LOGIC
PROPOSITIONAL LOGIC
FIRST ORDER LOGIC

Fundamentals of Artificial Intelligence

Session 18

Pramod Sharma
pramod.sharma@prasami.com

2

Agenda

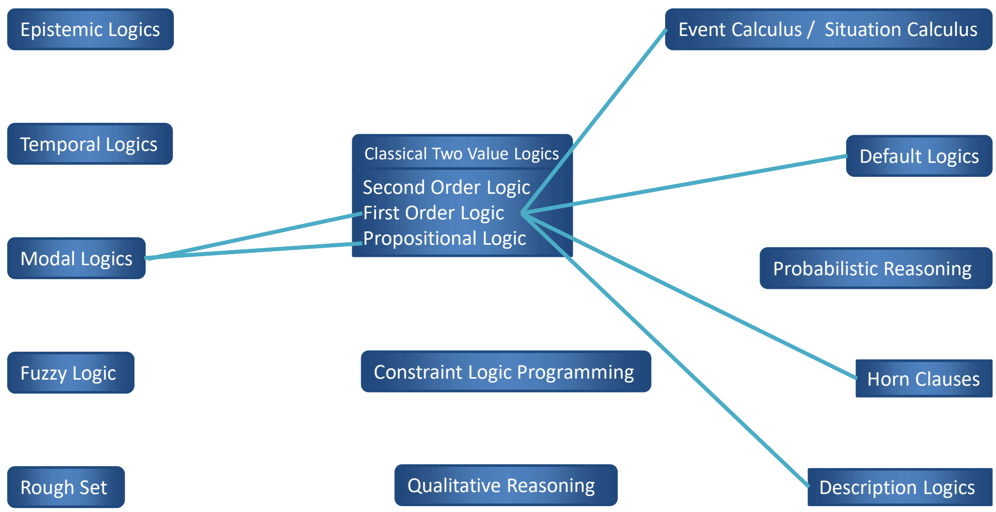
- Logic – Continues...
- Inference and Resolution for Problem Solving
- Propositional Logic
- First Order Logic

4/16/2024

pra-sâmi

3

Logic



4

Syntax and Semantics

- Syntax
  - Rules for constructing legal sentences in the logic
  - Which symbols we can use (English: letters, punctuation)…
  - How we are allowed to combine symbols…
- Semantics
  - How we interpret (read) sentences in the logic…
  - Assigns a meaning to each sentence
- Example: “All lecturers are seven feet tall”
  - A valid sentence (syntax)
  - And we can understand the meaning (semantics)
  - This sentence happens to be false (it’s a counterexample)

5

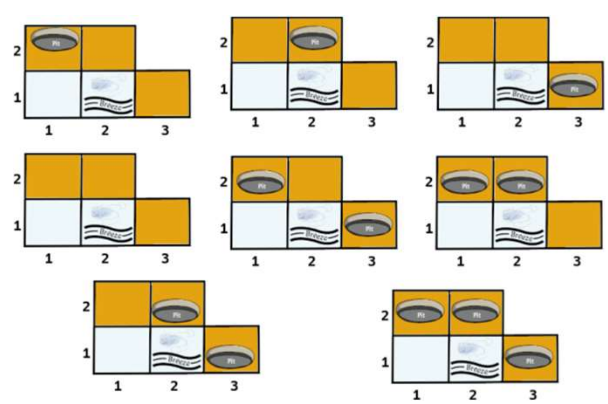
Models

- Models are mathematical abstractions of “Possible World”
- “Possible Worlds ⇒ (potentially) real environments that the agent might or might not be in
  - Models are mathematical abstractions, each of which simply fixes the truth or falsehood of every relevant sentence
- Having x men and y women sitting at a table playing bridge, and the sentence x + y = 4 is true when there are four people in total
  - Possible models are models with all real values of x and y
- If a sentence α is true in model m ⇒ m satisfies α
  - Notation M(α) is often used to denote all models of alpha
- Entailment
  - A sentence β follows logically another sentence α
  - α ⊨ β (sentence α entails the sentence β), If and only if in every model in which α is true, β is also true
  - Complete representation : α ⊨ β if and only if M(α) ⇒ M(β)

6

Applying Model to Wumpus World

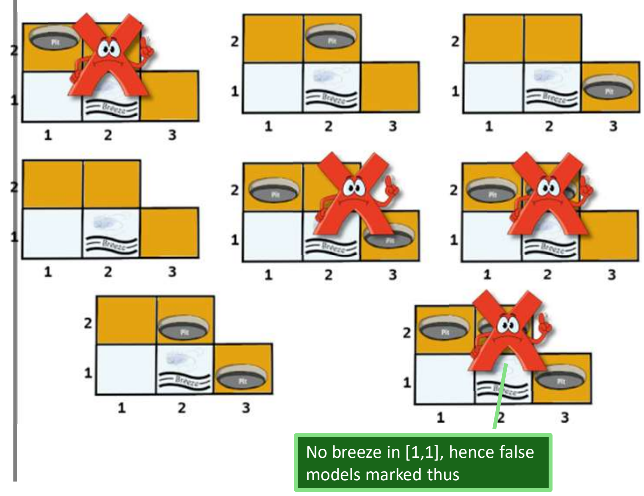
- Consider the situation:
  - The agent has detected nothing in [1 , 1] and,
  - A breeze in [1, 2]
- Knowledge Base:
  - Above percepts, and agent’s knowledge of the rules of the wumpus world
- Adjacent squares may or may not contain Pit
  - # of adjacent squares = 3
  - Two option; pit exists or not (True or False)
  - Total 2^3= 8 possible models



