: B) It prunes inconsistent values from the domains of variables

- ❑ Which of the following is a limitation of the hill climbing algorithm in combinatorial optimization?

- A. It always finds the global optimum
- B. It is susceptible to getting stuck in local optima
- C. It is computationally expensive
- D. It requires complete knowledge of the search space

❑ Ans: B) It is susceptible to getting stuck in local optima

- ❑ Which of the following is NOT a step in the typical problem-solving process for Constraint Satisfaction Search?

- A. Formulating the problem as a CSP
- B. Propagating constraints to reduce the search space
- C. Randomly assigning values to variables
- D. Searching for a solution using a suitable algorithm

❑ Ans: C) Randomly assigning values to variables

4/13/2024

pra-sâmi

75

## Next Session



4/13/2024

*pra-sâmi*

76



4/13/2024

*pra-sâmi*

## EXTRA MATERIAL

*pra-sâmi*

State of Games

78

## The State of Play

- ❑ Checkers:
  - ❖ Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994.
- ❑ Chess:
  - ❖ Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997.
- ❑ Othello:
  - ❖ Human champions refuse to compete against computers: they are too good.
- ❑ Go:
  - ❖ Human champions refuse to compete against computers: they are too bad  $b > 300$  (!)

4/13/2024

*pra-sâmi*

79

## The University of Alberta; CA

**The University of Alberta GAMES Group**

Game-playing,  
Analytical methods,  
Minimax search and  
Empirical  
Studies

[Projects](#) | [News](#) | [What We Do](#) | [People](#) | [Links](#) | [Publications](#)

**Announcements**

- GAMES group meetings are often from 4-5pm on Thursdays at CSC333.

**Projects**

|  |   |  |  |
|--|---|--|--|
| <b>Checkers</b><br>Chinook is the official world checkers champion.  | <b>Poker</b><br>Poki is the strongest poker AI in the world.  | <b>Lines of Action</b><br>YL & Mona are two of the best LoA programs in the world.   | <b>Hex</b><br>Queenbee is one of the best Hex programs in the world. Research on computer Hex is ongoing.  |
| <b>Go</b><br>The Computer Go group has developed the top program Fuego.  | <b>Real-Time Strategy</b><br>We are trying to apply AI to real-time strategy games.   | <b>Othello</b><br>Logistella defeated the human world Othello champion, 6-0, in 1997. Keyvane is another strong program.         | <b>Sokoban</b><br>Rolling Stone pushes the boundaries of single agent search.  |
| <b>RoShamBo</b><br>Home of the International RoShamBo Programming Competition  | <b>Amazons</b><br>Three programs, and several theoretical contributions.  | <b>Spades &amp; Hearts</b><br>Spades & Hearts are the test beds for the research on multi-player games.                          | <b>Shogi</b><br>IShogi has won the world computer Shogi championship many times (currently inactive).  |
| <b>Real-time Heuristic Search</b><br>We are developing heuristic search methods where the planning time per action is limited. | <b>Emotion and Culture Modeling</b><br>It is important to understand the role of emotions in intelligence both theoretically and practically. | <b>Player Modeling and Adaptive Story Generation</b><br>We explore interactive storytelling and player modelling in video games. | <b>Hide and Seek</b><br>We are investigating hide and seek human behavior in a pen-and-paper as well as a 3D environment created with the Source engine. |

**Previous Projects:**

Chess   Awari   Chinese Chess   Post's Correspondence Problem   Domineering

4/13/2024

pra-sâmi

80

## Deep Blue

- ❑ 1957: Herbert Simon
  - ❖ “within 10 years a computer will beat the world chess champion”
- ❑ 1997: Deep Blue beats Kasparov
- ❑ Parallel machine with 30 processors for “software” and 480 VLSI processors for “hardware search”
- ❑ Searched 126 million nodes per second on average
  - ❖ Generated up to 30 billion positions per move
  - ❖ Reached depth 14 routinely
- ❑ Uses iterative-deepening alpha-beta search with transpositioning
  - ❖ Can explore beyond depth-limit for interesting moves

4/13/2024

pra-sâmi