7

Applying Model to Wumpus World

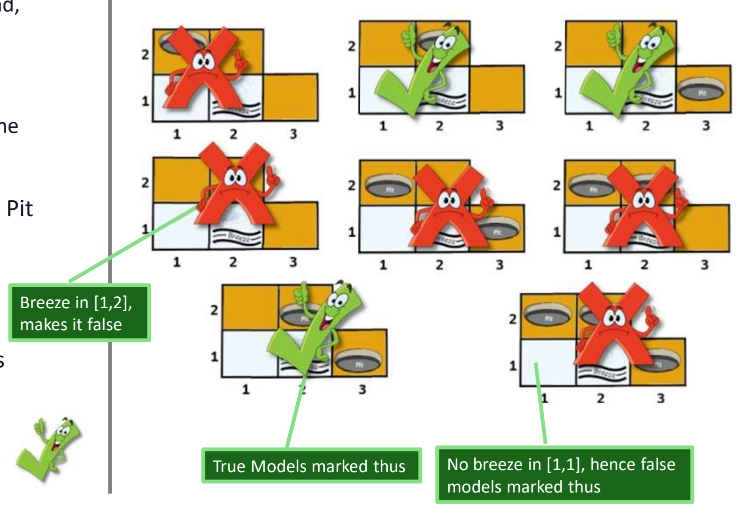
- Consider the situation:
  - The agent has detected nothing in [1 , 1] and,
  - A breeze in [1, 2]
- Knowledge Base:
  - Above percepts, and agent's knowledge of the rules of the wumpus world
- Adjacent squares may or may not contain Pit
  - # of adjacent squares = 3
  - Two option; pit exists or not (True or False)
  - Total 2^3= 8 possible models
- No Breeze in [1 , 1] ➔ KB "False" models marked thus



8

Applying Model to Wumpus World

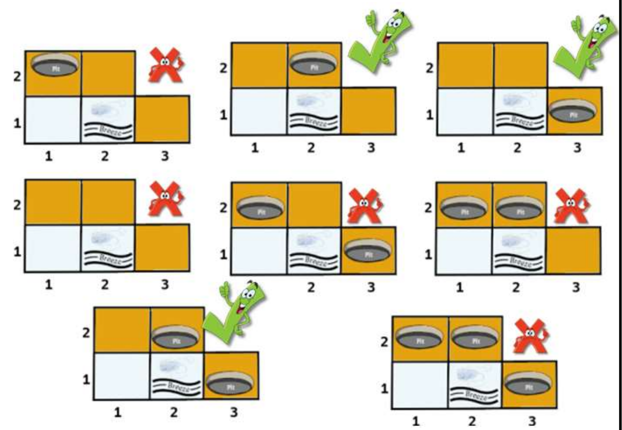
- Consider the situation:
  - The agent has detected nothing in [1 , 1] and,
  - A breeze in [1, 2]
- Knowledge Base:
  - Above percepts, and agent's knowledge of the rules of the wumpus world
- Adjacent squares may or may not contain Pit
  - # of adjacent squares = 3
  - Two option; pit exists or not (True or False)
  - Total 2^3= 8 possible models
- No Breeze in [1 , 1] ➔ KB "False" models marked thus
- True models are three only marked thus



9

Applying Model to Wumpus World

- Consider the two possible Sentences:
  - $\alpha 1$ = "There is no pit in [2, 1]"
  - $\alpha 2$ = "There is no pit in [2, 2]"
- In every model in which KB is true, $\alpha 1$ is also true.
  - KB $\models \alpha 1$: there is no pit in [2, 1]
- In some models in which KB is true, $\alpha 2$ is false
  - KB $\not\models \alpha 2$: the agent cannot conclude that there is no pit in [2, 2]
  - Nor can it conclude that there is a pit in [2, 2]



4/16/2024

pra-sâmi

10

Logical Inference or Model Checking

- An inference algorithm that derives only entailed sentences is called sound or truth preserving
- An inference algorithm is complete if it can derive any sentence that is entailed

  if KB is true in the real world, then any sentence α derived from KB by a sound inference procedure is also true in the real world
- Grounding: how do we know that KB is true in the real world?
  - After all its just Math in some storage location or in memory
- Simple Answer:
  - The agent's sensors create the connection

4/16/2024

pra-sâmi

11

Propositional Logic

- A proposition is a declarative statement which is either true or false
- It is also called Boolean logic:
  - Works on 0 or 1 i.e. True or False
- Simplest form of logic
  - All the statements are made by propositions
- It's a technique of knowledge representation in logical and mathematical form
- Examples:
  - It is morning
  - Sun is rising from North
  - Five is a prime number

4/16/2024

pra-sâmi

12

Propositional Logic

- Symbolic variables to represent the logic
  - Use any symbol for a representing a proposition, such A, B, C, P, Q, R, etc.
  - Or even P_{11}, B_{12}
  - Symbols are read in toto; break-down does not mean anything
- Propositional can be either true or false
  - But it cannot be both
  - Also, there is no "May be"
- Statements which are questions, commands, or opinions are **not** propositions
  - Examples:
    - "How are you"
    - "What is your name"
    - "Go to Connaught Place"

4/16/2024

pra-sâmi

13

Propositional Logic

- Propositional logic consists of:
  - an object,
  - relations or function, and
  - logical connectives
- These connectives are also called logical operators
- The propositions and connectives are the basic elements of the propositional logic

14

Connectives

- Connectives can be said as a logical operator which connects two sentences
- A proposition formula which is always true is called **Tautology**, and it is also called a valid sentence.
- A proposition formula which is always false is called **Contradiction**
- A proposition formula which has both true and false values is called **Contingency**

15

Syntax of Propositional Logic

- Syntax of propositional logic defines the allowable sentences for the knowledge representation
- There are two types of Propositions:
  - Atomic Propositions
  - Compound propositions
- Atomic Proposition: Atomic propositions are the simple propositions.
  - It consists of a single proposition symbol
  - These are the sentences which must be either true or false
    - 2+2 is 4, it is an atomic proposition as it is a true fact.
    - "The Sun rises in the west" is also a proposition as it is a false fact.
- Compound proposition: Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives
  - "It is raining today" (P), and "Street is wet" (Q). ➔ P ∧ Q
  - "Mohan is an Engineer" (R), and "Mohan works for Amazon" (S) ➔ R ∧ S

pra-sâmi

16

Logical Connectives

- Connectives: and, or, not, implies, iff (equivalent)

  $\wedge \quad \vee \quad \neg \quad \longrightarrow \quad \longleftrightarrow$

  - Brackets, T (true) and F (false)

Negation	¬ (not)	A sentence such as $\neg W_{3,1}$ is called the negation of $W_{3,1}$. A literal is either an atomic sentence (a positive literal) or a negated atomic sentence (a negative literal)
Conjunction	∧ (and)	A sentence whose main connective is ∧, such as $W_{3,1} \wedge P_{1,3}$ is called a conjunction; its parts are the conjuncts.
Disjunction	∨ (or)	A sentence using ∨, such as $(W_{3,1} \wedge P_{1,3}) \vee W_{2,2}$, is a disjunction of the disjuncts $(W_{3,1} \wedge P_{1,3})$ and $W_{2,2}$
Implication	⇒ (implies)	A sentence such as $(W_{3,1} \wedge P_{1,3}) \Rightarrow \neg W_{2,2}$ is called an implication (or conditional). Its premise or antecedent is $((W_{3,1} \wedge P_{1,3})$, and its conclusion or consequent is $\neg W_{2,2}$. Implications are also known as rules or if–then statements. Some literatures write it as books as ⍰ or →
Biconditional	⇔ (if and only if)	The sentence W1,3 ⇔ ¬W2,2 is a bi-conditional. Some literatures write this as ≡.

pra-sâmi

17

Logical Connectives

- Conjunction:
  - A sentence which has ∧ (and) connective e.g. p ∧ q.
  - Example: Mohan is intelligent and hardworking

$$\begin{cases} p \;=\; Mohan\ is\ intelligent \\ q \;=\; Mohan\ is\ hardworking \end{cases} \rightarrow p \wedge q$$

- Disjunction:
  - A sentence which has ∨ (or) connective e.g. p ∨ q
  - Example: Bhavna is either a doctor or an Engineer

$$\begin{cases} p \;=\; Bhavna\ is\ Doctor \\ q \;=\; Bhavna\ is\ an\ Engineer \end{cases} \rightarrow p \vee q$$

- Implication:
  - A sentence such as P → Q, is called an implication
  - Implications are also known as if-then rules
  - Example: If it is raining, then the pitch is wet

$$\begin{cases} p \;=\; It\ is\ raining \\ q = The\ pitch\ is\ wet \end{cases} \rightarrow P \rightarrow Q$$

- Bi-conditional:
  - A sentence such as p ⇔ q is a Bi-conditional sentence
  - Example: If I am breathing, then I am alive

$$\begin{cases} p \;=\; I\ am\ breathing \\ q = I\ am\ alive \end{cases} \rightarrow P \Leftrightarrow Q$$

- Negation:
  - A sentence such as ¬p is called negation of P
  - Example:

  Mohan is studying → p then ¬p → Mohan is not studying

4/16/2024

pra-sâmi

18

Logical Connectives

- Conjunction:
- Implication:

Connective symbols	English	Technical term	Example
∧	And	Conjunction	p ∧ q
∨	or	Disjunction	p ∨ q
⇒ Or →	If-then	Implication	p ⇒ q
⇔ Or ↔	If and only if (iff)	Bi-conditional	p ⇔ q
¬ or ~	Not	negation	¬p
∀	In all	Universal	∀(x)
∃	There exists	presence	∃(y)

4/16/2024

pra-sâmi

19

Propositional Logic - Semantics

- The semantics for propositional logic must specify how to compute the truth value of any sentence, given a model
- Define how connectives affect truth – true or false
  - "P and Q" is true if and only if P is true and Q is true
- Use truth tables to work out the truth of statements

4/16/2024

pra-sâmi

20

Five Rules

For complex sentences, we have five rules, which hold for any sub-sentences P and Q in any model m (here "iff" means "if and only if"):

- ¬P is true iff P is false in m
- P ∧ Q is true iff both P and Q are true in m
- P ∨ Q is true iff either P or Q is true in m
- P ⇒ Q is true unless P is true and Q is false in m
  - If P is true, then I am claiming that Q is true. Otherwise I am making no claim
- P ⇔ Q is true iff P and Q are both true or both false in m
- XOR : A different connective, called "exclusive or", yields false when both disjuncts are true.
  - There is no consensus on the symbol for exclusive or; some choices are $\dot{V}$ or $\neq$ or $\oplus$.

4/16/2024

pra-sâmi

21

Truth Table

p	q	p ∧ q	p ∨ q	p → q	p ⇔ q	¬p
T	T	T	T	T	T	F
T	F	F	T	F	F	F
F	T	F	T	T	F	T
F	F	F	F	T	T	T

4/16/2024

pra-sâmi

22

Truth Table with Three Propositions

p	q	r	¬r	p ∧ q	p ∨ q	p ∨ q → ¬r
T	T	T	F	T	T	F
T	T	F	T	T	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
F	T	T	F	F	T	F
F	T	F	T	F	T	T
F	F	T	F	F	F	T
F	F	F	T	F	F	T

4/16/2024

pra-sâmi

23

Precedence of connectives

Precedence	Operators
First	Parenthesis
Second	Negation
Third	Conjunction(AND)
Fourth	Disjunction(OR)
Fifth	Implication
Six	Bi-conditional

4/16/2024

pra-sâmi

24

Logical Equivalence

- Two propositions are said to be logically equivalent if and only if the columns in the truth table are identical to each other

p	q	¬p	¬p v q	p → q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

4/16/2024

pra-sâmi

26
Properties of Operators

- Commutative:
  - $p \wedge q = q \wedge p$, or
  - $p \vee q = q \vee p$
- Associativity:
  - $(p \wedge q) \wedge r \rightarrow p \wedge (q \wedge r)$, or
  - $(p \vee q) \vee r \rightarrow p \vee (q \vee r)$
- Identity element:
  - $p \wedge \text{true} \rightarrow p$, or
  - $p \vee \text{true} \rightarrow \text{true}$.
- Distributive:
  - $p \wedge (q \vee r) \rightarrow (p \wedge q) \vee (p \wedge r)$, or
  - $p \vee (q \wedge r) \rightarrow (p \vee q) \wedge (p \vee r)$
- DE Morgan's Law:
  - $\neg (p \wedge q) = (\neg p) \vee (\neg q)$, or
  - $\neg (p \vee q) = (\neg p) \wedge (\neg q)$
- Double-negation elimination:
  - $\neg (\neg p) = p$.

27
Limitations of Propositional logic

- We cannot represent relations like ALL, some, there exists or none with propositional logic.
  - All humans are mortals
  - Some apples are sweet
  - Everybody loves somebody
- Propositional logic has limited expressive power
- In propositional logic, we cannot describe statements in terms of their properties or logical relationships
- Take following two sentences
  - All students are intelligent
  - Mohan is a student
- Logical interpretation is that Mohan is intelligent.
  - How to represent this is Propositional logic

28

Conjunctive Normal Form or Clausal Normal Form

- A formula is in conjunctive normal form (CNF) or clausal normal form if it is a conjunction of one or more clauses, where a clause is a disjunction of literals
- Otherwise put, it is a product of sums or an AND of ORs
- As a canonical normal form, it is useful in automated theorem proving and circuit theory
- The resolution rule applies only to clauses (that is, disjunctions of literals), so it would seem to be relevant only to knowledge bases and queries consisting of clauses
  - To use resolution, all statements must be in Conjunctive Normal Form

4/16/2024

pra-sâmi

29

WFF to CNF

- Well-Formed Formula to Conjunctive Normal Form
  - Helps in auto analyzing the problems
- Eliminate bi-conditional
  - a ⇔ b becomes (a → b) ∧ (b → a)
- Eliminate Implications
  - (a → b) becomes (¬a ∨ b)
- Reduce scope of each ¬ (*not*) to single term
  - ¬(a ∨ b) becomes ¬a ∧ ¬ b
  - ¬(a ∧ b) becomes ¬a ∨ ¬ b
  - ¬(∀ x a) becomes ∃ x ¬ a
  - ¬(∃ x a) becomes ∀ x ¬ a
  - ¬ ¬ a becomes a

4/16/2024

pra-sâmi

30

WFF to CNF

- Standardize variables
  - Each quantifier binds a unique variable
  - $(\forall x\ p(x)) \wedge (\forall x\ q(x))$ becomes $(\forall x\ p(x)) \wedge (\forall y\ q(y))$
- Move all quantifiers to front (left), also called Prenex Normal form
  - $(\forall x\ p(x)) \wedge (\forall y\ q(y))$ becomes $\forall x\ \forall y : p(x) \wedge q(y)$
  - Keep the order same
- Skolemization : eliminate existential quantifiers.
  - $\exists$ x Rich (x) becomes Rich (g1) where g1 is a constant
- Drop universal quantifiers
  - $\forall$ x person (x) becomes person(x)
- Apply distributive law (no OR outside bracket)
  - $a \vee (b \wedge c)$ becomes $(a \vee b) \wedge (a \vee c)$

4/16/2024 pra-sâmi

31

WFF to CNF - Example 1

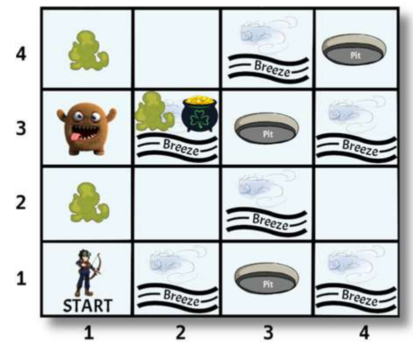
Convert $((p \to q) \to r)$

- $((p \to q) \to r)$
- becomes $((\neg p \vee q) \to r)$
- becomes $(\neg(\neg p \vee q) \vee r)$
- becomes $((\neg\neg p \wedge \neg q) \vee r)$
- becomes $((p \wedge \neg q) \vee r)$
- becomes $((p \vee r) \wedge (\neg q \vee r))$

4/16/2024 pra-sâmi

32

WFF to CNF - Example 2



- Convert B11 ⇔ (P12 ∨ P21) to CNF
- Eliminate ⇔, replacing α ⇔ β with (α ⇒ β) ∧ (β ⇒ α):
  - (B11 ⇒ (P12 ∨ P21)) ∧ ((P12 ∨ P21) ⇒ B11) .
- Eliminate ⇒, replacing α ⇒ β with ¬α ∨ β:
  - (¬B11 ∨ P12 ∨ P21) ∧ (¬(P12 ∨ P21) ∨ B11)
- CNF requires ¬ to appear only in literals, so we "move ¬ inwards" by application of De Morgan :
  - (¬B11 ∨ P12 ∨ P21) ∧ ((¬P12 ∧ ¬P21) ∨ B11)
- Now we have a sentence containing nested ∧ and ∨ operators applied to literals. We apply the distributivity law distributing ∨ over ∧ wherever possible.
  - (¬B11 ∨ P12 ∨ P21) ∧ (¬P12 ∨ B11) ∧ (¬P21 ∨ B11) ➔ Conjunction of three Clauses

34

Simple Knowledge Base

- Having defined the semantics for propositional logic, let's construct a knowledge base for the wumpus world
- Focus first on the immutable aspects
  - $P_{x,y}$ is true if there is a pit in [x, y]
  - $W_{x,y}$ is true if there is a wumpus in [x, y], dead or alive
  - $B_{x,y}$ is true if the agent perceives a breeze in [x, y]
  - $S_{x,y}$ is true if the agent perceives a stench in [x, y]
  - $V_{x,y}$ is true if [x, y] is visited.
  - $G_{x,y}$ true if there is gold in [x, y].
  - $OK_{x,y}$ true if [x, y] is safe.
- Sentence to denote no pit at x, y will be ¬ $P_{x,y}$
- A square is breezy if and only if there is a pit in a neighboring square
  - $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

35

Simple

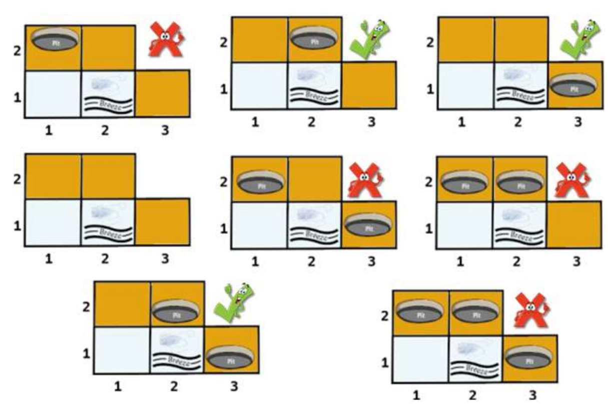
- Sentence to denote no pit at x, y will be ¬ $P_{x,y}$
  - R1 : ¬ $P_{1,1}$
- A square is breezy if and only if there is a pit in a neighboring square
  - R2 : $B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$
  - R3 : $B_{1,2} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{1,3})$
- Breeze percept of first two squares will be:
  - R4 : $\neg B_{1,1}$
  - R5 : $B_{1,2}$Knowledge Base

4/16/2024 pra-sâmi

36

Simple Inference Procedure

- Whether KB ⊨ α for some sentence α
  - Is ¬ $P_{2,1}$ entailed by our KB?
- Model-checking approach:
  - Direct implementation of the definition of entailment: enumerate the models, and check that α is true in every model in which KB is true
- Wumpus-world example:
  - Relevant proposition symbols : $B_{1,1}$, $B_{1,2}$, $P_{1,1}$, $P_{1,2}$, $P_{1,3}$, $P_{2,1}$, $P_{2,2}$
  - 7 symbols ➔ 2^7 = 128 models
  - KB is true in 3 models and ¬ $P_{2,1}$ is true in all 3
  - $P_{2,2}$ is true only in 2 and false in 1 ➔ cannot say whether there is a pit in 2,2



4/16/2024 pra-sâmi

37

Propositional Theorem Proving

- Apply rules of inference directly to the sentences in our knowledge base to construct a proof of the desired sentence without consulting models

- A few concepts:
  - Logical Equivalence: two sentences α and β are logically equivalent if they are true in the same set of models ($\alpha \leftrightarrow \beta$)
  - Validity: A sentence is valid if it is true in all models ($P \lor \neg P$)
    - Valid sentences are also known as tautologies—they are necessarily true
  - Satisfiability: if it is true in, or satisfied by, some model.
    - For example, the knowledge base given earlier, ($R1 \land R2 \land R3 \land R4 \land R5$), is satisfiable because there are three models in which it is true

4/16/2024 pra-sâmi

38

Inference Rules

- A chain of conclusions that leads to the desired goal
  - The best-known rule is called Modus Ponens

$$\frac{\alpha \Rightarrow \beta,\ \alpha}{\therefore\ \beta}$$

  - For example, if ("WumpusAhead" ∧ "WumpusAlive") ⇒ "Shoot" and ("WumpusAhead" ∧ "WumpusAlive") are given, then "Shoot" can be inferred

4/16/2024 pra-sâmi

39

Inference Rules and Equivalences in the Wumpus World

Knowledge Base

- No pit at x, y
  - R1 : $\neg P_{1,1}$
- A square is breezy if and only if there is a pit in a neighboring square
  - R2 : $B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$
  - R3 : $B_{1,2} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{1,3})$
- Breeze percept of first two squares will be:
  - R4 : $\neg B_{1,1}$
  - R5 : $B_{1,2}$

Using R1 to R5

- Apply bi-conditional elimination to R2 to obtain
  - R6 : $(B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})$
- Apply And-Elimination to R6 to obtain
  - R7 : $B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})$
- Logical equivalence for contrapositives gives
  - R8 : $(\neg B_{1,1} \Rightarrow \neg(P_{1,2} \lor P_{2,1}))$
- From the percept R4 (i.e., $\neg B_{1,1}$)
  - R9 : $\neg(P_{1,2} \lor P_{2,1})$
- Apply De Morgan's Rule
  - R10 : $\neg P_{1,2} \land \neg P_{2,1}$

But if the available inference rules are inadequate, then the goal is not reachable

Neither [1,2] nor [2,1] contains a pit

4/16/2024 pra-sâmi

40

Proof by Resolution

- Consider: the agent returns from [1,2] to [1,1] and then goes to [2,1], where it perceives a stench, but no breeze.
- Add the following facts to the knowledge base:
  - R11 : $\neg B_{2,1}$
  - R12 : $B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{1,3})$

4/16/2024 pra-sâmi

41

Prove that Wumpus is in [3, 1]

- We know that $S_{1,1} \rightarrow W_{1,1} \vee W_{1,2} \vee W_{2,1}$ and also agent did not detect any stench [1, 1] so $\neg S_{1,1}$
  - Hence: $\neg S_{1,1} \rightarrow \neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,1}$
  - And-Elimination Rule : $\neg W_{1,1}$, $\neg W_{1,2}$, $\neg W_{2,1}$

- We know that $S_{1,2} \rightarrow W_{1,2} \vee W_{2,2} \vee W_{1,3}$ and also agent did not detect any stench [1, 2] so $\neg S_{1,2}$
  - Hence : $\neg S_{1,2} \rightarrow \neg W_{1,2} \wedge \neg W_{2,2} \wedge \neg W_{1,3}$
  - And-Elimination Rule:$\neg W_{1,2}$, $\neg W_{2,2}$, $\neg W_{1,3}$

- We know that $S_{2,1} \rightarrow W_{1,1} \vee W_{2,1} \vee W_{2,2} \vee W_{3,1}$ and agent did detect stench in the [2,1]
  - Hence$S_{2,1} \rightarrow W_{1,1} \vee W_{2,1} \vee W_{2,2} \vee W_{3,1}$

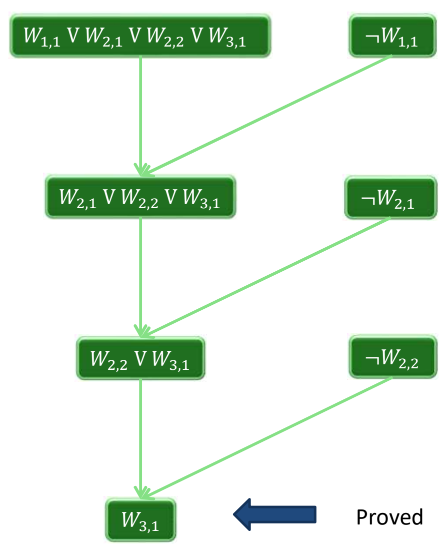
4/16/2024

pra-sâmi

42

Apply Unit Resolution

- 1 : $\neg W_{1,1}$, $\neg W_{1,2}$, $\neg W_{2,1}$
- 2 :$\neg W_{1,2}$, $\neg W_{2,2}$, $\neg W_{1,3}$
- 3 : $W_{1,1} \vee W_{2,1} \vee W_{2,2} \vee W_{3,1}$



4/16/2024

pra-sâmi

43

Beyond Propositional Logic

- In propositional logic, we can only represent the facts
- Facts which are either true or false
- Not sufficient to represent the complex sentences or natural language statements
  - Example: Every rose has a thorn
- Has very limited expressive power
- So we use First Order Logic
  - First-order logic is also known as Predicate logic or First-order predicate logic

44

Next Session … First Order Logic

45

4/16/2024

pra-sâmi

ADDITIONAL MATERIAL

Inference Rules

pra-sâmi

47

Arguments in Propositional Logic

- An argument in propositional logic is a sequence of propositions
- All but the final proposition are called premises.
  - The last statement is the conclusion
- The argument is valid if the premises imply the conclusion
- An argument form is an argument that is valid no matter what propositions are substituted into its propositional variables
- If the premises are $p_1, p_2, \dots, p_n$ and the conclusion is q then
  - $(p_1 \wedge p_2 \wedge \dots \wedge p_n) \rightarrow q$
- Inference rules are all argument simple argument forms that will be used to construct more complex argument forms.

Discover some useful inference rules!

4/16/2024

pra-sâmi

48

Modus Ponens or Law of Detachment

- Modus Ponens ➔ mode that affirms

$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore\ q \end{array}$$

- Let p be "It is snows."
- Let q be "I will study ."
- then p → q becomes "If it is snows, then I will study."
- "It is snowing."
  - "Therefore , I am studying ."

- Corresponding Tautology:

$$(p \wedge (p \rightarrow q)) \rightarrow q$$

- Proof using Truth Table:

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

4/16/2024

pra-sâmi

49

Modus Tollens or Denying the Consequent

$$\frac{\neg q \quad p \rightarrow q}{\therefore \neg p}$$

- Let p be "It is snowing."
- Let q be "I will study."
- then $p \rightarrow q$ becomes "If it is snowing, then I will study ."
- "I will not study " is True
  - Therefore , "it is not snowing" is also True

- Corresponding Tautology:

  $(\neg q \land (p \rightarrow q)) \rightarrow \neg p$

- Proof using Truth Table:

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

4/16/2024

pra-sâmi

50

Hypothetical Syllogism or Transitivity of Implication or Chain Argument

$$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$$

- Let p be "It is snowing".
- Let q be "I will study" .
- Let r be "I will get an A".
- If "it is snowing" is true then "I will study", if "I will study" is true then "I will get an A"
  - Therefore "it is snowing" then "I will get an A"

- Corresponding Tautology:

  $((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r)$

p	q	r	p → q	q → r	p → r
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

4/16/2024

pra-sâmi

51

Disjunctive Syllogism or Disjunction Elimination or Elimination

$$\begin{array}{l} p \lor q \\ \neg p \\ \hline \therefore q \end{array}$$

- Corresponding Tautology:

$((p \lor q) \land \neg p) \to q$

- Let p be "It is a banana".
- Let q be "It is an Apple".
- If I pull out one fruit out of the basket and it's not a banana ➔ it will be an apple.

4/16/2024

pra-sâmi

52

Addition or Disjunction Introduction

$$\begin{array}{l} p \\ \hline \therefore p \lor q \end{array}$$

- Corresponding Tautology:

$p \to (p \lor q)$

- Let p be "I will study AI".
- Let q be "I will visit Mumbai".
- If "I will study AI" is true.
  - Therefore, "I will study AI or I will visit Mumbai" is also true.

4/16/2024

pra-sâmi

53

Simplification or Conjunction Elimination

$$\frac{p \land q}{\therefore\ p}$$

- Corresponding Tautology:

$(p \land q) \rightarrow p$

- Let p be “It's raining”.
- Let q be “ It's pouring”.
- If “It's raining and it's pouring” is true.
  - Therefore, “It's raining” is also true.

4/16/2024

pra-sâmi

54

Conjunction or Conjunction Introduction

$$\frac{\begin{array}{c} p \\ q \end{array}}{\therefore\ p \land q}$$

- Corresponding Tautology:

$((p) \land (q)) \rightarrow (p \land q)$

- Let p be “It's raining.”
- Let q be “ It's pouring.”
- If “It's raining” is true and “it's pouring” is true.
  - Therefore, “It's raining and it’s pouring” is also true.

4/16/2024

pra-sâmi

55

Resolution

$$\begin{array}{l} p \lor q \\ \underline{\neg p \lor r} \\ \therefore q \lor r \end{array}$$

- Let p be "I will study discrete math."
- Let q be "I will study databases."
- Let r be "I will study English literature.
- "I will study discrete math or I will study databases."
  "I will not study discrete math or I will study English literature."
  - "Therefore, I will study databases or English literature."

- Corresponding Tautology:

$$((p \lor q) \land (\neg p \lor r)) \rightarrow (q \lor r)$$

Resolution plays an important role in Artificial Intelligence and is used in the programming language Prolog.

Let's Work out another example!

4/16/2024

pra-sâmi

56

Resolution

- Problem 1: Use resolution to show that the hypothesis
  "Mohan is a bad boy or Bhavna is a good girl"
  and
  "Mohan is a good boy or Gunjan is happy"
  implies the conclusion
  "Bhavna is a good girl or Gunjan is happy"

- Solution: let p denotes "Mohan is a good boy"
  q denotes "Bhavna is a good girl" and
  r denotes "Gunjan is happy".

C1 : $\neg p \lor q$
C2 : $p \lor r$

$\therefore$ C3 : $q \lor r$
Negating C3
C3 : $\neg(q \lor r)$
$= \neg q \land \neg r$

C1 : $\neg p \lor q$
C2 : $p \lor r$
C3 : $\neg q$
C4 : $\neg r$

From C1 and C2 → C5 : $q \lor r$
From C3 and C5 → C6 : r
From C6 and C4 → C6 : □ (empty)

Hence conclusion is proven!

4/16/2024

pra-sâmi

57

Proof by Cases or Disjunction Elimination

$$\begin{array}{c} p \rightarrow q \\ r \rightarrow q \\ \underline{p \lor r} \\ \therefore q \end{array}$$

- Corresponding Tautology:

$$((p \rightarrow q) \land (r \rightarrow q) \land (p \lor r)) \rightarrow q$$

- Apple is a fruit, banana is a fruit, I have apple or banana ➔ I have a fruit!

- Example:
  - Let p be “I will study discrete math.”
  - Let q be “I will study Computer Science.”
  - Let r be “I will study databases.”
  - “If I will study discrete math, then I will study Computer Science.”
  - “If I will study databases, then I will study Computer Science.”
  - “I will study discrete math or I will study databases.”
- “Therefore, I will study Computer Science.”

4/16/2024

pra-sâmi

58

Constructive Dilemma or Disjunction of modus ponens

$$\begin{array}{c} p \rightarrow q \\ r \rightarrow s \\ \underline{p \lor r} \\ \therefore q \lor r \end{array}$$

- Corresponding Tautology:

$$((p \rightarrow q) \land (r \rightarrow s) \land (p \lor r)) \rightarrow (q \lor s)$$

- Example:
  - Let p be “I will study discrete math.”
  - Let q be “I will study computer science.”
  - Let r be “I will study protein structures.”
  - Let s be “I will study biochemistry.”

  - “If I will study discrete math, then I will study computer science.”
  - “If I will study protein structures, then I will study biochemistry.”
  - “I will study discrete math or I will study protein structures.”
- “Therefore, I will study computer science or biochemistry.”

4/16/2024

pra-sâmi

59

Destructive Dilemma or Disjunction of modus tollens

$$\begin{array}{l} p \rightarrow q \\ r \rightarrow s \\ \neg q \lor \neg s \\ \hline \therefore \neg p \lor \neg r \end{array}$$

- Corresponding Tautology:

$$((p \rightarrow q) \land (r \rightarrow s) \land (\neg q \lor \neg s)) \rightarrow (\neg p \lor \neg r)$$

- Example:
  - Let p be “I will study discrete math.”
  - Let q be “I will study computer science.”
  - Let r be “I will study protein structures.”
  - Let s be “I will study biochemistry.”
  - “If I will study discrete math, then I will study computer science.”
  - “If I will study protein structures, then I will study biochemistry.”
  - “I will not study computer science or I will not study biochemistry.”
- “Therefore, I will not study discrete math or I will not study protein structures.”

pra-sâmi

60

Absorption

$$\begin{array}{l} p \rightarrow q \\ \hline \therefore p \rightarrow (p \land q) \end{array}$$

- Corresponding Tautology:

$$(p \rightarrow q) \rightarrow (p \rightarrow (p \land q))$$

- Example:
  - Let p be “I will study discrete math.”
  - Let q be “I will study computer science.”
  - “If I will study discrete math, then I will study computer science.”
- “Therefore, if I will study discrete math, then I will study discrete mathematics and I will study computer science.”

pra-sâmi

61

Building Valid Arguments

- A valid argument is a sequence of statements where each statement is either a premise or follows from previous statements (called premises) by rules of inference. The last statement is called conclusion.
- A valid argument takes the following form:
  - Premise 1
  - Premise 2
  - Premise 3
  - ●
  - ●
  - ●
  - Premise n

  - Conclusion

62

Valid Arguments

- Example: From the single proposition

$$p \land (p \rightarrow q)$$

Shows that q is conclusion

- Solution:

No	Step	Reason
1	$p \land (p \rightarrow q)$	Premise
2	p	Conjunction using (1)
3	$p \rightarrow q$	Conjunction using (1)
4	q	Modus Ponens using (2) and (3)

63

Valid Arguments

Example

- With these hypotheses:
  - "It is not sunny this afternoon and it is colder than yesterday."
  - "We will go swimming only if it is sunny."
  - "If we do not go swimming, then we will take a canoe trip."
  - "If we take a canoe trip, then we will be home by sunset."
- Using the inference rules, construct a valid argument for the conclusion:
  - "We will be home by sunset."

Solution

- Choose propositional variables:
  - p : "It is sunny this afternoon."
  - q : "It is colder than yesterday."
  - r : "We will go swimming."
  - s : "We will take a canoe trip."
  - t : "We will be home by sunset."
- Translation into propositional logic:

```
Hypotheses:
((¬p ∧  q) ∧ ( p →  r ) ∧ (¬r → s)
∧ (s → t)

Conclusion: t
```

4/16/2024 pra-sâmi

64

Valid Arguments

- Example:

```
Hypotheses:
((¬p ∧  q) ∧ ( r →  p ) ∧ (¬r → s) ∧ (s → t)

Conclusion: t
```

No	Step	Reason
1	¬p ∧ q	Premise
2	¬p	Simplification using (1)
3	p → r	Premise
4	¬r	Modus tollens using (2) and (3)
5	¬r → s	Premise
6	s	Modus Ponens using (4) and (5)
7	s → t	Premise
8	t	Modus Ponens using (6) and (7)

p : "It is sunny this afternoon."
q : "It is colder than yesterday."
r : "We will go swimming."
s : "We will take a canoe trip."
t : "We will be home by sunset."

4/16/2024 pra-sâmi

65

Programming domain for AI

- Artificial intelligence (AI) is a broad area of computer applications
  - Very heavy on logic and interpretations
  - Characterized by the use of symbolic rather than numeric computations.
  - Implying that symbols, consisting of names rather than numbers, are manipulated
  - More convenient to use linked lists of data rather than arrays.
- Lisp (McCarthy et al., 1965):
  - Functional language
  - First widely used programming language developed for AI applications was
  - Most AI applications developed prior to 1990 were written in Lisp or one of its close relatives
- Prolog (Clocksin and Mellish, 2013) :
  - logic programming
  - Alternative approach appeared in early 1970s
- More recently, some AI applications have been written in systems languages such as C. Scheme (Dybvig, 2009), a dialect of Lisp, and Prolog

4/16/2024

pra-sâmi